



Reliable hierarchical multimodal hub location problem: Models and Lagrangian relaxation algorithm

E. Korani^a, A. Eydi^{b,*}, and I. Nakhai Kamalabadi^b

a. *Department of Industrial Engineering, University of Kurdistan, Sanandaj, Iran.*

b. *Faculty of Engineering, University of Kurdistan, Sanandaj, Iran.*

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Abstract. Hub facility location problems involve the establishment of strategic hub facilities and the allocation of demand nodes to them. Meanwhile, with the emergence of new transport and distribution networks with multi-level structures, the design of such networks has evolved. The diversity of transportation system has added multimodality to these problems. Considering the strategic nature and long-term implications of decision-making in this field, the decisions shall be of high reliability. The present study is an attempt to consider the reliability of communication paths in proportion to the volume of transactions through them while covering all of the above-mentioned issues in hub location problems at the same time. Incorporating hierarchical characteristics of the problem into the model, one can obtain a significantly enhanced model in terms of multimodality and reliability. A Lagrangian solution method was developed considering the strategic level of the problem and the importance of the solution accuracy. The model was then validated in terms of time and quality.

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1. Introduction

Hub location problems follow the same approach as that applied to network optimization problems; in this approach, hubs are considered as collection and distribution centers [1]. In these problems, instead of a direct connection between each pair of points, fewer indirect connections are used for economic gain, and the parcels with the same destination are distributed together. The importance of these problems is doubled when it comes to the economy of scale and transportation or

communication costs. One of the clear examples of the economy of scale is the use of larger and more efficient aircrafts along the lines that connect hubs across an air network. In telecommunication networks, the use of higher-capacity optical fibers represents an example of connecting a pair of hubs.

In addition to transportation and telecommunication systems, many other applications have been introduced for hub facility location models including production planning, retail management, wholesale management, and healthcare service provision, as investigated by Teo and Shu [2], Jia et al. [3], Revelle et al. [4], Melo et al. [5], Gelareh et al. [6], Gelareh and Nickel [7], Yaman and Elloumi [8], Korani and Sahraeian [9], Alumur et al. [10], and An et al. [11]. Numerous research works have been conducted in this area. Disruption and/or cancellation of a service stream not only keeps the firm from achieving its

*. *Corresponding author. Tel.: +988733660073;*

Fax: +988733668513

E-mail addresses: Ehsan.korani@gmail.com (E. Korani);

Alireza.eydi@uok.ac.ir (A. Eydi); nakhai.isa@gmail.com (I. Nakhai Kamalabadi)

economic goals, but also incurs additional costs. For example, the cancellation of a flight in the aviation industry incurs several costs to an airline company, including commission costs to be paid to distribution and ticket sale centers, passenger-related costs as per regulations (accommodation and catering), and damages caused by a breach of promise. In addition to the direct and indirect material costs, the exacerbation of resentment and the undermining of the legitimacy of the system should be added to the costs incurred to the company [11]. Reliability of communication paths across the network and the approach to the deployment of hub facilities based on the reliability are important. However, Snyder et al. [12] believed that natural disasters, labor strikes, or terrorist threats were also among the factors that affected reliability. With the growth of industrial community and the resultant increase of communication volume, the importance of this issue has grown considerably, thereby increasing scientific research in the field of reliability of hub location. Kim and O'Kelly [13] were the first to step in this area; they focused on improving the reliability of communication network paths in hub location problems. An et al. [11] improved the validity of the services provided by hub facilities by defining the concept of backup facility. They believed that, given the strategic level of hub location problems, solution method was very important; therefore, they proposed two Lagrangian and branch-and-bound methods for their model. Zarandi et al. [14] presented a broad review of the literature that examined the hub location problem and its applications. Focusing on marine transpiration, they used capacitated models to investigate the most common applications of marine transportation. In order to simultaneously model pickup and deliveries based on observations from real-life hub networks with the aim of efficiency improvement, Kartal et al. [15] proposed the single allocation p-hub median location-routing problem, where the objective was to minimize total routing cost. The model was solved by several mixed-integer programming formulations and two heuristic approaches based on multi-start simulated annealing and ant colony heuristics. Roni et al. [16] designed a supply chain management model for biofuels to optimize CO₂ emissions due to transportation-related activities. They suggested a multi-objective, mixed-integer linear programming model to capture the trade-offs among costs, environmental impacts, and social impacts of delivering the biofuels. Finally, Roni et al. [16] encouraged the protection of renewable energy production.

In this paper, the actual condition of hub networks is considered and their outstanding features are recognized before defining the main research problem based on them. One of these features is the hierarchical structure of the problem, as pointed out by Yaman [17],

Alumur et al. [10], and Korani and Sahraeian [9]. Another important feature is the multimodality of transportation. Competition for expediting service is a key characteristic of the modern global trade; it has led to the inefficiency of specialized and individual transportation modes [18]. Therefore, multimodal transportation was emphasized in hub-related problems, as investigated by Chen et al. [19], Alumur et al. [10], Alumur et al. [20], Onyemechi [21], and SteadieSeifi et al. [22]. However, the simultaneous consideration of these three features (hierarchical structure, multimodal transportation, and reliability of communication network paths) gives rise to new requirements that need to be considered in the design of the model.

Figure 1 shows a hierarchical hub network with two levels of service provision. In this network, the first- and second-level hubs are called central hub and hub, respectively [17]. In this figure, the central hub is located at node 10 and hubs are located at nodes 3, 6, and 9; in addition, it is assumed that the hubs and the air central hub (*a*) and other links are connected via ground (*g*) transportation. Accordingly, one may identify 19 different paths with the reliability of each path calculated differently using different variables (see Table 1 for more details). These paths are different based on the number and type of hubs, the type of origin and destination nodes, and the number of hub and non-hub links.

Different approaches to calculating the reliability were used based on two reasons: absolute volume of flow along the entire path and the variable-based path specifications. Regarding the first reason, it is required to define limits for the balancing process so as to include all modes clearly, separate flows precisely, and prevent recalculation or omission of even one flow to detect flows efficiently; however, it should be taken into account that the difficulty of the problem depends on the number of constraints. The latter also is of special importance because the links constituting each path form a series on the network; therefore, defining variables with the minimum number of counters and indices is of great importance to avoid plurality of dimensions of the problem and the possible points. A closer look at Figure 1 shows that this seemingly simple network needs at least seven indices to account for the paths. This adds particular complexity to the problem.

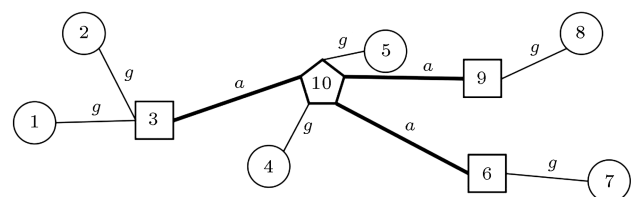


Figure 1. A hierarchical hub network with two modes of transportation: air and ground transportation.

Table 1. The flow passing through each path proportional to the reliability of the path in the hub network with a central hub.

No.	Type	Condition	Reliable flow	Route fig
1	1	—	$r_{ik}^a x_{ikl}^{ab} \sum_{j \in H} f_{ij}$	
2	2	—	$r_{kk}^b (r_{kl}^b)^{1-\gamma^b} z_{kkl}^{bb}$	
3	2	—	$r_{ik}^a (r_{kl}^b)^{1-\gamma^b} z_{ikl}^{ab}$	
4	3	$k \neq j$	$r_{ik}^a r_{kj}^d x_{ikl}^{ab} y_{ikj}^{db}$	
5	3	$k \neq j$	$r_{kk}^b r_{kj}^d x_{kkl}^{bb} y_{kkj}^{db}$	
6	4	$k \neq m$	$r_{ik}^a (r_{kl}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{ikl}^{ab} y_{lmj}^{cd}$	
7	4	$k \neq m$	$r_{ik}^a (r_{kl}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mm}^c x_{ikl}^{ab} y_{lmm}^{cc}$	
8	4	$k \neq m$	$r_{kk}^b (r_{kl}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mm}^c x_{kkl}^{bb} y_{kmm}^{cc}$	
9	4	$k \neq m$	$r_{kk}^b (r_{kl}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{kkl}^{bb} y_{kmj}^{dc}$	
10	4	—	$r_{ll}^b (r_{ll}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{lll}^{bb} y_{lmj}^{dc}$	
11	4	—	$r_{ll}^b (r_{ll}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mm}^c x_{lll}^{bb} y_{lmm}^{cc}$	
12	3	$l \neq j$	$r_{ll}^b r_{lj}^d x_{lll}^{bb} y_{llj}^{db}$	
13	1	—	$r_{il}^a x_{ill}^{aa} \sum_{j \in I} f_{ij}$	
14	3	$l \neq j$	$r_{il}^a r_{lj}^d x_{ill}^{ab} y_{ilj}^{db}$	
15	4	—	$r_{ik}^a (r_{kl}^b)^{1-\gamma^b} (\hat{r}_{ll}^d)^{1-\gamma^d} r_{lj}^d x_{ikl}^{ab} y_{llj}^{dd}$	
16	4	—	$r_{kk}^b (r_{kl}^b)^{1-\gamma^b} (r_{ll}^d)^{1-\gamma^d} r_{lj}^d x_{kkl}^{bb} y_{klj}^{dd}$	
17	4	—	$r_{il}^a (r_{ll}^a)^{1-\gamma^a} (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{ill}^{aa} y_{lmj}^{dc}$	
18	4	—	$r_{il}^a (r_{ll}^a)^{1-\gamma^a} (r_{lm}^c)^{1-\gamma^c} r_{mm}^c x_{ill}^{aa} y_{lmm}^{cc}$	
19	3	$m \neq j$	$r_{mm}^b r_{mj}^d x_{mml}^{bb} y_{mmj}^{db}$	

