Drivers of Cross-Country Vehicles

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Abstract

This work offers a new chassis design, namely the wheel-track, represents a mathematical model of this chassis, and also proves the advantage of the proposed design when driving the vehicle on arbitrary terrain - rough off-road. The proposed approach can find application in the design of unmanned research mechanisms for other planets (Mars Rovers, Lunar Rovers etc.), also for design of robots and transport of rescuers at liquidation of consequences of natural or technological disasters. The article presents the analysis of the requirements for the chassis of extraterrestrial research unmanned mechanism demonstrated high compliance of the proposed approach. The analysis of the requirements for the chassis of extraterrestrial research unmanned mechanism demonstrated high compliance of the proposed approach. We proposed and investigated a
mathematical model of wheel-track, demonstrated the optimization of the proposed mathematical model to machine computing, demonstrated the flexibility and scalability of the mathematical model. The proposed design is an attempt to combine the advantages of a walking and wheel types of travel in one mechanism.

**Key Words:** drivers; chassis; transport of rescuers; unmanned research mechanisms; wheel-track.

1. **Introduction**

Humankind has created many machines and mechanisms in its attempt to solve the transport problem. In most cases, wheel transport gets the job done on Earth’s surface and even the surface of other planets. However, wheel transport is not universal, which raises the question – what can surpass wheel transport? The answer is – movement with the help of legs, since many animal species move with the help of legs across terrain, which no wheel mechanism can traverse. What makes leg movement universal? Firstly, consider the wheel and the boundaries of its applicability. The limits of the applicability of the wheel lies in its interaction with the ground during movement across a surface that differs from solid road surface; when moving across agricultural land, sand, snow, etc. the wheel loses its superiority [1]. It is necessary to highlight several key moments: on an unsolid surface, the wheel causes deformation of the ground surface, which will increased during the rotational motion of the wheel, which is necessary for the vehicle to move forward, i.e. inevitable rotational motion. Secondly, solid wheel disks are used only in rail transport, while the soft rubber of pneumatic tires does not transfer the pressure from the
base to the wheel evenly. Simple model simulations show that the response reactions of the base that act from the surface onto the wheel are distributed unevenly across the “wheel-surface” contact patch: the response of the base at the center of the contact patch is at its maximum and tends to zero on the edges of the contact patch. This uneven distribution of forces exacerbates the deformation of the ground surface, thus weakening adherence. All these negative factors will only exacerbate during the rotational motion of the wheel.

Figure 1 shows the scheme of a tire with straight lugs in the elastic operating mode. The ground surface will apply pressure on both the tread pattern and the tire body between the lugs of the tread pattern. The pressure of the ground surface of the tread pattern and the tire body can be described using the following equations:

\[ P_t = (k_{c/b} + k_{\varphi})z_L^n \]  \( (1) \)

\[ P_{cr} = (k_{c/b} + k_{\varphi})(z - h_L)^n \]  \( (2) \)

[Figure 1 here]

How will the forces be distributed under the bearing surface of the foot in case of leg movement? A flat foot, when stepping on the ground surface in a downward vertical direction, will have an even distribution of forces under its surface even in motion, i.e. step-based movement will not deform the ground, which can be regarded as a key to steady travel across unsolid substrates.

There have been many projects of “walkers” (vehicles that move on legs), but none of them managed to get into batch production. Apparently, multi-hinge mechanisms are too complicated for modern engineering. How can one combine the advantages of a stepping
propelling device with the simplicity and constructability that are required for batch production? There is a widely reproduced variant – the track-type propelling device, which is used in military and agricultural machinery. The tracks are rigid, hence they provide an even distribution of response forces of the base under their surface; in addition, unlike the wheel, they drop onto the substrate surface without sliding. The popularity of the track-type propelling device is well deserved, but such devices are massive. Light polymeric tracks could solve the weight problem (and weight related problems of drag loss and loss due to rerolling of massive tracks); however, when turning, a track-type vehicle faces problems of substrate deformation – the need to turn a vehicle that rests on two rails of elastic track bands requires substrate deformation, while when moving on a solid surface, mechanical strain can cause early wear or even deformation of the track-type propelling device. Nevertheless, the need to ensure the maneuverability of vehicles forces inventors to create complex designs based on wheels, for example, [2] use the Mecanum wheel to ensure the maneuverability of the vehicle, but this wheel type, while providing exceptional maneuverability on a solid surface, is unsuitable for off-road conditions. The research [3], among other things, presents an analysis and statistics of damage from contact with the substrate surface, especially in case of an irregular surface, and discusses the high requirements to materials.

