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Estimation of mixed-mode fracture parameters by gene expression programming

A. Khademalrasoul* and A. Adib

Department of Civil Engineering, Engineering Faculty, Shahid Chamran University of Ahvaz, Ahvaz, Iran.

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Stress Intensity Factors (SIFs);
Extended finite element method.

Abstract. The linear elastic fracture phenomenon is characterized by Stress Intensity Factors. In this study, a general function was obtained in order to predict the fracture parameters. The numerical calculation of the SIFs in a mixed-mode condition is a cumbersome task. In this research, more than 6800 numerical analyses using the extended finite element method were conducted to simulate the fracture problem. States were considered for a plate with an arbitrary edge or center crack. Mixed-mode SIFs were calculated by the interaction integral. Then, Gene Expression Programming (GEP) method was utilized to extract a function. Results showed acceptable correlations between numerical calculations and genetic programming functions. R-square (R^2) values are in the range of 0.91 to 0.96, which guarantee the accuracy of the inferred functions.

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1. Introduction

Stress intensity factors are important parameters of the linear elastic fracture mechanics [1–3]. SIFs are useful for life prediction and fracture initiation in the structures. Further, crack path prediction is based on the concept of the SIF. Therefore, it is possible to arrest a crack according to the SIFs and the stress conditions in the vicinity of the crack tips. In general, there are three independent modes of fracture in the structural elements. Opening, shearing, and tearing are admissible displacements of crack surfaces with respect to each other. In two-dimensional problems, opening and shearing modes are considered [4]. Crack propagation could be determined with a combination of these two modes of fracture. Maximum tangential stress, minimum strain density, and maximum energy release rate are the most important criteria in the crack propagation. However, crack path prediction is characterized by the SIFs that demonstrate the stress conditions in the vicinity of the crack tips. Therefore, many efforts have been made on different approaches to calculating the SIFs [5–14]. Numerical, analytical, and experimental studies have been conducted on the principles of the fracture parameters [15].

Finite element method, boundary element approach, meshless methods, and isogeometric analysis are significant numerical methods in fracture mechanics. Among them, the Extended Finite Element Method (XFEM) has emerged as a flexible approach in the fracture mechanics framework [16–18]. Meshless and isogeometric analysis methods have some difficulties in essential boundary conditions [19–22]. Moreover, the stiffness matrix in the boundary element method is fully developed and, hence, matrix calculus is somewhat time consuming [23]. In other words, since the XFEM is based on the conventional finite element analysis, all benefits of finite element method are preserved. In general, XFEM mathematically is based on the concept of the partition of unity ap-

*. *Corresponding author.*
E-mail addresses: ag.khadem@scu.ac.ir (A. Khademalrasoul); arashadib@scu.ac.ir (A. Adib)

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proximation finite element spaces [24–26]. Additional degrees of freedom are introduced in the finite element approximation spaces and, eventually, any kind of discontinuities is implicitly imposed on the solution space. In this way, the influences of the discontinuities are considered in the stress distributions.

Unlike other extensive studies on fracture parameters, limited studies have performed on the concept of the genetic algorithm and artificial neural networks to estimate the fracture parameters [27]. This may arise from the fact that these methods need a lot of input data [28,29], whereas collecting a large number of the SIFs in different bodies with various geometries is a cumbersome task. Direct and indirect approaches have been used to calculate the SIFs. Stress extrapolation and displacement extrapolation are direct approaches, and the energy method is indirect method [30]. In the framework of the energy methods, since the SIFs are calculated using remote data from the crack tip, higher accuracy can be achieved.

Gene Expression Programming (GEP) method states the best equation for calculating the SIF based on different geometries and loading conditions. GEP was first invented by Ferreira and is a development of GP [31]. Although GEP uses the same kind of expression tree as GP, the entities evolved by ET are the expression of genomes.