As previously mentioned, usually, real-world hierarchical hub networks have different transportation modes at a given level. For example, consider an online shopping company established in a capital city. Now, if this company is willing to economically serve an order with a delivery address in a small town in a township, firstly, the ordered goods should be transferred to the township along with other goods via air transportation. Then, the goods ordered from

each small town should be transferred to that town through ground transportation. This holds true in different fields with different types of service at each level of the hierarchy, e.g., distribution, health care supplement, telecommunication, cargo delivery, public service, etc. [1].

Adopted in this paper are the following concepts: median objective, single allocation strategy, multi-modal structure, path reliability, bound time delivery,

hierarchical network, certain number of hubs at each level, and nested feature (“in a nested hierarchy, a higher-level facility provides all the services provided by a lower-level facility together with at least one additional service [23].” see Şahin and Süral [24] for more details). Based on these features, a model was designed according to a mathematical formulation called Reliable Hierarchical Multimodal Hub Location Problem (RHMHL). RHMHL applies to telecommunication networks and transportation systems along a supply chain administered by distribution and logistics companies. This model follows three objectives simultaneously when making decisions:

1. Maximizing the flow of service that can be provided;
2. Creating a reliable network with an appropriate number of hubs at optimal locations and linking paths with the highest reliability coefficient;
3. Ensuring timely delivery of the services.

In addition, an efficient approach to problem solving has been developed by proposing a Lagrangian relaxation algorithm and providing an upper bound for a large-scale problem.

A review of related literature is presented in the next section. Section 3 deals with a problem description and design of the mathematical model. In Section 4, different steps of solving the Lagrangian relaxation problem are discussed. Computational results and performance evaluation of the solutions are presented in Section 5. Conclusion and practical recommendations are presented in detail in Section 6.

2. Literature review

Hakimi [25] and O’Kelly [26] proposed the concept of hub and, accordingly, presented a mathematical model of an air transportation network. Campbell [27] suggested a number of mathematical models for single- and multiple-hub median, center, and covering problems, and Skorin-Kapov et al. [28] introduced a new mixed-integer formulation for the problem. In addition to the new formulation, Ernst and Krishnamoorthy [29] added the discount factor, α , to the literature; the factor was inspired by the operation of Australia’s post. Alumur and Kara [30] and Farahani et al. [1] carried out two review studies on hub problems, appropriately describing the evolution of the problems.

2.1. Hierarchical hub network

Location-allocation problems represent a subset of hierarchical hub location problems [31]. Elmastas [32] developed a model for a three-level cargo network in a Turkish distribution company with a hub median approach that aims to minimize hub establishment costs. Yaman [17] enhanced his model and introduced

a new approach to routing costs minimization with a single-allocation structure. With the same incentive combined with the reduction of the length of the longest path, Yaman and Elloumi [8] introduced a new model in which the quality of delivered services was of maximum importance at different levels. With the development of multi-modal transportation, Alumur et al. [10] presented a hierarchical multimodal hub location model, in which the truck transportation mode was considered at the first level and transportation of higher levels went through the air transportation mode. The justice-oriented view is a priority of public transportation in today’s societies; in this regard, Korani and Sahraeian [9] developed a hierarchical hub location model with emphasis on maximal covering approach. Zhong et al. [33] presented a foundation for integrating urban and rural public transport in the form of a hierarchical hub-and-spoke network. They proposed a mixed-integer programming model to achieve minimum total routing cost, wherein a Genetic-Tabu search hybrid optimization algorithm was adopted for validation and analysis purposes. Shavarani et al. [34] used a distance-constrained mobile hierarchical facility location problem to find the optimal number and sites of launch and recharge stations with the aim of minimizing the total costs incurred to the system.

2.2. Reliable facility location

Hub facility location is an operational and important concept in telecommunication networks, and defensive strategies to prevent interruptions in communication network performance depend strongly on the facility location [35]. In addition, internet hubs are usually located close to each other to achieve enhanced efficiency [36]. The vulnerability of hub facilities in telecommunication and transportation systems has led to the introduction of a reliable hub network design [37,38]. Murray and Grubescic [39] addressed the geographical and operational factors that affect the protection of high-speed and broad-bandwidth telecommunication networks. Reliability of communication paths in hub networks was stressed by Kim and O’Kelly [13] who located hubs with the goal of achieving the highest probability of success in each trip. Hub facility failure was taken into account in defining backup hubs, as proposed by An et al. [11]. Given the nonlinearity of their proposed model, An et al. employed Lagrangian and branch-and-bound methods in various dimensions. The study of Masoumzadeh et al. [40] can serve as a guide for managers and decision-makers who are looking for optimum hub locations under budget restrictions. Concentrating on the critical role of hubs in a telecommunication network, they stressed the necessity of taking precautions to protect such networks against possible disruptions. Accordingly, they suggested a protective

multi-objective hub location model to protect a sustainable hub network design. Shishebori et al. [41] considered an integrated, budget-constrained multi-objective location-routing problem under uncertainty. They proposed a fuzzy-based model that sought to minimize total expected costs including transshipment costs, facility location costs, and fixed cost of the path. The model was then solved by an efficient sub-gradient-based Lagrangian relaxation algorithm. An integrated hub location and revenue management problem was considered by Tikani et al. [42]. The problem was demonstrated in the case of an airline industry with a two-fold objective of maximizing the revenue made out of transportation networks and minimizing hub installation costs. They presented a capacitated model where a limited capacity was allocated to stochastic demands of customer classes following a revenue management approach. De Sá et al. [43] focused on a multiple allocation incomplete hub location problem, where uncertain origin-destination pairs and fixed hub-related costs were assumed. For the sake of optimization, they proposed Benders decomposition frameworks and a hybrid heuristic. Zetina et al. [44] studied the uncapacitated hub location problem under uncertainty, where demand and transportation costs were subject to interval uncertainty. They examined three cases: uncertain demand, uncertain transportation cost, and both simultaneously. They further proposed formulations based on mixed-integer programming and branch-and-cut algorithm for solving the model.

2.3. Multimodal hub location

StadieSeifi et al. [22] investigated a case study concerning multimodal transportation, bolding the importance of this research topic in recent years. Related studies have been classified by time horizon (strategic, planning, and operational) and network type (direct transportation network or transportation network with product integration), with location-routing problems receiving a great deal of attention. Li [45] played an important role in the introduction of multimodal terminal routing with direct transportation networks. Kumar et al. [46] proposed routing in a direct transportation network with the aim of completing the network. The logistic hub location in multimodal transportation networks was investigated by Chen et al. [19] who sought to minimize transportation costs at the service-delivery level. Ishfaq and Sox [47] generalized the logistic multimodal structure for a multi-allocation p-hub problem. Alumur et al. [20] modeled multimodal hub locations across a network hosting ground and air transportation modes with an approach to minimizing flow costs. Theoretical concepts of multimodal hub networks and their dimensions and influential factors were discussed by Liu et al. [48] who classified the hypotheses proposed in this

field. Marufuzzaman and Eksioglu [49] presented the development of supply chain of environmental cargo in multimodal hub problems. They placed an emphasis on different scenarios that arise from the trend of seasonal changes. Onyemechi [21] analyzed the concept of multimodal hub network in three areas: short-term shipping, pipeline transport, and rail connectivity, enumerating their theoretical benefits and concepts.

