A Mathematical Model for Competitive Location Problem with Product Selection

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Abstract

In this paper, a new competitive location problem for a chain is considered. The chain’s owner can offer a variety of products. The model’s objective is to determine both the location of the new facilities and the optimal product type for each opened facility. The patronizing behavior of the customers is based on Huff rule and the location of new facilities is selected from a set of potential sites. As a result, the model is a nonlinear integer programming problem and for solving the proposed model, the problem is reformulated as a mixed integer linear programming and therefore a standard optimization solver can be used for obtaining the optimal solutions for small and medium-size problems. To cope with large-size problems, we develop two methods: 1) a heuristic method for a special case and 2) a hybrid heuristic-firefly algorithm for general cases. By using the proposed model, it is shown numerically that in multi-product industries in which owner of the facilities is able to offer different types of products, in addition to the optimal location, it is necessary to determine the best products. In the end, a real-world case study for locating a new bakery is presented.

Keywords: Competitive location, product variety, Huff rule, mixed integer linear programming, location-product, hybrid heuristic-firefly algorithm
1. Introduction

Facility location is a critical part of strategic planning for different firms, because location decisions are costly and difficult to reverse, and their impact spans a long-time horizon [1]. Location problem has different types. For example, the covering location [2] and hub location [3] are some of the well-known problems in this field. The readers who are interested in learning about different location models are referred to the survey papers [1, 4, 5].

Competitive facility location (CFL) problems have developed classical location models under a more complicated situation, in which different owners of facilities compete for increasing their market shares and in this case, different preferences of customers, possible reactions of competitors, and many other factors must be noticed. Early works on the CFL problems were accomplished by Hotelling [6], Hakimi [7, 8, 9, 10], Drezner [11] and Huff [12, 13]. There are at least four papers reviewing the literature regarding CFL models [14, 15, 16, 17].

1.1. Literature Review

The difference between CFL models is based on their different types of components. For example, the space for locating new facilities can be plane, network, or discrete. The type of competition can be considered static, with foresight, or dynamic. Depending on whether the product is necessary or not, the demand can be elastic or inelastic. One of the other important issues in the CFL problems is the patronizing behavior of the customers. In some problems, the choice of customers is based on the deterministic rule. That is, all customers’ demand is provided by the facility that has attracted the most interest. In
another case, the customers divide their demands among various facilities based on a probabilistic approach.

Another difference between CFL models is the decision variable of the model. In many problems, the only decision variable is the location (See for instance [18]), while some others considered other variables in addition to the “location”. One of the mostly used variables is the “design” of new facilities (See for instance [19]) and the problems deal with this kind of variables are called as “Location-Design” models. At Table 1, we give an overview of CFL recently published papers with more than one variable in their models.

Table 1.

One of the underlying assumptions that have been considered in the majority of CFL models is the uniqueness of the product (or service) that is offered by the facilities and in fact, there is no difference between the various products. Hence, it is assumed that the demand is only for one product while, a variety of products can be offered by the facilities.

Serious analysis of product variety, has been recently conducted in the economic literature, while it has not yet been taken seriously in CFL models. Lancaster [61] separated the product variety problems into three categories:

1- Those concentrated on the production part and the cost benefits of joint production. (See for instance [62])

2- Those concentrated on the demand part, highlighting the balance between the increased revenue possible from multi-product production and the loss of economies of scale for producing each individual product. (See for instance [63])
3- Those concentrated on the strategic considerations (See for instance [54, 55, 64, 65, 66])

The first and the second categories investigated the impact of product diversity on production and demand variables. In this paper, we focus on the third category because the locational decisions are part of strategic planning. Although the economics literature is full of works that consider the product variety in the strategic considerations (See for instance [64]), few researchers have considered product variety in CFL models. Table 2 lists the multi product CFL studies and classifies them in terms of model variables and patronizing behavior.

**Table 2-**

The papers [54, 55, 65] investigated both the optimal location of the new facilities and the optimal price of various products. The reason that we noticed them is that they paid attention to multi-product concept in their studies although they did not consider the product type as a decision variable. Beresnev and Suslov [66] proposed a model in which both product types and their price are considered.

As can be seen in Table 2, no work has been accomplished to determine the optimal location and product determination, simultaneously specially with probabilistic patronizing behavior.

**1.2. The Contribution of This Paper**

In some situations, or environments, different types of products can be offered and delivered by a facility. Depending on which product is to be offered, the optimal location of the facility is affected. For example, suppose a bakery owner can produce three
different kinds of breads but the equipment and methods of production for each bread is
different. Given the fact that each bread has its own market and customers, the new
optimal facility location can vary depending on which bread to choose for production.
Therefore, in this paper, we add a new decision variable to the competitive location
problems, which is the optimal product types offered by each new facility. We show that
this variable has a great impact on the location variable and vice versa. It is shown
numerically that in industries where multi products can be produced, both variables must
be considered, simultaneously.

The vast majority of models in literature do not consider the variety of products, and in
very limited cases, multi-product is considered and the product selection is not taken as
the decision variable (except [66]). In the only work in which the product selection is
considered as the variable [66], the location is not part of the problem variables.
Therefore, to the best of authors’ knowledge, this paper is the first work in the field of
competitive location to consider the optimal location and the product, simultaneously.
We call this problem “Location-Product” model.

The rest of the paper is structured as follows. The proposed model is described in Section
2. Section 3 presents the solution methods. The computational experiments are provided
in Section 4 and Section 5 presents the conclusion and recommendations for future
research.

2. The Proposed Model

Consider a competitive market where \( p \) products or \( p \) different types of one product
group (such as food or bread) are offered by different competitors. These competitors
have already opened up facilities on the market, in which they may offer one or more types of these $p$ products. At present, there are $m$ existing facilities, in which $f$ of them belong to the chain and the remaining ones belong to the competitors.

There are $n$ customers in this market and each customer may face with different demands for different products where the demand for customer $j$ for the product $t$ is $b_{jt}$. The products are assumed to be necessary and therefore the demand is inelastic. Thus, all of the customers’ demands are met from the existing facilities. When a new facility enters the market for offering a given product, some parts of the existing facilities’ market share will be cannibalized. The patronizing behavior of the customers is considered according to Huff’s rule. In this rule, the facility’s attraction for a given customer is determined by the facility quality (design) divided by the distance (or a function of distance) between them. Obviously, the more the attraction of one facility, the higher the probability of attracting the customers by the mentioned facility. The quality parameters of a given facility for a specific product includes a variety of components such as the product quality, the size of facility, the number of personnel, the queue created for product receipt, the cleanliness of the facility, the availability of the park, the access to facility, the personnel behavior in relation with customers, etc.

