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Trapezoidal neutrosophic aggregation operators and their application to the multi-attribute decision-making process

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Abstract. The aim of this paper is to introduce an interval trapezoidal neutrosophic set, which is a combination of trapezoidal fuzzy numbers, and an interval neutrosophic set. This paper presents some operational rules and the score and accuracy functions of interval trapezoidal neutrosophic numbers. Then, some aggregation operators based on interval trapezoidal neutrosophic information are proposed: the Interval Trapezoidal Neutrosophic Number Weighted Arithmetic Averaging (ITNNWAA) operator and the Interval Trapezoidal Neutrosophic Number Weighted Geometric Averaging (ITNNWGA) operator; in addition, the properties of these operators are investigated in detail. Furthermore, a multi-attribute decision-making method is developed based on the operators. Finally, a numerical example is presented to illustrate the applicability and effectiveness of the proposed method.

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1. Introduction

In order to deal with various types of uncertainty, the theory of fuzzy sets [1] was introduced by Zadeh in 1965. Fuzzy set theory has been utilized in various fields with imprecise information. Although the theory is a useful tool for modeling problems including uncertainty information, determining the membership function, which characterizes a fuzzy set, can be too difficult in some cases. To avoid this difficulty, the concept of interval-valued fuzzy set [2] was proposed. Subsequently, to cope with the lack of knowledge of non-membership degrees, Intuitionistic Fuzzy Sets (IFSs) [3,4] were defined. The concept of vague set [5] was introduced in 1993, and Bustince and Burillo [6] showed that the vague set and Atanassovs IFSs were equivalent mathematically.

Although the FSs theory and IFSs theory have an important role in dealing with problems including uncertain information, they are not enough for modeling problems including indeterminate and inconsistent information in real decision-making. Therefore, the theory of Neutrosophic Set (NS) [7] was first introduced by Smarandache as a generalization of fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued

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intuitionistic fuzzy set, paraconsistent set, paradoxist set, and tautological set. An NS is identified by three functions, namely truth-membership function, indeterminacy-membership function, and falsity-membership function, which are independent of each other. In some engineering and scientific applications, modeling problems with NSs is a cumbersome task characterized by difficulty. In order to overcome difficulties, a Single-Valued Neutrosophic Set (SVNS) [8,9], which is a subclass of NS, was proposed. While indeterminacy and falsity membership values are a real or non-real subset of $]^{-}0, 1^{+}[$ in an NS truth, indeterminacy and falsity membership values are real values at the interval $[0, 1]$ in the SVNS truth. SVNS is quite a useful tool in scientific and engineering fields containing uncertain, imprecise, incomplete, and inconsistent information. Many researchers have studied the decision-making method under the single-valued environment. For example, Şahin and Liu [10] introduced the correlation coefficient of a single-valued neutrosophic hesitant fuzzy set and its application to decision-making. Ye [11] studied multi-criteria decision-making by the weighted correlation coefficient of SVNS. Ye [12] also developed single-valued neutrosophic cross-entropy for multi-criteria decision-making problems. An interval-valued neutrosophic set INS is a generalization of the NS. In 2005, for this purpose, Wang et al. [13] defined the concept of INS. Many MCDM methods were developed under the interval neutrosophic environment. For example, Tian et al. [14,15] provided an MCDM method based on a cross-entropy with interval NSs. Ye [16] proposed similarity measures between interval neutrosophic sets and presented their applications to MCDM. For more information on the NS and its applications to decision-making problems, See the following [17–23].

Since the end solution must be obtained through the synthesis of performance degrees of criteria in any MCDM problem [24], the aggregation of information is a fundamental factor. Therefore, many decision-makers have introduced various types of aggregation operators for MCDM and MADM problems with the assessment at $[0, 1]$ (see [25–33]), proportional assessment at [1/9,9], and linguistic assessment [34–36]. In general, decision-makers working in scientific and engineering fields should apply the weighted arithmetic average operator and the weighted geometric average operator [37,38]. Therefore, these aggregation operators are important tools for aggregating fuzzy information, intuitionistic fuzzy information, interval-valued fuzzy information, and interval-valued intuitionistic fuzzy information in the decision-making problems recently (see [39–42]).

The importance of scientific and engineering points of view comes to fore when most of the decision-makers commonly use the weighted arithmetic average operator and the weighted geometric average operator [43,44], which have been applied to decision-making problems. Therefore, these aggregation operators are important tools for aggregating fuzzy information, intuitionistic fuzzy information, interval-valued fuzzy information, and interval-valued intuitionistic fuzzy information in the decision-making problems in [45,46]. Although the NS generalizes the above sets from a philosophical point of view, the NS and set-theoretic operators are selected from scientific and engineering points of view.

Ye [47] suggested the application of an MCDM method using aggregation operators for the simplified NS. Moreover, some decision-making problems have been developed using the method of aggregation operators for triangular intuitionistic fuzzy sets proposed by Liu and Yuan [48]. The triangular intuitionistic fuzzy set is characterized in a way that its membership and nonmembership functions are triangular fuzzy numbers rather than exact numbers. Later, Wang [49] developed some MADM methods by using the method of aggregations operators of triangular intuitionistic fuzzy numbers. Liu and Jin [50] studied the multi-attribute decision-making method based on the weighted geometric averaging operator for interval-valued trapezoidal fuzzy numbers as a generalization of the triangular intuitionistic fuzzy number. Liu [51] introduced a weighted aggregation operator for the MAGDM method based on interval-valued trapezoidal fuzzy numbers. Wu and Liu [52] developed an approach to multi-attribute group decision-making problems with interval-valued intuitionistic trapezoidal fuzzy numbers. Further, Ye [53] extended the triangular intuitionistic fuzzy set to the trapezoidal intuitionistic fuzzy set in which, as its main characteristic, the values of the membership and nonmembership functions are trapezoidal fuzzy numbers rather than triangular fuzzy numbers. He proposed the trapezoidal intuitionistic fuzzy prioritized weighted averaging operator and the trapezoidal intuitionistic fuzzy prioritized weighted geometric operator to determine MCDM problems with respect to different priority levels. Since NSs are models with incomplete, indeterminate, and inconsistent information, they are suitable tools for modeling human thinking. From this point of view, some recent studies by Ye [54] have introduced the trapezoidal NS as a combination of trapezoidal fuzzy numbers and NS; Liang et al. [55] proposed the application of the MCDM through the trapezoidal neutrosophic preference relation; Ji et al. [56] introduced the fuzzy decision-making framework for treatment selection based on the combined QUALIFLEX-TODIM method in the trapezoidal neutrosophic environment; Liang et al. [57] developed the evaluation of e-commerce website management systems under the environment of trapezoidal neutrosophic information. Li et al. [58] utilized power aggregation operators based on the distance from the average solution (EDAS) method under linguistic neutrosophic environments to develop MAGDM problems. Peng et al. [59] proposed a single-valued neutrosophic hesitant fuzzy geometric weighted Choquet integral Heronian mean operator based on the combination of Heronian mean and Choquet integral operator and, then, utilized this operator to develop MACD problems for the applicability of the defined operator. Wang et al. [60] introduced interval neutrosophic probability with neutrosophic information and investigated several properties. Then, they developed an MCDM problem with interval neutrosophic information based on regret theory. Ji et al. [61] proposed the selection of an outsourcing provider based on the combined MABAC-ELECTRE method using single-valued neutrosophic linguistic sets. They finally provided an illustrative example to explain the utility and feasibility of their proposed method. Garg and Nancy [62] proposed linguistic prioritized aggregation operators in which the priority among the attributes and the uncertainty in linguistic terms under the Linguistic Single-Valued Neutrosophic Set (LNSVNS) and the utilization of this operator were considered to develop a multi-attribute decision-making approach. Motivated by the above works and to the best of our knowledge, there is no work available on interval trapezoidal neutrosophic numbers. Therefore, this study defines the concept of the interval trapezoidal NS and the score and accuracy functions of the interval trapezoidal NS. To propose a multi-attribute decision-making method under interval trapezoidal neutrosophic information, the interval trapezoidal neutrosophic number weighted arithmetic averaging operator ITNNWAA and the interval trapezoidal neutrosophic number weighted geometric averaging operator ITNNWGA are defined.

The rest of the paper is organized as follows: In Section 2, some fundamental concepts related to the NS and trapezoidal intuitionistic fuzzy set are given. In Section 3, a trapezoidal NS is defined as a generalization of the trapezoidal intuitionistic fuzzy set, and its operational behaviors are introduced. Moreover, the score and accuracy functions of the trapezoidal NS are defined. In Section 4, the operational rules and some properties of the score and accuracy functions of interval trapezoidal neutrosophic numbers are investigated. In Section 5, the aggregation operators, ITNNWAA and ITNNWGA, are proposed for aggregating interval trapezoidal neutrosophic information and investigating their properties in detail. In Section 6, based on ITNNWAA and ITNNWGA operators and the score and accuracy functions of interval trapezoidal neutrosophic numbers, a multi-attribute decision-making method for the interval trapezoidal neutrosophic information is developed. In Section 7, a numerical example is given to describe the application of the developed method. In Section 8, a conclusion and suggestions for future works are given.

2. Preliminaries

In this section, some basic concepts related to NSs [7] and interval NSs in [13] are briefly given, which are necessary for this work.

NS is a part of neutrosophy, which includes the origin, nature, scope of neutralities, and their interaction with different ideational spectra [7], and is a powerful general formal framework, which generalizes the above-mentioned sets from a philosophical point of view. The definition of a NS is given in [7] as follows:

Definition 2.1 [7]. Let X be a space of points (objects) with a generic element in X denoted by x. An NS a in X is defined by:

$$a = \{\langle T_a(x), I_a(x), F_a(x)\rangle | x \in X\},$$

where $T_a(x)$ is the truth-membership function, $I_a(x)$ is the indeterminacy-membership function, and $F_a(x)$ is the falsity-membership function. $T_a(x)$, $I_a(x)$, and $F_a(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ such that $T_a(x) : X \to]0^-, 1^+[$, $I_a(x) : X \to]0^-, 1^+[$ and $F_a(x) : X \to]0^-, 1^+[$. There is no restriction on the sum of $T_a(x)$, $I_a(x)$, and $F_a(x)$; thus, $0^- \leq T_a(x) + I_a(x) + F_a(x) \leq 3^+$.

The concept of the SVNS was defined by Smarandache [8] and Wang et al. [9] and is presented in the following.

Definition 2.2 [9]. Let X be a space of points (objects) with a generic element in X denoted by x. SVNS is defined as follows:

$$a = \{\langle T_a(x), I_a(x), F_a(x)\rangle | x \in X\},$$

where $T_a(x) : X \to [0,1]$ is the truth-membership degree, $I_a(x) : X \to [0,1]$ is the indeterminacy membership degree, and $F_a(x) : X \to [0,1]$ is the falsity-membership degree of x to A with the condition $0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3$.

An INS is a generalization of an NS, which has real scientific and engineering applications. In 2005, [9] gave the following definition of the INS.

Definition 2.3 [13]. Let X be a space of points (objects), with a generic element in X denoted by x. An interval NS is characterized by truth-membership function $T_A(x)$, indeterminacy membership function $I_A(x)$, and falsity-membership function $F_A(x)$ such that $T_a(x) = [\inf T_a(x), \sup T_a(x)]$, $I_a(x) = [\inf I_a(x), \sup I_a(x)]$, and $F_a(x) = [\inf F_a(x), \sup F_a(x)]$ with $0 \leq \sup T_a(x) + \sup I_a(x) + \sup F_a(x) \leq 3$ for $x \in X$.

Definition 2.4 [13]. Let $a = \langle[\inf T_a(x), \sup T_a(x)], [\inf I_a(x), \sup I_a(x)], [\inf F_a(x), \sup F_a(x)]\rangle$ and $b = \langle[\inf T_b(x), \sup T_b(x)], [\inf I_b(x), \sup I_b(x)], [\inf F_b(x), \sup F_b(x)]\rangle$ be as Interval Neutrosophic Numbers (INNs).

Then, the following operational rules hold true on INNs:

1. $a + b = \langle[\inf T_a(x) + \inf T_b(x) - \inf T_a(x) \inf T_b(x), \sup T_a(x) + \sup T_b(x) - \sup T_a(x) \sup T_b(x)], [\inf I_a(x) \inf I_b(x), \sup I_a(x) \sup I_b(x)], [\inf F_a(x) \inf F_b(x), \sup F_a(x) \sup F_b(x)]\rangle$;
2. $a.b = \langle[\inf T_a(x) \inf T_b(x), \sup T_a(x) \sup T_b(x)], [\inf I_a(x) + \inf I_b(x) - \inf I_a(x) \inf I_b(x), \sup I_a(x) + \sup I_b(x) - \sup I_a(x) \sup I_b(x)] [\inf F_a(x) + \inf F_b(x) - \inf F_a(x) \inf F_b(x), \sup F_a(x) + \sup F_b(x) - \sup F_a(x) \sup F_b(x)]\rangle$;
3. $\lambda a = \langle[1 - (1 - \inf T_a(x))^\lambda, 1 - (1 - \sup T_a(x))^\lambda], [\inf I_a^\lambda(x), \sup I_a^\lambda(x)], [\inf F_a^\lambda(x), \sup F_a^\lambda(x)]\rangle$;
4. $a^\lambda = \langle[\inf T_a^\lambda(x), \sup T_a^\lambda(x)], [1-(1-\inf I_a(x))^\lambda, 1-(1 - \sup I_a(x))^\lambda], [1 - (1 - \inf F_a(x))^\lambda, 1 - (1 - \sup F_a(x))^\lambda]\rangle$.