Given the three research areas involved in the present study, its prominent contribution to related literature is as follows. First, reliability is incorporated into a new and facilitated modeling scheme wherein two features are considered simultaneously: multimodal hub location and time coverage radius, as presented in the studies by Alumur et al. [10] and Alumur et al. [20], respectively. In addition, the Lagrangian relaxation method is herein used to solve the problem more comprehensively. In contrast to the studies reported by Kim and O'Kelly [13] and An et al. [11], the present study considers multi-level and multimodal dimensions of the problem to maximize reliability. A hierarchical multimodal hub covering network with a service time bound was proposed by Dukkanci and Kara [50], wherein three different types of a vehicle were considered at each level: airplanes, big trucks, and pickups. They proposed a heuristic algorithm for solving the problem on a Turkish network dataset. In the next section, the problem statement and mathematical model are presented.

3. Problem statement and mathematical model

3.1. Problem description

In this study, a mathematical model of a hierarchical network composed of hub link nodes is presented. Through this network, products can be delivered to the corresponding destinations via paths composed of different transportation modes at the highest level of reliability with the lowest deal of disorder along the path. Herein, all demand nodes must receive full service and be connected through links with the highest reliability. Therefore, the present model should select a number of hubs from a pool of candidate hubs so that the most reliable communication paths can be established among the hubs, provided that a path exists between each origin and destination. The aim of designing such a model is to achieve direct and indirect objectives that show the superiority of RHMHL model over other models presented in the literature so far. Direct objectives can be the design of a hub network with a hierarchical multimodal structure where reliability is taken into account at each level. Indirect objectives may refer to economic and cultural dimensions of the problem, such as no delay and on-time service delivery at minimum possible costs across

the distribution networks. Given the involvement of the volume of flow passing through each path in the objective function and dependency of the path reliability on the distance passed, investment costs are practically minimized by minimizing the path length. This is because of the lower probability of failure with a shorter path than that with a longer path. The presented model in this research is more comprehensive than similar models as it considers the flow passing through each path and the distance passed.

The following assumptions have been considered for developing the research model:

- Potential locations of facilities are known as discrete points;
- Cargo unit is fixed between different transportation modes;
- The number of hub facilities established at each level is fixed, and the problem is approached as a p-hub problem;
- Hub facilities at each level are homogenous; however, facilities at higher levels have greater capabilities with a nested condition and are more heterogeneous than those at other levels [24]. Therefore, various services are available for demand centers at each service-delivery level;
- The travel time between different cargo transportation nodes with a special transportation mode is considered to be fixed and determined;
- Demands are fixed and known, such that the modeling is performed deterministically;
- Customers' locations in designed hub networks are predetermined, and facility location-allocation is optimized;
- The first level has a central hub facility, which is assumed to be fixed based on the particular case considered in this study;
- Facilities transfer paths are of unlimited capacity for any type of service;
- The hubbing effect is considered as a constant value between 0 and 1, indicating the reliability of hub facilities and the discount rate of travel time at different levels (see [13]);
- All hubs are connected to the central hub;
- There is no direct connection between two non-hub nodes or between two second-level hub nodes;
- There is only one central hub.

3.2. Mathematical model

In order to model the problem, several nomenclatures were used. Accordingly, I is the reference set of all nodes under study. H and C are potential hub nodes and central hub, respectively ($C \subseteq H \subseteq I$). The

indices i and j refer to the origin and destination, respectively. In addition, k and m are second-level origin and destination nodes, respectively, and O refers to all of the transportation modes considered. The index l indicates the central hub. The parameters and variables used to build the research model are listed in the following.

Parameters:

P_H	The number of second-level hub facilities
f_{ij}	The flow of demand between origin $i \in I$ and destination $j \in I$
α^b	Discount rate of hubbing effect on time for $b \in O$ transportation mode ($0 \leq \alpha^b < 1$)
γ^b	Coefficient of hubbing effect on reliability for $b \in O$ transportation mode ($0 \leq \gamma^b < 1$)
β	Maximum delivery time between each pair of origin and destination nodes
t_{ij}^b	Travel time for the path connecting $i \in I$ and $j \in I$ nodes via $b \in O$ transportation mode
r_{ij}^b	Reliability of the path connecting $i \in I$ and $j \in I$ nodes via $b \in O$ transportation mode

Variables:

x_{ikl}^{ab}	It is 1 if node $i \in I$ is allocated to hub $k \in H$ and hub $k \in H$ is allocated to central hub $l \in C$ via $a \in O$ and $b \in O$ transportation modes, respectively. Otherwise, it is 0. In addition, if $x_{kkl}^{ab} = 1$, a second-level hub is established at node $k \in H$, and if $x_{lll}^{ab} = 1$, a central hub is established at node $l \in C$;
z_{ikl}^{ab}	The amount of flow from the origin node $i \in I$ to the hub node $k \in H$ via the $a \in O$ transportation mode and further to the central hub node $l \in C$ via the $b \in O$ transportation mode;
y_{imj}^{ab}	The amount of flow from the origin node $i \in I$ to the nodes $m \in H$ and $j \in I$ via the $a \in O$ transportation mode and further to the destination node $j \in I$. This is while the $b \in O$ transportation mode is utilized to establish a link between hub $m \in H$ and central hub;
T_l	Maximum time within which the demand flow of each origin reaches the central hub node $l \in C$.

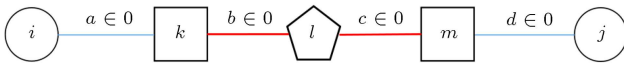


Figure 2. Reliability of two non-hub origin and destination nodes, two hub nodes, and central hub along the path (adopted from Kim and O’Kelly, 2009 [13]).

3.2.1. Mathematical model of RHMHL

In this subsection, different components of the RHMHL model are presented. Given that the flow passing through each path and the path reliability were considered in this model, it was necessary to identify possible paths across the network and determine a method for calculating the path reliability. Figure 2 shows the longest possible path along which two non-hub nodes $i \in I$ (origin) and $j \in I$ (destination) encounter the hub nodes $k \in H$ and $m \in H$ and the central hub node $l \in C$. Along this multimodal path, transportation modes between each pair of nodes are displayed on the ridges. Calculated reliability of the path is further reported in Figure 2, where the effect of the coefficient of hubbing, γ , on reliability is applied to the air path. Other paths, which are the subsets of the path shown in Figure 2, are detailed in Table 1. According to the table, four general types of paths are introduced.

The present model is nested [9,17,24]. Therefore, the central hub provides services to all second-level hubs. In addition, the value of reliability for all $i \in I$ and $a \in O$ is $r_{ii}^a = 1$. In other words, a node can certainly reach itself. Table 1 presents different methods used to calculate the path reliability for different pairs of origin and destination.

The objective function of the mathematical model of this study is to maximize the amount of flow passing through each path in proportion to the path reliability, as expressed in Eq. (1):

$$\begin{aligned} \max \sum_{a \in O} \sum_{b \in O} & \left(\sum_{i \in I} \sum_{k \in H} \sum_{l \in C} r_{ik}^a x_{ikl}^{ab} \sum_{j \in I} f_{ij} \right. \\ & + \sum_{i \in I} \sum_{k \in H} \sum_{l \in C} r_{ik}^a (r_{kl}^b)^{1-\gamma^b} z_{ikl}^{ab} \\ & + \sum_{d \in O} \sum_{j \in I} \left(\sum_{i \in I} \sum_{k \in H \setminus \{j\}} \sum_{l \in C} r_{ik}^a r_{kl}^d x_{ikl}^{ab} y_{ikj}^{db} \right. \\ & + \sum_{c \in O} \sum_{i \in I} \sum_{k \in H} \sum_{m \in H \setminus \{k\}} \sum_{l \in C} r_{ik}^a (r_{kl}^b)^{1-\gamma^b} \\ & \left. \left. (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{ikl}^{ab} y_{imj}^{dc} \right) \right). \end{aligned} \quad (1)$$

The objective function is formulated in four parts, namely parts (1), (2), (3), and (4); therefore, the

problem is now broken into different parts to explain each part separately:

$$\sum_{a \in O} \sum_{b \in O} \sum_{i \in I} \sum_{k \in H} \sum_{l \in C} r_{ik}^a x_{ikl}^{ab} \sum_{j \in I} f_{ij}, \quad (1-1)$$

$$\sum_{a \in O} \sum_{b \in O} \sum_{i \in I} \sum_{k \in H} \sum_{l \in C} r_{ik}^a (r_{kl}^b)^{1-\gamma^b} z_{ikl}^{ab}, \quad (1-2)$$

$$\sum_{a \in O} \sum_{b \in O} \sum_{d \in O} \sum_{i \in I} \sum_{k \in H \setminus \{j\}} \sum_{l \in C} \sum_{j \in I} r_{ik}^a r_{kl}^d x_{ikl}^{ab} y_{ikj}^{db}, \quad (1-3)$$

$$\sum_{a \in O} \sum_{b \in O} \sum_{d \in O} \sum_{c \in O} \sum_{i \in I} \sum_{k \in H} \sum_{m \in H \setminus \{k\}} \sum_{l \in C} \sum_{j \in I} r_{ik}^a (r_{kl}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{ikl}^{ab} y_{imj}^{dc}. \quad (1-4)$$

According to Figure 2 and the second column of Table 1, there are four types of paths across the considered network. Accordingly, the objective function is explained in the form of four relationships, namely Eqs. (1-1), (1-2), (1-3), and (1-4). Eq. (1-1) calculates the amount of path reliability-proportional flow passing toward a destination hub. In this equation, all flows originate from the node i as the origin and pass through the hub node k . Eq. (1-2) calculates the amount of path reliability-proportional flow passing through an origin hub to the central hub as the destination. Eq. (1-3) calculates the amount of path reliability-proportional flow passing toward a non-hub destination node. Finally, Eq. (1-4) calculates the amount of path reliability-proportional flow passing through a path with two hubs and a central hub, where the flow passes through the central hub to reach a destination.