In this investigation, a large number of numerical analysis cases are used to predict the fracture parameters. In order to generate the input data, more than 6800 numerical models using the XFEM are produced. In this study, the XFEM in combination with the level set method is adopted to simulate any kind of discontinuous media. Two level set functions were utilized to simulate the crack tips (crack tip function) and crack body (Heaviside function). Among different methods for the SIF calculation, the interaction integral is used. The interaction integral (M-integral) is based on the well-known J-integral. M-integral is the dual form of the J-integral. Interaction integral is formulated in the XFEM computer code using MATLAB programming language. All numerical calculations of the M-integral implemented in the computer code are solved automatically. All procedures include equivalent domain selection around the crack tips, detection of the crack tip elements, and the mathematical solutions of the integrals. By applying interaction integral, both the first and second stress intensity factors are obtained in one solution step. Then, influencing parameters for the SIFs are chosen as the input data for genetic programming models. These parameters consist of loading conditions, geometry specifications, and crack configurations. Crack configurations such as the length of the crack and the crack inclination angles are chosen. Geometry specifications include the width and height of the plates. Finally, loading conditions are remote stress acting whether in the x (shear mode) and y (tension mode). Then, genetic programming is used to predict a function for SIFs in mixed-mode conditions. However, the functions that have been proposed for the single edge-crack plate are mostly focused on the pure mode. This study considers the general mixed-mode conditions. Functions are inferred for plates with edge and center cracks with arbitrary inclinations up to 60 degrees with respect to the horizon. Quadratic finite element meshes are considered fine enough to achieve accurate results. Further, in order to increase the numerical integration in the vicinity of the crack tips, the sub triangulation is performed on the crack tip elements. Then, in order to establish the obtained functions, T-pair test is conducted. h and p values are calculated to be an approval for the GEP calculations. In addition, the Root Mean Square Error (RMSE) is chosen as the fitness function.

This paper is outlined as follows: Section 2 is dedicated to explaining the principles of the XFEM. Section 3 shows the calculation of the mixed-mode SIFs in one step using the interaction integral. Section 4 explains the genetic programming method in function finding problems. Section 5 presents the numerical results by the XFEM and expresses the unique formulas for both SIFs in edge and center cracked plates.

2. Principles of the XFEM

The XFEM was firstly introduced [32]. It is characterized by some special features in fracture mechanics. These special features of the extended finite element result from the partition of unity finite element property of XFEM, of which the most prominent features include:

1. The ability to include the local behavior of the solution in the finite element space;
2. The ability to construct finite element spaces of any desired regularity.

The XFEM can be assumed to be a classical FEM capable of handling arbitrary discontinuities. In fact, in the XFEM, any types of discontinuities are modeled implicitly onto the solution space [33,34]. In this method, by introducing additional degrees of freedom, any kind of discontinuities can be modeled. The XFEM approximates the displacement of point x as follows:

$$\mathbf{u}(\mathbf{x}) : \mathbb{R}^2 \to \mathbb{R}^2,$$

$$\mathbf{u}^h(\mathbf{x},t) = \sum_{i\in I} \mathbf{u}_i(t) N_i(\mathbf{x}) + \sum_{j\in J} \mathbf{b}_j(t) N_j(\mathbf{x}) H(\psi(\mathbf{x},t))$$

$$+\sum_{k\in K} N_k(\mathbf{x})\left(\sum_{l=1}^{4}\mathbf{a}_k^l(t)B_l(r,\theta)\right), \tag{1}$$

where $N_i(x)$ is the standard basis function of the finite element for the ith node, and t is time. Time is used for each parameter, which increases monotonically. Therefore, the whole solution steps include equilibrium equations with no dynamic effects. J and K represent nodal point sets for crack body and crack tip, respectively. $\mathbf{u}_i$, $\mathbf{b}_j$, and $\mathbf{a}_k$ demonstrate degrees of freedom. Further, $H(\psi(\mathbf{x},t))$ and $B_l(r,\theta)$ are the enrichment functions of the XFEM. These two functions are called Heaviside and crack tip enrichment functions, respectively. The enrichment functions consist of two series of functions. By introducing these functions to the influenced finite element nodes, the implicit additional degrees of freedom are added to the solution space. On the other hand, the effects of the considered discontinuity are simulated numerically.