Definition 2.5 [13]. Let $a = \langle[\inf T_a(x), \sup T_a(x)], [\inf I_a(x), \sup I_a(x)], [\inf F_a(x), \sup F_a(x)]\rangle$, $b = \langle[\inf T_b(x), \sup T_b(x)], [\inf I_b(x), \sup I_b(x)], [\inf F_b(x), \sup F_b(x)]\rangle$ and $c = \langle[\inf T_c(x), \sup T_c(x)], [\inf I_c(x), \sup I_c(x)], [\inf F_c(x), \sup F_c(x)]\rangle$ be INNs. Then, the following will be equivalent with respect to the INNs:

1. $a + b = b + a$,
2. $a.b = b.a$,
3. $\lambda(a + b) = \lambda a + \lambda b$, for $\lambda > 0$,
4. $(a.b)^\lambda = a^\lambda + b^\lambda$, for $\lambda > 0$,
5. $\lambda_1 a + \lambda_2 a = (\lambda_1 + \lambda_2)a$, for $\lambda_1 > 0$ and $\lambda_2 > 0$,
6. $a^{\lambda_1}.a^{\lambda_2} = a^{\lambda_1+\lambda_2}$, for $\lambda_1 > 0$ and $\lambda_2 > 0$,
7. $(a + b) + c = a + (b + c)$,
8. $(a.b).c = a.(b.c)$.

3. Trapezoidal NS

The trapezoidal IFS is a generalization of the triangular IFS, defined by Liu et al. [48]. Recently, Ye [54] introduced a trapezoidal neutrosophic set based on the combination of trapezoidal fuzzy numbers and a SVNS. The following definition is a notion of the trapezoidal neutrosophic set, as introduced by Ye [54].

Definition 3.1 [54]. Let X be a space of points (objects), with a generic element in X denoted by x.

A trapezoidal NS A in X defined in the form is:

$$\bar{A}_N = \{\langle T_N(x), I_N(x), F_N(x)\rangle | x \in X\},$$

where $T_N(x) \subset [0, 1]$, $I_N(x) \subset [0, 1]$, and $F_N(x) \subset [0, 1]$ are the three trapezoidal fuzzy numbers $T_N(x) = (t_N^1(x), t_N^2(x), t_N^3(x), t_N^4(x)) : X \to [0, 1]$, $I_N(x) = (i_N^1(x), i_N^2(x), i_N^3(x), i_N^4(x)) : X \to [0, 1]$ and $F_N(x) = (f_N^1(x), f_N^2(x), f_N^3(x), f_N^4(x)) : X \to [0, 1]$ under the condition $0 \le t_N^4(x) + i_N^4(x) + f_N^4(x) \le 3$, for $x \in X$.

For the sake of convenience, let $T_N(x) = (t_1, t_2, t_3, t_4)$, $I_N(x) = (i_1, i_2, i_3, i_4)$, and $F_N(x) = (f_1, f_2, f_3, f_4)$ be three trapezoidal neutrosophic numbers. The basic element of a trapezoidal NS is denoted by $\bar{n} = \langle(t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4)\rangle$. If $t_2 = t_3$, $i_2 = i_3$, and $f_2 = f_3$ hold in a trapezoidal neutrosophic number $\bar{n}$, then trapezoidal neutrosophic number $\bar{n}$ is reduced to a triangular neutrosophic number.

Here, some operations between two trapezoidal neutrosophic numbers are described.

Definition 3.2 [54]. Let $\bar{n}_1 = \langle(t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4)\rangle$ and $\bar{n}_2 = \langle(T_1, T_2, T_3, T_4), (I_1, I_2, I_3, I_4), (F_1, F_2, F_3, F_4)\rangle$ be two trapezoidal neutrosophic numbers. Then:

1. $\bar{n}_1 \oplus \bar{n}_2 = \langle(t_1 + T_1 - t_1T_1, t_2 + T_2 - t_2T_2, t_3 + T_3 - t_3T_3, t_4 + T_4 - t_4T_4), (i_1I_1, i_2I_2, i_3I_3, i_4I_4), (f_1F_1, f_2F_2, f_3F_3, f_4F_4)\rangle$;
2. $\bar{n}_1 \otimes \bar{n}_2 = \langle(t_1T_1, t_2T_2, t_3T_3, t_4T_4), (i_1 + I_1 - i_1I_1, i_2 + I_2 - i_2I_2, i_3 + I_3 - i_3I_3, i_4 + I_4 - i_4I_4), (f_1 + F_1 - f_1F_1, f_2 + F_2 - f_2F_2, f_3 + F_3 - f_3F_3, f_4 + F_4 - f_4F_4)\rangle$;
3. $\lambda\bar{n}_1 = \langle(1-(1-t_1)^\lambda, (1-(1-t_2)^\lambda, (1-(1-t_3)^\lambda, (1-(1 - t_4)^\lambda), (i_1^\lambda, i_2^\lambda, i_3^\lambda, i_4^\lambda), (f_1^\lambda, f_2^\lambda, f_3^\lambda, f_4^\lambda)\rangle$ for $\lambda > 0$;
4. $\bar{n}_1^\lambda = \langle(t_1^\lambda, t_2^\lambda, t_3^\lambda, t_4^\lambda), (1 - (1 - i_1)^\lambda, 1 - (1 - i_2)^\lambda, 1 - (1-i_3)^\lambda, 1-(1-i_4)^\lambda), (1-(1-f_1)^\lambda, 1-(1-f_2)^\lambda, 1-(1 - f_3)^\lambda, 1 - (1 - f_4)^\lambda)\rangle$ for $\lambda \ge 0$.

Definition 3.3 [54]. Let $\bar{n} = \langle(t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4)\rangle$ be a trapezoidal neutrosophic number. Then, the score function of trapezoidal neutrosophic number is defined by Ye [54] as follows:

$$S(\bar{n}) = \frac{1}{3}\left(2 + \frac{t_1 + t_2 + t_3 + t_4}{4} - \frac{i_1 + i_2 + i_3 + i_4}{4} - \frac{f_1 + f_2 + f_3 + f_4}{4}\right), \qquad S(\bar{n}) \in [0, 1], \quad (1)$$

where the larger the value of $S(\bar{n})$, the larger the trapezoidal neutrosophic number $\bar{n}$. For a particular case, when $t_2 = t_3$, $i_2 = i_3$, and $f_2 = f_3$, then the score function of trapezoidal neutrosophic number $\bar{n}$ of Eq. (1) is reduced to the following score function of the triangular neutrosophic number:

$$S(\bar{n}) = \frac{1}{3}\left(2 + \frac{t_1 + 2t_2 + t_4}{4} - \frac{i_1 + 2i_2 + i_4}{4} - \frac{f_1 + 2f_2 + f_4}{4}\right), \qquad S(\bar{n}) \in [0, 1]. \quad (2)$$

Definition 3.4 [54]. Let $\bar{n} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ be a trapezoidal neutrosophic number. Then, the accuracy function of the trapezoidal neutrosophic number is defined by Ye [54] as follows:

$$H(\bar{n}) = \left(\frac{t_1 + t_2 + t_3 + t_4}{4} - \frac{f_1 + f_2 + f_3 + f_4}{4} \right),$$

$$H(\bar{n}) \in [-1, 1], \tag{3}$$

where the larger the value of $H(\bar{n})$, the higher the accuracy degree of the trapezoidal neutrosophic number $\bar{n}$. In addition, when $t_2 = t_3$ and $f_2 = f_3$, then the accuracy function of trapezoidal neutrosophic number $\bar{n}$ of Eq. (3) is reduced to the following accuracy function of triangular neutrosophic number:

$$H(\bar{n}) = \left(\frac{t_1 + 2t_2 + t_4}{4} - \frac{f_1 + 2f_2 + f_4}{4} \right),$$

$$H(\bar{n}) \in [-1, 1], \tag{4}$$

which is the special case for Eq. (3).

4. Interval trapezoidal NS

In this section, this study extends the trapezoidal interval-valued intuitionistic fuzzy set to an INNS to introduce the interval trapezoidal neutrosophic set based on the connection between trapezoidal fuzzy numbers and the INNs and its score and accuracy functions. Let us propose a definition of the interval trapezoidal NS as a generalization of interval-valued intuitionistic trapezoidal fuzzy numbers.

Definition 4.1. Let X be a space of points (objects), with a generic element in X denoted by x. An interval trapezoidal NS A in X is defined in the following form:

$$\bar{A}_N = \left\{ \left\langle \left[T_N^L(x), T_N^U(x)\right], \left[I_N^L(x), I_N^U(x)\right], \left[F_N^L(x), F_N^U(x)\right] \right\rangle | x \in X \right\},$$

where $T_N^L(x), T_N^U(x) \subset [0,1]$, $I_N^L(x), I_N^U(x) \subset [0,1]$ and $F_N^L(x), F_N^U(x) \subset [0,1]$ are six trapezoidal fuzzy numbers such that $T_N^L(x) = (t_1^L(x), t_2^L(x), t_3^L(x), t_4^L(x)) : X \to [0,1]$, $T_N^U(x) = (t_1^U(x), t_2^U(x), t_3^U(x), t_4^U(x)) : X \to [0,1]$, $I_N^L(x) = (i_1^L(x), i_2^L(x), i_3^L(x), i_4^L(x)) : X \to [0,1]$, $I_N^U(x) = (i_1^U(x), i_2^U(x), i_3^U(x), i_4^U(x)) : X \to [0,1]$, $F_N^L(x) = (f_1^L(x), f_2^L(x), f_3^L(x), f_4^L(x)) : X \to [0,1]$ $F_N^U(x) = (f_1^U(x), f_2^U(x), f_3^U(x), f_4^U(x)) : X \to [0,1]$ and under the condition $0 \leq \sup t_4^U(x) + \sup i_4^U(x) + \sup f_4^U(x) \leq 3$ for $x \in X$.

Therefore, an interval trapezoidal neutrosophic number is denoted by $\bar{n} = \langle [(t_1^L, t_2^L, t_3^L, t_4^L), (t_1^U, t_2^U, t_3^U, t_4^U)], [(i_1^L, i_2^L, i_3^L, i_4^L), (i_1^U, i_2^U, i_3^U, i_4^U)], [(f_1^L, f_2^L, f_3^L, f_4^L), (f_1^U, f_2^U, f_3^U, f_4^U)] \rangle$, which is the basic element of the interval trapezoidal NS. An interval trapezoidal neutrosophic number ITNN consists of 24 real numbers that all belong to the interval $[0,1]$. If $t_2^L = t_3^L$, $t_2^U = t_3^U$, $i_2^L = i_3^L$, $i_2^U = i_3^U$, $f_2^L = f_3^L$ and $f_2^U = f_3^U$ hold in the interval trapezoidal neutrosophic number n, then it is reduced to an interval triangular neutrosophic number, which is the special case of an interval trapezoidal neutrosophic number.

Definition 4.2. Let $\bar{n}_1 = \langle [(t_1^L, t_2^L, t_3^L, t_4^L), (t_1^U, t_2^U, t_3^U, t_4^U)], [(i_1^L, i_2^L, i_3^L, i_4^L), (i_1^U, i_2^U, i_3^U, i_4^U)], [(f_1^L, f_2^L, f_3^L, f_4^L), (f_1^U, f_2^U, f_3^U, f_4^U)] \rangle$ and $\bar{n}_2 = \langle [(T_1^L, T_2^L, T_3^L, T_4^L), (T_1^U, T_2^U, T_3^U, T_4^U)], [(I_1^L, I_2^L, I_3^L, I_4^L), (I_1^U, I_2^U, I_3^U, I_4^U)], [(F_1^L, F_2^L, F_3^L, F_4^L), (F_1^U, F_2^U, F_3^U, F_4^U)] \rangle$ be two interval trapezoidal neotrosophic numbers. Then, the following operations hold:

1. $\bar{n}_1 \oplus \bar{n}_2 = \langle [(t_1^L + T_1^L - t_1^L T_1^L, t_2^L + T_2^L - t_2^L T_2^L, t_3^L + T_3^L - t_3^L T_3^L, t_4^L + T_4^L - t_4^L T_4^L), (t_1^U + T_1^U - t_1^U T_1^U, t_2^U + T_2^U - t_2^U T_2^U, t_3^U + T_3^U - t_3^U T_3^U, t_4^U + T_4^U - t_4^U T_4^U], [(i_1^L I_1^L, i_2^L I_2^L, i_3^L I_3^L, i_4^L I_4^L), (i_1^U I_1^U, i_2^U I_2^U, i_3^U I_3^U, i_4^U I_4^U)], [(f_1^L F_1^L, f_2^L F_2^L, f_3^L F_3^L, f_4^L F_4^L), (f_1^U F_1^U, f_2^U F_2^U, f_3^U F_3^U, f_4^U F_4^U)] \rangle$.
2. $\bar{n}_1 \otimes \bar{n}_2 = \langle [(t_1^L T_1^L, t_2^L T_2^L, t_3^L T_3^L, t_4^L T_4^L), (t_1^U T_1^U, t_2^U T_2^U, t_3^U T_3^U, t_4^U T_4^U], [(i_1^L + I_1^L - i_1^L I_1^L, i_2^L + I_2^L - i_2^L I_2^L, i_3^L + I_3^L - i_3^L I_3^L, i_4^L + I_4^L - i_4^L I_4^L), (i_1^U + I_1^U - i_1^U I_1^U, i_2^U + I_2^U - i_2^U I_2^U, i_3^U + I_3^U - i_3^U I_3^U, i_4^U + I_4^U - i_4^U I_4^U)], [(f_1^L + F_1^L - f_1^L F_1^L, f_2^L + F_2^L - f_2^L F_2^L, f_3^L + F_3^L - f_3^L F_3^L, f_4^L + F_4^L - f_4^L F_4^L), (f_1^U + F_1^U - f_1^U F_1^U, f_2^U + F_2^U - f_2^U F_2^U, f_3^U + F_3^U - f_3^U F_3^U, f_4^U + F_4^U - f_4^U F_4^U)] \rangle$.
3. $\lambda \bar{n}_1 = \langle [(1-(1-t_1^L)^\lambda, 1-(1-t_2^L)^\lambda, 1-(1-t_3^L)^\lambda, 1-(1-t_4^L)^\lambda), (1-(1-t_1^U)^\lambda, 1-(1-t_2^U)^\lambda, 1-(1-t_3^U)^\lambda, 1-(1-t_4^U)^\lambda)], [((i_1^L)^\lambda, (i_2^L)^\lambda, (i_3^L)^\lambda, (i_4^L)^\lambda), ((i_1^U)^\lambda, (i_2^U)^\lambda, (i_3^U)^\lambda, (i_4^U)^\lambda)], [((f_1^L)^\lambda, (f_2^L)^\lambda, (f_3^L)^\lambda, (f_4^L)^\lambda), ((f_1^U)^\lambda, (f_2^U)^\lambda, (f_3^U)^\lambda, (f_4^U)^\lambda)] \rangle$ for $\lambda > 0$.
4. $\bar{n}_1^\lambda = \langle [((t_1^L)^\lambda, (t_2^L)^\lambda, (t_3^L)^\lambda, (t_4^L)^\lambda), ((t_1^U)^\lambda, (t_2^U)^\lambda, (t_3^U)^\lambda, (t_4^U)^\lambda)], [(1-(1-i_1^L)^\lambda, 1-(1-i_2^L)^\lambda, 1-(1-i_3^L)^\lambda, 1-(1-i_4^L)^\lambda), (1-(1-i_1^U)^\lambda, 1-(1-i_2^U)^\lambda, 1-(1-i_3^U)^\lambda, 1-(1-i_4^U)^\lambda)], [(1-(1-f_1^L)^\lambda, 1-(1-f_2^L)^\lambda, 1-(1-f_3^L)^\lambda, 1-(1-f_4^L)^\lambda), (1-(1-f_1^U)^\lambda, 1-(1-f_2^U)^\lambda, 1-(1-f_3^U)^\lambda, 1-(1-f_4^U)^\lambda)] \rangle$ for $\lambda \geq 0$.