Constraints of the mathematical model RHMHL are categorized and described as follows:

$$\sum_{a \in O} \sum_{b \in O} \sum_{k \in H} \sum_{l \in C} x_{ikl}^{ab} = 1 \quad \forall i \in I, \quad (2)$$

$$\begin{aligned} x_{ikl}^{ab} & \leq x_{kkl}^{bb} & \forall i \in I, \quad k \in H, \quad l \in C, \\ a, b & \in O, \quad k \neq i, \end{aligned} \quad (3)$$

$$\sum_{m \in H} x_{kml}^{bb} \leq x_{lll}^{bb} \quad \forall b \in O, \quad k \in H, \quad l \in C, \quad l \neq k, \quad (4)$$

$$\begin{aligned} x_{ikl}^{ab} & = 0 & \forall a, b \in O, \quad k \in H, \quad l \in C, \\ & & l \neq k, \end{aligned} \quad (5)$$

$$\begin{aligned} x_{ikl}^{ab} & \in \{0, 1\} \\ \forall a, b & \in O, \quad i \in I, \quad k \in H, \quad l \in C. \end{aligned} \quad (6)$$

Given that a single-allocation model is discussed herein, all non-hub nodes and hub nodes should be

allocated to exactly one hub facility and one central hub facility, respectively. Constraints (2), (4), and (6) ensure the single-allocation characteristic of the model. Constraint (4) confirms that each hub cannot be linked to other nodes unless the node is the central hub node. Constraint (3) states that a demand node can be assigned to another node only if that node is a hub node. Constraint (5) is basically unnecessary; however, it makes the LP relaxation stronger.

$$\sum_{j \in H} \sum_{l \in C} \sum_{b \in O} x_{jjl}^{bb} = p_H, \quad (7)$$

$$\sum_{l \in C} \sum_{b \in O} x_{lll}^{bb} = 1. \quad (8)$$

Since the model was based on p-hub median problem, the number of hubs and central hubs was represented with Constraints (7) and (8), respectively.

$$z_{ikl}^{ab} \geq \sum_{j \in I: j \neq k} (f_{ij} + f_{ji})(x_{ikl}^{ab} - x_{jkl}^{db})$$

$$\forall a, b, d \in O, \quad i \in I, \quad k \in H, \quad l \in C, \quad l \neq k, \quad (9)$$

$$z_{ikl}^{ab} \leq x_{ikl}^{ab} \sum_{j \in I} (f_{ij} + f_{ji})$$

$$\forall i \in I, \quad k \in H, \quad l \in C, \quad a \in O, \quad b \in O, \quad (10)$$

$$\sum_{c \in O} \sum_{d \in O} \sum_{i \in I} y_{ikj}^{dc} \leq \sum_{a \in O} \sum_{b \in O} x_{jkl}^{ab} \sum_{i \in I} f_{ij}$$

$$\forall j \in I, \quad k \in H, \quad l \in C, \quad (11)$$

$$\sum_{k \in H} \sum_{a \in O} \sum_{b \in O} y_{ikj}^{ba} = f_{ij} \quad \forall i \in I, \quad j \in I, \quad (12)$$

$$z_{ikl}^{ab} \geq 0 \quad \forall i \in I, \quad k \in H, \quad l \in C, \quad a, b \in O, \quad (13)$$

$$y_{ikj}^{ab} \geq 0 \quad \forall i \in I, \quad j \in I, \quad k \in H, \quad a, b \in O. \quad (14)$$

Constraint (9) calculates the outflow from the origin node i and that passing through the link between hub k and central hub l . This constraint is used as a balancing constraint in studies of Yaman [17], Alumur et al. [10], and Korani and Sahraeian [9]. Hence, for each link from a hub to the central hub, the amount of demand could be computed by Constraints (9) and (10).

Constraint (10) ensures that the rate of the flow originating from the origin node i and passing through the path-connecting hub node k to the central hub l does not exceed the total flow originating from the origin node i . It is noteworthy that a similar constraint was introduced by Correia et al. [51] called cutting capacity constraint. Inspired by Ernst and Kirshnamoorthy [52] and Karimi and Setak [53], Constraints (11) and (12) are designed to ensure that the

destination node j receives the flow originating from the origin node i . Constraints (13) and (14) reinforce the LP model.

$$T_l \geq \sum_{a \in O} \sum_{b \in O} \sum_{k \in H} (t_{ik}^a + \alpha^b t_{kl}^b) x_{ikl}^{ab}$$

$$\forall i \in I, \quad l \in C, \quad (15)$$

$$T_l + \sum_{c \in O} \sum_{d \in O} \sum_{r \in H} (\alpha^c t_{lr}^c + t_{rj}^d) x_{jrl}^{dc} \leq \beta$$

$$\forall j \in I, \quad l \in C, \quad (16)$$

$$T_l \geq 0 \quad \forall l \in C, \quad (17)$$

Constraints (15) to (17) are time-bound constraints. These constraints ensure that the travel time between each pair of origin and destination nodes may not exceed the predetermined value of β . Constraint (15) records the travel time from each origin node to the central hub of the node, and Constraint (16) overshadows the total travel time from the origin node to the destination node under the upper bound β .

The time-bound constraints were explained using the ideas presented in Ebery [54], Ernst et al. [55], Yaman [17], and Korani and Sahraeian [9].

3.3. Linearization of the objective function

As stated before, the third and fourth parts of the formulated objective function are nonlinear, as indicated by variables x_{ikl}^{ab} and y_{imj}^{dc} . As such, a method was presented to linearize this formulation. To this end, $x_{ikl}^{ab} \cdot y_{imj}^{dc}$ was replaced by w_{ikmj}^{abcd} in the third and fourth parts of the objective function, respectively, and Constraints (18), (19), (20), and (21) were added to the model. The outcome is a linear model called Linear Reliable Hierarchical Multimodal Hub Location Problem (L-RHMHLP) with the following definition:

$$\sum_{c \in O} \sum_{d \in O} w_{ikmj}^{abcd} \leq \sum_{l \in C} x_{ikl}^{ab} \sum_{q \in I} f_{iq}$$

$$\forall a, b \in O, \quad i \in I, \quad k \in H, \quad m \in H, \quad j \in I, \quad (18)$$

$$\sum_{a \in O} \sum_{b \in O} w_{ikmj}^{abcd} \leq y_{imj}^{dc}$$

$$\forall c, d \in O, \quad i \in I, \quad k \in H, \quad m \in H, \quad j \in I, \quad (19)$$

$$w_{ikmj}^{abcd} \geq y_{imj}^{dc} - (1 - x_{ikl}^{ab}) \sum_{q \in I} f_{iq} \quad \forall a, b, c, d \in O,$$

$$i \in I, \quad k \in H, \quad l \in C, \quad m \in H, \quad j \in I, \quad (20)$$

$$w_{ikmj}^{abcd} \geq 0 \quad \forall i \in I, \quad k \in H, \quad m \in H, \quad j \in I,$$

$$a, b, c, d \in O. \quad (21)$$

By attempting to validate the linearization, the upper bound y_{imj}^{dc} in Constraint (11), which is equal to all flows originated from origin node i , was employed, and the upper limit of the variable replaced by multiplication of y_{imj}^{dc} by x_{ikl}^{ab} was considered to be w_{ikmj}^{abcd} .

Theorem 1. *Any feasible solution of RMHL is a feasible solution of L-RMHP.*

Proof: Let \bar{X} be a feasible solution of RMHL and x_{ikl}^{ab} and y_{imj}^{dc} be the fixed parameters; accordingly, the theorem is proved if one can show that w_{ikmj}^{abcd} is equal to $x_{ikl}^{ab} \cdot y_{imj}^{dc}$. Accordingly, one may consider four cases when it comes to linearization based on x_{ikl}^{ab} and y_{imj}^{dc} values.