3. Interaction integral

Behavior of a body with a discontinuity, such as crack, is generally characterized by a parameter such as SIFs or path independent J-integral in linear elastic fracture mechanics. Further, during the last decades, much effort has been made for SIFs calculation. Theoretical, numerical, and experimental methods have been employed for determining the SIFs in the vicinity of the crack tips.

Interaction integral (M-integral) has been used for mixed-mode of fracture problems. M-integral was introduced by Yau et al. [35] for isotropic materials. In fact, M-integral is the dual form of the J-integral for the cracked body. In this method, an auxiliary field is introduced and imposed on the solution space. The auxiliary stresses and displacement derived by Westergaard and Williams have been used. Displacement and stress auxiliary fields have been chosen in a situation to satisfy the equilibrium equations and boundary conditions in the problem of the traction-free crack surfaces. The mixed-mode of SIFs has been calculated in one solution by conducting a computer subroutine in the extended finite element framework. This integral is numerically calculated in the equivalent area in the vicinity of the crack tips. The interaction integral is defined as follows:

$$M^{(1,2)}=\int_A\left(\sigma_{ij}^{(1)}\frac{\partial u_i^{(2)}}{\partial x_1}+\sigma_{ij}^{(2)}\frac{\partial u_i^{(1)}}{\partial x_1}-W^{(1,2)}\delta_{1j}\right)\frac{\partial q_1}{\partial x_j}dA, \tag{2}$$

where A is the integration area, q_1 is the smoothing function with a value of 0 or 1 for different nodes, δ_{ij} is the Kronecker delta, x_1 is the local coordinate axis in the crack line direction, $W^{(1,2)}$ is the interaction strain energy density, σ_{ij} is the stress tensor, and u_i stands for the displacements vector. It should be noted that by introducing the q_1 function, the equivalent domain around the crack tip moves like a rigid body. Superscripts "1" and "2" demonstrate the real and auxiliary fields, respectively.

4. Principles of GEP method

GEP was first invented by Ferreira and is a development of GP [31]. Although GEP uses the same kind of expression tree as GP, the entities evolved by expression tree are the expression of genomes. In basic GEP, genes (individuals) are often selected and copied into the next generation based on their fitness by roulette-wheel sampling with elitism [31]. This guarantees the survival and cloning of the best individual to the next generation. The variation in the population is introduced by applying one or more genetic operators to select chromosomes. Most genetic operators used in genetic algorithms can also be implemented in GEP with minor changes, including crossover, mutation, and rotation. The flowchart of GEP is shown in Figure 1. The algorithm begins with an initial population with many genes. After generations of evolution, the best chromosome will be selected and its decoding process can be expressed [31]. According to the GEP rules, the genes will be expressed as ETs and the ETs can also be easily decoded as an algebraic equation. A more detailed description of GEP can be found in the referenced study [31].

In our work, the procedure of construction for handgrip force prediction is as follows:

Step 1: Population initialization
The set of functions F and the set of terminals T were selected to create the chromosomes. The edge-crack and center-crack plates are different in terms of chosen elements. Five elements were chosen

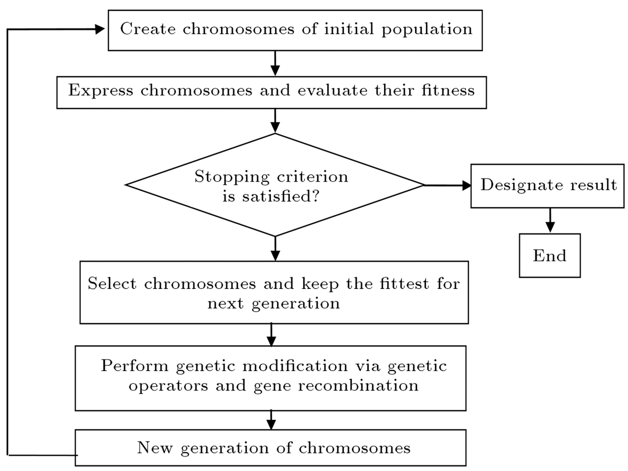

Figure 1. The flowchart of gene expression algorithm.