Thus, an ideal vehicle for off-road conditions should be a mechanism that incorporates the simplicity of the wheel and a solid bearing surface that steps on the substrate without tangential displacements and ensures high maneuverability when compared to the track-type propelling device.
At the moment, difficult conditions of roadless areas encourage designers to experiment with the number of wheels, suspension, etc. using a well-known concept of the wheel [4]. However, in the majority, these are attempts to improve the suspension, but not the wheels. See, for example: [5-7]. Meanwhile, it was with the wheels that many of the problems of the Martian rovers were concerned.

Another important aspect is the distribution of forces during turning. Ideally, the wheel should transform and change its shape in proportion to the lateral response force. This deformation rolls the tire towards its longitudinal direction at the angle of its lateral slip. However, in this case, the term “slip” implies not exactly slipping, but rather a deformation, during which the tire “goes” into the outer radius of the turn. In this case, maximum adherence with the surface will be achieved at a lateral angle size of more than zero.

It has been many centuries since the invention of the wheel, and mankind does not leave attempts to improve it. Therefore, our work is not unique in this series. As an example of ongoing attempts to improve and "reinvent" the wheel, we can name the following design projects: http://www.yankodesign.com/2009/03/10/split-your-wheel-into-eight/ and http://www.michelinchallengedesign.com/the-challenge-archives/2009-brave-bold/2009-showcase-of-selected-entrants/transforming-multifunctional-wheels-by-su-yang-park-and-chang-hoe-heo-south-korea/

Given projects are just a design in its purest form. Unfortunately, a pretty picture does not have the necessary calculations, the mathematical model of the concepts and the behavior of the proposed projects on the surface is unclear, especially on the off-roads – we can only guess. Meanwhile, on the surface of other planets, which are forced to use research machines, there is off-road in the extreme phase.
On this complex surface, comprising loose silty sands, the outputs of shattered sedimentary rocks with inclusions of relatively hard pebbles, it is impossible to move straightforward, and there is no need to move fast. The way of research machine will be forced winding, especially if it meets the outcrops on its way – which it will need to come around, or breeds of interest to the study, which it will have to approach for sampling and measurements in situ (Figure 2).

[Figure 2 here]

2. Results and Discussion

Recently in the press, the design of the walking wheel began to appear based on the process of interaction of wheel elements with irregularities of homogeneous reference surface [8]. The geometric parameters of the walking wheel are determined based on the conditions of interaction with frontal obstacles of the soil. The advantage of these walking wheels equipped with curved feet, is that when riding it excludes the impact of the rim of the front obstacles of the soil. This is achieved by removing part of the rim of the round wheels. Thus obtained walking wheel is very simple in design (one-piece made of the same material), easy to move, rolls into the off-road conditions and has a high relief flotation on hard ground. The authors [8-19] of walking wheels has developed methods for the synthesis and analysis of the scheme of the walking wheel and four varieties that they investigated in layout. A common drawback of the walking-wheel is considered the drop of the coming wheel feet from height of 7-10 mm (at the wheel parameter is D=1200 mm) on the
supporting surface. The height of the drop, although not large in magnitude, but creates discomfort when riding on the asphalt road and at high speeds it creates vibration.

When riding on the off-road, the defect of the drive wheels becomes invisible.

Last modification of the walking wheel is made consisting of two elements – shin-foot and hip, which are connected by a hinge with limited movement relative to each other and connected through a damping elastic element [9].

The amount of relative rotation of the shin and hip in the knee joint is of the order of 40-60.

In this way, they received walking wheel "Kanbak" (Salsola paulsenii in Kazakh) is stable on a supporting surface, do not destroy it and saves the straightness of the movement of the wheel hub with soft damping.

