Damage detection in frame structures using noisy accelerometers and Damage Load Vectors (DLV)

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Abstract. In the domain of damage detection, many notable methods have been introduced over the past years. Damage Load Vectors (DLV) is among the most powerful methods, which computes a set of load vectors from variations in flexibility matrices of a frame in undamaged and damaged conditions. These flexibility matrices are derived from acceleration responses of the frame that can be captured using accelerometers. The DLV method then scrutinizes this shift among flexibility matrices, which ultimately enables locating the damaged member(s). This study holistically conducted seven experimental tests with seven damage scenarios of a test frame installed on a semi-harmonic shaking table. The DLV method was subsequently employed to locate the damaged members using recorded frame vibration data that were obtained from ‘noisy’ accelerometers positioned on the frame at eight predefined locations. The Eigen Realization Algorithm (ERA) alongside Pandy’s recommendations was adopted herein to facilitate the generation of accurate flexibility matrices derived from the noisy accelerometers. The outcome is very encouraging with the accurate identification of damaged members in all seven damage scenarios without any ‘positive-false’ and ‘negative-false’ findings. Additionally, there is a decrease (from 0.045 to 0.289) in the accuracy of Weighted Stress Indices (WSI) index when the number of damaged members is increased.

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1. Introduction

Detecting damaged members of civil structures is one of the main concerns of engineers and asset owners alike. Among the methods in this niche, Experimental Modal Analysis (EMA) is widely utilized in the civil engineering industry for detecting impairment to a structural member. EMA, which is arguably the most cost-efficient method, generically functions by detecting drops of stiffness in the structural system during a damage-event. This decrease in stiffness directly affects modal characteristics of the structure and theoretically enables the EMA to identify and locate the damaged member(s) [1–10]. However, in real practice, it is difficult to generate an accurate stiffness matrix from acquired responses of a system since both lower and higher order modes are required [11–13]. As the flexibility matrix is the inverse of the stiffness matrix, this leads to an intuitive phenomenon wherein the flexibility matrix converges quickly when the system frequency increases. This results in a good approximation of only a few numbers of lower modal frequencies of the system [13,14]. Pandy and Biswas...
generated the flexibility matrix and evaluated its sensitivity in the undamaged and damaged conditions of a beam. Although the aforementioned method was introduced 25 years ago, it is still one of the widely used and researched methods for generating a flexibility matrix [3, 4, 13, 15, 16].

While the flexibility matrix can be generated through well-established methods, the task of computing accurate modal parameters derived from physical sensor nodes remains a daunting challenge. Bernal [17] illustrated that the mode shape of a structural system could be determined by the minimum order state space realization of the input and output signals. Although this method is reasonably accurate, it is required for the acceleration time series or signals to be clean and clean. Modal parameters may also be computed through an indirect method of the Eigen Realization Algorithm (ERA), which was introduced by Anh [18] and Juang and Pappas [19] to estimate Markov parameters that are precursors to computing modal parameters. Although this method was introduced around 30 years ago, it is still regarded to be among the relevant methods for studying structural health monitoring and vibration data analysis [20, 21].

Damage Load Vectors (DLV), which was first introduced by Bernal, generates a set of load vectors from flexibility matrices of a frame in both of the undamaged and damaged conditions. The algorithm essentially identifies members with zero internal forces as damaged members [22]. The accuracy of the DLV method was determined through a set of experimental tests on a 3D truss structure, where its capability to detect reasonably small damages amounting to 40% stiffness reduction has been demonstrated. Nonetheless, several studies reported “false negative” identification of damaged members, whenever the number of damaged members exceeded three members [23]. On the other hand, there are also works in the literature that reported no “false positives” and “false negatives”, indicating the possibility of detecting error-less damaged members [24–27]. It can be observed that the DLV algorithms applied in almost all of the previous studies are very similar to each other. In general, the application of various methods proposed for extracting structural flexibility matrices showed the uniqueness involved. Wang and Ong extended the stochastic DLV method for multi-degree of input data with a symmetric strain flexibility matrix. This effectively resolves the limitations of the DLV, whereby only the strain in damaged elements is accepted as input [28]. Through an experimental test, Wang et al. proved that the DLV method was capable to detect all damages including single and multiple damaged members if the system was sufficiently calibrated with observation data. Their results indicated that if up to 60% of the observation data were processed, only single damaged scenarios were detectable [29]. In another study, the DLV method was evaluated for inline and diagonal members, wherein it was shown that the corresponding finite element model did not need to be precise, and the damage detection results could rely primarily on the acceleration input data. It is remarked here that the DLV method can be applied to a part of a system, given that sensors are installed appropriately. Finally, it has been proven that there are no differences between the inline and diagonal members with respect to damage detection by DLV algorithm [30].