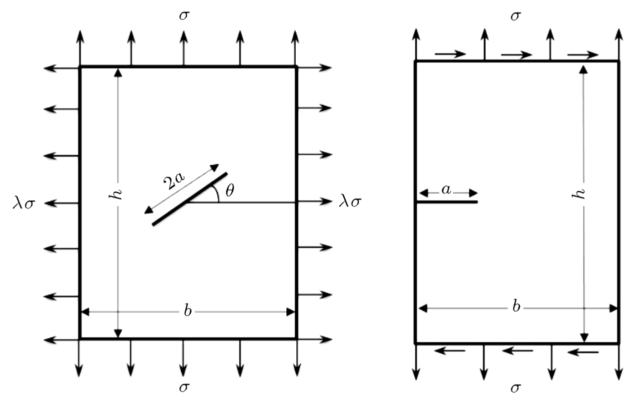

Figure 2. Geometric configuration of cracked plates.

as in a mathematical function set for edge-crack problems: $F = [+, -, \times, \div, \text{power}]$. The terminal set $(b, h, a, \sigma_{yy}, \sigma_{xx})$ was selected. Lengths of head and tail are 8 and 30, respectively, and five genes per chromosomes were employed. In addition, nine elements were chosen as in a mathematical function set for center-crack problems: $F = [+, -, \times, \div, \text{power}, \text{sqrt}, \exp, \sin, \cos]$. The terminal set $(b, h, a, \theta, \sigma_{yy}, \sigma_{xx})$ was selected. Lengths of head and tail are 8 and 30, and six genes per chromosomes were employed. Figure 2 shows the assumed plate with an edge or a center crack;

Step 2: Genetic operation

Basic genetic operators were applied for each generation including mutation, inversion, IS (Insertion Sequence) transposition, RIS (Root Insertion Sequence) transposition, one-point recombination, two-point recombination, gene recombination, and gene transposition. The details about how these operators are implemented can be seen in the referenced study [31];

Step 3: Fitness calculation

The maximum fitness (fmax) was set to 1000 based on the default of GEP method and the suitable magnification of maximum fitness; then, the fitness was calculated as follows:

$$f_{fitness} = 1000 \times \frac{1}{MSE_i + 1}, \tag{3}$$

where:

$$MSE_i = \frac{1}{m}\sum_{j=1}^{m}(F_{ij} - T_j)^2.$$

MSE represents the mean square error, m is the total number of fitness cases, F_{ij} is the value output by the individual program i for the fitness case j (out of m fitness cases), and T_j is the target value for the fitness case j. For a perfect fit, $F_{ij} = T_j$ [31];

Step 4: Termination criterion

There are two termination criteria:

1. $f_{fitness} = f_{\max}$;
2. The maximum number of generations reached 2000.

If either criterion is satisfied, stop; else, go to Step 2 [31].

The flowchart of this research is outlined in Figure 3.

5. Results

In the first step, in order to determine the accuracy of the finite element modeling, the stress distributions for edge and center cracked plates are shown. Quadratic finite element mesh is considered fine enough to obtain an exact solution. Rectangular finite element mesh was constructed with 0.02×0.02 elements in width and height. Further, because of the importance of the numerical integration in the vicinity of the crack tips, sub-triangulation is conducted on elements, which are located in the integration area. Figure 4 demonstrates the stress distributions for cracked plates containing edge and center cracks with different inclinations.

5.1. SIFs calculation

This section is dedicated to making a comparison between current numerical analysis using XFEM and existing experimental-analytical solutions for evaluating SIFs. Details of the numerical calculations for edge-crack plates are illustrated in Table 1.

In addition, Table 2 illustrates the computational SIFs for center-crack plates under uniaxial tension. Since previous experimental-analytical solutions in finite plates are mostly in pure mode, results are shown for the pure-mode problems.

