Damage detection in frame structures using noisy accelerometers and Damage Load Vectors (DLV)

Iraj Toloue*1, Mohd Shahir Liew2, Indra Sati Hamonangan Harahap3, Hsiu Eik Lee4

1-Ph.D, Civil Engineering department, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 32610 Perak, Malaysia. Email: toloue. iraj@gmail.com Phone: +60 134469684

2-Professor, Civil Engineering department, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 32610 Perak, Malaysia. Email: shahir_kew@utp.edu.my

3-Associate Professor, Civil Engineering department, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 32610 Perak, Malaysia. Email: indrasati@utp.edu.my

4-Ph.D. candidate, Civil Engineering department, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 32610 Perak, Malaysia. Email: aaronhe@gmail.com

Abstract

In the area of damage detection, there have been many notable methods introduced in the past years. Damage Load Vectors (DLV) is among the most powerful methods, which computes a set of load vectors from variations in flexibility matrices of a frame in the undamaged and damaged conditions. These flexibility matrices are derived from acceleration responses of the frame which can be captured using accelerometers. The DLV method then scrutinizes this shift among the flexibility matrices, which ultimately enables locating the damaged member(s). This study holistically conducted seven experimental tests, with seven damage scenarios of a test frame installed on a semi-harmonic shaking table. The DLV method was subsequently employed to locate the damaged members using recorded frame vibration data obtained from ‘noisy’ accelerometers positioned on the frame at eight predefined locations. The Eigen Realization Algorithm (ERA) alongside Pandy’s recommendations were adapted herein to facilitate generation of accurate flexibility matrices derived from the noisy accelerometers. The outcome is very encouraging with accurate identification of damaged members in all seven damage scenarios without any ‘positive-false’ and ‘negative-false’ findings. Additionally, there is a decrease (from 0.045 to 0.289) in the accuracy of WSI index when the number of damaged members is increased.

Keywords: Vibration, Damage detection, load vector, flexibility matrix, modal analysis

1. Introduction

Detecting damaged members in civil structures is one of the main concerns of engineers and asset owners alike. Among the methods in this niche, Experimental Modal Analysis (EMA) is widely utilized in the civil engineering industry for detection of impairment to a structural member. EMA, which is arguably the most cost efficient method, generically functions by detecting drops of stiffness in the structural system during a damage-event. This decrease in stiffness directly affects modal characteristics of the structure and theoretically enables the EMA to identify and locate the damaged member(s) [1]–[10]. However, in real practice, it is difficult to generate an accurate stiffness matrix from acquired responses of a system, due to the fact that both lower and higher order modes are required [11]–[13]. As the flexibility matrix is the inverse of the stiffness matrix, this leads to an intuitive phenomenon wherein the flexibility matrix converges quickly when the system frequency is increased. This results in a good approximation of only a few numbers of lower modal frequencies of the system [13], [14]. Pandy and Biswas generated the flexibility matrix and evaluated its sensitivity in the undamaged and damaged conditions of a beam. Although the aforementioned method was introduced 25 years ago, it is still one of the widely used and researched methods in the generation of Flexibility matrix [13], [3], [4], [15], [16].

While the flexibility matrix can be generated through well-established methods, the task of computing accurate modal parameters derived from physical sensor nodes remains a daunting challenge. Bernal [17] illustrated that the mode shape of a structural system can be determined by the minimum order state space realization of the input and output signals. Although this method is reasonably accurate, it requires that the acceleration time series or signals are clear and clean. Modal parameters may also be computed through an indirect method of the Eigen Realization Algorithm (ERA) which was introduced by Juang and Pappa to estimate Markov parameters that are precursors to computing modal parameters [18], [19]. Although this
method was introduced around 30 years ago, it is still regarded to be among the relevant methods in the study of structural health monitoring and vibration data analysis [20], [21].