On this competitive market, the chain wants to open $r$ new facilities (among $o$ potential locations) and selects the best product types for each opened facility. The maximum number of new facilities that offer product $t$ is $NP_t$. Also, since it is not possible that the potential sites (in terms of the size of the facility) offer an equal number of products, $NF_k$ is the maximum number of products that can be offered at potential location $k$. 
Fig. 1 illustrates the proposed model with a typical example.

As we can observe from the example shown in Fig. 1, there are 25 customers who have different buying power and therefore the size of them are different. The chain has 2 existing facilities, while other competitors have 5 existing facilities. There are 4 different product types available in these existing facilities. Facilities 1 and 7 deliver two products and the other facilities offer one product. There are 10 potential locations for new facilities and the size of them is different and so the number of products offered by each potential facility can vary.

The chain seeks to find the answer for the following two questions:

1) What are the optimal locations of new facilities?

2) In each opened facility, which type of products should be offered?

The notations are used in the proposed model are as follows:

**Indices:**

\( i \) : Index of existing facility; \( i = 1, 2, ..., f \) chain’s existing facilities and \( i = f + 1, f + 2, ..., m \) competitors’ existing facilities

\( j \) : Index of customers; \( j = 1, 2, ..., n \)

\( k \) Index of potential locations; \( k = 1, 2, ..., o \)

\( t \) : Index of products; \( t = 1, 2, ..., p \)

**Parameters:**

\( m \) : The number of existing facilities
\( n \): The number of customers

\( o \): The number of potential locations

\( p \): The number of products

\( b_{jt} \): Demand of customer \( j \) for the product \( t \)

\( d_{ij} \): The distance between existing facility \( i \) and customer \( j \)

\( d_{kj} \): The distance between the new facility opened at potential location \( k \) and customer \( j \)

\( \gamma_j \): Weight for the quality of the new facilities as perceived by customer \( j \)

\( \alpha_a \): Quality of product \( t \) at existing facility \( i \)

\( q_{kt} \): Quality of product \( t \) at new facility at potential location \( k \)

\( pr_t \): The profit per unit of product \( t \) sold

\( r \): The number of new facilities

\( NP_t \): The maximum number of new facilities that offer product \( t \)

\( NF_k \): The maximum number of products to be delivered at potential location \( k \)

**Variables:**

\( y_k \): A binary variable that is equal to 1 if a new facility is opened at potential location \( k \), 0 otherwise

\( x_{kt} \): A binary variable that is equal to 1 if product \( t \) is produced at new facility opened at potential location \( k \), 0 otherwise

Based on Huff rule, the attraction of the product \( t \) at facility \( i \) for customer \( j \) can be as follows [67]:

\[
\begin{align*}
\text{Attraction} \propto & \exp\left(-\frac{d_{ij}}{d_{ij}^0}\right) \\
\text{where} & \quad d_{ij}^0 \text{ is a constant related to customer effort.}
\end{align*}
\]
The attraction of new facilities is calculated similar to eq. (1). As mentioned before, the demand for each customer is spread across all facilities. That is, each facility meets a part of the demand of a given customer that is directly related to its attraction. Therefore, the greater the attraction of a facility for a given customer, the greater the share of the facility from the customer's demand. If this share is considered by dividing the facility’s attraction for the customer on the overall attraction of the facilities, then the chain’s share on the demand of a particular customer equals the total attraction of the chain’s facilities (existing and new) divided by the attraction of all facilities [12, 13, 26, 44, 67]. Therefore, the chain’s market share on the demand of the product \( t \) for customer \( j \) can be as follows:

\[
MS_{jt} = \frac{\sum_{i=1}^{f} U_{ijt} + \sum_{k=1}^{o} U_{kjt} x_{kt}}{\sum_{i=1}^{m} U_{ijt} + \sum_{k=1}^{o} U_{kjt} x_{kt}}
\]  

(2)

The chain's profit is equal to the total sales of products multiplied by the profit of each unit sold. So, the “Location-Product” problem (P1) is as follows:

\[
Maxz = \sum_{j=1}^{n} \sum_{t=1}^{p} Pr_{jt} b_{jt} \left( \frac{\sum_{i=1}^{f} \alpha_{ij} \gamma_{ij}}{(\varepsilon + d_{ij})^2} + \sum_{k=1}^{o} \frac{q_{ik} \gamma_{ik}}{(\varepsilon + d_{ik})^2} x_{kt} \right)
\]

(3)
where, the Eq. (3) represents the chain’s profit, which must be maximized. The number of new facilities is determined by constraint (4), and Eq. (5) ensures that the maximum number of new facilities that offer product $t$ is equal to $NP_t$. Relation (6) shows how many products can be offered in each potential location. By constraint (7) we can make sure that the products can be offered only if the facility is opened up.

This model is an integer nonlinear programming problem, in which the objective function is a sum of ratios with a particular structure: numerators and denominators of a ratio differ by the constants only. Similar problems with a sum of ratios as objective, are studied by Hansen et al. [68] and Benati & Hansen [69]. Benati [70] proved that the problem is NP-hard. In the following, we will see that problem “Location-Product” can
be reformulated to a mixed integer linear programming problem and therefore the optimal solutions can be obtained using standard optimization software (like CPLEX, Gurobi, Mosek, Xpress-MP, others).

3. Solution Methods

In this section, we introduce three solution methods for problem P1. The first one exploits directly the formulation given above. The other two methods are heuristics.