Case 1. If $x_{ikl}^{ab} = 0$ and $y_{imj}^{dc} = 0$, then Constraints (18) and (20) imply that $w_{ikmj}^{abcd} \leq 0$; on the other hand, $x_{ikl}^{ab} \cdot y_{imj}^{dc} = 0$, and Constraints (21) implies that $w_{ikmj}^{abcd} \geq 0$. Therefore, both w_{ikmj}^{abcd} and $x_{ikl}^{ab} \cdot y_{imj}^{dc}$ are equal to zero;

Case 2. If $x_{ikl}^{ab} = 0$ and $y_{imj}^{dc} > 0$, then Constraint (18) implies that $w_{ikmj}^{abcd} \leq 0$ and Constraints (21) show that $w_{ikmj}^{abcd} \geq 0$; thus, w_{ikmj}^{abcd} is equal to zero; on the other hand, $x_{ikl}^{ab} \cdot y_{imj}^{dc} = 0$. Thus, both of w_{ikmj}^{abcd} and $x_{ikl}^{ab} \cdot y_{imj}^{dc}$ will be equal to zero;

Case 3. If $x_{ikl}^{ab} = 1$, $y_{imj}^{dc} > 0$, and $y_{imj}^{dc} \leq \sum_{q \in I} f_{iq}$, then Constraints (18) and (19) imply that $w_{ikmj}^{abcd} \leq \sum_{q \in I} f_{iq}$ and $w_{ikmj}^{abcd} \leq y_{imj}^{dc}$, respectively. Therefore, only $w_{ikmj}^{abcd} \leq y_{imj}^{dc}$ will be applied. It should be noted that Constraints (18) indicate that the term $\sum_{c \in O} \sum_{d \in O} w_{ikmj}^{abcd}$ takes the same value for all c, d, i, k, m , and j values. According to Constraint (20), $w_{ikmj}^{abcd} \geq y_{imj}^{dc}$; thus, $w_{ikmj}^{abcd} = y_{imj}^{dc}$. On the other hand, $x_{ikl}^{ab} \cdot y_{imj}^{dc} = y_{imj}^{dc}$. Therefore, both w_{ikmj}^{abcd} and $x_{ikl}^{ab} \cdot y_{imj}^{dc}$ are equal to y_{imj}^{dc} ;

Case 4. If $x_{ikl}^{ab} = 1$ and $y_{imj}^{dc} = 0$, then Constraints (18) and (20) imply that $w_{ikmj}^{abcd} \leq \sum_{q \in I} f_{iq}$ and $w_{ikmj}^{abcd} \leq 0$, respectively. Therefore, only $w_{ikmj}^{abcd} \leq 0$ will be considered. On the other hand, $x_{ikl}^{ab} \cdot y_{imj}^{dc} = 0$ and Constraint (21) gives $w_{ikmj}^{abcd} \geq 0$. Therefore, both of w_{ikmj}^{abcd} and $x_{ikl}^{ab} \cdot y_{imj}^{dc}$ are equal to zero.

4. Lagrangian relaxation method

Lagrangian relaxation represents a mathematical method for relaxing constraints of a model before proceeding to solve the model so as to improve its bound as much as possible. Fisher [56] thoroughly examined the application of this method to the hub location problem. Other studies on the application of this method to such

problems include those reported by Aykin [57], Lee et al. [58], Marín [59], Contreras et al. [60], Ishfaq and Sox [47], Mohammadi et al. [61], Karimi and Setak [53], He et al. [62], and Neamatian Monemi et al. [63]. Given the complexity of the RHMHL model, the Lagrangian relaxation has been used to identify the best possible bound for the problem.

A review of related literature shows that, in most of such works, the constraints with the greatest effect on processing time have been identified to define an alternative relaxed problem. This could effectively speed up the solution pace (see more in Contreras et al. [60], Karimi and Setak [53], and An et al. [11]). By investigating the model of Constraints (18) and (20), a relationship can be established between the binary variable x_{ikl} and the non-negative variable w_{ikmj}^{abcd} that significantly affected the processing time. Therefore, two Lagrange multipliers (λ_{iklmj}^{ab} and μ_{iklmj}^{abcd}) are defined for Constraints (18) and (20), respectively ($\lambda_{iklmj}^{ab} \geq 0$ and $\mu_{iklmj}^{abcd} \geq 0$). The relaxed model (herein referred to as LR) is defined by Eq. (22) as shown in Box I. It should be noted that $LR(\bar{\lambda}, \bar{\mu})$ designates the objective function for the Lagrangian approach to an upper bound for RHMHL. Therefore, an attempt has been made to find the greatest value of Eq. (22) by identifying the Lagrange multipliers. According to the relevant literature, the subgradient optimization method can provide a solution for this problem [53,62,63]. This method operates by iterating a series of steps until the solution is converged. The subgradient algorithm can be summarized in 6 steps. In this algorithm, $\tilde{\psi}$, LB, and UB are possible solution, lower bound, and upper bound, respectively. $\tilde{\psi}$ is a random solution obtained by solving the relaxed model. The iteration index is defined as $iter$. The Lagrangian relaxation outcome of each iteration is denoted by lr_{iter} . In addition, π_{iter} and θ_{iter} are the subgradients of Constraints (18) and (20) in the $iter$ th iteration, respectively. Step size in each iteration is designated as s_{iter} .

Herein, ρ_{iter} is a constant value used as the dimming coefficient in each iteration.

Step 0. Once finished with correcting the research variables, the relaxed RHMHL is solved, with dual variables corresponding to each constraint calculated at the same time;

Step 1. In this step, the initial values of the algorithm parameters are determined. The maximum number of iterations is set to 100 ($\max.iter = 100$), and indices π_{iter} and θ_{iter} are the values of the dual variables corresponding to Constraints (18) and (20) of the relaxed RHMHL, respectively (because these two indices are dual variables of the initial problem and are considered as reasonable solutions, which are very effective in speeding up the algorithm). The

$$\begin{aligned}
LR(\bar{\lambda}, \bar{\mu}) : \quad & \max + \sum_{i \in I} \sum_{k \in H} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \left(r_{ik}^a x_{ikl}^{ab} \sum_{j \in I} f_{ij} + r_{ik}^a (r_{kl}^b)^{1-\gamma^b} z_{ikl}^{ab} \right) \\
& + \sum_{i \in I} \sum_{k \in H \setminus \{j\}} \sum_{j \in I} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \sum_{d \in O} r_{ik}^a r_{kj}^d x_{ikl}^{ab} y_{ikj}^{db} \\
& + \sum_{i \in I} \sum_{k \in H} \sum_{m \in H \setminus \{k\}} \sum_{j \in I} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \sum_{c \in O} \sum_{d \in O} r_{ik}^a (r_{kl}^b)^{1-\gamma^b} (r_{lm}^c)^{1-\gamma^c} r_{mj}^d x_{ikl}^{ab} y_{imj}^{dc} \\
& + \sum_{i \in I} \sum_{k \in H} \sum_{m \in H} \sum_{j \in I} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \sum_{c \in O} \sum_{d \in O} \mu_{iklmj}^{abcd} w_{ikmj}^{abcd} \\
& + \sum_{i \in I} \sum_{k \in H} \sum_{m \in H} \sum_{j \in I} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \lambda_{iklmj}^{ab} x_{ikl}^{ab} \sum_{q \in I} f_{iq} \\
& + \sum_{i \in I} \sum_{k \in H} \sum_{m \in H} \sum_{j \in I} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \sum_{c \in O} \sum_{d \in O} \mu_{iklmj}^{abcd} (1 - x_{ikl}^{ab}) \sum_{q \in I} f_{iq} \\
& - \sum_{i \in I} \sum_{k \in H} \sum_{m \in H} \sum_{j \in I} \sum_{l \in C} \sum_{a \in O} \sum_{b \in O} \lambda_{iklmj}^{ab} \sum_{c \in O} \sum_{d \in O} w_{ikmj}^{abcd} \\
& - \sum_{i \in I} \sum_{k \in H} \sum_{m \in H} \sum_{j \in I} \sum_{a \in O} \sum_{b \in O} \sum_{c \in O} \sum_{d \in O} \sum_{l \in C} \mu_{iklmj}^{abcd} y_{imj}^{dc}
\end{aligned}$$

S.T. (2)-(17), (19) and (21). (22)

Box I

$$s_{iter} = \frac{\rho_{iter}(lr_{iter} - LB)}{\sum_{i \in I} \sum_{k \in H} \sum_{l \in C} \sum_{m \in H} \sum_{j \in I} \sum_{a \in O} \sum_{b \in O} \left(\left(\pi_{iter}^{iklmjab} \right)^2 + \sum_{c \in O} \sum_{d \in O} \left(\theta_{iter}^{iklmjabcd} \right)^2 \right)}. \quad (25)$$

Box II

lower bound is denoted by $\tilde{\psi}$ and the upper bound is considered to be infinite with regard to the positive maximization approach;

Step 2. An iterative scheme is adopted in this step. If the value of $LR(\bar{\lambda}, \bar{\mu})$ is better than the upper bound, then solving the problem as LR will be considered as the upper bound. Otherwise, a predetermined discount factor ρ_{iter} is applied (0.8 in this study);

Step 3. Subgradients of Constraints (18) and (20) are calculated using Eqs. (23) and (24), respectively:

$$\pi_{iter}^{iklmjab} = \sum_{c \in O} \sum_{d \in O} w_{ikmj}^{abcd} - x_{ikl}^{ab} \sum_{q \in I} f_{iq}, \quad (23)$$

$$\theta_{iter}^{iklmjabcd} = y_{imj}^{dc} - (1 - x_{ikl}^{ab}) \sum_{q \in I} f_{iq} - w_{ikmj}^{abcd}. \quad (24)$$

Step 4. The value of s_{iter} is obtained via Eq. (25) as shown in Box II.