This study proposes a definition of score and accuracy functions for an interval trapezoidal neutrosophic number based on the expected value of an interval trapezoidal fuzzy number defined by Ye [63] and the score and accuracy functions of a trapezoidal neutrosophic number defined by Ye [54].

Definition 4.3. Let $\bar{n}_1 = \langle [(t_1^L, t_2^L, t_3^L, t_4^L), (t_1^U, t_2^U, t_3^U, t_4^U)], [(i_1^L, i_2^L, i_3^L, i_4^L), (i_1^U, i_2^U, i_3^U, i_4^U)], [(f_1^L, f_2^L, f_3^L, f_4^L), (f_1^U, f_2^U, f_3^U, f_4^U)] \big\rangle$ be an interval trapezoidal neutrosophic number. Then, the score function of an interval trapezoidal neutrosophic number is defined as follows:

$$S(\bar{n})=\left(\frac{t_1^U+t_2^U+t_3^U+t_4^U}{4}+\frac{t_1^L+t_2^L+t_3^L+t_4^L}{4}\right)$$
$$+\left(\frac{i_1^U+i_2^U+i_3^U+i_4^U}{4}-\frac{i_1^L+i_2^L+i_3^L+i_4^L}{4}\right)$$
$$+\left(\frac{f_1^U+f_2^U+f_3^U+f_4^U}{4}-\frac{f_1^L+f_2^L+f_3^L+f_4^L}{4}\right),$$
$$S(\bar{n})\in[-1,1], \tag{5}$$

where the larger the value of $S(\bar{n})$, the larger the interval trapezoidal neutrosophic number $\bar{n}$. When $t_2^L=t_3^L$, $t_2^U=t_3^U$, $i_2^L=i_3^L$, $i_2^U=i_3^U$, $f_2^L=f_3^L$ and $f_2^U=f_3^U$ hold in an interval trapezoidal neutrosophic number $\bar{n}$ in Eq. (5), then Eq. (5) is reduced to the score function of an interval triangular neutrosophic number:

$$S(\bar{n})=\left(\frac{t_1^U+2t_2^U+t_4^U}{4}+\frac{t_1^L+2t_2^L+t_4^L}{4}\right)$$
$$+\left(\frac{i_1^U+2i_2^U+i_4^U}{4}-\frac{i_1^L+2i_2^L+i_4^L}{4}\right)$$
$$+\left(\frac{f_1^U+2f_2^U+f_4^U}{4}-\frac{f_1^L+2f_2^L+f_4^L}{4}\right),$$
$$S(\bar{n})\in[-1,1], \tag{6}$$

which is the special case of Eq. (5).

Definition 4.4. Let $\bar{n}_1=\langle[(t_1^L,t_2^L,t_3^L,t_4^L),(t_1^U,t_2^U,t_3^U,t_4^U)],\ [(i_1^L,i_2^L,i_3^L,i_4^L),\ (i_1^U,i_2^U,i_3^U,i_4^U)],\ [(f_1^L,f_2^L,f_3^L,f_4^L),\ (f_1^U,f_2^U,f_3^U,f_4^U)]\rangle$ be an interval trapezoidal neutrosophic number. Then, the accuracy function of an interval trapezoidal neutrosophic number can be defined as:

$$H(\bar{n})=\left(\frac{t_1^U+t_2^U+t_3^U+t_4^U}{4}+\frac{t_1^L+t_2^L+t_3^L+t_4^L}{4}\right)$$
$$+\left(\frac{f_1^U+f_2^U+f_3^U+f_4^U}{4}-\frac{f_1^L+f_2^L+f_3^L+f_4^L}{4}\right),$$
$$H(\bar{n})\in[0,1], \tag{7}$$

where the larger the value of $H(\bar{n})$, the larger the interval trapezoidal neutrosophic number $\bar{n}$. When $t_2^L=t_3^L$, $t_2^U=t_3^U$, $i_2^L=i_3^L$, $i_2^U=i_3^U$, $f_2^L=f_3^L$, and $f_2^U=f_3^U$ hold in an interval trapezoidal neutrosophic number $\bar{n}$ in Eq. (7), then Eq. (7) is reduced to the accuracy function of an interval triangular neutrosophic number as:

$$H(\bar{n})=\left(\frac{t_1^U+2t_2^U+t_4^U}{4}+\frac{t_1^L+2t_2^L+t_4^L}{4}\right)$$
$$+\left(\frac{f_1^U+2f_2^U+f_4^U}{4}-\frac{f_1^L+2f_2^L+f_4^L}{4}\right),$$
$$H(\bar{n})\in[0,1], \tag{8}$$

which is the special case of Eq. (7).

Definition 4.5. Let $\bar{n}_1=\langle[(t_1^L,t_2^L,t_3^L,t_4^L),(t_1^U,t_2^U,t_3^U,t_4^U)],\ [(i_1^L,i_2^L,i_3^L,i_4^L),\ (i_1^U,i_2^U,i_3^U,i_4^U)],[(f_1^L,f_2^L,f_3^L,f_4^L),(f_1^U,f_2^U,f_3^U,f_4^U)]\rangle$ and $\bar{n}_2=\langle[(T_1^L,T_2^L,T_3^L,T_4^L),(T_1^U,T_2^U,T_3^U,T_4^U)],\ [(I_1^L,I_2^L,I_3^L,I_4^L),\ (I_1^U,I_2^U,I_3^U,I_4^U)],[(F_1^L,F_2^L,F_3^L,F_4^L),(F_1^U,F_2^U,F_3^U,F_4^U)]\rangle$ be two interval trapezoidal neutrosophic numbers. Therefore, $S(\bar{n}_1)$ and $S(\bar{n}_2)$ are the scores of $\bar{n}_1$, $\bar{n}_2$, respectively, and $H(\bar{n}_1)$, $H(\bar{n}_2)$ are the accuracy degrees of $\bar{n}_1$ and $\bar{n}_2$. respectively.

Then, the order relation of the two interval trapezoidal neutrosophic numbers is defined in the following form:

1. If $S(\bar{n}_1)>S(\bar{n}_2)$, then $\bar{n}_1>\bar{n}_2$,
2. If $S(\bar{n}_1)=S(\bar{n}_2)$, and
   (i) If $H(\bar{n}_1)=H(\bar{n}_2)$, then $\bar{n}_1=\bar{n}_2$,
   (ii) If $H(\bar{n}_1)>H(\bar{n}_2)$, then $\bar{n}_1>\bar{n}_2$.

5. Weighted aggregation operators of interval trapezoidal neutrosophic numbers

In aggregation information, for the decision-making problem, the weighted arithmetic averaging operator and geometric averaging operator are usually used. Here, based on Definition 4.3, these two operators are proposed here for the interval trapezoidal neutrosophic numbers.

5.1. Interval trapezoidal neutrosophic numbers with the weighted arithmetic averaging operator

Definition 5.1. Let $\bar{n}_{1j}=\langle[(t_{1j}^L,t_{2j}^L,t_{3j}^L,t_{4j}^L),(t_{1j}^U,t_{2j}^U,t_{3j}^U,t_{4j}^U)],\ [(i_{1j}^L,i_{2j}^L,i_{3j}^L,i_{4j}^L),\ (i_{1j}^U,i_{2j}^U,i_{3j}^U,i_{4j}^U)],\ [(f_{1j}^L,f_{2j}^L,f_{3j}^L,f_{4j}^L),(f_{1j}^U,f_{2j}^U,f_{3j}^U,f_{4j}^U)]\rangle$ $(j=1,2,\cdots,n)$ be an interval trapezoidal neutrosophic number. Then, an Interval Trapezoidal Neutrosophic Number with Weighted Arithmetic Averaging (ITNNWAA) operator is defined as follows:

$$ITNNWAA(\bar{n}_{11},\bar{n}_{12},\bar{n}_{13},\ldots,\bar{n}_{1n})$$
$$=w_1\bar{n}_{11}\oplus w_2\bar{n}_{12}\oplus\cdots\oplus w_n\bar{n}_{1n}=\bigoplus_{j=1}^{n}(w_j\bar{n}_{1j}), \tag{9}$$

where w_j $(j=1,2,\cdots,n)$ is the weight of the jth interval trapezoidal neutrosophic number $\bar{n}_{1j}$ $(j=1,2,\cdots,n)$ with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$.

By the operation rules of the interval trapezoidal neutrosophic numbers in Definition 4.2, the following theorem is given.

Theorem 5.2. *Let* $\bar{n}_{1j} = \langle[(t_{1j}^L, t_{2j}^L, t_{3j}^L, t_{4j}^L), (t_{1j}^U, t_{2j}^U, t_{3j}^U, t_{4j}^U)], [(i_{1j}^L, i_{2j}^L, i_{3j}^L, i_{4j}^L), (i_{1j}^U, i_{2j}^U, i_{3j}^U, i_{4j}^U)], [(f_{1j}^L, f_{2j}^L, f_{3j}^L, f_{4j}^L), (f_{1j}^U, f_{2j}^U, f_{3j}^U, f_{4j}^U)]\rangle$ $(j = 1, 2, \cdots, n)$ *be a collection of interval trapezoidal neutrosophic numbers. Then, their aggregated value by using the ITNNWAA operator is also an interval trapezoidal neutrosophic number. Then, we have:*

$$ITNNWAA\,(\bar{n}_{11}, \bar{n}_{12}, \bar{n}_{13}, \ldots, \bar{n}_{1n})$$

$$= w_1\bar{n}_{11} \oplus w_2\bar{n}_{12} \oplus \ldots \oplus w_n\bar{n}_{1n} = \bigoplus_{j=1}^{n}(w_j\bar{n}_{1j})$$

$$= \Bigg\langle \Bigg[\Bigg(1 - \prod_{j=1}^{n}\left(1 - t_{1j}^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - t_{2j}^L\right)^{w_j},$$

$$1 - \prod_{j=1}^{n}\left(1 - t_{3j}^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - t_{4j}^L\right)^{w_j} \Bigg),$$

$$\Bigg(1 - \prod_{j=1}^{n}(1 - t_{1j}^U)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - t_{2j}^U\right)^{w_j},$$

$$1 - \prod_{j=1}^{n}\left(1 - t_{3j}^U\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - t_{4j}^U\right)^{w_j} \Bigg) \Bigg],$$

$$\Bigg[\Bigg(\prod_{j=1}^{n}\left(i_{1j}^L\right)^{w_j}, \prod_{j=1}^{n}(i_{2j}^L)^{w_j}, \prod_{j=1}^{n}\left(i_{3j}^L\right)^{w_j},$$

$$\prod_{j=1}^{n}\left(i_{4j}^L\right)^{w_j} \Bigg), \Bigg(\prod_{j=1}^{n}\left(i_{1j}^U\right)^{w_j}, \prod_{j=1}^{n}\left(i_{2j}^U\right)^{w_j},$$

$$\prod_{j=1}^{n}\left(i_{3j}^U\right)^{w_j}, \prod_{j=1}^{n}\left(i_{4j}^U\right)^{w_j} \Bigg) \Bigg], \Bigg[\Bigg(\prod_{j=1}^{n}(f_{1j}^L)^{w_j},$$

$$\prod_{j=1}^{n}\left(f_{2j}^L\right)^{w_j}, \prod_{j=1}^{n}\left(f_{3j}^L\right)^{w_j}, \prod_{j=1}^{n}\left(f_{4j}^L\right)^{w_j} \Bigg),$$

$$\Bigg(\prod_{j=1}^{n}\left(f_{1j}^U\right)^{w_j}, \prod_{j=1}^{n}\left(f_{2j}^U\right)^{w_j}, \prod_{j=1}^{n}\left(f_{3j}^U\right)^{w_j},$$

$$\prod_{j=1}^{n}\left(f_{4j}^U\right)^{w_j} \Bigg) \Bigg] \rangle,$$

where w_j $(j = 1, 2, \cdots, n)$ *is the weight of the* j*th interval trapezoidal neutrosophic number* $\bar{n}_{1j}$ $(j = 1, 2, \cdots, n)$ *with* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$.