The present study deals with the DLV method to detect damaged members on a 3D frame structure through seven experimental tests. In order to record the frame acceleration data, noisy or 'consumer-grade' accelerometers (ADXL335) were employed. The ERA algorithm with Pandy's recommendations was utilized to generate the flexibility matrices under both undamaged and damaged conditions. The frame acceleration responses in all of the degrees of freedom were measured by eight accelerometers. The aim of this study is henceforth to investigate the reliability and applicability of the DLV method for damage member detection of a 3D frame when subjected to un-clean input signals from noisy and cheap accelerometers.

2. Theory

2.1. Computing difference in flexibility matrix

It must be emphasized herein that the acceleration data obtained by the ADXL335 sensors in this study are inherently noisy. Thus, it is preemptive that the application of direct methods such as Frequency Response Functions (FRF) to determine modal responses without substantial prior filtering or algorithm modification would not be advisable or perhaps downright impossible, as FRF techniques typically require strong signals and clean input data. Henceforth, this study applies the so-called indirect method in order to detect the modal characteristics of the 3D frame. The Markov matrix, as shown in Eqs. (1)-(3), plays a central role in the indirect method, as presented by Juang and Pappas [19]:

\[ [x(k)]_{p \times q} = [\text{Trans}]_{p \times 2N} [A]_{2N \times 2N}^{k-1} [B]_{(2N \times q)} \]

\[ [B] = - \int_0^{\Delta t} e^{[A]t'} [E] dt' \]

\[ [E] = \left[ \begin{bmatrix} 0 & [\Delta C] \end{bmatrix} \right]_T \]

where \( x(k) \) is the Markov matrix for \( q \) input signals at \( p \) physical coordinates. \( N \) is the number of Degrees of Freedom (DoF). \([\text{Trans}]\) is the transformation matrix,
and $[A]$ is the state matrix that is computed based on the latest update on the system state, as shown in Eq. (4):

$$A = \begin{bmatrix} \frac{[0]}{[M]} & \frac{[J]}{[H]} \\ -\frac{[K]}{[M]} & -\frac{[C]}{[H]} \end{bmatrix},$$  

(4)

where $[M]$, $[K]$, and $[C]$ are the mass, stiffness, and damping matrices, respectively.

Further building on the definition of $[E]$ in Eq. (3), the matrices $[I]$ and $[C]$ represent the coefficients for input load vectors. The Hankel matrix of Eq. (1) can thus be written as in Eq. (5):

$$[H(k-1)] = \begin{bmatrix} x(k) & x(k+1) & \ldots & x(k+m) \\ x(k+1) & x(k+2) & \ldots & x(k+m+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(k+r) & \ldots & \ldots & x(k+m+r) \end{bmatrix}.$$  

(5)

By reducing dimensions of the Hankel matrix using Singular Value Decomposition (SVD), the state space matrices $A$, $B$, and $C$ are obtained from input acceleration signals. The composite equation of the post-SVD Hankel matrix and Markov matrix is shown in Eq. (6):

$$[x(k-1)] = \left[ [E_p]^T [U_{2N}] [\Theta_{2N}]^{-1} \right]^T 
\times \left[ [\Theta_{2N}]^T [U_{2N}]^T [H(1)] [\Theta_{2N}] \right] \times \left[ [\Theta_{2N}]^T [V_{2N}]^T [E_d] \right].$$  

(6)

where:

$$[E_p]^T_{p\times p} = \begin{bmatrix} [I]_{p\times p} & [0]_{p\times p} & \cdots & [0]_{p\times p} \end{bmatrix}.$$  

(7a)

$$[E_{q\times q}]^T_{q\times p} = \begin{bmatrix} [I]_{q\times q} & [0]_{q\times q} & \cdots & [0]_{q\times q} \end{bmatrix}.$$  

(7b)