Thus, the authors believe that they solved the problem task of creating a generic driver for motor vehicles, which can be used instead of pneuma-wheel when driving on the asphalt road inside the city and you can go for a walk on the off-road.

We told about the universal driver – the walking wheel, which is intended for driving on a firm supporting surface with obstacles in the form of projections or pits, the height of which does not exceed the value of the radius of the wheel.

Off-roads are different. They can be in the form of nomadic sand, in the form of loose snow banks, in swamps (temporary wetlands and natural), in the form rock blockages, in mountain valleys, ridges and gorges.

In any case, the reference surface (soil) and geometric shape of the foot of the vehicle interact while riding. Our task is to pick up their options when designing musculoskeletal system of the vehicle, so that the machine could move without stopping under the action of
the driving torque of the engine. For this purpose, in the beginning of the design, we should establish or select features of the reference surface.

These characteristics are the permissible values of specific pressure of environment and friction coefficient between "the support surface and environment" ["p"] and ["f_o"].

And then from these interactive parameters, we should determine the desired shape and parameter values of the musculoskeletal system with the condition that the movement exists.

These parameters are as follows:

\[
[p] = \frac{G}{W} \left( \frac{N}{cm^2} \right)
\]  

where: [p]- the allowable specific pressure of the material of support surface \( \frac{N}{cm^2} \), \( G \) – the total weight of the machine falling on one support apparatus (wheel), \( W \) is the contact area of the foot of musculoskeletal apparatus (cm\(^2\)), [p] is given in the beginning of the design, environmental parameter, \( G \) – part of the machine weight falling on one musculoskeletal apparatus (specified parameter), \( W \) is determined parameter of the foot, contact area of the foot.

\[
F_{mp} = f_o \cdot p \cdot W(N)
\]

where: \( F_{mp} \) – the friction force between the contacting elements (N), \( f_o \) - the coefficient of friction between the materials of the contacting elements (dimensionless), \( G=N \) – pressure or reaction of the support (N).

The value of determined interacting elements must ensure the movement of the center of the wheel hub towards to the reference surface, i.e.:

\[
M_g \geq M_{res}
\]
where: $M_{res} = F_{mp} \cdot r, r$ - the radius of the wheel (moment of resistance, acting towards to the wheel contact N·m), $M_g$ – torsion torque acting on the wheel from the driver side (N·m).

When the condition (1.3), musculoskeletal system moves the wheel hubs towards to the support surface.

If $M_{dr} - M_{res} > 0$, the center of the wheel hub will move rapidly, i.e. there is acceleration of the wheel.

If $M_{dr} - M_{res} = 0$, the center of the wheel hub will move uniformly, i.e. there will be established regime of movement.

If $M_{res} > M_g$, there will be a slowdown, i.e. coasting.

Pliable is called a support surface, on which we are forced to travel in vehicles due to life circumstances, the mechanical properties of which are similar on several characteristics.

For example, snow, sand, swamp and water have weak internal communications, each resist in their own way, but with similar forces. It is permissible resistance "[p]" of environment to a vertically acting load. You can hardly find in dictionaries the exact value of this parameter for each case at different places.

Therefore, while designing, we should specify at least in laboratory conditions, the exact value of the specific pressure "[p]". In addition, it is important to know when designing structures, an accurate value of the coefficient of friction "[f]" for materials of supporting surface and foot of musculoskeletal apparatus of the machine. Knowing and [f] at a known weight G (N), we can determine the value of the area W of the contact support surface of the musculoskeletal apparatus (cm²), i.e.:
\[ W = \frac{G}{[p]} \text{ (cm)}^2 \]  \hspace{1cm} (6)

where \( G \) – weight falling on one foot (average)

In addition to weight \( G \), on the support surface applies a projection of the \( P_{dr} \) – the driving force of the actuator, which must be balanced by a friction force \( F_{fr} \), i.e.:

\[ P_{dr} - F_{fr} = 0 \]  \hspace{1cm} (7)

Only under these conditions, there will be movement of the center of the wheel hub of the vehicle.

where \( F_{fr} \) equals:

\[ F_{fr} = \frac{G}{[f]} \text{; } (G = N) \]  \hspace{1cm} (8)

where: \( N \) – the normal reaction support surface (N).