Proof. The above theorem can be proved through the mathematical induction method. When $n = 2$, then:

$$w_1\bar{n}_{11} = \Big\langle [(1 - (1 - t_{11}^L)^{w_1}, 1 - (1 - t_{21}^L)^{w_1},$$

$$1 - (1 - t_{31}^L)^{w_1}, 1 - (1 - t_{41}^L)^{w_1}),$$

$$(1 - (1 - t_{11}^U)^{w_1}, 1 - (1 - t_{21}^U)^{w_1},$$

$$1 - (1 - t_{31}^U)^{w_1}, 1 - (1 - t_{41}^U)^{w_1})],$$

$$[((i_{11}^L)^{w_1}, (i_{21}^L)^{w_1}, (i_{31}^L)^{w_1}, (i_{41}^L)^{w_1}),$$

$$((i_{11}^U)^{w_1}, (i_{21}^U)^{w_1}, (i_{31}^U)^{w_1}, (i_{41}^U)^{w_1})],$$

$$[((f_{11}^L)^{w_1}, (f_{21}^L)^{w_1}, (f_{31}^L)^{w_1}, (f_{41}^L)^{w_1}),$$

$$((f_{11}^U)^{w_1}, (f_{21}^U)^{w_1}, (f_{31}^U)^{w_1}, (f_{41}^U)^{w_1})] \Big\rangle,$$

$$w_2\bar{n}_{12} = \Big\langle [(1 - (1 - t_{12}^L)^{w_2}, 1 - (1 - t_{22}^L)^{w_2},$$

$$1 - (1 - t_{32}^L)^{w_2}, 1 - (1 - t_{42}^L)^{w_2}),$$

$$(1 - (1 - t_{12}^U)^{w_2}, 1 - (1 - t_{22}^U)^{w_2},$$

$$1 - (1 - t_{32}^U)^{w_2}, 1 - (1 - t_{42}^U)^{w_2})],$$

$$[((i_{12}^L)^{w_2}, (i_{22}^L)^{w_2}, (i_{32}^L)^{w_2}, (i_{42}^L)^{w_2}),$$

$$((i_{12}^U)^{w_2}, (i_{22}^U)^{w_2}, (i_{32}^U)^{w_2}, (i_{42}^U)^{w_2})],$$

$$[((f_{12}^L)^{w_2}, (f_{22}^L)^{w_2}, (f_{32}^L)^{w_2}, (f_{42}^L)^{w_2}),$$

$$((f_{12}^U)^{w_1 2}, (f_{22}^U)^{w_2}, (f_{32}^U)^{w_2}, (f_{42}^U)^{w_2})] \Big\rangle.$$

Therefore:

$$ITNNWAA(\bar{n}_{11}, \bar{n}_{12}) = w_1\bar{n}_{11} \oplus w_2\bar{n}_{12} = \langle[(1$$

$$-(1-t_{11}^L)^{w_1} + 1 - (1-t_{12}^L)^{w_2} - (1-(1-t_{11}^L)^{w_1})(1$$

$$-(1-t_{12}^L)^{w_2}), 1-(1-t_{21}^L)^{w_1} + 1 - (1-t_{22}^L)^{w_2} - (1$$

$$-(1-t_{21}^L)^{w_1})(1-(1-t_{22}^L)^{w_2}), 1-(1-t_{31}^L)^{w_1} + 1$$

$$-(1-t_{32}^L)^{w_2} - (1-(1-t_{31}^L)^{w_1})(1-(1-t_{32}^L)^{w_2}), 1$$

$$-(1-t_{41}^L)^{w_1} + 1 - (1-t_{42}^L)^{w_2} - (1-(1-t_{41}^L)^{w_1})(1$$

$$-(1-t_{42}^L)^{w_2})), (1-(1-t_{11}^U)^{w_1} + 1 - (1-t_{12}^U)^{w_2} - (1$$

$$-(1-t_{11}^U)^{w_1})(1-(1-t_{12}^U)^{w_2}), 1-(1-t_{21}^U)^{w_1} + 1$$

$$-(1-t_{22}^U)^{w_2}-(1-(1-t_{21}^U)^{w_1})(1-(1-t_{22}^U)^{w_2}),1$$
$$-(1-t_{31}^U)^{w_1}+1-(1-t_{32}^U)^{w_2}-(1-(1-t_{31}^U)^{w_1})(1$$
$$-(1-t_{32}^U)^{w_2}),1-(1-t_{41}^U)^{w_1}+1-(1-t_{42}^U)^{w_2}-(1$$
$$-(1-t_{41}^U)^{w_1})(1-(1-t_{42}^U)^{w_2}))],[((i_{11}^L)^{w_1}(i_{12}^L)^{w_2},$$
$$(i_{21}^L)^{w_1}(i_{22}^L)^{w_2},(i_{31}^L)^{w_1}(i_{32}^L)^{w_2},(i_{41}^L)^{w_1}(i_{42}^L)^{w_2}),$$
$$((i_{11}^U)^{w_1}(i_{12}^U)^{w_2},(i_{21}^U)^{w_1}(i_{22}^U)^{w_2},(i_{31}^U)^{w_1}$$
$$(i_{32}^U)^{w_2},(i_{41}^U)^{w_1}(i_{42}^U)^{w_2})],[((f_{11}^L)^{w_1}(f_{12}^L)^{w_2},$$
$$(f_{21}^L)^{w_1}(f_{22}^L)^{w_2},(f_{31}^L)^{w_1}(f_{32}^L)^{w_2},(f_{41}^L)^{w_1}$$
$$(f_{42}^L)^{w_2}),((f_{11}^U)^{w_1}(f_{12}^U)^{w_2},(f_{21}^U)^{w_1}(f_{22}^U)^{w_2},$$
$$(f_{31}^U)^{w_1}(f_{32}^U)^{w_2},(f_{41}^U)^{w_1}(f_{42}^U)^{w_2})]\rangle=\langle[(1$$
$$-(1-t_{11}^L)^{w_1}(1-t_{12}^L)^{w_2},1$$
$$-(1-t_{21}^L)^{w_1}(1-t_{22}^L)^{w_2},1$$
$$-(1-t_{31}^L)^{w_1}(1-t_{32}^L)^{w_2},1$$
$$-(1-t_{41}^L)^{w_1}(1-t_{42}^L)^{w_2}),(1$$
$$-(1-t_{11}^U)^{w_1}(1-t_{12}^U)^{w_2},1$$
$$-(1-t_{21}^U)^{w_1}(1-t_{22}^U)^{w_2},1$$
$$-(1-t_{31}^U)^{w_1}(1-t_{32}^U)^{w_2},1$$
$$-(1-t_{41}^U)^{w_1}(1-t_{42}^U)^{w_2})],$$
$$\left[\left(\prod_{j=1}^{2}(i_{1j}^L)^{w_j},\prod_{j=1}^{2}(i_{2j}^L)^{w_j},\prod_{j=1}^{2}(i_{3j}^L)^{w_j},\prod_{j=1}^{2}(i_{4j}^L)^{w_j}\right),\right.$$
$$\left.\left(\prod_{j=1}^{2}(i_{1j}^U)^{w_j},\prod_{j=1}^{2}(i_{2j}^U)^{w_j},\prod_{j=1}^{2}(i_{3j}^U)^{w_j},\prod_{j=1}^{2}(i_{4j}^U)^{w_j}\right)\right],$$
$$\left[\left(\prod_{j=1}^{2}(f_{1j}^L)^{w_j},\prod_{j=1}^{2}(f_{2j}^L)^{w_j},\prod_{j=1}^{2}(f_{3j}^L)^{w_j},\prod_{j=1}^{2}(f_{4j}^L)^{w_j}\right),\right.$$
$$\left.\left.\left(\prod_{j=1}^{2}(f_{1j}^U)^{w_j},\prod_{j=1}^{2}(f_{2j}^U)^{w_j},\prod_{j=1}^{2}(f_{3j}^U)^{w_j},\prod_{j=1}^{2}(f_{4j}^U)^{w_j}\right)\right]\right\rangle.$$

When $n=k$ holds, then:

$ITNNWAA\,(\bar{n}_{11},\bar{n}_{12},\cdots,\bar{n}_{1k})$

$$=w_1\bar{n}_{11}\oplus w_2\bar{n}_{12}\oplus\cdots\oplus\bar{n}_{1k}$$
$$=\bigoplus_{j=1}^{k}(w_j\bar{n}_{1j})=\left\langle\left[\left(1-\prod_{j=1}^{k}(1-t_{1j}^L)^{w_j},1\right.\right.\right.$$
$$-\prod_{j=1}^{k}(1-t_{2j}^L)^{w_j},1-\prod_{j=1}^{k}\left(1-t_{3j}^L\right)^{w_j},1$$
$$\left.-\prod_{j=1}^{k}(1-t_{4j}^L)^{w_j}\right),\left(1-\prod_{j=1}^{k}(1-t_{1j}^U)^{w_j},1\right.$$
$$-\prod_{j=1}^{k}(1-t_{2j}^U)^{w_j},1-\prod_{j=1}^{k}(1-t_{3j}^U)^{w_j},1$$
$$\left.\left.-\prod_{j=1}^{k}(1-t_{4j}^U)^{w_j}\right)\right],\left[\left(\prod_{j=1}^{k}(i_{1j}^L)^{w_j},\prod_{j=1}^{k}(i_{2j}^L)^{w_j},\right.\right.$$
$$\left.\prod_{j=1}^{k}(i_{3j}^L)^{w_j},\prod_{j=1}^{k}(i_{4j}^L)^{w_j}\right),\left(\prod_{j=1}^{k}(i_{1j}^U)^{w_j},\prod_{j=1}^{k}(i_{2j}^U)^{w_j},\right.$$
$$\left.\left.\prod_{j=1}^{k}(i_{3j}^U)^{w_j},\prod_{j=1}^{k}(i_{4j}^U)^{w_j}\right)\right],\left[\left(\prod_{j=1}^{k}(f_{1j}^L)^{w_j},\right.\right.$$
$$\prod_{j=1}^{k}(f_{2j}^L)^{w_j},\prod_{j=1}^{k}(f_{3j}^L)^{w_j},\prod_{j=1}^{k}(f_{4j}^L)^{w_j}),\left(\prod_{j=1}^{k}(f_{1j}^U)^{w_j},\right.$$
$$\left.\left.\left.\prod_{j=1}^{k}(f_{2j}^U)^{w_j},\prod_{j=1}^{k}(f_{3j}^U)^{w_j},\prod_{j=1}^{k}(f_{4j}^U)^{w_j}\right)\right]\right\rangle.$$

When $n=k+1$ holds, then by applying:

$ITNWAA\,(\bar{n}_{11},\bar{n}_{12},\cdots,\bar{n}_{1k+1})$

$$=\left\langle\left[\left(1-\prod_{j=1}^{k}(1-t_{1j}^L)^{w_j}+1-(1-t_{1k+1}^L)^{w_k+1}\right.\right.\right.$$
$$-\left(1-\prod_{j=1}^{k}\left(1-t_{1j}^L\right)^{w_j}\right)\left(1-(1-t_{1k+1}^L)^{w_k+1}\right),1$$
$$-\prod_{j=1}^{k}(1-t_{2j}^L)^{w_j}+1-(1-t_{2k+1}^L)^{w_{k+1}}$$
$$-\prod_{j=1}^{k}(1-(1-t_{2j}^L)^{w_j})(1-(1-t_{2k+1}^L)^{w_{k+1}}),1$$

$$-\prod_{j=1}^{k}(1-t_{3j}^{L})^{w_j}+1-(1-t_{3k+1}^{L})^{w_{k+1}}$$

$$-\big(1-\prod_{j=1}^{k}(1-t_{3j}^{L})^{w_j}\big)(1-(1-t_{3k+1}^{L})^{w_{k+1}}),1$$

$$-\prod_{j=1}^{k}(1-t_{4j}^{L})^{w_j}+1-(1-t_{4k+1}^{L})^{w_{k+1}}$$

$$-\big(1-\prod_{j=1}^{k}(1-t_{4j}^{L})^{w_j}\big)(1-(1-t_{4k+}^{L})^{w_{k+1}})\Bigg),\Bigg(1$$

$$-\prod_{j=1}^{k}(1-t_{1j}^{U})^{w_j}+1-(1-t_{1k+1}^{U})^{w_{k+1}}$$

$$-\big(1-\prod_{j=1}^{k}(1-t_{1j}^{U})^{w_j}\big)(1-(1-t_{1k+1}^{U})^{w_{k+1}}),1$$

$$-\prod_{j=1}^{k}(1-t_{2j}^{U})^{w_j}+1-(1-t_{2k+1}^{U})^{w_{k+1}}$$

$$-\big(1-\prod_{j=1}^{k}(1-t_{2j}^{U})^{w_j}\big)(1-(1-t_{2k+1}^{U})^{w_{k+1}}),1$$

$$-\prod_{j=1}^{k}(1-t_{3j}^{U})^{w_j}+1-(1-t_{3k+1}^{U})^{w_{k+1}}$$

$$-\big(1-\prod_{j=1}^{k}(1-t_{3j}^{U})^{w_j}\big)(1-(1-t_{3k+1}^{U})^{w_{k+1}}),1$$

$$-\prod_{j=1}^{k}(1-t_{4j}^{U})^{w_j}+1-(1-t_{4k+1}^{U})^{w_{k+1}}$$

$$-\big(1-\prod_{j=1}^{k}(1-t_{4j}^{U})^{w_j}\big)(1-(1-t_{4k+1}^{U})^{w_{k+1}}))\Bigg]\,,$$

$$\Bigg[\Bigg(\prod_{j=1}^{k+1}(i_{1j}^{L})^{w_j},\prod_{j=1}^{k+1}(i_{2j}^{L})^{w_j},\prod_{j=1}^{k+1}(i_{3j}^{L})^{w_j},\prod_{j=1}^{k+1}(i_{4j}^{L})^{w_j}\Bigg),$$

$$\Bigg(\prod_{j=1}^{k+1}(i_{1j}^{U})^{w_j},\prod_{j=1}^{k+1}(i_{2j}^{U})^{w_j},\prod_{j=1}^{k+1}(i_{3j}^{U})^{w_j},\prod_{j=1}^{k+1}(i_{4j}^{U})^{w_j}\Bigg)\Bigg],$$

$$\Bigg[\Bigg(\prod_{j=1}^{k+1}(f_{1j}^{L})^{w_j},\prod_{j=1}^{k+1}(f_{2j}^{L})^{w_j},\prod_{j=1}^{k+1}(f_{3j}^{L})^{w_j},\prod_{j=1}^{k+1}(f_{4j}^{L})^{w_j}\Bigg),$$

$$\Bigg(\prod_{j=1}^{k+1}(f_{1j}^{U})^{w_j},\prod_{j=1}^{k+1}(f_{2j}^{U})^{w_j},\prod_{j=1}^{k+1}(f_{3j}^{U})^{w_j},\prod_{j=1}^{k+1}(f_{4j}^{U})^{w_j}\Bigg)\Bigg]\Bigg\rangle$$