It is further clarified that the matrices $[U_{2N}]$, $[\Theta_{2N}]$, and $[V_{2N}]$ are the direct outputs from SVD operations on the Hankel matrix. The realization matrices with respect to Eq. (6) are shown in Eqs. (8a)-(8c):

$$[Trans] = \left[ [E_p]^T [U_{2N}] [\Theta_{2N}]^{-1} \right]^T,$$  

(8a)

$$[F_s] = \left[ [\Theta_{2N}]^T [U_{2N}]^T [H(1)] [\Theta_{2N}] \right]^T,$$  

(8b)

$$[B] = \left[ [\Theta_{2N}]^T [V_{2N}]^T [E_d] \right].$$  

(8c)

The modal values $\Omega$ and vectors $\{ \Phi_u \}$ of the recorded data can be determined by solving the Eigen problem of matrix $[F_s]$, as shown in Eq. (9):

$$[F_s] \{ \Phi_u \} = \Omega \{ \Phi_u \}.$$  

(9)

The mode shape may be finally computed based on the recorded data through Eq. (10):

$$\{ \Phi_x \} = [Trans] \{ \Phi_u \}. $$  

(10)

Then, by following Pandy’s recommendations [13], the flexibility matrix can be determined, as shown in Eq. (11):

$$F = \Phi \Omega^{-1} \Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T,$$  

(11)

where $\phi_i$ is the $i$th mode shape, $\omega_i$ is the $i$th modal frequency, and $n$ is the number of DoF.

The results of this method are accurate as the poor and noisy signals have been effectively eliminated from the Markov matrix. Given that $F_i$ and $F_d$ represent undamaged and damaged conditions of the structural system, respectively, the changes of the flexibility matrix are calculated through Eq. (12):

$$\Delta F = F_i - F_d.$$  

(12)

2.2. Locating damage members

In order to locate damaged members, the benchmark is set as the flexibility matrix of the system in its undamaged condition, which is denoted by $[F_i]$. The ‘updated’ flexibility matrix, $[F_d]$, is computed in every time-step loop for damage monitoring using data from the $m$-number of sensors and $n$-number of DoFs. It is further assumed that a similar number of load vectors in each of the sensor locations lead to identical deformations in both damaged and undamaged conditions. Considering all of the load vectors that can satisfy this condition to be gathered in a “L” matrix, Eq. (13) showcases the homogenous relation:

$$(F_i - F_d)L = \Delta F L = 0.$$  

(13)

Only two conditions can satisfy Eq. (12). This can be either that $F_i - F_d = 0$, which translates into the absence of damage in the system, or that the “L” matrix is the basis of the null space. While $F_i - F_d \neq 0$, SVD should be operated on $\Delta F$ to compute the vectors assigned to the null space in $\Delta F$ [22]. This operation can be written, as shown in Eq. (14):

$$\Delta F = SVD^T = [S] \begin{bmatrix} V_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{L}^T \\ \hat{L} \end{bmatrix},$$  

(14)

where $[S]$ contains singular values (s-numbers) of $\Delta F$ as diagonal entries, and $[V]$ contains vectors in null space and row space in the orthogonal form. In order to detect damaged members, the matrix $[V]$ should be computed in accordance with the corresponding sensor node locations. The stresses for all the elements are computed with respect to each load vector. The Normalized Stress Index (nsi) can thus be defined as in Eq. (15):

$$nsi_i = \frac{\sigma_i}{\sigma_{i(max)}},$$  

(15)
Of note, all of the calculated load vectors are not directly representative of DLV. To separate null and row space vectors, the $svn_i$ index of all load vectors should be calculated through Eq. (16):

$$svn_i = \sqrt{\frac{S_i C_i^2}{S_q C_q^2}}$$

(16)

In Eq. (16), $S$ is the strain energy of the structural members, and $C$ is a constant which enforces that the largest “$\sigma_i$” in the physical domain becomes equal to unity. Finally, by calculating the Weighted Stress Indices (WSI) index via “nsi” for all load vectors, arbitrary damages in a structural system can be detected through Eq. (17):

$$ws_i = \sum_{v=1}^{nvd} wsi_{vi}$$

(17)

Based on the recommendation of Bernal [22], the $wsi$ index should not take on a value of less than 0.015. The general rule states that members with wsi index less than unity are to be considered as damaged members.