Damage Load Vectors (DLV) which was first introduced by Bernal, generates a set of load vectors from flexibility matrices of a frame in both the undamaged and damaged conditions. The algorithm essentially identifies members with zero internal forces as damaged members [22]. The accuracy of the DLV method was determined through a set of experimental tests on a 3D truss structure where its capability to detect reasonably small damages amounting to 40% stiffness reduction has been demonstrated. Nonetheless, several studies reported “false negative” identification of damaged members, whenever the number of damaged members exceeded three members [23]. On the other hand, there are also works in the literature that reported no “false positives” and “false negatives”, indicating possibility of error-less damaged members detection [24]-[27]. It can be observed that the DLV algorithm applied in almost all of the previous studies are very similar to each other. In general, their uniqueness was in the various methods proposed for the extraction of structural flexibility matrices. Wang and Ong extended the stochastic DLV method for multi-metric input data with a symmetric strain flexibility matrix. This effectively resolved limitations of the DLV, whereby only the strain in damaged elements are accepted as inputs [28]. Wang et al. through an experimental test, proved that the DLV method is capable to detect all damages, including single and multiple damaged members, if the system is sufficiently calibrated with observation data. Their results have indicated that if up to 60% of the observation data is processed, only single damaged scenarios were detectable [29]. In another study, the DLV method was evaluated for inline and diagonal members wherein it was shown that the corresponding Finite Element Model (FEM) does not need to be precise, and the damage detection results can rely primarily on the acceleration input data. It is also remarked here that the DLV method can be applied on a part of a system, given that sensors are installed appropriately. Finally, it has been proven that there are no differences between the inline and diagonal members with respect to damage detection by DLV algorithm [30].

The present study deals with the Damage load vectors (DLV) method to detect damaged members on a 3D frame structure through seven experimental tests. In order to record the frame’s acceleration data, ‘noisy’ or ‘consumer-grade’ accelerometers (ADXL335) were employed. The ERA algorithm with Pandy’s recommendations were utilized to generate the flexibility matrices for both undamaged and damaged conditions. The frame’s acceleration responses in all of the degrees of freedom were measured by eight accelerometers. The aim of this study is henceforth to investigate the reliability and applicability of the DLV method for damage member detection of a 3D frame when subjected to un-clean input signals from noisy and cheap accelerometers.

2. Theory

2.1 Computing Difference in Flexibility Matrix

It must be emphasized herein that the acceleration data obtained by the ADXL335 sensors in this study are inherently noisy. Thus, it is preemptive that employing direct methods such as Frequency Response Functions (FRF) to determine modal responses without substantial prior filtering or algorithm modification would be not advisable or perhaps downright impossible, as FRF techniques typically require strong signals and clean input data. Henceforth, this study applies the so called indirect method in order to detect the modal characteristics of the 3D frame. The Markov matrix, as shown in Eqs. (1-3) plays a central role in the indirect method as presented by Juang and Pappa [19].

\[
[x(k)]_{p×q} = [Trans]_{p×2N} [A]_{2N×2N}^{k-1} [B]_{2N×q}^{(2)}
\]

(1)

\[
[B] = -\frac{\Delta t}{0} e^{-\frac{\Delta t}{t}} d\tau'[E]
\]

(2)

\[
[E] = \begin{bmatrix} 0 \\ \frac{I_c}{M} \end{bmatrix}^T
\]

(3)
Where \( x(k) \) is the Markov matrix for \( q \) input signals at \( p \) physical coordinates. \( N \) is the number of degrees of freedom. \([\text{Trans}]\) is the transformation matrix and \([A]\) is the state matrix which is computed based on the latest update on the system state as shown in Eq. (4).

\[
A = \begin{bmatrix}
0 & I \\
K & C \\
M & M
\end{bmatrix}
\]

(4)

Where \([M], [K] \) and \([C]\) are mass, stiffness and damping matrices, respectively.