3.1. Reformulation of the model to a mixed integer linear programming problem:

Let

\[ A_{jt} = \sum_{i=1}^{l} \frac{\alpha_{i} \gamma_{j}}{e + d_{jt}} \]

and

\[ A'_{jt} = \sum_{i=1}^{m} \frac{\alpha_{i} \gamma_{j}}{e + d_{jt}} \]

for \( j = 1, 2, ..., n \); \( k = 1, 2, ..., o \) and

\[ B_{jk} = \frac{q_{ik} \gamma_{j}}{e + d_{jt}} \]

for \( j = 1, 2, ..., n \); \( t = 1, 2, ..., p \) and \( k = 1, 2, ..., o \). The objective function can be expressed as the following:

\[
\sum_{t=1}^{p} \sum_{j=1}^{n} \sum_{r=1}^{n} \Pr_{r} b_{jr} \left( \frac{A_{jt} + \sum_{k=1}^{o} B_{jk} x_{kt}}{A'_{jt} + \sum_{k=1}^{o} B'_{jk} x_{kt}} \right) = \text{(9)}
\]

3.1.1. Concavity of the objective function:

**Theorem 1.** The continuous relaxation of the objective function P1 is concave.

**Proof.** Since, there are existing facilities in the market, therefore \( A'_{jt} \), the attraction of the existing facilities, is definitely not equal to zero and so the function does not have singularity points over the domain of its continuous relaxation, hence the second cross-
derivative can be computed by standard methods. Since the objective function is a sum of ratios, it is sufficient to prove that each term is concave.

Consider the term \( j \): 

\[
f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} b_{ji} \frac{A_{ji} + \sum_{k=1}^{o} B_{jkr} x_{kt}}{A_{ji}'} + \sum_{k=1}^{o} B_{jkr} x_{kt}
\]  

(10)

The Hessian matrix is \( H = [h_{lw}] \), where

\[
h_{lw} = \frac{\partial^2 f(x_1, x_2, \ldots, x_n)}{\partial x_l \partial x_w} = -2 \prod_{i=1}^{n} b_{ji} B_{jlt} B_{jkt} \frac{A'_{ji} - A_{ji}}{(A_{ji}' + \sum_{k=1}^{o} B_{jkr} x_{kt})^3}
\]

(11)

Since \( A'_{ji} - A_{ji} \geq 0 \), \( h_{lw} \) is always negative. It can be seen easily that the determinant of every submatrix of order two is equal to 0, while the elements on the diagonal are negative. So \( f(x_1, x_2, \ldots, x_n) \) is concave. ■

Note that the continuous relaxation of the domain is a convex set, so an upper bound can be computed by unconstrained nonlinear optimization algorithms like gradient methods.

**3.1.2. Integer linear formulation:**

Consider variable \( Z_{ji} \) as follows:
\[
Z_{jt} = \frac{A_{jt} + \sum_{k=1}^{o} B_{jkt} x_{kt}}{A'_{jt} + \sum_{k=1}^{o} B_{jkt} x_{kt}}
\]  
(12)

as the denominator is positive this is equivalent to:

\[
Z_{jt} \left( A'_{jt} + \sum_{k=1}^{o} B_{jkt} x_{kt} \right) = A_{jt} + \sum_{k=1}^{o} B_{jkt} x_{kt}
\]  
(13)

Assume the following variable:

\[
w_{jkt} = Z_{jt} x_{kt}
\]  
(14)

for which the following constraints are equivalent:

\[
w_{jkt} \leq x_{kt}
\]  
(15)

And

\[
w_{jkt} \geq Z_{jt} - (1 - x_{kt})
\]  
(16)

With these substitutions, we can write

\[
Z_{jt} A'_{jt} + \sum_{k=1}^{o} B_{jkt} w_{jkt} - A_{jt} - \sum_{k=1}^{o} B_{jkt} x_{kt} = 0 ,
\]  
(17)

Where

\[
Z_{jt} = \frac{1}{A'_{jt}} \left( \sum_{k=1}^{o} B_{jkt} x_{kt} - \sum_{k=1}^{o} B_{jkt} w_{jkt} + A_{jt} \right)
\]  
(18)
Finally, Problem P1 can be reformulated to problem P2 as follows:

$$\text{Max} \ z = \sum_{j=1}^{n} \sum_{t=1}^{p} \frac{Pr_{j} b_{jt}}{A'_{jt}} \left( \sum_{k=1}^{o} B_{jkt} x_{kt} - \sum_{k=1}^{o} B_{jkt} w_{jkt} + A_{jt} \right)$$ \hspace{1cm} (19)

S.t.

$$w_{jkt} \geq \frac{1}{A'_{jt}} \left( \sum_{k'=1}^{o} B_{jk't} x_{jt} - \sum_{k'=1}^{o} B_{jk't} w_{jk't} + A_{jt} \right) - (1 - x_{kt}) \hspace{1cm} j=1,2,\ldots,n; \ t=1,2,\ldots,p \ \text{and} \ k=1,2,\ldots,o$$ \hspace{1cm} (20)

$$\sum_{k=1}^{o} y_{k} = r$$ \hspace{1cm} (21)

$$\sum_{k=1}^{o} x_{kt} \leq NP_{t} \hspace{1cm} t=1,2,\ldots,p$$ \hspace{1cm} (22)

$$\sum_{t=1}^{p} x_{kt} \leq NF_{k} \hspace{1cm} k=1,2,\ldots,o$$ \hspace{1cm} (23)

$$y_{k} \geq x_{kt} \hspace{1cm} t=1,2,\ldots,p \ \text{and} \ k=1,2,\ldots,o$$ \hspace{1cm} (24)

$$x_{kt}, y_{k} \in \{0,1\} \ \text{and} \ w_{jkt} \geq 0 \hspace{1cm} j=1,2,\ldots,n; \ t=1,2,\ldots,p \ \text{and} \ k=1,2,\ldots,o$$ \hspace{1cm} (25)

As $w_{jkt} \leq x_{kt}$ is a redundant constraint, it can be removed from the model. This formulation is a MILP formulation and therefore the optimal solution can be obtained by a standard MIP solver.

3.2. A heuristic method (for a special case)
Since the problem is NP-hard, heuristic methods must be used for large-scale problems. In this section we present a heuristic method for a special case of P1. The case happens when the right-hand side of the constraint 6 is equal to 1. That is, only on product can be offered by each potential location. Since the facilities usually have a space limitation in practice, this assumption is not unrealistic and the mentioned case is applicable in many situations.