$$=\Bigg\langle\Bigg[\Bigg(1-\prod_{j=1}^{k+1}(1-t_{1j}^{L})^{w_j},1-\prod_{j=1}^{k+1}(1-t_{2j}^{L})^{w_j},1$$

$$-\prod_{j=1}^{k+1}(1-t_{3j}^{L})^{w_j},1-\prod_{j=1}^{k+1}(1-t_{4j}^{L})^{w_j}\Bigg),\Bigg(1$$

$$-\prod_{j=1}^{k+1}(1-t_{1j}^{U})^{w_j},1-\prod_{j=1}^{k+1}(1-t_{2j}^{U})^{w_j},1$$

$$-\prod_{j=1}^{k+1}(1-t_{3j}^{U})^{w_j},1-\prod_{j=1}^{k+1}(1-t_{4j}^{U})^{w_j}\Bigg)\Bigg],$$

$$\Bigg[\Bigg(\prod_{j=1}^{k+1}(i_{1j}^{L})^{w_j},\prod_{j=1}^{k+1}(i_{2j}^{L})^{w_j},\prod_{j=1}^{k+1}(i_{3j}^{L})^{w_j},\prod_{j=1}^{k+1}(i_{4j}^{L})^{w_j}\Bigg),$$

$$\Bigg(\prod_{j=1}^{k+1}(i_{1j}^{U})^{w_j},\prod_{j=1}^{k+1}(i_{2j}^{U})^{w_j},\prod_{j=1}^{k+1}(i_{3j}^{U})^{w_j},\prod_{j=1}^{k+1}(i_{4j}^{U})^{w_j}\Bigg)\Bigg],$$

$$\Bigg[\Bigg(\prod_{j=1}^{k+1}(f_{1j}^{L})^{w_j},\prod_{j=1}^{k+1}(f_{2j}^{L})^{w_j},\prod_{j=1}^{k+1}(f_{3j}^{L})^{w_j},\prod_{j=1}^{k+1}(f_{4j}^{L})^{w_j}\Bigg),$$

$$\Bigg(\prod_{j=1}^{k+1}(f_{1j}^{U})^{w_j},\prod_{j=1}^{k+1}(f_{2j}^{U})^{w_j},\prod_{j=1}^{k+1}(f_{3j}^{U})^{w_j},\prod_{j=1}^{k+1}(f_{4j}^{U})^{w_j}\Bigg)\Bigg]\Bigg\rangle.$$

Thus, through the mathematical induction method, the proof of the theorem is completed. □

It is observed that when $W=(1/n,1/n,\cdots,1/n)^T$, then ITNNWAA operator is reduced to an interval trapezoidal neutrosophic number arithmetic averaging operator.

It is obvious that the ITNNWAA operator is characterized by the following properties:

Theorem 5.3. Idempotency property: *Let $\bar{n}_{1j}=\langle[(t_{1j}^L,t_{2j}^L,t_{3j}^L,t_{4j}^L),(t_{1j}^U,t_{2j}^U,t_{3j}^U,t_{4j}^U)],[(i_{1j}^L,i_{2j}^L,i_{3j}^L,i_{4j}^L),(i_{1j}^U,i_{2j}^U,i_{3j}^U,i_{4j}^U)],[(f_{1j}^L,f_{2j}^L,f_{3j}^L,f_{4j}^L),(f_{1j}^U,f_{2j}^U,f_{3j}^U,f_{4j}^U)]\rangle$ $(j=1,2,\cdots,n)$ be a collection of interval trapezoidal neutrosophic numbers. If each $\bar{n}_{1j}$ where $j\in\{1,2,\cdots,n\}$ is equal to $\bar{n_1}$, $\bar{n}_{1j}=\bar{n_1}$ for $j=1,2,\cdots,n$, then:*

$$ITNNWAA(\bar{n}_{11},\bar{n}_{12},\ldots,\bar{n}_{1n})=\bar{n_1}. \tag{10}$$

Theorem 5.4. Boundness property: *Let $\bar{n}_{1j}$ $(j=1,2,\cdots,n)$ be a collection of ITNNs. Let:*

$$(\bar{n}_1)^- = \Big\langle \Big(\min_j t_{1j}^L, \min_j t_{2j}^L, \min_j t_{3j}^L, \min_j t_{4j}^L\Big), \Big(\max_j i_{1j}^L, \max_j i_{2j}^L, \max_j i_{3j}^L, \max_j i_{4j}^L\Big), \max_j f_{1j}^L, \max_j f_{2j}^L, \max_j f_{3j}^L, \max_j f_{4j}^L\Big)\Big\rangle,$$

$$(\bar{n}_1)^+ = \Big\langle \Big(\max_j t_{1j}^U, \max_j t_{2j}^U, \max_j t_{3j}^U, \max_j t_{4j}^U\Big), \Big(\min_j i_{1j}^U, \min_j i_{2j}^U, \min_j i_{3j}^U, \min_j i_{4j}^U\Big), \min_j f_{1j}^U, \min_j f_{2j}^U, \min_j f_{3j}^U, \min_j f_{4j}^U\Big)\Big\rangle.$$

Then:

$$\bar{n}_1^- \leq ITNNWAA(\bar{n}_{11}, \bar{n}_{12}, \cdots, \bar{n}_{1n}) \leq \bar{n}_1^+. \tag{11}$$

Theorem 5.5. Monotonicity property: *Let $\bar{n}_{1j}$ $(j = 1, 2, \cdots, n)$ and $\bar{n}_{1j}$ $(j = 1, 2, \ldots, n)$ be two collections of the interval trapezoidal neutrosophic numbers. If $\bar{n}_{1j} \leq \bar{n}_{1j}^*$ for $j \in \{1, 2, \cdots, n\}$, then:*

$$ITNNWAA(\bar{n}_{11}, \bar{n}_{12}, \ldots, \bar{n}_{1n}) \leq ITNNWAA(\bar{n}_{11}^*, \bar{n}_{12}^*, \ldots, \bar{n}_{1n}^*). \tag{12}$$

Proof. Let:

(A) $\bar{n}_{1j} = \bar{n}_1$ where $j = 1, 2, \cdots, n$, then we have:

$$\begin{aligned}
&ITNNWAA(\bar{n}_{11}, \bar{n}_{12}, \cdots, \bar{n}_{1n}) \\
&= (w_1 \oplus \bar{n}_{11}^* \oplus w_2\bar{n}_{12}^*, \cdots, \oplus w_n\bar{n}_{1n}^*) \\
&= \bigoplus_{j=1}^n (w_j\bar{n}_{1j}) = \Big\langle \Big[\Big(1 - \prod_{j=1}^n (1 - t_{1j}^L)^{w_j}, 1 - \prod_{j=1}^n (1 - t_{2j}^L)^{w_j}, 1 - \prod_{j=1}^n (1 - t_{3j}^L)^{w_j}, 1 - \prod_{j=1}^n (1 - t_{4j}^L)^{w_j}\Big), \\
&\Big(1 - \prod_{j=1}^n (1 - t_{1j}^U)^{w_j}, 1 - \prod_{j=1}^n (1 - t_{2j}^U)^{w_j}, 1 - \prod_{j=1}^n (1 - t_{3j}^U)^{w_j}, 1 - \prod_{j=1}^n (1 - t_{4j}^U)^{w_j}\Big)\Big], \\
&\Big[\Big(\prod_{j=1}^n (i_{1j}^L)^{w_j}, \prod_{j=1}^n (i_{2j}^L)^{w_j}, \prod_{j=1}^n (i_{3j}^L)^{w_j}, \prod_{j=1}^n (i_{4j}^L)^{w_j}\Big), \Big(\prod_{j=1}^n (i_{1j}^U)^{w_j}, \prod_{j=1}^n (i_{2j}^U)^{w_j}, \prod_{j=1}^n (i_{3j}^U)^{w_j}, \prod_{j=1}^n (i_{4j}^U)^{w_j}\Big)\Big], \\
&\Big[\Big(\prod_{j=1}^n (f_{1j}^L)^{w_j}, \prod_{j=1}^n (f_{2j}^L)^{w_j}, \prod_{j=1}^n (f_{3j}^L)^{w_j}, \prod_{j=1}^n (f_{4j}^L)^{w_j}\Big), \Big(\prod_{j=1}^n (f_{1j}^U)^{w_j}, \prod_{j=1}^n (f_{2j}^U)^{w_j}, \prod_{j=1}^n (f_{3j}^U)^{w_j}, \prod_{j=1}^n (f_{4j}^U)^{w_j}\Big)\Big]\Big\rangle, \\
&= \Big\langle \Big[\Big(1 - (1 - t_1^L)^{\sum_{j=1}^n w_j}, 1 - (1 - t_2^L)^{\sum_{j=1}^n w_j}, 1 - (1 - t_3^L)^{\sum_{j=1}^n w_j}, 1 - (1 - t_4^L)^{\sum_{j=1}^n w_j}\Big), \\
&\Big(1 - (1 - t_1^U)^{\sum_{j=1}^n w_j}, 1 - (1 - t_2^U)^{\sum_{j=1}^n w_j}, 1 - (1 - t_3^U)^{\sum_{j=1}^n w_j}, 1 - (1 - t_4^U)^{\sum_{j=1}^n w_j}\Big)\Big], \\
&\Big[\Big((i_1^L)^{\sum_{j=1}^n w_j}, (i_2^L)^{\sum_{j=1}^n w_j}, (i_3^L)^{\sum_{j=1}^n w_j}, (i_4^L)^{\sum_{j=1}^n w_j}\Big), \Big((i_{1j}^U)^{\sum_{j=1}^n w_j}, (i_{2j}^U)^{\sum_{j=1}^n w_j}, (i_{3j}^U)^{\sum_{j=1}^n w_j}, (i_{4j}^U)^{\sum_{j=1}^n w_j}\Big)\Big], \\
&\Big[\Big((f_1^L)^{\sum_{j=1}^n w_j}, (f_2^L)^{\sum_{j=1}^n w_j}, (f_3^L)^{\sum_{j=1}^n w_j}, (f_4^L)^{\sum_{j=1}^n w_j}\Big), \Big((f_1^U)^{\sum_{j=1}^n w_j}, (f_2^U)^{\sum_{j=1}^n w_j}, (f_3^U)^{\sum_{j=1}^n w_j}, (f_4^U)^{\sum_{j=1}^n w_j}\Big)\Big]\Big\rangle, \\
&= \big\langle \big[(t_1^L, t_2^L, t_3^L, t_4^L), (t_1^U, t_2^U, t_3^U, t_4^U)\big], [(i_1^L, i_2^L, i_3^L, i_4^L), (i_1^U, i_2^U, i_3^U, i_4^U)], [(f_1^L, f_2^L, f_3^L, f_4^L), (f_1^U, f_2^U, f_3^U, f_4^U)]\big\rangle = \bar{n}_1.
\end{aligned}$$

(B) Since $\bar{n}_1^- \leq \bar{n}_{1j} \leq \bar{n}_1^+$ for $j = 1, 2, \cdots, n$. Thus,

$\sum_{j=1}^{n} w_j\bar{n}_1^- \leq \sum_{j=1}^{n} w_j\bar{n}_{1j} \leq \sum_{j=1}^{n} w_j\bar{n}_1^+$. Therefore, according to (A), $\bar{n}_1^- \leq \sum_{j=1}^{n} w_j\bar{n}_{1j} \leq \bar{n}_1^+$, that is, $\bar{n}_1^- \leq ITNNWAA(\bar{n}_{11},\bar{n}_{12},\cdots,\bar{n}_{1n}) \leq \bar{n}_1^+$;

(C) Since $\bar{n}_1 \leq \bar{n}_1^*$ for $j = 1,2,\cdots,n$, then $\sum_{j=1}^{n} w_j\bar{n}_1 \leq \sum_{j=1}^{n} w_j\bar{n}_1^*$, hence obtained $ITNNWAA(\bar{n}_{11},\bar{n}_{12},\cdots,\bar{n}_{1n}) \leq ITNNWAA(\bar{n}_{11}^*,\bar{n}_{12}^*,\cdots,\bar{n}_{1n}^*)$.

Hence, the proof is completed. □

5.2. Interval trapezoidal neutrosophic number weighted geometric averaging operator

In this section, the interval trapezoidal neutrosophic number weighted geometric averaging operator is proposed, and its properties are discussed.

Definition 5.6. Let $\bar{n}_{1j} = \langle[(t_{1j}^L,t_{2j}^L,t_{3j}^L,t_{4j}^L),(t_{1j}^U,t_{2j}^U,t_{3j}^U,t_{4j}^U)],[(i_{1j}^L,i_{2j}^L,i_{3j}^L,i_{4j}^L),(i_{1j}^U,i_{2j}^U,i_{3j}^U,i_{4j}^U)], [(f_{1j}^L,f_{2j}^L,f_{3j}^L,f_{4j}^L),(f_{1j}^U,f_{2j}^U,f_{3j}^U,f_{4j}^U)]\rangle$ $(j = 1,2,\cdots,n)$ be interval trapezoidal neutrosophic numbers. Then, an interval trapezoidal neutrosophic number with a weighted geometric averaging ITNNWGA operator is defined as:

$$ITNNWGA(\bar{n}_{11},\bar{n}_{12},\bar{n}_{13},\cdots,\bar{n}_{1n})$$

$$=\bar{n}_{11}^{w_1}\otimes\bar{n}_{12}^{w_2}\otimes\cdots\otimes\bar{n}_{1n}^{w_n}=\bigotimes_{j=1}^{n}(\bar{n}_{1j}^{w_j}), \tag{13}$$

where w_j $(j = 1,2,\cdots,n)$ is the weight of the jth interval trapezoidal neutrosophic number $\bar{n}_{1j}$ $(j = 1,2,\cdots,n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

By the operation rules of interval trapezoidal neutrosophic numbers in Definition 4.2, the following theorem is given.