Step 5. The values of two Lagrange multipliers (λ_{iklmj}^{ab} and μ_{iklmj}^{abcd}) are updated using the step size s_{iter} and Eqs. (26) and (27), followed by increasing the $iter$ by 1 ($iter = iter + 1$):

$$\lambda_{iklmj-iter+1}^{ab} = \max \left\{ 0, \lambda_{iklmj-iter}^{ab} + s_{iter} \pi_{iter}^{iklmjab} \right\}, \quad (26)$$

$$\mu_{iklmj-iter+1}^{abcd} = \max \left\{ 0, \mu_{iklmj-iter}^{abcd} + s_{iter} \theta_{iter}^{iklmjabcd} \right\}. \quad (27)$$

Step 6. Three stopping criteria of the algorithm are investigated:

- (1) The maximum number of iterations reaches ($iter \leq 100 \max.iter$);
- (2) $s_{iter} \leq \varepsilon$ (where ε is a very small value);
- (3) $|UB - LB| \leq \varepsilon$ (where ε is a very small value).

Upon meeting at least one of the criteria, the algorithm is terminated; otherwise, it operates iteratively by going back to Step 2.

In the next section, open-source well-known datasets published in the literature are used to evaluate the model and investigate the performance of the Lagrangian algorithm. In addition, sensitivity analysis is conducted, too.

5. Numerical results

In this section, the performances of the proposed model and solution method were evaluated with respect to two well-known datasets presented in the literature on the hub location problem. For this purpose, firstly, a brief overview of the data used and analysis tools is presented. Then, preprocessing techniques are employed for reducing unnecessary dimensions of the model. Finally, the trend of changes in the outputs is discussed by performing a sensitivity analysis. In this numerical study, the same multimodal paths as those investigated in the work by Alumur et al. [10] are used, considering ground communication between non-hub and hub nodes and air communication links between hub and central hub nodes.

5.1. Datasets for model evaluation

For validating the model, the *Civil Aeronautics Board (CAB) dataset* and *Iranian Aviation Dataset (IAD)* were used. The CAB dataset contained transportation information of 25 cities in the USA; it was used by O'Kelly [64] for hub location. IAD is a dataset that contains air transportation information on 37 important cities of Iran [65]. The CAB and IAD datasets include all parameters required in the presented model except for the path reliabilities, which were calculated herein using the calculation method applied to CAB data in the literature [11,13] while taking into account the direct relationship between the chance of success in each path and the distance traveled from the origin to the destination. Accordingly, the reliability associated with each path connecting a particular pair of origin and destination nodes was equal to the difference between the ratio of the distance between them to the length of the longest distance between all origin-destination pairs and 1, i.e., a value between zero and one. Given that the odds of air industry are larger than those of ground transportation, reliability values of the ground transportation are adjusted by an odds ratio of

50%. Respecting the high speed of air transportation mode, travel times of this mode were reduced by 50%. According to IAD, Tehran, as the capital city of Iran, has the largest international airport in Iran. Therefore, it has been considered as the central hub of the air transportation mode [9,20]. However, this case was different for CAB dataset since making-decision for 25 important cities in the USA was difficult. Therefore, Group 17 with the greatest number of communications with other groups, i.e., 1447732, was selected as the central hub.

Each of the studied datasets was divided into three subsets: small, medium, and large airports. The number of second-level hub facilities was considered to be one-fourth of the number of nodes under study. The second option for each dataset was to take the number of hubs twice as large as the number of nodes under study so as to evaluate changes in the output upon changing the number of hubs. β was obtained from the feasible states of the RHMHP problem in the restricted mode, as discussed in detail by Yaman [17], Alumur et al. [10], and Korani Sahraeian [9]. Accordingly, the value of the parameter for CAB and IAD data was found to be 2640 and 2880, respectively.

Moreover, given the dimension of the problem, the index ρ_{iter} was initialized with a constant value. By using the IAD, the value of the index for 10-, 20-, 30-, and 3-node systems was determined to be 9, 9, 90, and 90, respectively. Of course, for the 37-node system and 18-hub sets, the value of the index increased to 180 to investigate its effect on the solution growth.

5.2. Preprocessing

The assumption of fixed values for some of the variables increased the rate of convergence to proper solution significantly [53]. However, some of the variables were yet to be evaluated. Since the variable y_{ikj}^{cd} gives the flow from origin node i to destination node j and given that no flow will return to the node when the origin and destination nodes are the same, $y_{ikj}^{ab} = 0$ for all $a \in O$, $b \in O$, $i \in I$, and $k \in H$. On the other hand, the equation $w_{ikmi}^{abcd} = 0$ holds true for all $a \in O$, $b \in O$, $c \in O$, $d \in O$, $i \in I$, $k \in H$, and $m \in H$ according to Constraints (23) and (24). The variable z_{ikl}^{ab} estimates the flow originating from node $i \in I$ and passing through the link between hub $k \in H$ and the central hub $l \in C$; therefore, if the origin node i and central hub node l are the same, there will be no flow from the hub to the central hub given the uniqueness of the central hub, i.e., $z_{ikl}^{ab} = 0$ for all $a \in O$, $b \in O$, $l \in C$, and $k \in H$.

5.3. Performance of the Lagrangian method

In this section, the results obtained upon evaluating the performance of the Lagrangian approach via sensitivity analysis are presented. For this purpose,

Table 2. Computational results for IAD data with $\beta = 2880$.

N	P_H	α	$1 - \gamma$	Obj	$\%Imp$	Cpu time	Hub
10	3	0.2	0.2	16.25	28.00	76.59	1,9,31
			0.8	13.97	29.00	80.33	3,5,31
		0.8	0.2	16.10	28.00	82.61	2,5,31
			0.8	13.85	29.30	80.15	2,5,31
	6	0.2	0.2	16.97	26.40	76.77	2,4,5,6,7,31
			0.8	14.41	25.80	75.92	2,4,5,6,7,31
		0.8	0.2	16.83	22.30	77.86	1,3,5,7,9,31
			0.8	14.33	26.40	47.67	2,4,6,7,9,31
20	5	0.2	0.2	113.77	22.80	992.93	4,7,9,10,31
			0.8	90.47	15.50	878.75	3,7,13,15,31
		0.8	0.2	113.28	19.10	924.36	1,7,14,18,31
			0.8	90.72	15.90	905.64	4,7,9,10,31
	10	0.2	0.2	108.48	18.80	894.08	4,5,6,9,11,14,15,17,18,31
			0.8	86.56	19.00	877.76	1,3,5,6,9,13,14,17,18,31
		0.8	0.2	107.74	20.90	934.16	1,3,4,6,7,11,13,14,15,31
			0.8	86.50	19.60	915.60	3,5,6,7,9,13,14,15,19,31
30	7	0.2	0.2	308.96	15.30	5369.29	9,23,24,25,27,29,31
			0.8	251.80	18.80	4554.38	7,9,11,14,21,24,31
		0.8	0.2	306.57	15.90	4031.19	5,6,9,12,14,27,31
			0.8	249.15	19.70	3699.43	9,14,16,19,20,28,31
	14	0.2	0.2	295.25	18.40	5355.85	3,6,8,9,12,14,15,17,18,21,24,28,29,31
			0.8	237.44	21.80	5845.37	4,6,8,9,11,13,14,17,18,20,21,24,26,31
		0.8	0.2	294.50	18.60	5425.37	1,4,5,6,8,9,14,15,16,17,19,21,27,31
			0.8	238.37	21.50	5326.30	5,6,8,9,14,16,17,18,21,23,26,28,29,31
37	9	0.2	0.2	518.06	12.01	19527.87	2,3,9,24,27,31,33,34,37
			0.8	415.57	16.00	17651.64	1,9,13,17,22,26,31,33,34
		0.8	0.2	205.76	11.84	17561.84	8,9,14,17,18,24,31,35,378,9,14,17,18,24,31,35,37
			0.8	413.99	16.30	13816.28	3,9,11,24,26,29,31,33,37
	18	0.2	0.2	498.80	20.48	21220.68	2,5,6,7,8,9,13,14,20,21,25,26,31,33,34,35,36,37
			0.8	383.84	20.48	23277.40	2,5,6,7,8,9,13,14,20,21,25,26,31,33,34,35,36,37
		0.8	0.2	484.75	20.48	21729.58	2,5,6,7,8,9,13,14,20,21,25,26,31,33,34,35,36,37
			0.8	383.98	20.48	20924.31	2,5,6,7,8,9,13,14,20,21,25,26,31,33,34,35,36,37

four indicators were considered: the objective function value (Obj), response improvement rate ($\% Imp$), processing time ($Time$, in seconds), and hub facility location (Hub). The $\% Imp$ was obtained as the ratio of the difference between the absolute relaxed solution

and the Lagrangian lower bound to the absolute relaxed solution.