5.2. The results of GEP method for edge-crack problems

GEP method states an equation for predicting mode I (K_{I}) and mode II of fracture (K_{II}) in edge-crack problems based on width of domain (b), height of

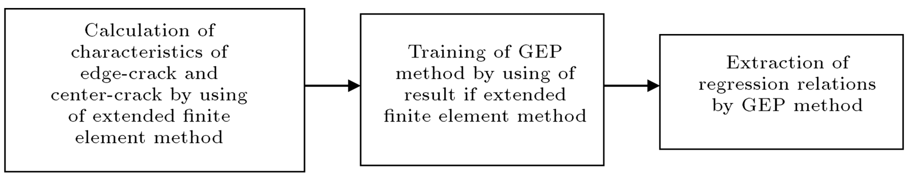

Figure 3. The flowchart of research methodology.

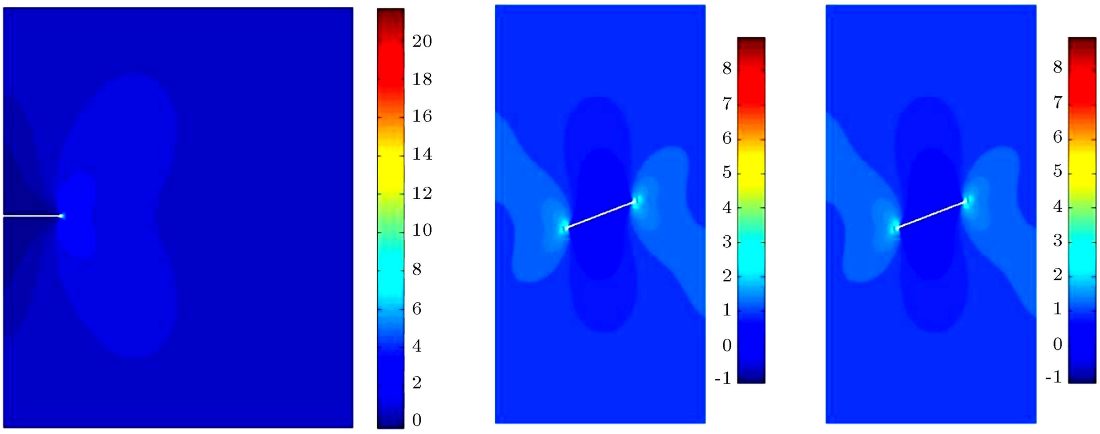

Figure 4. Stress distribution for a plate containing an edge crack and center cracks.

Table 1. Stress intensity factors for single edge-crack plate under uniaxial tension.

Crack length	Analytical-experimental K_I	Numerical K_I	Numerical K_{II}	K_I (numerical)/ K_I (analytical)
0.1	0.6296	0.6251	0.0018	0.993
0.2	0.9082	0.9135	0.0025	1.006
0.3	1.1493	1.1547	0.0032	1.005
0.4	1.3846	1.3869	0.0038	1.001
0.5	1.6266	1.6258	0.0044	0.999
0.6	1.8825	1.8789	0.0051	0.998
0.7	2.1581	2.1535	0.0058	0.998
0.8	2.4591	2.4532	0.0008	0.998
0.9	2.7927	2.7905	0.0009	0.999
1.0	3.1674	3.1708	0.001	1.001
1.5	6.1407	6.1340	0.0161	0.999
2.0	13.1032	13.4695	0.0038	1.028

Table 2. Stress intensity factors for center-crack plate under uniaxial tension.

Crack length	Analytical-experimental K_I	Numerical K_I	Numerical K_{II}	K_I (numerical)/ K_I (analytical)
0.1	0.3979	0.3982	0.0000	1.001
0.2	0.5648	0.5434	0.0000	1.01
0.3	0.6943	0.6787	0.0000	0.976
0.4	0.8050	0.7919	0.0000	0.984
0.5	0.9043	0.8988	0.0000	0.994
0.6	0.9963	0.9810	0.0000	0.985
0.7	1.0838	1.0701	0.0000	0.987
0.8	1.1687	1.1566	0.0000	0.990
0.9	1.2528	1.2422	0.0000	0.992
1.0	1.3375	1.3278	0.0000	0.993
1.5	1.8153	1.7940	0.0000	0.988
2.0	2.4977	2.4517	0.0000	0.982

Table 3. The Root Mean Square Error (RMSE) and R^2 of training and validating the developed equations by Gene Expression Programming (GEP) method for edge crack.