3. Experimental setup

As illustrated in Figure 1, an experimental two-floor 3D aluminum frame structure with dimensions of 144 cm (height), 41 cm (width), and 41 cm (length) is utilized for experimental tests in this study. The square hollow section material properties as extracted through tensile testing at the mechanical engineering laboratory of Universiti Teknologi PETRONAS are shown in Table 1.

The aluminum frame comprises 12 nodes and 16 members including beams and columns, which are all parallel to the main reference axes. The frame is connected to the shaking table through 4 fixed supports.

Seven damage scenarios shown in Table 2 were reflected on the frame. Each test duration on the shaking table is sustained for at least 3 minutes. Acceleration data for all 24 DoFs, comprised of 3 DoFs, from each of nodes #5 through #12 were then recorded.

It is noted herein that the motion of the shaking table is physically limited to global X and Y directions. However, based on the recorded shaking table base.

![Figure 1](image-url)
movement data shown in Figure 2, it is clear that vertical accelerations somewhat exist and cannot be ignored. It can also be observed that regardless of the orientation of shaking, there exist response contributions from all three DoFs (X, Y, Z). Henceforth, throughout the shaking table experiments in both X and Y directions, it is pertinent that the accelerometers are set up to record the complete 3 DoFs in all of the predefined node locations.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Damaged element(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>2(c)</td>
</tr>
<tr>
<td>T2</td>
<td>2(c) - 7(b)</td>
</tr>
<tr>
<td>T3</td>
<td>2(c) - 7(b) - 3(c)</td>
</tr>
<tr>
<td>T4</td>
<td>2(c) - 7(b) - 3(c) - 10(c)</td>
</tr>
<tr>
<td>T5</td>
<td>2(c) - 7(b) - 3(c) - 10(c) - 16(b) -</td>
</tr>
<tr>
<td>T6</td>
<td>2(c) - 7(b) - 3(c) - 10(c) - 16(b) - 12(c) -</td>
</tr>
<tr>
<td>T7</td>
<td>2(c) - 7(b) - 3(c) - 10(c) - 16(b) - 12(c) - 1(c).</td>
</tr>
</tbody>
</table>

Note: (c) for column and (b) for beam element

4. Results and discussion

Figures 3 to 5 illustrate the results of nsi, svn, and wsi indices for all members of the frame in all seven experimental tests. For T1 in which member #2 is damaged, the nsi matrix is observed to be dense which makes it trivial to calculate the corresponding svn index. According to Figure 4(a), the first vector shows the full value of a singular response, which indicates that the first vector cannot be a DLV. However, based on Demal’s recommendation to promote robustness of the algorithm, the value of svn should not be taken to be more than 0.2 [22]. Therefore, by amending the first vector to 0.2, the wsi index is able to correctly identify member #2 as a damaged member with an index value of 0.0455.

In the second test, in addition to the damaged member #2, member #7 is intentionally cut using a rotary blade on all four faces. While the nsi graph would seem to be more obscure than the first test, the signal thereof is strong enough to enable valid computation of the svn indices. As revealed by svn in Figure 4(b), the number of vectors that exceeds the

Figure 2. Recorded ground accelerations in the undamaged condition: (a) Ground acceleration for shaking table in global X direction (b) ground acceleration for shaking table in global Y direction, and (c) ground acceleration for the shaking table in the global X and Y directions.
Figure 3. Normalized Stress Index (NSI) of all members: (a) NSI for T1, (b) NSI for T2k, (c) NSI for T3k, (d) NSI for T4, (e) NSI for T5, (f) NSI for T6, and (g) NSI for T7.

The recommended SVN cut-off value of 0.2 is noted to have increased. Similar to the first case, the SVN vectors were amended to 0.2 as necessary prior to the calculation of the WSI index. It is noted in both cases that the damaged members were identified accurately without “false positive” and “false negative” outcomes.

An important observation throughout all tests as the number of damaged members increases is that the number of vectors which pass the cut-off value increases. This is an indication of decrement in the sensitivity of the damage detection algorithm.

Compared to the WSI index of the damage scenario (Figure 3(g)) with the NSI in the damage scenario (Figure 3(a)), it can be observed that the NSI matrix becomes inherently sparse after damaging almost 44% of the elements in the frame. Consequently, the number of vectors which should be reduced to the recommended cut-off SVN index increased.