Observing these conditions, the design of parameters of the musculoskeletal apparatus of the machine is created.

Basically, the ingenuity of the designer manifests itself when creating a structural diagram of the musculoskeletal system, because there can be quite a lot of the varieties of designs.

Below, we present one design of the musculoskeletal system, which would be called the walking wheel – track, which is designed to drive on highly challenging support surface.

Fig. 3 shows the current model of the wheel-track, which consists of track, closed with its links around the traffic generator of triangular shape and pivotally connected with the generator in three points. The traffic generator is coaxially and pivotally connected with the drive shaft of the actuator. The track links are made as a foot with the calculated reference area.
and are rectangular in shape and are interconnected pivotally by means of rods, forming a hexagonal shape.

Fig. 4 shows a diagram of the walking wheel-track with sync ratchet device. Wheel-track contains of six equal parts of 1 caterpillar, covering the outside of the traffic generator of triangular shape 2, which in the middle part at three points is pivotally connected with the covering elements of a caterpillar by the fingers 3. This forms a strictly symmetrical system of three four-bar mechanisms, a ratchet disk 4, having six teeth coaxially and fixedly connecting in the disk center with traffic generator 2, clutch 5, rotating on an axis fixed to the drum 6 of the drive shaft of the driver of motor vehicle.

The mechanism of the wheel – track is composed as follows: the shaft of the motor 7 is rigidly associated with the drum 6, connected to a ratchet disk 4 assembled with the traffic generator 2 and is fixed towards to the drive shaft with the possibility of free rotation. Clutch 5 is worn on finger of the drum 6, it is also fixed with a possibility of free rotation and the other end is pulled by a spring to the drum so that the first shoulder of the clutch 5 was pulled against the teeth of the ratchet disk 4. In this form, the wheel – track is set at one end of the drive shaft of the engine in the form of a wheel of one-way traffic. On the other end of the drive shaft of the engine similarly, we set the second wheel – track. As a result, we formed the mechanism of the drive axle of the vehicle, which has both wheel-tracks that rotate together with the drive shaft of the engine only forward in the direction of movement as a pair of wheels of railway carriages.

If we rotate the housing of the axle together with the drive shaft in the plane of the
support surface, one wheel of the caterpillar stops and the other rotates towards to the stopped wheel – track with twice the angular speed as the differential drive axle.

Such a device is structurally simple and provides precise turning of the vehicle, securely by rectilinear motion of the chassis not susceptible to all sorts of drifts in the direction of in slippery roads. Fig. 5 shows a structural diagram of a vehicle equipped with four wheel – tracks, of which two are leading and rotate synchronously as a pair of wheels, other two are driven, they are installed at the ends of the horizontal axis managed a cross-shaped rear axle. The chassis comprises a frame 1; front drive axle 2, on drive shaft of which two wheel – tracks are mounted that rotate together with the drive shaft synchronously through six-tooth ratchet clutch; and the cruciform rear axle 3 connected by its vertical stand with the common frame 1 with the possibility to rotate relative to the total frame on a plane supporting surface. At the ends of the horizontal axis of the cross-shaped axle, two wheel – tracks 3 and 4 are pivotally mounted. The vehicle operates as follows: the movement of the actuator (motor) is transmitted via the transmission mechanism to the drive shaft and from there via the included ratchet clutch to both wheel – tracks and they synchronously rotate in the direction of rotation of the drive shaft and is straight-ahead movement of the vehicle. The turning of the vehicle is performed by turning the vertical posts of the rear axle by means of a steering device.