Parameter	Training		Validation	
	RMSE	R^2	**RMSE**	R^2
K_{I}	94.22	0.988	68.29	0.976
K_{II}	382.98	0.971	315.77	0.953

domain (h), length of crack (a), tension stress (σ_{yy}), and shear stress (σ_{xx}). The number of considered geometries and loading conditions and values is 2124 for edge-crack problems. In this research, GEP method is used from power, $+$, $-$, $\times$ and / functions. The number of genes and chromosomes is 5 and 30, respectively, and the size of head is 8. Results of the numerical method (for 2124 states) were applied for the training and validation of the developed equations by GEP method (80% of data for training and 20% of data for validation).

The significance of these equations was determined by t-test. This test showed that there were significant relations between calculated K_{I} and K_{II} by the numerical models and predicted K_{I} and K_{II} by GEP method. The significant level is 1% and p-values are 0.7083 and 0.7813 for K_{I} and K_{II}, respectively. Degrees of Freedom (Df) of the t-test is 4246. The values of the test statistics are 0.3742 and -0.2777 for K_{I} and K_{II}, respectively. In addition, the values of standard deviation are 84.7846 and 9.7566 for K_{I} and K_{II}, respectively. The RMSE and R^2 of training and validating the developed equations by GEP method are illustrated in Table 3.

The developed equations by GEP method for Mode one of fracture in edge-crack plates for the mixed mode of fracture are as follows:

$$
\begin{aligned}
&G1C8 = -2.28021990608709,\\
&G1C5 = 1.28783142637904,\\
&G2C9 = 4.84246639304335,\\
&G2C1 = -4.95895260475478,\\
&G2C4 = 7.80024855522324,\\
&G3C3 = 3.20169682912687,\\
&G3C2 = -0.878299089422313,\\
&G3C1 = 1.36135438765005,\\
&G4C5 = -9.95291498316064,\\
&G5C1 = -0.772524115659732,\\
&y = 0.0,\\
&y = (\sigma_{yy}/(G1C8/(b\\
&\quad - \mathrm{realpow}(\mathrm{realpow}(a, G1C5), G1C5)))),\\
&y = y + ((((b+b) \times \sigma_{yy}) + \mathrm{realpow}(a, G2C9))\\
&\quad /(\mathrm{realpow}(h, a) + (G2C1 + G2C4))),\\
&y = y + (a \times ((G3C3\\
&\quad + \mathrm{realpow}(\mathrm{realpow}(G3C1, b), G3C2)) \times \sigma_{yy}));\\
&y = y + ((d(4)/(d(1) + ((G4C5/d(1)) - d(2)))) \times d(3)),\\
&y = y + (a \times \mathrm{realpow}((((a/b) \times \sigma_{yy})/(G5C1 + b)), a)),\\
&K\mathrm{Ipredict} = y.
\end{aligned}
\tag{4}
$$

Moreover, the developed equations by GEP method for Mode two of fracture in edge-crack plates are as follows:

$$
\begin{aligned}
&G1C2 = -0.634809827115886,\\
&G1C4 = 3.24860522578786,\\
&G2C6 = 7.96888487182226,\\
&G2C5 = 7.9865730202641,\\
&G2C3 = -2.82639945239753,\\
&G2C0 = -0.579835990882018,\\
&G3C5 = 0.82648295819544,\\
&G3C7 = -7.66337909045983,\\
&G4C0 = 93.0965971274657,\\
&G4C6 = -10.8990646128312,\\
&G5C1 = -0.766952253388981,\\
&G5C2 = 3.13861613436109,\\
&y = 0.0,\\
&y = ((((\sigma_{xx} \times b) - \sigma_{xx}) - G1C2)\\
&\quad /\mathrm{realpow}(\exp(h), (G1C4 * a))),\\
&y = y + ((((\sigma_{xx}/G2C5)/(G2C3/b))\\
&\quad /((h + G2C0)/a)) * G2C6),\\
&y = y + ((\sigma_{xx} - (((h+a)/G3C7) + (\sigma_{xx}/h))) - G3C5),\\
&y = y + (\mathrm{realpow}(G4C0, ((\exp(\sigma_{yy}) \times G4C6)\\
&\quad /\mathrm{realpow}(b, \sigma_{yy}))) - \sigma_{xx}),
\end{aligned}
$$