Given the fact that the time to solve the problem P1 and P2 in terms of the number of new facilities increases exponentially, we have realized that (by solving several examples) for the mentioned special case, the near optimal solution of the multi facility problem consists of solving the single facility problem different times. That is to say, for example, if we want to open 2 new facilities, one can first solve the single-facility problem and then, remove the optimal potential location as well as the optimal product and once again solve the single facility problem. Combining the optimal solutions of two sub problems will be the optimal answer to the main problem. This is entirely logical, because two different facilities provide two different products and do not cannibalize the market share of each other. Therefore, the first- and second-best solutions of the single-facility problem will be near optimal solution of the two-facility problem.

The flowchart of the proposed heuristic is shown at Fig. 2.

Fig 2

3.3. A hybrid heuristic-discrete firefly algorithm (HHDFA)

Firefly algorithm is inspired by social behavior of fireflies presented by Xin-She Yang [71]. Firefly is a powerful population-based meta-heuristic technique for solving
combinatorial optimization problems. Xin-She Yang [72] indicated that the Firefly algorithm is an efficient method in finding the global optima with high success rates.

In the Firefly algorithms, attractiveness of a firefly is proportion to its brightness. Thus, less bright firefly will move towards the brighter one for any flashing fireflies. For a maximization problem, brightness of a firefly can be determined by the objective function’s value. Attractiveness and brightness both increase as their distance decrease. If there is no brighter one than a particular firefly, it will move randomly [73]. Sayadi et al. [74] suggested a discrete firefly algorithm for flow shop scheduling problem. Sadjadi et al. [67] presented a hybrid continuous and discrete firefly algorithm for the competitive location-design model.

In this paper, discrete firefly is used for obtaining location variable. Once the location is obtained in each iteration, the optimal product type is selected using a heuristic method.

The developed HDDFA is described.

3.3.1. Representation scheme

A proper encoding scheme, which is indicative of the characteristics of a solution, has considerable influence on the performance of a meta-heuristic method. The encoding scheme of a solution for the location variable has been illustrated in Table 3. This scheme denotes the location of the new facilities for a special firefly, which is indicated by a \( o \times r \) matrix.

The location for the firefly \( i \) in the generation \( t \) can be denoted as \( X_i^t = (X_{i11}^t, X_{i12}^t, \ldots, X_{ior}^t) \). The value 1 shows the location of a new facility. For example,
$X_{kl}^t$ is a binary number and $X_{kl}^t = 1$ indicates that the new facility $l$ of firefly $i$ is placed in the $k$-th potential point at $t$-th generation and 0 otherwise.

Table 3-

3.3.2. Initialization

In this paper, the location of the new facilities is initialized randomly and a random product is selected for each facility.

3.3.3. The operators in HH DFA

In the firefly algorithm, the movement of firefly $i$ toward the more attractive (brighter) firefly $j$ is determined through the following equation [71].

$$X_i^t = X_i^t + \beta_0 e^{-\gamma r_{ij}^2} (X_j^t - X_i^t) + \lambda (\text{rand} - \frac{1}{2}), m \geq 1$$  (26)

where $\beta_0 e^{-\gamma r_{ij}^2}$ is the attractiveness function whose value decreases with the increase of the distance between two fireflies ($r_{ij}$). $\beta_0$ is the attractiveness at $r_{ij} = 0$, and $\gamma$ is the fixed light absorption coefficient in the environment. Expression $\lambda (\text{rand} - \frac{1}{2})$ is for the randomization of movement, in which, $\lambda$ is the randomization parameter, and “rand” is a function that generates random numbers with uniform distribution in the [0,1] interval.

By using the Cartesian distance, the distance between the two fireflies of $i$ and $j$ is obtained from the following relation [73]:

$$r_{ij} = \|X_i^t - X_j^t\| = \sqrt{\sum_{k=1}^{d} (X_{ik}^t - X_{jk}^t)^2}$$  (27)
where $X_{id}^t$ is the $d^{th}$ component of the $i^{th}$ firefly.

3.3.4. Discretization

When firefly $i$ moves towards firefly $j$ the position of the firefly $i$ change from a binary number to a real number. So, this real number must be replaced by a binary value. By using the sigmoid function, the position value is constrained to the interval $[0, 1]$ and then it is transformed to the binary number [74]:

3.3.5. Finding the optimal product

Once the locations have been selected on each iteration, it is time to determine products. In this phase, we start with the first facility and the objective function is calculated for different products, and the product that leads to the highest value of the objective function is selected and we go to the next facility. In this step, we repeat the same with the remaining products until all products are consumed. Then we redistribute the products again.

Once the first product of each opened facility is selected, the algorithms will repeat for assigning the next product until all facilities include products according to their capacities.

The flowchart of the proposed HH DFA is shown at Fig. 3.

4. Numerical Examples
To investigate the performance of the model, first a small example is solved by MIP solver and the results are analyzed. Then, the efficiency of the proposed algorithms is evaluated by solving several examples. Finally, a case study will also be presented.

All the computational results in this paper were obtained on a Core i7 with 3.5 GHz CPU and 8 GB memory. The heuristic method and HH DFA were coded in MATLAB R2018a.

4.1. An illustrative example

It is assumed that there are 16 customers, 4 existing facilities and 4 products in a competitive market. The location and the products of chain’s and competitors’ existing facilities and qualities of them are stated at Table 4.

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Table 5 includes the location, the demand and the weight for the new facilities’ quality of different customers.

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The chain wants to open new facilities. Table 6 shows 12 potential locations and their qualities for different products.

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Suppose the profit per unit of products 1, 2, 3 and 4 are 15, 11, 10 and 9, respectively and the distance between the customers and the new and existing facilities are assumed as city-block.

If the chain wants to open 1 new facility with 1 product, the optimal location for different products and the optimal product for different locations are quite different, which
indicates the impact of both variables on each other, and this is ignored in practice and in many mathematical models.

Table 7 and Table 8 show the optimal locations and products for different scenarios.

Table 7

Table 8

Table 7 and Table 8 show the optimal solution is location (2,1) in which the product 4 should be offered. If the product is given and fixed, the optimal location will be different and location (2,1) will no longer considered as the optimal location for product 1, 2 and 3.

Also note that if the owner of the chain has the ability to open the facility by offering 2 products, then the location (2,1) is not optimal and the location (2.2) is the best solution with delivering products 2 and 4. Table 9 shows the optimal location of the problem when the chain wants to open a facility with 2 products.