Tables 2 and 3 report the results of applying the Lagrangian method to the IAD and CAB data, respectively. These tables show that, under a given

Table 3. Computational results for CAB data with $\beta = 2640$.

N	P_H	α	$1 - \gamma$	Obj	$\%Imp$	Cpu time	Hub
5	1	0.2	0.2	1234845.52	36.40	26.74	17
			0.8	1170003.49	35.30	27.63	17
		0.8	0.2	1234845.52	36.40	25.78	17
			0.8	1170003.49	35.30	28.52	17
	3	0.2	0.2	1470844.17	23.60	29.99	3,4,17
			0.8	1375857.39	22.90	31.51	1,3,17
		0.8	0.2	1470844.17	23.60	30.97	3,4,17
			0.8	1375857.39	22.90	29.93	1,3,17
15	4	0.2	0.2	8581277.21	19.40	303.49	3,9,12,17
			0.8	6927494.39	21.80	298.22	9,12,14,17
		0.8	0.2	8344651.53	21.60	293.98	6,8,12,17
			0.8	6944090.14	20.90	290.83	2,8,12,17
	8	0.2	0.2	8302771.44	21.70	316.90	6,7,8,10,11,12,13,17
			0.8	6683215.42	23.20	296.31	2,3,5,6,8,13,14,17
		0.8	0.2	8184242.79	22.80	305.89	1,3,4,8,12,13,14,17
			0.8	6649183.94	23.10	283.40	1,2,4,11,12,13,14,17
	6	0.2	0.2	25413461.93	11.90	3835.42	13,16,17,19,20,23
			0.8	20202865.44	14.30	2876.64	2,3,8,17,19,24
		0.8	0.2	25109500.30	12.80	2020.93	5,12,17,19,22,23
			0.8	20236900.45	13.60	3476.64	9,11,17,19,22,23,
25	12	0.2	0.2	24983278.44	12.90	3132.29	7,8,10,11,13,15,16,17,18,19,21,23
			0.8	19702691.09	14.70	2601.04	1,2,3,5,9,10,13,14,17,19,22,23
		0.8	0.2	24774015.37	13.50	1940.60	7,8,10,11,13,15,17,18,19,21,22,23
			0.8	19478500.52	15.20	2104.40	2,6,8,11,12,13,16,17,19,22,23,24

set of conditions, an increase in the number of second-level hubs leads to a reduction in improvement rates and a longer processing time. Similarly, an increase in γ was found to be inversely related to the rate of improvement; however, the effect of α on this parameter was negligible. In all of the tables presented in this subsection, the reported processing times and improvement rates are measured in units of seconds and percent changes, respectively. The reliability of the output network increased with decreasing γ and α adjustment coefficients.

All in all, nodes 9, 5, and 6 were selected as hubs for IAD in the majority of the cases, and for the number of nodes, 14 nodes were selected as hubs in most cases. No significant conclusion could be drawn for the CAB data; however, node 3 followed by node 4 was found to play a strong role under different scenarios. In 25-node and 15-node sets of CAB data, nodes 19 and 12 were present in all cases, respectively. However, those were not selected only when γ and α were 0.8 and 0.2, respectively.

In order to better understand the effect of β on the processing time, hub location, and response improvement rate, Tables 4 and 5 are completed for CAB and IAD datasets, respectively. These tables are obtained for the specific $1 - \gamma$ and α values of 0.8 and 0.8, respectively, considering a 15-node set of each dataset. Four values of β were considered for each dataset, namely $\{2520, 2640, 2760, \infty\}$ and $\{2760, 2880, 3000, \infty\}$ for CAB and IAD, respectively. In Tables 4 and 5, ρ_{iter} is assumed to be 90 and the cases with no solution are not justified.

Table 4 and Figure 3 can develop a better understanding of the trend of changes in the results of CAB data. In the figure, the values presented in Table 4 are normalized (the number is divided by the maximum value along the column). This puts the three parameters of time, improvement rate, and reliability together in one figure.

An overview of Figure 4, which refers to the 15-node IAD dataset with 10 hub facilities, suggests that as long as the delivery time limit exists as a

Table 4. Model outputs on CAB dataset for different values of β with a 15-node set where $1 - \gamma = 0.8$ and $\alpha = 0.8$.

P_H	CAB data				
	β	<i>Obj</i>	% <i>Imp</i>	Cpu time	Hub
1	2520	—	—	—	—
	2640	—	—	—	—
	2760	—	—	—	—
	∞	4604627.62	48.51	221.90	17
5	2520	6726185.98	23.09	276.75	6,8,10,12,17
	2640	6762297.28	22.76	281.23	3,12,13,14,17
	2760	6811941.36	22.30	298.37	3,5,12,13,17
	∞	7051404.85	20.36	305.47	3,6,11,14,17
10	2520	6425424.80	24.95	272.44	1,2,3,5,8,10,11,12,13,17
	2640	6415723.26	24.91	282.04	2,3,5,8,9,10,12,13,14,17
	2760	6516025.71	23.86	294.36	2,3,5,8,10,11,12,13,14,17
	∞	6627741.76	22.92	295.09	1,2,3,5,8,10,12,13,14,15
15	2520	4894410.33	39.42	223.62	1–14,17
	2640	4894410.33	39.42	225.05	1–14,17
	2760	4894410.33	39.42	224.30	1–14,17
	∞	4894410.33	39.42	224.31	1–14,17

Table 5. Model outputs when applied to IAD for different values of β with a 15-node set where $1 - \gamma = 0.8$ and $\alpha = 0.8$.

P_H	IAD data				
	β	<i>Obj</i>	% <i>Imp</i>	Cpu time	Hub
1	2760	—	—	—	—
	2880	—	—	—	—
	3000	—	—	—	—
	∞	26.23	49.46	228.86	17
5	2760	36.05	40.02	302.02	10,11,12,14,31
	2880	36.65	39.45	249.01	3,4,7,14,31
	3000	36.56	39.59	292.08	3,7,9,10,31
	∞	37.85	39.86	330.36	3,6,11,14,31
10	2760	35.77	28.91	301.78	1,3,4,5,7,9,10,13,14,31
	2880	36.19	28.60	289.73	3,4,5,6,7,9,10,11,14,31
	3000	36.10	28.78	290.70	1,4,5,6,7,8,11,12,14,31
	∞	37.45	26.20	310.26	1,3,4,5,6,11,12,13,14,31
15	2760	48.94	39.42	224.30	11–14,17
	2880	29.31	38.86	233.02	11–14,17
	3000	29.31	38.86	235.21	11–14,17
	∞	29.31	38.86	226.54	11–14,17

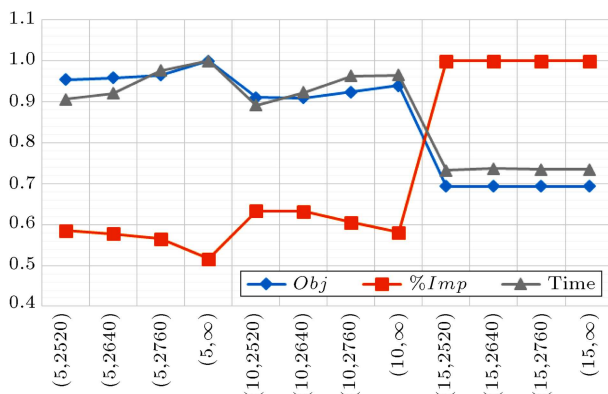


Figure 3. Trend of changes in time, improvement rate, and reliability for a 15-node set of CAB data.