Both rear wheel – tracks rotate around a vertical strut of the rear axle, and a shared frame 1 starts to rotate and releases the left or right of the ratchet clutch and one of the wheel – tracks stops. There will be a rotation of the vehicle relative to the instantaneous center (IC), which will lie in the intersection of the directions of the drive shaft and the horizontal axis of the rear axle. Depending on the angle of rotation horizontal axis of the
rear axle, the position of the IC can get close to the vertical axis of the hub of the wheel – tracks and can be the same. Then the vehicle can turn with minimum radius, towards to the vertical axis of one of the wheel – tracks. The uniqueness of the vehicle is determined using the formula anatomical (structural) of the structure. The total number of independent elements of the chassis are four, namely a front driving axle 2, total frame 1, a cross-shaped rear axle 3, the two rear wheel – tracks 3 and 4. Then the number of all elements (links) is:

\[ n = 5. \]

The number of one-directional compounds of these elements between P1 is four, namely, the junction of the common frame 1 with the drive shaft of the front axle 2, a cross-shaped rear axle 3 with the general frame 1 and the connection of both rear wheel – tracks with two axles of the rear axle.

That is \( P_1 = 4. \)

Temporary contacts of the wheel – track on the surface of the support are four movable joints P4,

That is \( P_4 = 3, \)

because the contact of the leading wheel – tracks with the surface of the support is considered as one contact, since they are rigidly connected to each other and have one connection with the surface of the support.

Contacts of the wheel – track on the surface of a support area are considered to be four movable parts due to the fact that the wheel – track as a separate element of the overall system has four mobilities towards to the reference surface.

They are as follows: two movements of the slide in the directions of two mutually
perpendicular directions along the axes OX and OY on the surface of the support area, one rotational movement about the axis of the drive shaft and one rotational motion around vertical axis OZ.

Of the six degrees of freedom of the wheel – track in space, there are four, and two degrees of freedom are limited.

As a result:

\[ n = 5. \]

\[ P_1 = 4. \]

\[ P_4 = 3, \]

then:

\[ W = 6 \cdot n - 5P_1 - 2P_4 = 6 \cdot 5 - 5 \cdot 4 - 2 \cdot 3 = 4 \]

They are as follows: two movements of the slide in the directions of two mutually perpendicular directions along the axes OX and OY on the surface of the support area, one rotational movement about the axis of the drive shaft and one rotational motion around vertical axis OZ. Of the four mobility chassis relative to the reference surface, two are controlled, the straight line movement by the actuator and control via the steering wheel. The remaining two mobilities of the vehicle are not operated. Since this vehicle on purpose is slow-speed, it makes no sense to eliminate unnecessary degrees of freedom.

Received extra mobilities are a consequence of the fact that the vehicle stays on three footings when driving. Therefore, the formula considers its mobility in two planes. The transition from one plane to another is threat at large velocities, when inertial factors have a dominant influence. Chassis of modern four-wheel cars have four unmanaged mobilities. But yet, nobody complains about this, although at higher speeds, these
mobilities are controlled by inertial factors, and can lead to tragic consequences. Scheme of the mechanism of wheel – track contains leading link generator 1 made in the form of triangle, six (Fig. 4a) support elements of equal size 2,3,4,5,6,7 are interconnected pivotally, forming a closed, planar kinematic chain. Generator 1 at the three corners are pivotally connected to three supporting elements 7, 3 and 5, forming three identical, symmetrically placed around the generator, the hinged four-bar mechanisms (Fig. 5).

The resulting mechanism consists of seven moving links, i.e.

\[ n = 7, \text{link O is the frame (fixed link) and of the ten one-direction kinematic pairs, i.e.} \]
\[ P_1 = 10. \]

The system is flat. Then:

\[ W = 3n - 2p_1 = 3 \cdot 7 - 2 \cdot 10 = 1, \]
i.e. the scheme of the mechanism has one mobility. The closed circuit (Fig. 6) has:

\[ W = 3n - 2p_1 = 3 \cdot 6 - 2 \cdot 9 = 0 \]
of the zero mobility.

The movement of the generator 1 uniquely determines the number of mobilities of the mechanism scheme of wheel – track.

Fig. 6 shows the kinematic chain of the wheel - tracks. To determine the motion pattern of the scheme mechanism when moving one of the units (master), we need to determine the transfer function that describes the relative position of all links of the mechanism.