Theorem 5.7. *Let $\bar{n}_{1j} = \langle[(t_{1j}^L,t_{2j}^L,t_{3j}^L,t_{4j}^L),(t_{1j}^U,t_{2j}^U,t_{3j}^U,t_{4j}^U)], [(i_{1j}^L,i_{2j}^L,i_{3j}^L,i_{4j}^L), (i_{1j}^U,i_{2j}^U,i_{3j}^U,i_{4j}^U)],[(f_{1j}^L,f_{2j}^L,f_{3j}^L,f_{4j}^L),(f_{1j}^U,f_{2j}^U,f_{3j}^U,f_{4j}^U)]\rangle$ $(j = 1,2,\cdots,n)$ be a collection of interval trapezoidal neutrosophic numbers. In this respect, their aggregated value by using the ITNNWGA operator becomes also an interval trapezoidal neutrosophic number. Then, we have:*

$$ITNNWGA(\bar{n}_{11},\bar{n}_{12},\bar{n}_{13},\cdots,\bar{n}_{1n})$$

$$=\bar{n}_{11}^{w_1}\otimes\bar{n}_{12}^{w_2}\otimes\cdots\otimes\bar{n}_{1n}^{w_n}=\bigotimes_{j=1}^{n}(\bar{n}_{1j}^{w_j})$$

$$=\Bigg\langle\Bigg[\Bigg(\prod_{j=1}^{n}(t_{1j}^L)^{w_j},\prod_{j=1}^{n}(t_{2j}^L)^{w_j},\prod_{j=1}^{n}(t_{3j}^L)^{w_j},\prod_{j=1}^{n}(t_{4j}^L)^{w_j}\Bigg),$$

$$\Bigg(\prod_{j=1}^{n}(t_{1j}^U)^{w_j},\prod_{j=1}^{n}(t_{2j}^U)^{w_j},\prod_{j=1}^{n}(t_{3j}^U)^{w_j},\prod_{j=1}^{n}(t_{4j}^U)^{w_j}\Bigg)\Bigg],$$

$$\Bigg[\Bigg(1-\prod_{j=1}^{n}(1-i_{1j}^L)^{w_j},1-\prod_{j=1}^{n}(1-i_{2j}^L)^{w_j},1$$

$$-\prod_{j=1}^{n}(1-i_{3j}^L)^{w_j},1-\prod_{j=1}^{n}(1-i_{4j}^L)^{w_j}\Bigg),$$

$$\Bigg(1-\prod_{j=1}^{n}(1-i_{1j}^U)^{w_j},1-\prod_{j=1}^{n}(1-i_{2j}^U)^{w_j},1$$

$$-\prod_{j=1}^{n}(1-i_{3j}^U)^{w_j},1-\prod_{j=1}^{n}(1-i_{4j}^U)^{w_j}\Bigg)\Bigg],$$

$$\Bigg[\Bigg(1-\prod_{j=1}^{n}(1-f_{1j}^L)^{w_j},1-\prod_{j=1}^{n}(1-f_{2j}^L)^{w_j},1$$

$$-\prod_{j=1}^{n}(1-f_{3j}^L)^{w_j},1-\prod_{j=1}^{n}(1-f_{4j}^L)^{w_j}\Bigg),$$

$$\Bigg(1-\prod_{j=1}^{n}(1-f_{1j}^U)^{w_j},1-\prod_{j=1}^{n}(1-f_{2j}^U)^{w_j},1$$

$$-\prod_{j=1}^{n}(1-f_{3j}^U)^{w_j},1-\prod_{j=1}^{n}(1-f_{4j}^U)^{w_j}\Bigg)\Bigg]\Bigg\rangle,$$

where w_j $(j = 1,2,\cdots,n)$ is the weight of the jth interval trapezoidal neutrosophic number $\bar{n}_{1j}$ $(j = 1,2,\cdots,n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Proof. The proof can be made in similar way to that of Theorem 5.2. □

It is observed that when $W = (1/n,1/n,\cdots,1/n)^T$, the ITNNWGA operator is reduced to an interval trapezoidal neutrosophic number geometric averaging operator.

The ITNNWGA operator satisfies the following properties.

Theorem 5.8. Idempotency property: *Let $\bar{n}_{1j} = \langle[(t_{1j}^L,t_{2j}^L,t_{3j}^L,t_{4j}^L), (t_{1j}^U,t_{2j}^U,t_{3j}^U,t_{4j}^U)], [(i_{1j}^L,i_{2j}^L,i_{3j}^L,i_{4j}^L), (i_{1j}^U,i_{2j}^U,i_{3j}^U,i_{4j}^U)],[(f_{1j}^L,f_{2j}^L, f_{3j}^L,f_{4j}^L),(f_{1j}^U,f_{2j}^U,f_{3j}^U,f_{4j}^U)]\rangle$ $(j = 1,2,\cdots,n)$ be a collection of interval trapezoidal neutrosophic numbers. If each $\bar{n}_{1j}$ where $j \in \{1,2,\cdots,n\}$ is equal to $\bar{n}$, $\bar{n}_{1j} = \bar{n}$ for $j = 1,2,\cdots,n$, then:*

$$ITNNWGA(\bar{n}_{11}, \bar{n}_{12}, \cdots, \bar{n}_{1n}) = \bar{n}. \tag{14}$$

Theorem 5.9. Boundedness property: *Let $\bar{n}_{1j}$ $(j = 1, 2, \cdots, n)$ be a collection of ITNNs. Let:*

$$(\bar{n})^{-} = \Big\langle \left(\min_j t_{1j}^L, \min_j t_{2j}^L, \min_j t_{3j}^L, \min_j t_{4j}^L\right), \left(\max_j i_{1j}^L, \max_j i_{2j}^L, \max_j i_{3j}^L, \max_j i_{4j}^L\right), \left(\max_j f_{1j}^L, \max_j f_{2j}^L, \max_j f_{3j}^L, \max_j f_{4j}^L\right) \Big\rangle,$$

$$(\bar{n})^{+} = \Big\langle \left(\max_j t_{1j}^U, \max_j t_{2j}^U, \max_j t_{3j}^U, \max_j t_{4j}^U\right), \left(\min_j i_{1j}^U, \min_j i_{2j}^U, \min_j i_{3j}^U, \min_j i_{4j}^U\right), \left(\min_j f_{1j}^U, \min_j f_{2j}^U, \min_j f_{3j}^U, \min_j f_{4j}^U\right) \Big\rangle.$$

Then:

$$\bar{n}^{-} \leq ITNNWGA(\bar{n}_{11}, \bar{n}_{12}, \cdots, \bar{n}_{1n}) \leq \bar{n}^{+}. \tag{15}$$

Theorem 5.10. Monotonicity property: Let $\bar{n}_{1j}$ $(j = 1, 2, \cdots, n)$ and $\bar{n}_{1j}$ $(j = 1, 2, \cdots, n)$ be two collections of interval trapezoidal neutrosophic numbers. If $\bar{n}_{1j} \leq \bar{n}_{1j}^*$ for $j \in \{1, 2, \cdots, n\}$, then:

$$ITNNWGA(\bar{n}_{11}, \bar{n}_{12}, \cdots, \bar{n}_{1n}) \leq (\bar{n}_{11}^*, \bar{n}_{12}^*, \cdots, \bar{n}_{1n}^*). \tag{16}$$

This property and the one presented in Subsection 5.1 can be similarly proved.

6. Multi-attribute decision-making method using ITNNWAA and ITNNWGA operators

In this section, the multi-attribute decision-making problem is solved using ITNNWAA and ITNNWGA operators with the score and accuracy functions based on an interval trapezoidal neutrosophic information.

In the multi-attribute decision-making problem, let $A = \{A_1, A_2, \cdots, A_m\}$ be a set of alternatives, and $G = \{G_1, G_2, \cdots, G_n\}$ be a set of attributes related to alternatives. In this problem, decision-makers evaluate each of alternatives with interval trapezoidal neutrosophic numbers according to each of criteria.

Thus, we can work out an interval trapezoidal neutrosophic decision matrix:

$$Q = (q_{jk})_{m\times n} = \Big(\Big\langle \left[\left(t_{1jk}^L, t_{2jk}^L, t_{3jk}^L, t_{4jk}^L\right), \left(t_{1jk}^U, t_{2jk}^U, t_{3jk}^U, t_{4jk}^U\right)\right], \left[\left(i_{1jk}^L, i_{2jk}^L, i_{3jk}^L, i_{4jk}^L\right), \left(i_{1jk}^U, i_{2jk}^U, i_{3jk}^U, i_{4jk}^U\right)\right], \left[\left(f_{1jk}^L, f_{2jk}^L, f_{3jk}^L, f_{4jk}^L\right), \left(f_{1jk}^U, f_{2jk}^U, f_{3jk}^U, f_{4jk}^U\right)\right]\Big\rangle\Big)_{m\times n},$$

where $t_{1jk}^L, t_{2jk}^L, t_{3jk}^L, t_{4jk}^L, t_{1jk}^U, t_{2jk}^U, t_{3jk}^U, t_{4jk}^U$ all belong to the interval $[0,1]$, $i_{1jk}^L, i_{2jk}^L, i_{3jk}^L, i_{4jk}^L, i_{1jk}^U, i_{2jk}^U, i_{3jk}^U, i_{4jk}^U$ all belong to the interval $[0,1]$, and $f_{1jk}^L, f_{2jk}^L, f_{3jk}^L, f_{4jk}^L, f_{1jk}^U, f_{2jk}^U, f_{3jk}^U, f_{4jk}^U$ all belong to the interval $[0,1]$, indicating that alternative A_j does not satisfy the attribute C_j under $0 \leq t_{4jk}^U + i_{4jk}^U + f_{4jk}^U \leq 3$, where $j = 1, 2, \cdots, m$ and $k = 1, 2, \cdots, n$.

Herein, the following algorithm is proposed to obtain the solution of the multi-attribute decision-making problem with the interval trapezoidal neutrosophic information by using ITNNWAA and ITNNWGA operators with score and accuracy functions.

Algorithm:

Input: To select the best alternative;

Output: Best alternative;

Step 1: We obtained q_j $(j = 1, 2, \cdots, m)$ by using the ITNNWAA operator:

$$q_j = \Big(\Big\langle \left[\left(t_{1j}^L, t_{2j}^L, t_{3j}^L, t_{4j}^L\right), \left(t_{1j}^U, t_{2j}^U, t_{3j}^U, t_{4j}^U\right), \left[\left(i_{1j}^L, i_{2j}^L, i_{3j}^L, i_{4j}^L\right), \left(i_{1j}^U, i_{2j}^U, i_{3j}^U, i_{4j}^U\right)\right], \left[\left(f_{1j}^L, f_{2j}^L, f_{3j}^L, f_{4j}^L\right), \left(f_{1j}^U, f_{2j}^U, f_{3j}^U, f_{4j}^U\right)\right]\Big\rangle = ITNNWAA(q_{j1}, q_{j2}, \cdots, q_{jn}),$$

or :

$$q_j = \Big(\Big\langle \left[\left(t_{1j}^L, t_{2j}^L, t_{3j}^L, t_{4j}^L\right), \left(t_{1j}^U, t_{2j}^U, t_{3j}^U, t_{4j}^U\right)\right], \left[\left(i_{1j}^L, i_{2j}^L, i_{3j}^L, i_{4j}^L\right), \left(i_{1j}^U, i_{2j}^U, i_{3j}^U, i_{4j}^U\right)\right], \left[\left(f_{1j}^L, f_{2j}^L, f_{3j}^L, f_{4j}^L\right), \left(f_{1j}^U, f_{2j}^U, f_{3j}^U, f_{4j}^U\right)\right]\Big\rangle = ITNNWGA(q_{j1}, q_{j2}, \cdots, q_{jn}),$$

where $j \in \{1, 2, \cdots, m\}$ to obtain interval trapezoidal neutrosophic numbers of q_j $(j = 1, 2, \cdots, m)$ for each alternative A_j $(j = 1, 2, \cdots, m)$;

Step 2: Next, the value of $S(q_j)$ $(j = 1, 2, \cdots, m)$ of the overall interval trapezoidal neutrosophic numbers of q_j $(j = 1, 2, \cdots, m)$ is obtained to rank the alternatives A_j $(j = 1, 2, \cdots, m)$. If the score values of $S(q_j)$ and (q_k) are equal for two alternatives A_j and A_k, then it is required to calculate accuracy degrees of $H(q_j)$ and $H(q_k)$ with respect to the overall collective interval trapezoidal neutrosophic numbers to rank the alternatives A_j and A_k, respectively, based on the aforementioned accuracy degrees $H(q_j)$ and $H(q_k)$;

Step 3: We select the best alternative from the rankings of all alternatives A_j $(j = 1, 2, \cdots, m)$ according to $S(q_j)$ $(H(q_j))$ $(j = 1, 2, \cdots, m)$;

Step 4: End.

7. Illustrative example

In this part, a numerical result is given to establish a probable depicted (see [64]) assessment of technology commercialization with trapezoidal neutrosophic data so as to reach the proposed approach in this article. There is a committee that selects five viable emerging technology enterprises A_t $(t = 1, 2, \cdots, 5)$. They choose four attributes to assess five possible rising technology enterprises as follows:

- G_1 : Technical advancement
- G_2 : Potential market and market risk
- G_3 : Industrialization framework, human resources, and financial investments
- G_4 : The employment formation and the progress of science and technology.