Table 9

Table 9 shows that the optimal location varies depending on the type of products. Optimal solutions at Tables 4-6, indicate that not only the location and product variables affect each other, but also the number of new facilities, as well as the number of products offered by each facility extremely affect the optimal solutions. Table 10 summarizes the optimal solution of the problem under different scenarios.

Table 10.

By comparing the answers, in addition to the results obtained from the impact of the variables on each other, two points are achievable.
1) Obviously, with the increase in the number of facilities or the number of products, the chain’s profit will increase in which in the mentioned example it is approximately 10% for each facility and 9% for each product. By solving several examples, we understand that increasing the number of branches influences on the profitability more than the increase in the number of products. The logical reason is that the new facility is located in a new region of customers and can better capture the market.

2) By comparing Table 10 and Table 7, we find that if the chain wants to open multiple facilities, the optimal locations appear in the order shown in Table 7. By observing this case in other examples, it was concluded that, given the longer time to solve for more new facilities, we can solve the single facility problem several times according to the number of new facilities. Thus, the CPU time will be considerably reduced. This method was described in section 3.2.

For investigating the importance of the “location-product” model in comparison with the models that do not consider the optimal product, we assume that the owner of the chain, chooses one product based on its experience and open a new facility in one of the potential sites. Table 11 shows the difference in profit (in percent) between the optimal location and other solutions for different products.

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<tbody>
<tr>
<td>Table 11 shows the necessity of using location models, where, in the absence of the model, an average of 2.5% less profit earning is resulted for the chain. We do the same with the products to see the impact of the optimal product for each potential location at Table 12.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 12.</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
</tr>
</tbody>
</table>
Table 12 shows the necessity of using the product model, where, in the absence of the model, an average 1.15% less profit earning is resulted for the chain. Although the average value is lower than the respective one for the location variable, but it is a significant value in many cases. Therefore, in multi-product industries in which owner of the facilities is able to offer different types of products, in addition to the optimal location, determining the best product should be considered.

4.2. Solution methods efficiency

We have provided some experimental results to investigate the efficiency of MIP solver, heuristic method and HH DFA. For this purpose, we have generated various problems, in which, the number of customers \((n = 25, 50, 100)\), the number of existing facilities \((m = 5, 10)\), the number of potential locations \((o = 25, 50, 100)\), the number of products \((p = 5, 10)\) and the number of new facilities \((r = 2, 4, 10)\) are changed.

In all cases, the number of chain’s facilities is \(f = 2\) for \(m = 5\) and \(f = 4\) for \(m = 10\); the maximum number of new facilities that offer a given product equals to the number of new facilities and the maximum number of products to be delivered at potential locations is 1 (the assumption for using heuristic method).

For each setting, five examples have been created, in which, the parameters of the problems are randomly chosen from the following intervals:

\[
\begin{align*}
  b_{jt} &\sim U(1, 100), & d_{ij} &\sim U(1, 10), & d_{kj} &\sim U(1, 10), & \gamma_j &\sim U(0.01, 1), & \alpha_{it} &\sim U(1, 10) , \\
  q_{kt} &\sim U(5, 10), & pr_t &\sim U(10, 20), & \varepsilon & = 0.05 .
\end{align*}
\]

4.2.1. The value of the HH DFA’s parameters
The $\gamma$ should be related to the scales of design variables of a problem. For example, one possible choice is to use $\gamma = \frac{1}{\sqrt{L}}$ where $L$ is the average scale of the problem. After comparing different values for $\gamma$, the value 0.6 is selected. For most cases we set $\beta_0 = 1$ and $\lambda \in [0,1]$. Comparing different values for $\lambda$, the value 0.2 is suitable.

The optimal solution can be found after about 500 evaluations for most cases. So, 25 fireflies and 20 generations have been selected in the computational experiment.

### 4.2.2. The results

To evaluate the solution quality of the proposed algorithm, HH DFA with 10 runs and heuristic method for each problem have been compared with the optimal solver for the small and medium size instances.

First, we show the results for a typical setting in details. The difference between the optimal value obtained by the optimal solver and the solution obtained by the heuristic and HH DFA and also the CPU time spent by the MINLP solver, MIP solver, heuristic method and HH DFA for the five generated examples of the case $n = 25$, $m = 5$, $o = 25$, $p = 5$, $r = 2$ are presented in Table 13. The last two lines show the total average and total standard deviation.

<table>
<thead>
<tr>
<th>Table 13-</th>
</tr>
</thead>
</table>

The effectiveness (the capability of the method for obtaining the optimal solution) of the proposed algorithms in small instances is depicted in Table 13.

From now on, only the total average values are shown for investigating the results. In terms of different number of potential locations, a summarizing table (like the last two
lines of Table 13) has been produced and depicted in Table 14. The values in the table is related to the average and the values in brackets are standard deviations. In the last line, the average for the setting, regardless of the number of potential locations is given.

Table 14-

Note that, when the number of potential locations increases, the solution time for the optimal solvers specially MINLP solver increases more than the proposed methods do. In fact, HH DFA is much faster than the other two solvers and heuristic for large size problems. The quality of the heuristic is slightly better than HH DFA but the quality of the results of HH DFA is still good enough. This issue is more clearly seen in Table 15, in which the number of new facilities is increased.

Table 15-

According to Table 15, increasing the number of new facilities leads to increase the solution time of the MINLP and MIP Solvers, significantly while the heuristic method and HH DFA are less sensitive.

Since the difference in objective function of heuristic method, even in large size problems, is below 1%, and it has a reasonable time to solve, this method is very practical in the special case mentioned in section 3.2. Regarding the HH DFA method, it can be said that although its quality is somewhat less than the heuristic method, its percentage difference in objective function is still acceptable. Since it can be used in general cases and has a good time to solve, it is a good method for solving general examples, while for the special case, the heuristic method is suggested.
Table 16 depicts the results of large examples with up to 100 customers obtained by MIP solver, heuristic method, and HHDF which could not be worked out using MINLP solver.

Table 16-

According to Table 16, the increase in the number of customers and facilities will increase the CPU time for solving the model but the increase is not as large as the number of new facilities and the number of products, because they are decision variables and they increase the solution space and make the problem harder to find the optimal solution.