The mechanism of the wheel – track is a high-class mechanism (HCM). Described "High-Class Mechanism" can be represented as four-bar link mechanism with a closed kinematic linkage (four links are: two hinges on the supporting foot and two hinges on the triangular
"wheel", and all the "caterpillar" of the wheel rim – closed kinematic relationship).
Whereas this is a many-hinged mechanism, it is appropriate to refer to the well-known theory of many-hinged mechanisms. Approximate or approximating synthesis of hinge mechanisms originates in the works of P.L. Chebyshev. Chebyshev in the analysis and synthesis of articulated mechanisms used established mathematical apparatus of best approximation of functions, which has found wide application in many branches of mathematics, physics, theory of electrical circuits, etc. Created by Chebyshev methods of synthesis of mechanisms on the basis of approximating functions were further developed in the classical works of Z.Sh. Bloch (1943) [20], and N.I. Levitsky (1948) [21]. A great contribution to the development of methods of approximate synthesis of articulated mechanisms was made by F. Freudenstein (1955) [22], who on the basis of a simple system of equations has developed effective ways for accurate and approximate solution of the classical problem of kinematics of a flat hinge four-link chains. In the work [23], the method of Freudenstein or the method of displacement equations has been generalized and applied to other types of planar mechanisms, in particular for a crank-driven slide mechanism. The method was also extended to spatial four-link chains. To illustrate the method, we focused on the synthesis of four-link chains to reproduce functions of the form:

\[ f(x) = \log(x) \text{ and } f(x) = \frac{1}{x} \]  \tag{9}

In the work [24] for the approximate solution of the system of equations of Freudenstein, an efficient algorithm was proposed based on the use of orthogonalization of matrices, method of least squares. This algorithm is well suited to implement as rather complex many-hinged mechanisms relatively easy to calculate for an arbitrary playback functions.
Considered in this work method of calculation of many-hinged mechanism goes back to the solution of the classical problem of Freudenstein but it is original in the methodology. Further, we describe in details the math for a simple case of wheel-tracks, consisting of six reference surfaces. It is easy to show that the number of bearing surfaces can be increased, that may be important for some particular cases, however, it is clear that the mathematical description of such many-hinged mechanisms will be more complicated than the simplest case with six bearing surfaces. From Fig. 7, it is seen that one leading link generator $O_1 O_2 O_3$ during rotation towards to the center hub $O$, sets in motion three four-link mechanism 1, 2 and 3 at the same time. Although all the units of four-bar linkage are the same, i.e. the relevant lengths of the links have the same length:

$$l = \frac{r}{2} \quad \text{and} \quad R = 1.5r \quad \text{and} \quad L = l = \frac{r}{2}$$

(10)

They on some section impede the relative motion of each other. When stationary (mundane) position of link a-c, the center the wheel hub $O$ can move from position 1 and 2 to the right and can make the same movement to the left, when the changed direction of rotation of the leading link, i.e. it has limited mobility. When in the extreme positions, the scheme wedged, becomes rigid, the motion of the center of the hub is continuing, revolving either towards point "c" or point "a" of a circuit. Then it lands on another side of the hexagonal track, and the center hub similarly will start to move towards to the new base. Although the trajectory of the hub $O_{12}$ in the stillness of the grounds is small but it very well softens the movement of the hub, because it is caused by the interaction of three four-bar chains. At this point, the basis is not valid by tangent component of the driving force of the actuator. For pliable support surface, this motion effect provides an extremely high smoothness, because reproducible function of the whole complex system of
conjugate four-bar linkage and the kinematic connection of the wheel rim-of the caterpillar is a straight line parallel to the surface. However, it is worth emphasizing that this condition exists on a malleable surface, as it requires a certain shift of the edges of the bearing surfaces below the level of the supporting ground surface. This circumstance may cause unwanted vibration while driving on smooth and hard surfaces at high speeds i.e. may be negative. But at movement with small speed on a pliable surface, that will be not only non-negative phenomena, but even reverse – the penetration of the edges of the bearing surfaces in the loose soil will contribute to increasing the traction of the moving crew with the surface, on which it is moving. If we assume movement on the slope or terrain feature, that is very positive. When moving from one base to another base, a sharp angle "c" of the hexagon deepens to a pliable surface. This creates favorable conditions, on the one hand, it eliminates the slipping of the wheel – track on the supporting surface, and on the other hand, it reduces the height of the trajectory "C" of the hub in this moment, bringing it closer to straight. For analytical determination of the characteristics of the mechanism scheme, we need the formula of kinematic model of the scheme. Let us try to define this function. We should replace closed contours 1 and 2 in Fig. 7 with the vectors \( \vec{r}, \vec{l} \) and \( \vec{R} \) in the following way:

\[
\vec{l} + \vec{r} + \vec{l} = \vec{R}
\]  

(11)

Let us represent it on the coordinate axes OXY and we will get the:

\[
l + r \cos \varphi + l \cos \alpha = R \cos \Psi
\]  

(12)

\[
r \sin \varphi + l \sin \alpha = R \sin \Psi
\]  

(13)

Let us represent it on the coordinate axes OXY and we will get the:

\[
l + r \cos \varphi + l \cos \alpha = R \cos \Psi
\]  

(14)
\[ r \sin \varphi + l \sin \alpha = R \sin \Psi \]  
(15)

As \( l = 0,5r \) and \( R = 1,5r \), let us insert them:

\[
\begin{align*}
0,5r + r \cos \varphi + r \cos \alpha &= 1,5r \cos \varphi \\
r \sin \varphi + r \sin \alpha &= 1,5r \sin \varphi
\end{align*}
\]  
(16)

We set different step values for angle \( \varphi \), given that \( \varphi \) is changed starting from \( 60^0 \). Let us square the system of equations (1.8) and sum them up. After conversion we will get:

\[ r^2 \cos \varphi + 2r^2 (0,5 + \cos \varphi) \cdot \cos \alpha = 2r^2 \sin \varphi \cdot \sin \alpha \]  
(17)

Let us denote the constants using:

\[ A = r^2 \cos \varphi, B = r^2 + 2A, C = 2r^2 \sin \varphi \]  
(18)

and we get:

\[ A + (r^2 + A) \cos \alpha = -C \sin \alpha \]  
(19)

or:

\[ A + (r^2 + A) \cos \alpha = -C \cdot \sqrt{1 - \cos^2 \alpha} \]  
(20)

Let us get rid square root:

\[ [(r^2 + A)^2 + C^2] \cos^2 \alpha + 2A(r + A) \cos \alpha + A^2 - C^2 = 0 \]  
(21)

Let us denote the constants:

\[ D = [(r^2 + A)^2 + C^2] \]  
(22)

\[ E = 2A(r^2 + A) \]  
(23)

and we will get:

\[ \cos \alpha = \frac{E}{D} \pm \sqrt{\frac{E^2}{D^2} - \frac{A^2 - C^2}{D}} = 0 \]  
(24)

or:
\[
\alpha = \arccos \left( -\frac{E}{D} \pm \sqrt{\frac{E^2}{D^2} - \frac{A^2 - C^2}{D}} \right)
\]  
(25)

from (1.9) we will get:

\[
\Psi = \arcsin \left( \frac{r \sin \varphi + r \sin \alpha}{1.5r} \right)
\]  
(26)

Further, for circuit 2 (see Fig. 7) similarly, we get the system of equations:

\[
\begin{align*}
r (0.5 \cos \varphi_1 + r \cos \alpha_1) &= 1.5r \cos(\Psi + 60^0) \\
r \sin \varphi_1 + r \sin \alpha_1 &= 1.5r \sin(\Psi + 60^0)
\end{align*}
\]  
(27)

from the obtained values of \(\Psi\), we will determine \(\alpha_1\) and \(\varphi_1\).

\[
a_1 = \arccos \left( -\frac{E_1}{D_1} \pm \sqrt{\frac{E_1^2}{D_1^2} - \frac{A_1^2 + C_1^2}{D}} \right)
\]  
(28)

from (1.11) we will get:

\[
\sin \varphi_1 = 1.5 \sin(\Psi + 60^0) - \sin \alpha
\]  
(29)

from this:

\[
\varphi_1 = \arcsin(1.5 \sin(\Psi + 60^0) - \sin \alpha)
\]  
(30)

Using equation (26) and (27), and (28) and (30), we plot the points "c" and "e" of the mechanism scheme.