Further building on the definition of \([E]\) in Eq. (3), the matrix \([I]\) and \([C]\) represents the coefficient for input load vectors. The Hankel matrix of Eq. (1) can thus be written as Eq. (5).

\[
[H(k-1)] = \begin{bmatrix}
x(k) & x(k+1) & \cdots & x(k+m) \\
x(k+1) & x(k+2) & \cdots & x(k+m+1) \\
\vdots & \vdots & \ddots & \vdots \\
x(k+r) & \cdots & \cdots & x(k+m+r)
\end{bmatrix}
\]

(5)

By reducing dimensions of the Hankel Matrix using Singular Value Decomposition (SVD), the state space matrices \( A, B, \) and \( C \) are obtained from input acceleration signals. The composite equation of the post-SVD Hankel matrix and Markov matrix is shown in Eq. (6).

\[
\begin{bmatrix}
x(k-1)
\end{bmatrix} = \left([E_p]^T[U_{2N}][\Theta_{2N}]^{-1}\right)^2 \left([\Theta_{2N}]^2[U_{2N}]^T[H(1)][\Theta_{2N}]^2\right)^k \left([\Theta_{2N}]^2[V_{2N}]^T[E_q]\right)
\]

(6)

Where

\[
\begin{bmatrix}
[E_p]^T
\end{bmatrix}_{p \times p} = \begin{bmatrix}
[I]_{p \times p} & [0]_{p \times p} & \cdots & [0]_{p \times p}
\end{bmatrix}
\]

(7-a)

\[
\begin{bmatrix}
[E_q]^T
\end{bmatrix}_{q \times q} = \begin{bmatrix}
[I]_{q \times q} & [0]_{q \times q} & \cdots & [0]_{q \times q}
\end{bmatrix}
\]

(7-b)

It is further clarified that the matrices \([U_{2N}], [\Theta_{2N}] \) and \([V_{2N}]\) are the direct outputs from SVD operations on the Hankel matrix. The realization matrices with respect to Eq. (6) are shown in Eqs. (8a-c).

\[
[\text{Trans}] = \left([E_p]^T[U_{2N}]\right)^{\frac{1}{2}}[\Theta_{2N}]^{-\frac{1}{2}}
\]

(8-a)

\[
[F_s] = \left([\Theta_{2N}]^2[U_{2N}]^T[H(1)]\right)^{\frac{1}{2}}[\Theta_{2N}]^2
\]

(8-b)

\[
[B] = \left([\Theta_{2N}]^2[V_{2N}]^T[E_q]\right)^{\frac{1}{2}}
\]

(8-c)

The modal values \( \Omega \) and vectors \( \{\Phi_u\} \) of the recorded data can be determined through solving the Eigen problem of matrix \([F_s]\) as shown in Eq. (9).

\[
[F_s]\{\Phi_u\} = \Omega\{\Phi_u\}
\]

(9)

The mode shape may be finally computed based on the recorded data through Eq. (10).
Then by following Pandy’s recommendations [13] the flexibility matrix can be determined as shown in Eq. (11).

\[
F = \Phi \Omega^{-1} \Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T
\]  

(11)

Where \( \phi_i \) is the \( i^{th} \) mode shape and \( \omega_i \) is the \( i^{th} \) modal frequency, and \( n \) is the number of Degree of Freedoms (DoF).

The results from this method are accurate as the poor and noisy signals have been effectively eliminated from the Markov matrix. Considering \( F_u \) to respectively represent undamaged and damaged conditions of the structural system, the changes of the flexibility matrix calculated as in Eq. (12).

\[
\Delta F = F_i - F_d
\]

(12)

2.2 Locating Damage Members

In order to locate damaged members, the benchmark is set as the flexibility matrix of the system in its undamaged condition, which is denoted by [\( F_i \)]. The ‘updated’ flexibility matrix [\( F_d \)], is computed in every time-step loop for damage monitoring using data from the m-number of sensors and n-number of DoFs. It is further assumed that a similar number of load vectors at each of the sensor locations leads to identical deformations in both damaged and undamaged conditions. Considering all of the load vectors which can satisfy this condition to be gathered in a “L” matrix, Eq. (13) showcases the homogenous relation.