In order to avoid any conflict or any sense of dominance over one another, decision-makers are required to allow the four possible emerging technology enterprises A_t $(t = 1, 2, \cdots, n)$ under the above attributes whose weight vector $(0.25, 0.25, 0.3, 0.2)^T$ are presented by decision-makers, where the decision matrix, $\tilde{Q} = (\beta_{ts})_{5\times 4}$, is given in Box I where β_{ts} is in the form of the ITrNs. For the proposed method, different parameters are calculated in the following:

Step 1. We applied the ITNNWAA operator to obtain collective overall interval trapezoidal neutrosophic numbers q_j $(j = 1, 2, 3, 4, 5)$ for A_j $(j = 1, 2, 3, 4, 5)$ as follows:

$$\begin{aligned}
q_1 =& \langle [(0.1028, 0.1841, 0.2566, 0.3569),\\
& (0.1896, 0.3171, 0.4025, 0.4764)],\\
& [(0.0, 0.1, 0.1414, 0.1565),\\
& (0.1231, 0.1464, 0.3257, 0.3279)],\\
& [(0, 0.1366, 0.1681, 0.1823),\\
& (0.1366, 0.2018, 0.3085, 0.3719)]\rangle,\\
q_2 =& \langle [(0.1261, 0.1514, 0.2338, 0.2907),\\
& (0.3003, 0.4032, 0.4940, 0.5747)],\\
& [(0.0, 0.1189, 0.1414, 0.1565)\\
& (0.1189, 0.1927, 0.2632, 0.3662)],\\
& [(0.0, 0.1149, 0.1625, 0.2431),\\
& (0.1366, 0.1824, 0.2595, 0.2713)]\rangle,
\end{aligned}$$

$$Q = \begin{array}{l}
\langle[(0.2, 0.3, 0.3, 0.4), (0.2, 0.4, 0.5, 0.5)], [(0.1, 0.1, 0.2, 0.2), (0.1, 0.1, 0.2, 0.3)], [(0.1, 0.1, 0.1, 0.1), (0.1, 0.1, 0.2, 0.2)]\rangle\\
\langle[(0.2, 0.2, 0.3, 0.4), (0.2, 0.3, 0.4, 0.5)], [(0.1, 0.2, 0.2, 0.2), (0.1, 0.3, 0.4, 0.5)], [(0.1, 0.1, 0.2, 0.2), (0.2, 0.2, 0.2, 0.2)]\rangle\\
\langle[(0.0, 0.1, 0.1, 0.1), (0.1, 0.1, 0.1, 0.2)], [(0.1, 0.2, 0.2, 0.2), (0.1, 0.1, 0.2, 0.3)], [(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6)]\rangle\\
\langle[(0.3, 0.4, 0.5, 0.5), (0.4, 0.5, 0.6, 0.7)], [(0.0, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.4)], [(0.0, 0.1, 0.1, 0.2), (0.1, 0.1, 0.1, 0.2)]\rangle\\
\langle[(0.0, 0.1, 0.2, 0.2), (0.1, 0.2, 0.2, 0.3)], [(0.0, 0.1, 0.1, 0.1), (0.2, 0.2, 0.2, 0.2)], [(0.2, 0.3, 0.4, 0.5), (0.4, 0.4, 0.6, 0.7)]\rangle\\
\\
\langle[(0.0, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.4)], [(0.0, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.3)], [(0.2, 0.2, 0.2, 0.2), (0.2, 0.3, 0.4, 0.5)]\rangle\\
\langle[(0.1, 0.2, 0.3, 0.4), (0.5, 0.6, 0.7, 0.8)], [(0.1, 0.1, 0.2, 0.3), (0.2, 0.2, 0.3, 0.4)], [(0.0, 0.1, 0.2, 0.2), (0.1, 0.1, 0.2, 0.2)]\rangle\\
\langle[(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.4, 0.6)], [(0.1, 0.1, 0.2, 0.2), (0.2, 0.3, 0.3, 0.3)], [(0.1, 0.2, 0.2, 0.2), (0.2, 0.2, 0.2, 0.3)]\rangle\\
\langle[(0.0, 0.1, 0.2, 0.3), (0.2, 0.3, 0.4, 0.5)], [(0.0, 0.1, 0.1, 0.3), (0.1, 0.1, 0.2, 0.3)], [(0.0, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.3)]\rangle\\
\langle[(0.2, 0.2, 0.3, 0.3), (0.6, 0.7, 0.7, 0.8)], [(0.0, 0.1, 0.1, 0.1), (0.1, 0.1, 0.1, 0.2)], [(0.0, 0.1, 0.1, 0.1), (0.1, 0.1, 0.2, 0.2)]\rangle\\
\\
\langle[(0.1, 0.2, 0.3, 0.4), (0.3, 0.4, 0.5, 0.6)], [(0.1, 0.1, 0.1, 0.1), (0.2, 0.2, 0.2, 0.2)], [(0.0, 0.1, 0.1, 0.2), (0.1, 0.2, 0.3, 0.4)]\rangle\\
\langle[(0.1, 0.1, 0.2, 0.2), (0.3, 0.4, 0.5, 0.5)], [(0.0, 0.1, 0.1, 0.1), (0.1, 0.2, 0.2, 0.3)], [(0.0, 0.1, 0.1, 0.1), (0.1, 0.2, 0.3, 0.3)]\rangle\\
\langle[(0.1, 0.2, 0.2, 0.3), (0.1, 0.2, 0.3, 0.4)], [(0.1, 0.1, 0.1, 0.1), (0.1, 0.1, 0.2, 0.2)], [(0.1, 0.2, 0.3, 0.3), (0.3, 0.4, 0.5, 0.6)]\rangle\\
\langle[(0.2, 0.2, 0.3, 0.3), (0.2, 0.3, 0.3, 0.4)], [(0.1, 0.1, 0.1, 0.1), (0.2, 0.2, 0.2, 0.2)], [(0.2, 0.3, 0.4, 0.5), (0.4, 0.5, 0.6, 0.7)]\rangle\\
\langle[(0.1, 0.1, 0.2, 0.2), (0.1, 0.2, 0.3, 0.3)], [(0.1, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.4)], [(0.1, 0.2, 0.3, 0.4), (0.5, 0.6, 0.7, 0.8)]\rangle\\
\\
\langle[(0.1, 0.1, 0.2, 0.3), (0.1, 0.2, 0.2, 0.3)], [(0.1, 0.1, 0.1, 0.1), (0.1, 0.1, 0.2, 0.3)], [(0.1, 0.2, 0.2, 0.3), (0.2, 0.3, 0.4, 0.5)]\rangle\\
\langle[(0.1, 0.1, 0.1, 0.1), (0.1, 0.2, 0.2, 0.3)], [(0.0, 0.1, 0.1, 0.1), (0.1, 0.1, 0.2, 0.3)], [(0.1, 0.2, 0.2, 0.3), (0.2, 0.3, 0.4, 0.5)]\rangle\\
\langle[(0.2, 0.3, 0.4, 0.5), (0.6, 0.7, 0.8, 0.9)], [(0.1, 0.1, 0.2, 0.3), (0.1, 0.1, 0.2, 0.3)], [(0.0, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.3)\rangle\\
\langle[(0.1, 0.1, 0.3, 0.3), (0.1, 0.2, 0.4, 0.4)], [(0.0, 0.2, 0.2, 0.3), (0.1, 0.2, 0.2, 0.3)], [(0.1, 0.1, 0.1, 0.1), (0.2, 0.2, 0.2, 0.2)]\rangle\\
\langle[(0.2, 0.3, 0.3, 0.4), (0.4, 0.4, 0.6, 0.7)], [(0.0, 0.1, 0.2, 0.2), (0.1, 0.2, 0.2, 0.2)], [(0.0, 0.1, 0.2, 0.3), (0.1, 0.2, 0.3, 0.4)]\rangle
\end{array}$$

Box I

$$\begin{aligned}q_3 =&\langle[(0.1237, 0.2241, 0.2761, 0.3593),\\ &(0.2813, 0.3698, 0.4417, 0.5929)],\\ &[(0.1, 0.1189, 0.1625, 0.1762),\\ &(0.1189, 0.1316, 0.2213, 0.2656)],\\ &[(0.0, 0.1927, 0.2686, 0.3080),\\ &(0.2176, 0.2928, 0.3590, 0.4392)]\rangle,\end{aligned}$$

$$\begin{aligned}q_4 =&\langle[(0.1624, 0.2150, 0.3548, 0.3565),\\ &(0.2378, 0.3390, 0.4321, 0.5179)],\\ &[(0.0, 0.1149, 0.1366, 0.2158),\\ &(0.1231, 0.1681, 0.2213, 0.2854)],\\ &[(0.0, 0.1390, 0.1803, 0.2536),\\ &(0.1741, 0.2214, 0.2588, 0.3223)]\rangle,\end{aligned}$$

$$\begin{aligned}q_5 =&\langle[(0.1237, 0.1690, 0.2467, 0.2695),\\ &(0.3224, 0.4090, 0.4764, 0.5680)],\\ &[(0.0, 0.1, 0.1414, 0.1597),\\ &(0.1189, 0.1682, 0.1899, 0.2462)],\\ &[(0.0, 0.1620, 0.2259, 0.2823),\\ &(0.2291, 0.2780, 0.4157, 0.4763)]\rangle.\end{aligned}$$

Again, the ITNNWGA operator is applied here to obtain the overall interval trapezoidal neutrosophic information q_j $(j = 1, 2, .., n)$ for A_j $(j = 1, 2, ..., n)$ as follows:

$$\begin{aligned}q_1 =&\langle[(0.0, 0.1620, 0.2500, 0.3514),\\ &(0.1653, 0.2928, 0.3664, 0.4509)],\\ &[(0.0760, 0.1, 0.1515, 0.1793),\\ &(0.1312, 0.1565, 0.2263, 0.2714)],\\ &[(0.0981, 0.1465, 0.1465, 0.1978),\\ &(0.1465, 0.2241, 0.3247, 0.4060)]\rangle,\end{aligned}$$

$$\begin{aligned}q_2 =&\langle[(0.1189, 0.1414, 0.2132, 0.2462),\\ &(0.2473, 0.3586, 0.4282, 0.5078)],\\ &[(0.0513, 0.1261, 0.1515, 0.1793),\\ &(0.1793, 0.2078, 0.2800, 0.3808)],\\ &[(0.04630, 0.1210, 0.1712, 0.1930),\\ &(0.1465, 0.1978, 0.2744, 0.3004)]\rangle,\end{aligned}$$

$$\begin{aligned}q_3 =&\langle[(0.0, 0.2018, 0.2297, 0.2869),\\ &(0.1883, 0.2569, 0.2980, 0.4378)],\\ &[(0.1, 0.1261, 0.1712, 0.1931),\\ &(0.1261, 0.1637, 0.2263, 0.2714)],\\ &[(0.1075, 0.2078, 0.2848, 0.3366),\\ &(0.2389, 0.3170, 0.3985, 0.4855)]\rangle,\end{aligned}$$

$$\begin{aligned}q_4 =&\langle[(0.0, 0.1741, 0.3080, 0.3409),\\ &(0.2071, 0.3143, 0.4061, 0.4865)],\\ &[(0.0311, 0.1210, 0.1465, 0.2452),\\ &(0.1312, 0.1760, 0.2263, 0.2989)],\\ &[(0.0843, 0.1654, 0.2262, 0.3120),\\ &(0.2216, 0.2844, 0.3528, 0.4235)]\rangle,\end{aligned}$$

$$\begin{aligned}q_5 =&\langle[(0.0, 0.1189, 0.2400, 0.1882),\\ &(0.2065, 0.3142, 0.3849, 0.4542)],\\ &[(0.0311, 0.1, 0.1515, 0.1848),\\ &(0.1261, 0.1761, 0.2084, 0.2661)],\\ &[(0.1075, 0.1841, 0.2634, 0.3457),\\ &(0.3182, 0.3772, 0.5120, 0.6100)]\rangle.\end{aligned}$$

Step 2. We calculated score values $S(q_j)$ and accuracy values $H(q_j)$ for the overall collective interval trapezoidal neutrosophic information q_j $(j = 1, 2, 3, 4, 5)$. Based on Definition 4.3, different score values are obtained and given in Table 1 for the operators ITNNWAA and ITNNWGA. Therefore, there is no need to compute the accuracy function value;

Step 3. Ranking of alternatives.

According to Definition 4.5 and the score values obtained in Step 2, it is clear that $S(q_5) > S(q_2) >$

Table 1. Score values of alternatives using ITNNWAA and ITNNWGA operators.

Alternative (A_i)	Score value (ITNNWAA)	Score value (ITNNWGA)
A_1	0.8357	0.7071
A_2	0.8571	0.7972
A_3	0.7895	0.6545
A_4	0.8376	0.6725
A_5	0.9089	0.7833

Table 2. Ranking order of the alternatives.

Aggregation operator	Ranking ordered
ITNNWAA	$A_5 \succ A_2 \succ A_4 \succ A_1 \succ A_3$
ITNNWGA	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$

$S(q_4) > S(q_1) > S(q_3)$ (see Table 1) can be obtained for the ITNNWAA operator. Therefore, the final ranking is A5 $A_5 \succ A_2 \succ A_4 \succ A_1 \succ A_3$ (see Table 2) for the ITNNWAA operator. Thus, the most desirable emerging technology is A_5, while the worst is A_3.

Now, the ITNNWGA operator is used to calculate score values as per Definition 4.5. The values obtained in Step 2 are shown in Table 1 (Second column). It is seen that $S(q_2) > S(q_5) > S(q_1) > S(q_4) > S(q_3)$. Therefore, as per operator ITNNWGA, the ranking of the technologies is $A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$ (see Table 2). Hence, as per this operator, the best technology is A_2, while the worst is A_3. According to the two obtained results above, the best technology includes A_5 and A_2 and the worst one is A_3 (in both cases).

In this paper, the proposed method is more valuable than that in the relevant papers (see [50,51,65]), which proposed multi-attribute decision-making method based on the weighted geometric averaging operator for interval-valued trapezoidal fuzzy numbers, weighted aggregation operator multi-attribute group decision-making method based on interval-valued trapezoidal fuzzy numbers, and an approach to multi-attribute group decision-making problems with interval-valued intuitionistic trapezoidal fuzzy numbers, whereas the decision in this proposed method is considered based on interval trapezoidal neutrosophic information. As mentioned earlier, the trapezoidal neutrosophic set is a generalization of the trapezoidal intuitionistic fuzzy set, and interval trapezoidal neutrosophic number is a generalization of interval-valued intuitionistic trapezoidal fuzzy numbers. Therefore, the proposed method is typical to apply to decision-making problems. This method solves not only triangular and trapezoidal interval-valued intuitionistic fuzzy information, but also triangular and trapezoidal neutrosophic information in decision-making; however, decision-making in [50,51,65] was only based on triangular and trapezoidal interval-valued intuitionistic fuzzy numbers. Thus, the proposed method in this paper is a more relevant generalization of the existing decision-making methods with triangular and trapezoidal interval-valued intuitionistic fuzzy information.

In most cases, to calculate the actual aggregation values of the alternatives, different aggregation operators have been used. Moreover, the two aforementioned aggregation operators, ITNNWAA operator or ITNNWGA operator, are all used to deal with different relationships of the aggregated arguments, which can provide decision-makers with more options.