$$y = y + ((\sigma_{xx}/((b + G5C1) \times (b \times G5C2))) \times \exp(a)),$$

$$K\text{IIpredict} = y. \quad (5)$$

5.3. The results of GEP method for center-crack problems

In this section, by conducting genetic programming principles, a unique formula is inferred from the data for an arbitrary center crack. In these problems, a center crack with any inclination angle with respect to the horizon up to 60 degrees is considered. In particular, for a plate with a center crack, there are two tips in the computational domain. The SIFs for both of the crack tips are calculated by implementing the interaction integral. The results demonstrated that the values of the SIFs were the same for two crack tips. Therefore, accordingly, the extracted GEP formula can be used for each desired crack tip. GEP method expresses an equation for the prediction of mode I (K_{I}) and mode II of fracture (K_{II}) in center-crack problems based on the width of domain (b), height of domain (h), length of crack ($2a/2$), crack orientation angle (θ), tension stress (σ_{yy}), and shear stress (σ_{xx}). The number of considered different geometries and loading conditions is 4710 for center-crack problems. In this research, $+, -, \times, \div$, power, sqrt, exp, sin, cos mathematical functions are utilized for GEP solution to generate a proper formula for the prediction of both SIFs in center-crack plates. By conducting a series of diagnostic analyses, there are 6 genes and 30 chromosomes, respectively. The size of head is selected to be 8. Results of numerical simulations (for 4710 states) were applied for training and validating the developed equations by GEP method (80% of data for training and 20% of data for validation).

Eventually, the significance of the extracted equations was determined by the t-test. This test shows that there are significant relations between the calculated K_{I} and K_{II} by the numerical models and the predicted K_{I} and K_{II} by GEP method. Therefore, from the implementation point of view, one can use the inferred formula instead of the numerical simulation. The significance level is considered 1% and p-values are 0.2511 and 0.7917 for K_{I} and K_{II}, respectively. The Degrees of Freedom (Df) for the t-test is 9420. The values of the test statistic are -1.1476 and -0.2641 for K_{I} and K_{II}, respectively. In addition, the values of standard deviation are 6.9121 and 3.6832 for K_{I} and K_{II}, respectively. The RMSE and R^2 of training and validating the developed equations by GEP method are illustrated in Table 4.

Results demonstrate inferred functional relations for estimating the fracture parameters. These results generate an accurate mixed-mode fracture initiation. Eq. (6) is obtained for K_{I} prediction for any kind of center-crack plates:

Table 4. The Root Mean Square Error (RMSE) and R^2 of training and validating the developed equations by Gene Expression Programming (GEP) method for center crack.

Parameter	**Training**		**Validation**	
	RMSE	R^2	**RMSE**	R^2
K_{I}	396.93	0.957	382.69	0.951
K_{II}	504.81	0.928	467.14	0.909

$$G2C9 = 0.757119862771529,$$

$$G3C9 = -4.54871208075198,$$

$$G3C5 = 1.27180460798975,$$

$$G3C3 = 5.29529709768975,$$

$$G4C3 = 1.41314424963903,$$

$$G4C2 = -3.84813610579428,$$

$$G5C2 = 0.397656178472243,$$

$$G5C9 = -3.15164718252734,$$

$$G5C4 = -1.22101216812634,$$

$$G6C7 = 0.635492167409079,$$

$$y = 0.0,$$

$$y = (\cos(((\cos(b) - (\sigma_{xx}/h)) + \cos(a))) \times a),$$

$$y = y + \text{realpow}((\sin((a/G2C9)) + ((\sigma_{yy} + \sigma_{yy}) \times a)), \cos(\theta)),$$