\[( F_i - F_d ) L = \Delta F L = 0\]

(13)

Only two conditions can satisfy Eq. (12). This can be either that \( F_i - F_d = 0 \), which translates to an absence of damage in the system, or that the “L” matrix is a basis of the null space. While \( F_i - F_d \neq 0 \), SVD should be operated on \( \Delta F \) to compute the vectors assigned to the null space in \( \Delta F \) [22]. This is operation can be written as shown in Eq. (14).

\[
\Delta F = SVD^T = [S] \begin{bmatrix} V & 0 \\ 0 & 0 \end{bmatrix} [D^T \ L^T]
\]

(14)

[S] contains singular values (s-numbers) of \( \Delta F \) as diagonal entries and [D] contains vectors in null space and row space in orthogonal form. In order to detect damaged members, the matrix [V] should be computed in accordance to the corresponding sensor node locations. The stresses for all the elements are computed with respect to each load vector. The Normalized Stress Index (nsi) can thus be defined as Eq. (15).

\[
n_{si} = \frac{\sigma_i}{\sigma_{i(max)}}
\]

(15)

It should be noted that all of the calculated load vectors are not directly representative of DLV. To separate null and row space vectors, the svn index of all load vectors should be calculated as Eq. (16).

\[
svn_i = \sqrt{\frac{S_i C_i^2}{S_q C_q^2}}
\]

(16)
In Eq. (16) \( S \) refers to strain energy of the structural members and \( C \) is a constant which enforces that the largest \( \sigma_i \) in the physical domain becomes equal to unity. Finally, by calculating the WSI index via “nsi” for all load vectors, arbitrary damages in a structural system can be detected via Eq. (17).

\[
WSI = \frac{\sum_{i=1}^{ndlv} nsi_i}{ndlv}
\]

Based on the recommendation of Bernal [22], the \( svn_i \) index should not take on a value of less than 0.015. The general rule states that members with WSI index less than unity are to be considered as damaged members.

3. Experimental Setup

As illustrated in Figure. 1, an experimental two-floors 3D aluminum frame structure with dimensions of 144 cm (height), 41 cm (width) and 41 cm (length) is utilized for experimental tests in this study. The square hollow section material properties as extracted through tensile testing at the mechanical engineering laboratory of Universiti Teknologi PETRONAS are shown in Table 1.

The aluminum frame comprises of 12 nodes and 16 members including beams and columns which all are parallel to the main reference axes. The frame is connected to the shaking table through 4 fixed supports.

Seven damage scenarios as shown in Table 2 were reflected on the frame. Each test duration on the shaking table is sustained for at least 3 minutes. Acceleration data for all 24 DoFs, which is comprised of 3 DoFs from each of the nodes #5 through #12 were then recorded.

It is noted herein that the motion of the shaking table is physically limited to global X and Y directions. However, based on the recorded shaking table base movement data as shown in Figure. 2, it is clear that vertical accelerations somewhat exist and cannot be ignored. It can also be observed that regardless of orientation of shaking, there exists response contributions from all three DoFs (X,Y,Z). Henceforth, throughout the shaking table experiments in both X and Y directions, it was pertinent that the accelerometers were set up to record the complete 3DoFs in all the predefined node locations.

4. Results and discussion

Figures. 3 to 5 illustrates the results of nsi, svn and wsi index for all members of the frame in all seven experimental tests. For T1 in which member #2 is damaged, the nsi matrix is observed to be dense which makes it trivial to calculate the corresponding svn index. As can be seen from Figure 4-a, the first vector shows the full value for singular response, which indicates that the first vector cannot be a DLV. However, based on Bernal’s recommendation to promote robustness of the algorithm, the value of svn should not be taken to be more than 0.2 [22]. Therefore, by amending the first vector to 0.2, the wsi index is able to correctly identify member #2 as a damaged member with an index value of 0.0455.