In Table 17, some problems are solved by considering the value 2 for constraint 6. Since the heuristic method is no longer able to solve the problem, the comparison is only between the CPU time of the MIP solver and HHDF.

Table 17-

Opening up to 10 new facilities with up to 10 different product groups can be a real problem, because many chains may not be able to open more than this number of new facilities. Therefore, Table 16 and 17 show the effectiveness of all three methods, especially the heuristic (for the special case) and HHDF (for general cases), which in less than 10 minutes (for heuristic) and 2 minutes (for HHDF) a problem with 100 customers, 100 potential locations, 10 existing facilities, 10 products and 10 new facilities with can be solved with reasonable solutions.

4.3. A real-word application
In this section, a real-word case study of the proposed model is presented. The model has been used for the new facility location for a bakery in sector Eram, district 5, Tehran as depicted with red line in Fig. 4.

Fig 4.

Various breads are produced in Iran, where, each of them has a different equipment and furnace for production and different bakeries often produce one of these breads. Three breads that are more popular than others are: Barbari, Sangak and Lavash.

We divided the sector into 10 areas and consider each one as a customer. Fig. 5 shows how the sector is divided.

Fig 5.

As the goods are necessary and the demand has no specific relationship with the level of income, the population of each area can be considered as \( b_j \). The latitude and longitude of different areas of this sector which are calculated by LatLon system and also normalized population for each area are depicted at Table 18.

Table 18.

The consumption rates of Barbari, Sangak and Lavash are 1.2, 1 and 3, respectively (these values are used to compute \( b_j \)). There are 4 existing facilities in this sector for offering three mentioned breads. Using LatLon system the latitude and longitude and the products of existing facilities have been depicted in Table 19.

Table 19.

The level of quality of the existing facilities have been measured from the customers through the questionnaire designed by SERVQUAL [75, 76].
The level of modern looking building and equipment, the level of staff training for treating customers, the levels of materials associated with service (promotional brushers, service tracking documents), providing gifts for customer as a promotion, allocating special personnel with high public relation to respond the customer’s probable questions which they face and keeping in contact with customers (e.g. informing them about the new products) are examples of quality dimension of the facilities which are evaluated by the customers for calculation of existing facilities quality.

From each existing facility, 400 customers have been chosen as a sample and the Cronbach’s alpha for the designed questionnaire was 0.83. The paired comparison matrix was used for extracting the weights of different criteria as well. Customer evaluations were aggregated for obtaining the total design (quality) score in scale of 1 to 10 depicted in Table 20.

Table 20.

There are 9 potential locations which are candidate and their latitude and longitude and maximum number of products to be offered are depicted in Table 21.

Table 21.

The profit per unit of products Barbari, Sangak and Lavash are 500, 600 and 200, respectively. The optimal solutions of different scenarios for this real application are depicted at Table 22.

Table 22.

5. Conclusion
In this paper a competitive facility location model has been formulated under a condition that a chain can offer different types of products. For this model, we need to determine the optimal location and the optimal products.

We have formulated this problem in a static competition in which the competitors of a given company are already on the market and the company was aware of the characteristics of the facilities. The customer patronizing behavior has been modeled according to Huff rule. The solution space of the model was discrete and the optimal locations should be selected from the potential locations set. Obviously, the product space was also discrete and as a result, the problem is an integer nonlinear programming problem. We have reformulated the problem into a mixed integer linear formulation. MIP solvers can be easily applied for solving the model at least for small and medium size problems with a reasonable CPU time.

We have developed a heuristic and a hybrid heuristic-firefly algorithm for solving the large-scale problems. The results have shown that the difference in objective value of their solutions with the optimal solvers was small even for large-size problems. The obtained results have indicated that in the multi-product industries the chain should consider the product variable in addition to the location. As it was seen in this paper, the chain’s optimal solution was different for various products. Therefore, neglecting the product selection may impact the optimal solution and cause a major damage.

As a future research, it is interesting to study this problem in a leader-follower situation. Furthermore, considering other patronizing behaviors assumption is suggested. Also, the tastes of each region (about the type of product) can be considered as a factor at the objective function for the further research.
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Figures:

Fig 1:
Fig 2:
Start

---

$r=1$ in constraint (4)
---

Solve the model

---

Has the number of facilities been provided?

---

Yes

Finish

---

No

Remove the optimal location and product from list of solution space

---

No

Have all products been used?

---

No

Add all removed products

---

Yes

Fig 3:
Start

Generate initial population of fireflies

Generate random products for each opened facility of fireflies

Determine objective function for each firefly

Set light absorption coefficients, randomization parameters and maximum iterations

Move fireflies to the more attractive fireflies

Discrete the location decision variable of i-th firefly

Calculate objective function for each product in the first/selected facility and assign the product with maximum objective function

Yes

Do all opened facilities have products according to their capacity?

No

Rank the fireflies and find the current best for the follower

Go to the first/selected facility and assign next products (if it can)

Yes

Do all opened facilities have product?

No

Add all removed products

Yes

Have all products been used?

No

Remove selected product and go to the next facility and repeat the previous step

Fig 4:
Tables:
### Table 1:

<table>
<thead>
<tr>
<th>Patronizing behavior &amp; Variables</th>
<th>Location and Design</th>
<th>Location and Price</th>
<th>Location and other variables</th>
</tr>
</thead>
<tbody>
<tr>
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<td>[47-55]</td>
<td>[57-59]</td>
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<tr>
<td>Probabilistic</td>
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<td>[56]</td>
<td>[60]</td>
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</tbody>
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### Table 2:

<table>
<thead>
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<th>Research</th>
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<th>Patronizing behavior</th>
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<td>Price</td>
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<td>✓</td>
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<tr>
<td>[65]</td>
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<td>✓</td>
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<tr>
<td>[66]</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Table 3:

- New facility \((l)\)
  - Potential point \((k)\)
    - \(X'_{i11} \quad X'_{i12} \quad \ldots \quad X'_{i1r}\)
    - \(X'_{i21} \quad X'_{i22} \quad \ldots \quad X'_{i2r}\)
    - \(\vdots \quad \vdots \quad \ldots \quad \vdots\)
    - \(X'_{iwl} \quad X'_{iwl2} \quad \ldots \quad X'_{iwr}\)