8. Conclusion and future scope of work

This study defined the concept of the interval trapezoidal neutrosophic set and its score and accuracy functions. Then, aggregating operators, ITNNWAA and ITNNWGA, along with the score and accuracy functions based on interval trapezoidal neutrosophic information were introduced, and a multi-attribute decision-making method was developed based on the operators ITNNWAA and ITNNWGA and the score and accuracy functions based on an interval trapezoidal neutrosophic information. The ITNNWAA and ITNNWGA operators were used to aggregate interval trapezoidal neutrosophic information corresponding to each of the alternatives to obtain the overall information of each alternative and, then, rank the alternatives of the values of the score and accuracy function to choose the most desirable one. Finally, an illustrative example was given to demonstrate the implementation process of the proposed method. The main advantage of this method is that it is a useful method for solving a multi-attribute decision-making problem with an interval trapezoidal neutrosophic environment including indeterminate and inconsistent information. All in all, the proposed method can be a useful tool to solve some other practical problems for aggregating information such as supply chain management system, software system selection problems, and water resource schedule problems.

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References

1. Zadeh, L.A. "Fuzzy sets", *Information and Control*, **8**, pp. 338–353 (1965).

2. Turksen, I.B. "Interval valued fuzzy sets based on normal forms", *Fuzzy Sets and Systems*, **20**, pp. 191–210 (1989).

3. Atanassov, K.T. "Intuitionistic fuzzy sets: Theory and applications", *Studies in Fuzziness and Soft Computing*, **35**, Physica-Verlag, Heidelberg/New York (1999).

4. Atanassov, K.T. and Gargov, G. "Interval valued intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, **31**, pp. 343–349 (1989).

5. Gau, W.L. and Buehrer, D.J. "Vague sets", *IEEE Trans Syst ManCybern*, **23**, pp. 610–614 (1993).

6. Bustince, H. and Burillo, P. "Vague sets are intuitionistic fuzzysets", *Fuzzy Sets and Systems*, **79**, pp. 403–405 (1996).

7. Smarandache, F., *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth (1999).

8. Smarandache, F., *Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information and Learning*, Ann Arbor, Michigan, USA, p. 105 (1998).

9. Wang, H., Smarandache, F., Zhang, Y.Q., and Sunderraman, R. "Single valued neutrosophic sets", *Multispace and Multistructure*, **4**, pp. 410–413 (2010).

10. Şahin, R. and Liu, P.D. "Correlation coefficients of single valued neutrosophic hesitant fuzzy sets and their applications in decision making", *Neural Computing and Applications*, **28**(6), pp. 1387–1395 (2017).

11. Ye, J. "Multi criteria decision-making method using the correlation coefficient under single-valued neutrosophic environment", *International Journal of General Systems*, **42**(4), pp. 386–394 (2013).

12. Ye, J. "Single valued neutrosophic cross entropy for multi criteria decision making problems, *Applied Mathematical Modelling*, **38**, pp. 1170–1175 (2013).

13. Wang, H., Smarandache, F., Zhang, Y.Q., and Sunderraman, R., *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*, Hexis, Phoenix, AZ (2005).

14. Tian, Z.P., Wang, J., Wang, J.Q., and Zhang, H.Y. "Simplified neutrosophic linguistic multi-criteria group decision-Making approach to green product development", *Group Decision and Negotiation*, **26**(3), pp. 597–627 (2017).

15. Tian, Z.P., Zhang, H., Wang, J., Wang, J.Q., and Chen, X. "Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets", *International Journal of Systems Science*, **47**(15), pp. 3598–3608 (2016).

16. Ye, J. "Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making", *Journal of Intelligent and Fuzzy Systems*, **26**, pp. 165–172 (2014).

17. Liang, R.X., Wang, J.Q., and Li, L. "Multi-criteria group decision making method based on interdependent inputs of single valued trapezoidal neutrosophic information", *Neural Computing and Applications*, **30**(1), pp. 241–260 (2018).

18. Peng, H.G., Zhang, H.Y., and Wang, J.Q. "Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems", *Neural Computing and Applications*, **30**(2), pp. 536–583 (2018).

19. Wang, J.Q., Yang, Y., and Lin, L. "Multi-criteria decision-making method based on single valued neutrosophic linguistic Maclaurin symmetric mean operators", *Neural Computing and Applications*, **30**(5), pp. 1529–1547 (2018).

20. Yu, S.M., Wang, J., and Wang, J.Q. "An extended TODIM approach with intuitionistic linguistic numbers", *International Transactions in Operational Research, Multiple Criteria Decision Making: Current Challenges and Future Trends*, **25**(3), pp. 781–805 (2018).

21. Karaaslan, F. "Possibility neutrosophic soft sets and PNS-decision making method", *Applied Soft Computing*, **54**, pp. 403–414 (2017).

22. Karaaslan, F. "Correlation coefficients of single-valued neutrosophic refined soft sets and their applications in clustering analysis", *Neural Computing and Applications*, **28**(9), pp. 2781–2793 (2017).

23. Karaaslan, F. "Multi-criteria decision making method based on similarity measures under single-valued neutrosophic refined and interval neutrosophic refined environments", *International Journal of Intelligent Systems*, **33**(5), pp. 928–952 (2018).

24. Herrera, F., Herrera-Viedma, E., and Chiclana, F. "A study of the origin and uses of the ordered weighted geometric operator in multicriteria decision making", *International Journal of Intelligent Systems*, **18**(6), pp. 689–707 (2003).

25. Gao, H., Wei, G.W., and Huang, Y. "Dual hesitant bipolar fuzzy Hamacher prioritized aggregation operators in multiple attribute decision making", *IEEE Access*, **6**(1), pp. 11508–11522 (2018).

26. Xu, Z.S. and Yager, R.R. "Some geometric aggregation operators based on intuitionistic fuzzy sets", *International Journal of General System*, **35**, pp. 417–433 (2006).

27. Xu, Z.S. "Intuitionistic fuzzy aggregation operators", *IEEE Transactions on Fuzzy Systems*, **15**(6), pp. 1179–1187 (2007).

28. Xu, Z.S. "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making", *Control and Decision*, **22**(2), pp. 215–219 (2007).

29. Liu, P., Zhang, X., and Jin, F. "A multi-attribute group decision making method based on interval-valued trapezoidal fuzzy numbers hybrid harmonic averaging operators", *Journal of Intelligent and Fuzzy Systems*, **23**(5), pp. 159–168 (2012).

30. Liu, H.W. and Wang, G.J. "Multi-criteria methods based on intuitionistic fuzzy sets", *European Journal of Operational Research*, **179**, pp. 220–233 (2007).

31. Zhi, P. and Li, Z. "A novel approach to multi-attribute decision making based on intuitionistic fuzzy sets", *Expert Systems with Applications*, **39**, pp. 2560–2566 (2007).

32. Wei, G.W., Gao, H., and Wei, Y. "Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making", *International Journal of Intelligent Systems*, **33**(7), pp. 1426–1458 (2018).

33. Wei, G.W., Lu, M., Tang, X., and Wei, Y. "Pythagorean hesitant fuzzy Hamacher aggregation operators and their application to multiple attribute decision making", *International Journal of Intelligent Systems*, **33**(6), pp. 1197–1233 (2018).

34. Liu, P. and Jin, F. "Methods for aggregating intuitionistic uncertain linguistic variables and their application to group decision making", *Information Sciences*, **205**, pp. 58–71 (2012).

35. Wang, J., Wei, G.W., and Wei, Y. "Models for green supplier selection with some 2-tuple linguistic neutrosophic number Bonferroni mean operators", *Symmetry*, **10**(5), p. 131 (2018). DOI: 10.3390/sym10050131.

36. Wu, S., Wang, J., Wei, G.W., and Wei, Y. "Research on construction engineering project risk assessment with some 2-tuple linguistic neutrosophic Hamy mean operators", *Sustainability*, **10**(5), p. 1536 (2018). https://doi.org/10.3390/su10051536.

37. Liu, P. "Some Hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making", *IEEE Transactions on Fuzzy Systems*, **22**(1), pp. 83–97 (2014). DOI: 10.1109/TFUZZ.2013.2248736

38. Liu, P. "Some generalized dependent aggregation operators with intuitionistic linguistic numbers and their application to group decision making", *Journal of Computer and System Sciences*, **79**(1), pp. 131–143 (2013).

39. Garg, H., Agarwal, N., and Tripathi, A. "Some improved interactive aggregation operators under interval-valued intuitionistic fuzzy environment and its application to decision making process", *Scientia Iranica E*, **24**(5), pp. 2581–2604 (2017).

40. Garg, H. "Generalized intuitionistic fuzzy multiplicative interactive geometric operators and their application to multiple criteria decision making", *International Journal of Machine Learning and Cybernetics*, **7**(7), pp. 1075–1092 (2016).

41. Garg, H. "A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes", *International Journal of Intelligent Systems*, **31**(12), pp. 1234–1253 (2016).

42. Garg, H. "A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making", *International Journal of Intelligent Systems*, **31**(9), pp. 886–920 (2016).

43. Garg, H. "Generalized intuitionistic fuzzy interactive geometric interaction operators using Einstein t-norm and t-conorm and their application to decision making", *Computer and Industrial Engineering*, **101**, pp. 53–69 (2016).

44. Arora, R. and Garg, H. "Prioritized averaging/geometric aggregation operators under the intuitionistic fuzzy soft set environment", *Scientia Iranica, E*, **25**(1), pp. 662–687 (2018).

45. Jana, C. and Pal, M. "Application of bipolar intuitionistic fuzzy soft sets in decision making problem", *International Journal of Fuzzy System Applications*, **7**(3), pp. 32–55 (2018).

46. Wei, G.W., Zhao, X.F. and Lin, R. "Some induced aggregating operators with fuzzy number intuitionistic fuzzy information and their applications to group decision making", *International Journal of Computational Intelligence Systems*, **3**(1), pp. 84–95 (2010).

47. Ye, J. "A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets", *Journal of Intelligent and Fuzzy Systems*, **26**, pp. 2459–2466 (2014).

48. Liu, F. and Yuan, X.H. "Fuzzy number intuitionistic fuzzy set", *Fuzzy Systems and Mathematics*, **21**(1), pp. 88–91 (2007).

49. Wang, X.F. "Fuzzy number intuitionistic fuzzy arithmetic aggregation operators", *International Journal of Fuzzy Systems*, **10**(2), pp. 104–111 (2008).

50. Liu, P. and Jin, F. "A multi-attribute group decision-making method based on weighted geometric aggregation operators of interval-valued trapezoidal fuzzy numbers", *Applied Mathematical Modelling*, **36**(6), pp. 2498–2509 (2012).

51. Liu, P. "A weighted aggregation operators multi-attribute group decision-making method based on interval-valued trapezoidal fuzzy numbers", *Expert Systems with Applications*, **38**(1), pp. 1053–1060 (2011).

52. Wu, J. and Liu, Y. "An approach for multiple attribute group decision making problems with interval-valued intuitionistic trapezoidal fuzzy numbers", *Computers and Industrial Engineering*, **66**, pp. 311–324 (2013).

53. Ye, J. "Prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multi criteria decision making", *Neural Computing and Applications*, **25**(6), pp. 1447–1454 (2014).

54. Ye, J. "Trapezoidal neutrosophic set and its application to multiple attribute decision making", *Neural Computing and Applications*, **26**(5), pp. 1157–1166 (2015).

55. Liang, R.X., Wang, J.Q., and Zhang, H.Y. "A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information", *Neural Computing and Applications* (2017). DOI: 10.1007/s00521-017-2925-8

56. Ji, P., Zhang, H.Y., and Wang, J.Q. "Fuzzy decision-making framework for treatment selection based on the combined QUALIFLEX-TODIM method", *International Journal of Systems Science*, **48**(14), pp. 3072–3086 (2017).

57. Liang, R.X., Wang, J.Q., and Zhang, H.Y. "Evaluation of e-commerce websites: An integrated approach under a single-valued trapezoidal neutrosophic environment", *Knowledge-Based Systems*, **135**, pp. 44–59 (2017).

58. Li, Y., Wang, J., Wang, T.A. "Linguistic neutrosophic multi-criteria group decision-making approach with EDAS method", *Arab J Sci Eng*, **44**, pp. 2737–2749 (2019). DOI: 10.1007/s13369-018-3487-5

59. Peng, J.J., Wang, J.Q., Hu, J.H., and Tian, C. "Multi-criteria decision-making approach based on single-valued neutrosophic hesitant fuzzy geometric weighted choquet integral heronian mean operator", *Journal of Intelligent and Fuzzy Systems*, **35**(3), pp. 3661–3674 (2018). DOI: 10.3233/IFS-18249

60. Wang, Y., Wang, J.Q., and Wang, T.L. "Fuzzy stochastic multi-criteria decision-making methods with interval neutrosophic probability based on regret theory", *Journal of Intelligent and Fuzzy Systems* (2018). DOI: 10.3233/JIFS-17622

61. Ji, P., Zhang, H.Y., and Wang, J.Q. "Selecting an outsourcing provider based on the combined MABAC-ELECTRE method using single-valued neutrosophic linguistic sets", *Computers and Industrial Engineering*, **120**, pp. 429–441 (2018).

62. Garg, H., Nancy, "Linguistic single-valued neutrosophic prioritized aggregation operators and their applications to multiple-attribute group decision-making", *Journal of Ambient Intelligence and Humanized Computing*, pp. 1–23 (2018). DOI:10.1007/s12652-018-0723-5.

63. Ye, J. "Multi criteria decision-making method using the Dice similarity measure between expected intervals of trapezoidal fuzzy numbers, *Journal of Decision Systems*, **21**(4), pp. 307–317 (2012).

64. Wei, G.W. "Picture fuzzy cross-entropy for multiple attribute decision making problems", *J. Bus. Econ. Manag.*, **17**(4), pp. 491–502, (2016).

65. Wu, J. and Liu, Y. "An approach for multiple attribute group decision making problems with interval-valued intuitionistic trapezoidal fuzzy numbers, *Computers and Industrial Engineering*, **66**, pp. 311–324 (2013).

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