In the second test, in addition to the damaged member #2, member #7 is intentionally cut using a rotary blade in all four faces. While the nsi graph would seem to be more obscure in comparison with the first test, the signal thereof is strong enough to enable valid computation of the svn indices. As revealed by svn in the Figure 4-b, the number of vectors which passes the recommended svn cut off value of 0.2 is noted to have increased. Liken to the first case, the svn vectors were amended to 0.2 as necessary prior to the calculation of the wsi index. It is noted in both cases that the damaged members were identified accurately without “false positive” and “false negative” outcomes.
An important observation in all tests as the number of damaged members is increased, is that the number of vectors which passes the cut-off value increased. This is an indication of decrement in the sensitivity of the damage detection algorithm.

By comparison of the nsi index of damage scenario 7 (Figure. 3-g) with the nsi in damage scenario 1(Figure. 3-a), it can be observed that the nsi matrix becomes inherently sparse after damaging almost 44% of the elements in the frame. Consequently, the number of vectors which should be reduced to the recommended cut-off svn index increased.

According to the predefined damaged scenario which is shown in Table 2, the results of the proposed algorithm from the experimental model are illustrated in Figure 5. The wsi index of damaged member shown by red color. In all tests, the damaged elements were identified accurately without any positive or negative error.

Scrutinizing member #2 post-damage, it is observed that the value of its wsi in subsequent damage scenarios (by which other members are damaged consecutively) fluctuated within the range of 0.0455 to 0.289, which requires further discussion. For completeness, the comprehensive tracking the wsi index value of member #2 for all seven damage scenarios is shown in Figure 6. Although the member’s wsi index in most of the cases is noted to increase, there are several cases where the wsi index of member #2 decreased. This fluctuation is not linear, and does not conform to any specific or observable pattern. Hence, insofar, the only practicable outcome of the calculated wsi index is to determine the occurrence of measurable damage of members in the frame, in real-time. Although the wsi for member #2 does not remain constant post-damage, its fluctuation had no effect in calculation of damage in the algorithm.

To quantify the severity of damage, it is recommended form a sensitivity relation between the flexibility matrix of the frame in undamaged condition and the corresponding matrices when it is damaged. As demonstrated in this study, the damaged members can be identified though the results of the wsi index.

5. Conclusions

In this study, a 3D aluminum frame is tested in seven damaged configurations. In each scenario, the frame is subjected to ground vibration through fixed attachments at a shaking table with consumer grade ADXL335 accelerometers utilized to record the acceleration data at eight predefined nodes. The recorded data is passed through a combination of ERA and Pandey’s recommendations in order to generate the flexibility matrices which led to damage member detection by the DLV method. The application of Markov matrix on the noisy accelerometer data enables the exclusive inclusion of the most useful signals. These signals were subsequently fed into Pandey’s method in order to generate flexibility matrices by using only the first few modes. This process ultimately enabled the utilization of noisy accelerometers for accurate damage detection. Evaluating the DLV results is indicative that decrease in severity of the nsi index will typically correspond to the increase in number of damaged members. The value of wsi, which would ultimately identify the damaged member, tends to increase when there is a triggering increase in the number of damaged members. The post-damage wsi value for member #2 was observed to fluctuate from 0.0455 swayed to 0.289 but this has no effect in the identification of damaged members. The outcome of this study is encouraging as the damaged members were identified accurately with no “false positives” and “false negatives”.