According to the predefined damaged scenario which is shown in Table 2, the results of the proposed algorithm from the experimental model are illustrated in Figure 5. The WSI index of the damaged member is shown by red color. In all tests, the damaged elements were identified accurately without any positive or negative error.

By scrutinizing member #2 after damage, it is observed that the value of its WSI in subsequent damage scenarios (by which other members are damaged consecutively) fluctuated within the range of 0.0455 to 0.289, which requires further discussion. For completeness, the comprehensive tracking of the WSI index value of member #2 for all seven damage scenarios is shown in Figure 6. Although the member’s WSI index in most of the cases is noted to increase, there are several cases where the WSI index of member #2 decreases. This fluctuation is not linear and does not conform to any


Figure 4. SVN index: (a) svn for T1, (b) svn for T2, (c) svn for T3, (d) svn for T4, (e) svn for T5, (f) svn for T6, (g) svn for T7.

Figure 5. Weighted Stress Indices (WSI): (a) wsi for T1, (b) wsi for T2, (c) wsi for T3, (d) wsi for T4, (e) wsi for T5, (f) wsi for T6, and (g) wsi for T7.
specific or observable pattern. Hence, insofar, the only practicable outcome of the calculated WSI index is to determine the occurrence of the measurable damage of members in the frame in real time. Although the WSI of member #2 does not remain constant after damage, its fluctuation has shown no effect on the calculation of damage in the algorithm.

To quantify the severity of damage, it is recommended that a sensitivity relation between the flexibility matrix of the frame in undamaged condition and the corresponding matrices be formed in case of any damage. As demonstrated in this study, the damaged members can be identified through the results of the WSI index.

5. Conclusions

In this study, a 3D aluminum frame was tested in terms of seven damaged configurations. In each scenario, the frame was subjected to ground vibration through fixed attachments in a shaking table with consumer-grade ADXL335 accelerometers utilized to record the acceleration data at eight predefined nodes. The recorded data were passed through a combination of Eigen Realization Algorithm (ERA) and Pandey’s recommendations in order to generate flexibility matrices, which led to damage member detection by the Damage Load Vectors (DLV) method. The application of Markov matrix to the noisy accelerometer data enabled the exclusive inclusion of the most useful signals. These signals were subsequently fed into Pandey’s method in order to generate flexibility matrices by using only the first few modes. This process ultimately facilitated the utilization of noisy accelerometers for accurate damage detection. Evaluating the DLV results showed that a decrease in severity of the WSI index would typically correspond to an increase in the number of damaged members. The post-damage WSI value for member #2 was observed to fluctuate from 0.0455 swayed to 0.289; however, this has no effect on the identification of damaged members. The outcome of this study is encouraging since the damaged members were identified accurately with no “false positives” and “false negatives”.

References


Biographies

Iraj Toloue is a Structural Engineer and obtained his MEng from Universiti Putra Malaysia and PhD from Universiti Teknologi PETRONAS, Malaysia. He is a member of Board of Engineering Malaysia (BEM), Malaysia Structural Steel Association (MSSA) and is involved in a number of consultancies to PETRONAS group in the oil and gas industry. He has also thought a variety of subjects in vibration and stochastic analysis of structures to undergraduates, graduates, and engineers. He has received international awards, 1 copyright, and 1 patent and published 14 scientific papers.

Mohd Shahir Liew is currently the Deputy Vice Chancellor of Research and Innovation at Universiti Teknologi PETRONAS. Prior to his engagement in academia in 2009, Shahir has spent more than twenty years in the engineering consulting industry as well as in engineering research. Shahir’s approach to research is centered heavily around adopting the quadruple helix model which focuses on close ties with the industry, government, and local community. In his current capacity, he focuses on core activities such as business development, research-based ventures, talent development, leadership mentoring, and strategic research/industry planning. In tandem with his portfolio, he has been instrumental in supporting PETRONAS’ core business through research solutions and has been responsible as a catalyst for the formation of several UTP - PETRONAS Skill Group Steering Committee and more recently further activating close cooperation between UTP and PRSB. Shahir also holds several strategic positions in industry-based NGOs such as the current Vice President of Malaysia Structural Steel Association (MSSA), current Fellow of Energy Institute UK, and former Honorary Secretary at Malaysian Oil and Gas Services Council (MOGSC). He also engages in national standards devel-
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