### Table 4:

<table>
<thead>
<tr>
<th>Location Coordinates</th>
<th>Chain’s existing facilities</th>
<th>Competitors’ existing facilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location Coordinates</td>
<td>(1,3) (0.3) (3,1) (0,0)</td>
<td>(1,3) (3,4) (2,4) (1,3)</td>
</tr>
<tr>
<td>Product Type</td>
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<td>3,4</td>
</tr>
<tr>
<td>Quality Value</td>
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### Table 5:
### Table 6:

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<th>(2,2)</th>
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<th>(3,1)</th>
<th>(3,2)</th>
<th>(3,3)</th>
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<tbody>
<tr>
<td>Quality's Weight Value</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>3</td>
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<td>3</td>
<td>5</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Demand of Product 1</td>
<td>9</td>
<td>24</td>
<td>31</td>
<td>7</td>
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<td>26</td>
<td>84</td>
<td>74</td>
<td>78</td>
<td>57</td>
<td>44</td>
<td>34</td>
<td>59</td>
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<tr>
<td>Demand of Product 2</td>
<td>20</td>
<td>31</td>
<td>33</td>
<td>76</td>
<td>81</td>
<td>18</td>
<td>61</td>
<td>77</td>
<td>52</td>
<td>90</td>
<td>45</td>
<td>17</td>
<td>4</td>
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<td>Demand of Product 3</td>
<td>10</td>
<td>87</td>
<td>68</td>
<td>79</td>
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<td>Demand of Product 4</td>
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<td>96</td>
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### Table 7:

<table>
<thead>
<tr>
<th>Product type</th>
<th>Optimal Location Coordinates</th>
<th>Objective function ($)</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>2</td>
<td>(2,2)</td>
<td>23,692</td>
</tr>
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<td>3</td>
<td>(1,0)</td>
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</tr>
<tr>
<td>4</td>
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</table>

### Table 8:

<table>
<thead>
<tr>
<th>Potential Location Coordinates</th>
<th>Optimal Location Coordinates</th>
<th>Objective function ($)</th>
</tr>
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<td>23,034</td>
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<tr>
<td>(0,2)</td>
<td>4</td>
<td>22,767</td>
</tr>
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</tr>
<tr>
<td>(1,1)</td>
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<td>23,207</td>
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<td>23,697</td>
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<td>(2,1)</td>
<td>4</td>
<td>23,742</td>
</tr>
<tr>
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<tr>
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### Table 9:
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
<th>Optimal Location Coordinates</th>
<th>Optimal Product Type</th>
<th>Profit ($)</th>
<th>Increase in Profit ($)</th>
<th>Increase Rate (%)</th>
</tr>
</thead>
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<tr>
<td>0</td>
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<td>-</td>
<td>21,501</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>Open 1 new facility with 1 product</td>
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<td>4</td>
<td>23,742</td>
<td>2,241</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>Open 2 new facilities with 1 product in each one</td>
<td>(2,0)</td>
<td>And (2,1)</td>
<td>25,937</td>
<td>4,436</td>
<td>21%</td>
</tr>
<tr>
<td>3</td>
<td>Open 1 new facility with 2 products</td>
<td>(2,2)</td>
<td>2 and 4</td>
<td>25,840</td>
<td>4,339</td>
<td>20%</td>
</tr>
<tr>
<td>4</td>
<td>Open 3 new facilities with 1 product in each one</td>
<td>(2,0)</td>
<td>And (2,1)</td>
<td>28,128</td>
<td>6,627</td>
<td>31%</td>
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<tr>
<td>5</td>
<td>Open 1 new facility with 3 products in each one</td>
<td>(2,1)</td>
<td>2 and 3 and 4</td>
<td>27,750</td>
<td>6,249</td>
<td>29%</td>
</tr>
<tr>
<td>6</td>
<td>Open 4 new facilities with 1 product in each one</td>
<td>(1,0)</td>
<td>And (2,0)</td>
<td>30,244</td>
<td>8,743</td>
<td>41%</td>
</tr>
<tr>
<td>7</td>
<td>Open 1 new facility with 4 products in each one</td>
<td>(2,1)</td>
<td>1 and 2 and 3 and 4</td>
<td>29,699</td>
<td>8,198</td>
<td>38%</td>
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Table 11:

<table>
<thead>
<tr>
<th>Product type</th>
<th>Worst solution (%)</th>
<th>Average solution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>5.13%</td>
<td>2.67%</td>
</tr>
<tr>
<td>Product 2</td>
<td>5.04%</td>
<td>2.47%</td>
</tr>
<tr>
<td>Product 3</td>
<td>4.91%</td>
<td>2.61%</td>
</tr>
<tr>
<td>Product 4</td>
<td>4.22%</td>
<td>2.24%</td>
</tr>
</tbody>
</table>

Table 12:

<table>
<thead>
<tr>
<th>Potential Location Coordinates</th>
<th>Worst solution (%)</th>
<th>Average solution (%)</th>
</tr>
</thead>
</table>
Table 13:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Difference in obj (%)</th>
<th>CPU Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heuristic</td>
<td>HH DFA</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Average</td>
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<td>0.02</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.00</td>
<td>0.01</td>
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</table>

Table 14:

<table>
<thead>
<tr>
<th>Number of Potential Locations</th>
<th>Difference in obj (%)</th>
<th>CPU Time (seconds)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Heuristic</td>
<td>HH DFA</td>
</tr>
<tr>
<td>25</td>
<td>0.00 (0.00)</td>
<td>0.02 (0.01)</td>
</tr>
<tr>
<td>50</td>
<td>0.00 (0.00)</td>
<td>0.34 (0.18)</td>
</tr>
<tr>
<td>100</td>
<td>0.01 (0.01)</td>
<td>0.83 (0.26)</td>
</tr>
<tr>
<td>All</td>
<td>0.00 (0.01)</td>
<td>0.39 (0.16)</td>
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</tbody>
</table>

Table 15:

<table>
<thead>
<tr>
<th>Number of New Facilities</th>
<th>Difference in obj (%)</th>
<th>CPU Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heuristic</td>
<td>HH DFA</td>
</tr>
<tr>
<td>2</td>
<td>0.01 (0.01)</td>
<td>0.83 (0.26)</td>
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<tr>
<td>4</td>
<td>0.51 (0.03)</td>
<td>1.37 (0.40)</td>
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<tr>
<td>10</td>
<td>1.27 (0.16)</td>
<td>3.93 (0.98)</td>
</tr>
<tr>
<td>All</td>
<td>0.59 (0.06)</td>
<td>2.17 (50.14)</td>
</tr>
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</table>