References


List of Captions

Table 1. Material and section properties
Table 2. Damage scenario details

Figure 1. The numerical and experimental model of the frame (a) Numerical model (b) Experimental model (c) Sketch of applied damage on a member (d) Sample of cut on damaged member
Figure 2. Recorded ground accelerations (a-c) in the undamaged condition (a) Ground acceleration for shaking table in global X direction (b) Ground acceleration for shaking table in global Y direction Ground acceleration for shaking table in the global X & Y directions
Figure 3. Normalized Stress Index (NSI) of all members (a) nsi for T1(b) nsi for T2(c) nsi for T3(d) nsi for T4(e) nsi for T5(f) nsi for T6(g) nsi for T7
Figure 4. SVN index (a) svn for T1(b) svn for T2(c) svn for T3(d) svn for T4(e) svn for T5(f) svn for T6(g) svn for T7
Figure 5. Weighted Stress Indices (WSI) (a) wsi for T1(b) wsi for T2(c) wsi for T3(d) wsi for T4(e) wsi for T5(f) wsi for T6(g) wsi for T7
Figure 6. Tracking changed if wsi for damaged member#2

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<td>T7</td>
<td>2(c) - 7(b) - 3(c) - 10(c) - 16(b) - 12(c) - 1(c)</td>
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</table>

(c) for column and (b) for beam element
Figure 1

Figure 2
Figure 3
Figure 4
Figure 5

Figure 6
Authors brief technical biography:

Dr Iraj Toloue is a structural engineer and obtained his M.Eng from Universiti Putra Malaysia and Ph.D. from Universiti Teknologi PETRONAS, Malaysia. He is a member of Board of Engineering Malaysia (BEM), Malaysia Structural Steel Association (MSSA), and involved in numbers of consultancies to PETRONAS group in the oil and gas industry. He has also thought variety of subjects in vibration and stochastic analysis of structures to undergraduates, graduates, and engineers. He has received international awards, 1 copyright, 1 patent, and published 14 scientific papers.

Prof Ir Dr Mohd Shahir Liew is currently the Deputy Vice Chancellor of Research and Innovation at Universiti Teknologi PETRONAS. Prior to his engagement in the academia in 2009, Shahir has spent more than twenty years in the engineering consulting industry as well as in engineering research. Shahir’s approach to research is centered heavily around adopting the quadruple helix model which focuses on close ties with the industry, government and local community. In his current capacity, he focuses on core activities such as business development, research-based ventures, talent development, leadership mentoring and strategic research/industry planning. In tandem with his portfolio, he has been instrumental in supporting PETRONAS’ core business through research solutions and has been responsible as a catalyst for the formation of several UTP – PETRONAS Skill Group Steering Committee and more recently further activating close cooperation between UTP and PRSB. Shahir also holds several strategic positions in industry-based NGOs such as the current Vice President of Malaysia Structural Steel Association (MSSA), current Fellow of Energy Institute UK and former Honorary Secretary at Malaysian Oil and Gas Services Council (MOGSC). He also engages in national standards developments and was formerly the Chairman of the Wind Engineering Development and National Standard for CIDB (MS1553) and is the current Deputy Chairman of the current Technical Committee at SIRIM to develop the MS standards for Offshore Structures.

Associate Prof Dr Harahap joint the Department of Civil and Environmental Engineering, Universiti Teknologi PETRONAS since 2005. He holds a bachelor degree in civil engineering from North Sumatera University, Indonesia, and obtained his master and doctoral degrees from Ohio University and Northwestern University, respectively. His research covers the topic range from slope risk management, geotechnical reliability analysis and renewable energy.

Mr. Lee Hsiu Eik obtained both his B.Eng and MSc. in Civil Engineering from Universiti Teknologi PETRONAS. A chartered member of IMaREST, Hsiu Eik now serves as a research engineer at the Offshore Engineering Centre-UTP. Aside from providing specialist consultancies to PETRONAS group and the O & G industry in general, he is a HRDF certified trainer actively involved in the provisioning of short courses to the engineering fraternity with training engagements at PETRONAS and several regional players. To date, he currently holds 2 copyrights, 1 trademark and 2 patents.