If the length of the connecting link of the "c e" is equal to a given length \(r\) of the sides of the hexagon, i.e.

\[
ce = ac,
\]  
(31)

then the scheme of mechanism exists and we continue to determine the following values of the angle of the leading link \(\varphi_1 = \varphi + \Delta\). \(\varphi = 60^0\) is the initial value, and then we decrease it in increments of \(\Delta\), until the found value will not diverge, i.e.
At the moment when \( ce \neq r \), the scheme of the mechanism wedges. Further rotation of the generator will cause rotation of scheme as a rigid system towards to the point "c" and the scheme will establish the following ground (Fig. 7). The first scheme position (Fig. 7) differs from the second position (Fig. 7), because in the first case, the generator makes a rotational movement towards the center O1 and in the second case, it makes complicated plane-parallel movement as the connecting rod. In the first case, the trajectory of the hub O_{12} is the arc of a circle, and in the second position 5, 6, O_1, 7, 8 – is part of the connecting rod curve. Therefore, the second stage of the analysis will start with an analysis of the position of the hinge of four-bar linkage \( \alpha f k_b \) \((\overline{r}, 0.5\overline{r}, \overline{R}, 0.5\overline{r})\), i.e. with the analysis of:

\[
\overline{r} + 0.5\overline{r} + \overline{R} = 0.5\overline{r}
\]  

(33)

That is:

\[
\begin{align*}
& r + 0.5r \cos \varphi + 1.5r \cos \psi = 0.5r \cos \varphi_1 \\
& 0.5r \sin \varphi + 1.5r \sin \psi = 0.5r \sin \varphi_1
\end{align*}
\]

(34)

[Figure 7 here]

from this system, we can determine the angles:

\( \psi \) and \( \varphi_1 \),

Therefore, the positions of the scheme points a and a_1.

Next, we determine the position of the points "e" and "c", which must lie on the same straight line with point t – the top of the generator.

Otherwise, the calculations are repeated for reciprocal to the measured value of the angle \( \varphi \).

Thus, by a two-stage calculation, we can determine the positions of the scheme of the
mechanism of wheel – track. For the following positions, we used again the calculations of the first stage.

As can be seen from the algorithm, the calculations for computation and building the positions of mechanism scheme can be performed using modern computers.

3. Conclusion

An offer to combine in one mechanism the advantages of a walking type travel and the wheels helped to create the concept of the wheel-tracks. The proposed design of the wheel-track was subjected to mathematical modeling, the result of mathematical modeling was the conclusion about the suitability of the playback features of the complex multi-link mechanism, wheels and tracks to ensure smooth motion without slip of a vehicle on a loose surface. This work has demonstrated the applicability of the proposed approach for the design of vehicles forced to move along a strong off-road. We have demonstrated the superiority of the proposed approach in terms of requirements to unmanned exploratory machines and robotized platforms compelled to act in the aftermath of natural and man-made disasters.

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List of figures

Figure 1. a - A tire with lugs in the elastic operating mode; b - Distribution of forces in the contact patch – ground surface; c - Distribution of forces in the contact patch – wheel.

Let us skip the intermediate calculations and immediately present the distribution of forces in the contact patch – ground surface/wheel.

Figure 2. A winding road near the outcrops, where presumably the traces of water flow were found [8].

Figure 3. Photo of maquette of the wheel – a) track, and b) 3D model of proposed wheel – track.

Figure 4. a - The scheme of walking wheel - track with the synchronizing ratchet device; b - Scheme of the vehicle with the chassis of the type wheel – track.

Figure 5. Structural diagram of the wheel – track.

Figure 6. Kinematics of the wheel – track.

Figure 7. To the kinematic calculation.
Figure 1
Figure 2
Figure 3
Figure 5
Figure 6

\[
r = 2l (l = 0.5r) \\
R = 1.5r \\
l = 0.5r
\]
Figure 7