$$y = y + \sin(\exp((G3C9 - (\cos(\text{realpow}(G3C5, G3C3)) \times (a + \sigma_{xx}))))),$$

$$y = y + \cos(\exp((\theta + \text{realpow}((\exp(G4C2)/h), (G4C3 - \theta))))),$$

$$y = y + ((\sigma_{xx} \times (a - G5C2)) \times (\cos(G5C9)/\cos(G5C4))),$$

$$y = y + \sin((a \times \sin((h - \exp((G6C7 - \sigma_{xx})))))),$$

$$K\text{Ipredict} = y. \quad (6)$$

Further, Eq. (7) is inferred from the GEP solution to predict KII of center-crack plates. In the presented relations, the unit of crack inclination angles, θ, is considered in radian.

$$G2C1 = 9.12337040144215,$$

$$G2C0 = 8.47272255623035,$$

$$G3C6 = 0.417246421040969,$$

$$G3C5 = 2.90015291132003,$$

$$G4C6 = 4.5951719718009,$$

$$G4C3 = -544.124293487823,$$

$$G5C3 = -8.96053060670797,$$

$$G5C7 = -4.82549630761725,$$

$$G5C5 = -7.21011415951111,$$

$$y = 0.0,$$

$$y = ((\cos(b - \sin(\sigma_{yy}))) \times a) \times a,$$

$$y = y + \sin((((a \times \theta) - (\sigma_{yy}/G2C1)) + (\sigma_{yy} \times (G2C0 * b)))),$$

$$y = y + ((\sin(\sin(((G3C5 \times \theta) - G3C6))) \times a) \times \sigma_{yy}),$$

$$y = y + (\cos(G4C6) - (\cos(\mathrm{realpow}(\sigma_{yy}, \theta)) \times \sin((b \times G4C3)))),$$

$$y = y + (\mathrm{realpow}(\exp((a/G5C5)), G5C3) /(\sin(h) + G5C7)),$$

$$y = y + \exp((a \times \mathrm{realpow}(\cos(\exp(\sin(b))), \sigma_{yy}))),$$

$$K\mathrm{II}\mathrm{predict} = y. \tag{7}$$

The main objective of this research is to extract appropriate equations. These extracted equations can be used for determining characteristics of edge and center cracks, and it is not necessary to apply numerical models following their extraction.

6. Conclusion

In this research work, unique formulas in the mixed-mode condition were inferred from the data for predicting fracture parameters. Data were generated by the numerical method. The Extended Finite Element Method (XFEM) was used to produce more than 6500 input data. In fact, by combining the interaction integral with the XFEM, the mixed-mode Stress Intensity Factors (SIFs) were calculated. Models consist of edge-crack plates and center-crack plates. Genetic programming was utilized to find appropriate functions for Modes one and two of fracture parameters. R-square and correlation coefficients were in a condition that demonstrated the significant relationships between predicted and numerical SIFs. Further, in order to examine the functions, the T-test was done on the results. T-test results proved the existence of a correlation between input data and the output results. Therefore, this study used the suggested functions instead of numerical solutions to estimate the fracture initiation.

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Biographies

Abdolghafour Khademalrasoul is an Assistant Professor at the Civil Engineering Department, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran. He received BSc degree in Civil Engineering in 2006 as the top student and MSc degree in Geotechnical Engineering from Shahid Chamran University of Ahvaz in 2009. His profession includes numerical methods and computational models. Moreover, he received PhD degree in Geotechnical

Engineering on the subject of computational fracture mechanics and the application and innovative notion in isogeometric analysis in 2015 from Shahroud University of Technology, Shahroud, Iran.

Arash Adib is a Professor at the Civil Engineering Department, Faculty of Engineering, Shahid Chamran University of Ahvaz. He received BSc degree in Civil Engineering from Shiraz University in 1997 and MSc degree in Hydraulic Structures from Shiraz University in 1999. Further, he received PhD degree in Water Engineering from K.N. Toosi University of Technology in 2006. Dr. Adib was a supervisor to four PhD students and almost 30 MSc students. He published more than 50 journal papers and 100 conferences papers.