Table 16:

<table>
<thead>
<tr>
<th>Number of Customers</th>
<th>Number of Existing Facilities</th>
<th>Number of Products</th>
<th>CPU Time (seconds)</th>
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<tbody>
<tr>
<td>25</td>
<td>5</td>
<td>5</td>
<td>509 (36)</td>
</tr>
<tr>
<td>Table 17:</td>
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<td></td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
<td>------------------------------------------------</td>
<td>------------------------------------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td><strong>Number of Existing Facilities</strong></td>
<td><strong>Number of Products</strong></td>
<td><strong>CPU Time (seconds)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>MIP Solver</strong></td>
<td><strong>HHDFIA</strong></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3323 (201)</td>
<td>71 (17)</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>4673 (253)</td>
<td>98 (22)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>4164 (198)</td>
<td>85 (19)</td>
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<tr>
<td></td>
<td>10</td>
<td>5594 (305)</td>
<td>105 (27)</td>
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<tr>
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<tbody>
<tr>
<td><strong># Area</strong></td>
<td><strong>Latitude (degree)</strong></td>
<td><strong>Longitude (degree)</strong></td>
<td><strong>Quality’s weight</strong></td>
</tr>
<tr>
<td>1</td>
<td>51.28374</td>
<td>35.73405</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>51.28519</td>
<td>35.73149</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>51.28873</td>
<td>35.73287</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>51.28699</td>
<td>35.72953</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>51.29054</td>
<td>35.73146</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>51.29346</td>
<td>35.73219</td>
<td>1</td>
</tr>
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<td>7</td>
<td>51.29012</td>
<td>35.72617</td>
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<td>8</td>
<td>51.29322</td>
<td>35.72750</td>
<td>1</td>
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<td>9</td>
<td>51.29345</td>
<td>35.72415</td>
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<thead>
<tr>
<th>Table 19:</th>
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<tbody>
<tr>
<td><strong># Facility</strong></td>
<td><strong>Latitude (degree)</strong></td>
<td><strong>Longitude (degree)</strong></td>
<td><strong>Products</strong></td>
</tr>
<tr>
<td>1</td>
<td>51.28362</td>
<td>35.73308</td>
<td>Lavash</td>
</tr>
<tr>
<td>2</td>
<td>51.28974</td>
<td>35.72937</td>
<td>Lavash-Barbari-Sangak</td>
</tr>
<tr>
<td>3</td>
<td>51.29225</td>
<td>35.72606</td>
<td>Lavash-Barbari-Sangak</td>
</tr>
<tr>
<td>4</td>
<td>51.28937</td>
<td>35.73171</td>
<td>Sangak</td>
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</table>

<table>
<thead>
<tr>
<th>Table 20:</th>
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<tbody>
<tr>
<td><strong># Facility</strong></td>
<td><strong>Barbari Quality value</strong></td>
<td><strong>Sangak Quality value</strong></td>
<td><strong>Lavash Quality value</strong></td>
</tr>
<tr>
<td>1</td>
<td>5.1</td>
<td>-</td>
<td>4.2</td>
</tr>
<tr>
<td>2</td>
<td>5.1</td>
<td>-</td>
<td>5.2</td>
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Table 21:

<table>
<thead>
<tr>
<th># Potential Location</th>
<th>Latitude (degree)</th>
<th>Longitude (degree)</th>
<th>Max. number of products</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51.28625</td>
<td>35.73080</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>51.28692</td>
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<tr>
<td>3</td>
<td>51.29063</td>
<td>35.73297</td>
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<tr>
<td>4</td>
<td>51.29224</td>
<td>35.73176</td>
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<tr>
<td>5</td>
<td>51.28868</td>
<td>35.73027</td>
<td>2</td>
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<tr>
<td>6</td>
<td>51.29062</td>
<td>35.72779</td>
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<tr>
<td>7</td>
<td>51.29034</td>
<td>35.72581</td>
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<td>8</td>
<td>51.29175</td>
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<tr>
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<td>35.72423</td>
<td>1</td>
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Table 22:

<table>
<thead>
<tr>
<th># Scenario</th>
<th>Description</th>
<th># Optimal Locations</th>
<th>Optimal Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open 1 new facility with 1 product</td>
<td>4</td>
<td>Barbari</td>
</tr>
<tr>
<td>2</td>
<td>Open 2 new facilities with 1 product in each one</td>
<td>4 And 3</td>
<td>Barbari And Lavash</td>
</tr>
<tr>
<td>3</td>
<td>Open 1 new facility with 2 products</td>
<td>5</td>
<td>Barbari and Lavash</td>
</tr>
<tr>
<td>4</td>
<td>Open 3 new facilities with 1 product in each one</td>
<td>4 And 3 And 1</td>
<td>Barbari And Lavash And Sangak</td>
</tr>
</tbody>
</table>
Technical Biography:

Seyed Jafar Sadjadi finished his PhD at the University of Waterloo, Canada. His research interest focused on solving different classes of optimization problems in industrial engineering areas, such as supply chain management, portfolio optimization, optimal pricing, etc. He has been serving at Iran University of Science and Technology since 2001.

Milad Gorji has trained in the field of Industrial Engineering at Iran University of Science and Technology. His PhD research focuses on the modeling and analysis of the facilities location in a competitive environment. His research interests include operations research, location theory, project planning, and supply chain management.

Ahmad Makui is a Professor of Industrial Engineering at Iran University of Science and Technology. He received his MS in 1991 and PhD in 1994 in Industrial Engineering at the Iran University of Science and Technology. He lectures in the field of scheduling, project and product planning and control, and MCDM. He has published three books in Persian and more than one hundred national and international papers on scheduling, etc. His main research interests consist of scheduling, MCDM, and supply chain management.

Reza Ramezanian is an assistant Professor of Industrial Engineering at K. N. Toosi University of Technology. He received B.S., M.S. and PhD degrees from Iran University of Science and Technology. His research interests include operations research, production planning, dynamic lot-sizing, scheduling and deterministic and stochastic optimization.