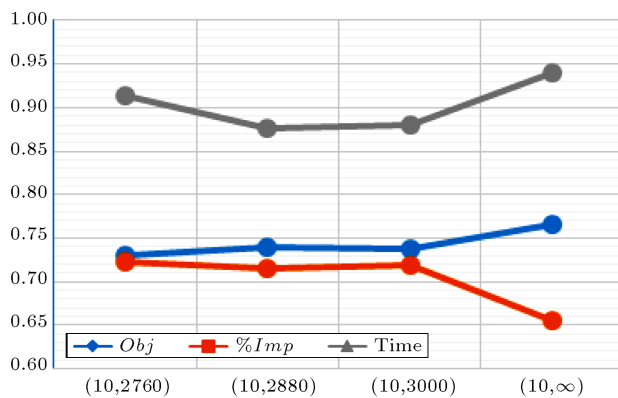


Figure 4. Variations of processing time, improvement rate, and reliability for a 15-node set of IAD.

constraint, the improvement rate and reliability exhibit proportional changes with respect to one another, and when the constraint is removed, the parameters break apart and diverge from one another at some point. In the two extreme β values (2760 and infinity), it took more time to solve the model than those of other β values. The cases with one hub and no solution indicated that the delivery time constraint had a significant effect on the output.

The considered problem had three different outputs, namely *obj*, *% Imp*, and *Time*, with different measurement units. Hence, in order to illustrate the trend of changes in these three parameters simultaneously, they are plotted in a diagram. However, respecting the difference between their ranges, the data were standardized before being plotted on a graph with the standardized values on its vertical axis. Therefore, the vertical axis in Figures 3 and 4 indicates normalization values of *obj*, *% Imp*, and *Time* as the problem outputs.

In order to investigate the relationship between P_H and *% Imp*, a 15-node set of IAD (Table 5) was used, and Figure 5 was obtained as a result. This figure shows that the trend of changes in P_H and *% Imp* follows a convex form.

Figure 5 shows that an increase in β reduces

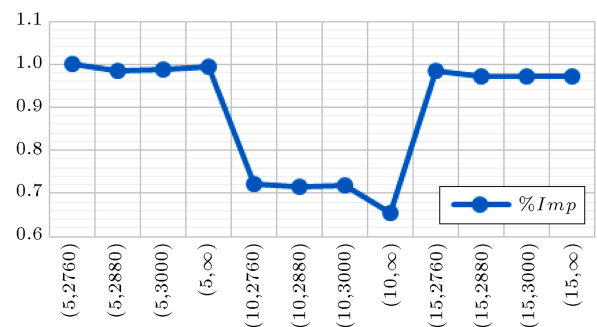


Figure 5. Variations of improvement rate for a 15-node set of IAD.

improvement rate, objective function value, and processing time. The significant point to note is that β has no effect on either of time, objective function, and improvement rate when all 15 nodes are selected as hubs.

In order to demonstrate the effect of ρ_{iter} on the variations of *Obj* and step size, Figure 6 was created by GAMS software for a 30-node set of IAD with $P_H = 7$, $1 - \gamma = 0.8$, and $\alpha = 0.8$. Two different cases with different values of ρ_{iter} were considered.

Figure 6 shows that an increase in the discount rate in large sets of nodes increases the accuracy of the calculations, contributes to the concave trend of changes in the discount rate, and changes the slope of step size from linear to stepwise, which helps achieve a better solution.

6. Conclusion and future research

In this paper, a new variant of the hub location problem in the field of transportation and telecommunication called Reliable Hierarchical Multimodal Hub Location Problem (RHMHL) was developed. The proposed model had outstanding hierarchical, multi-modal and path reliability features. With its new design, this model was unique in that the required number of variables was very small as available information in the literature was used to integrate a variety of flow constraints into the problem. Another novelty of this modeling was exact routing of demand from origin to destination so that the chain of locations passed by each demand unit could be carefully recognized. Due to high complexity and strategic nature of the problem, the Lagrangian method was developed to solve it in such a way that the results were more efficient than those of the relaxed linear model. For an accurate assessment of the problem, the most important parameters were subjected to detailed sensitivity analysis. In these assessments, trends of changes in time, solution quality, and improvement rate of the proposed algorithm were discussed, and the algorithm parameters were adjusted accordingly, which proved to be an outstanding outcome. All

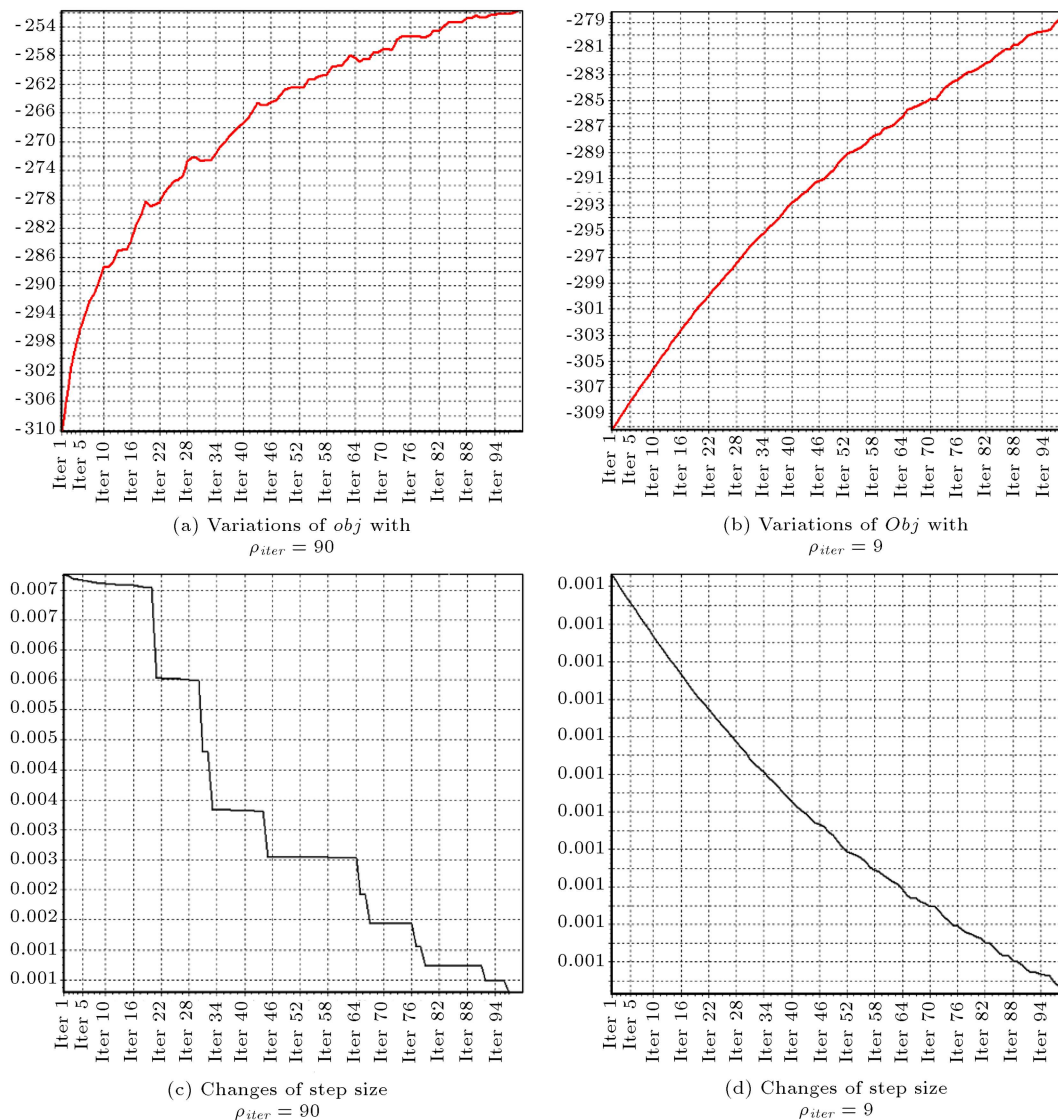


Figure 6. Variations of Obj and step size at different values of ρ_{iter} for a 30-node set of IAD with $P_H = 7$, $1 - \gamma = 0.8$, and $\alpha = 0.8$.

quantitative analyses were performed on well-known CAB and IAD datasets, thereby doubling the potential of applying the research outputs to real-world cases.

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Biographies

Ehsan Korani is a PhD student of Industrial Engineering in the faculty of Engineering at University

of Kurdistan, Iran. He received his MSC degree from Shahed University, Iran. His thesis subject focuses on the hub location problem. His main areas of research interests are location-allocation problems and urban transportation planning.

Alireza Eydi is an Associate Professor of Industrial Engineering in the Faculty of Engineering at University of Kurdistan, Iran. He received his PhD from the Department of Industrial Engineering at Tarbiat Modares University, Iran in 2009. His main areas of teaching and research interests include supply chain and transportation planning and network optimization problems including routing and location problems on networks.

Isa Nakhai Kamalabadi is a Professor of Industrial Engineering in the Faculty of Engineering at University of Kurdistan, Iran. He received his PhD from University of Toronto, Canada in 1995. His main areas of teaching and research interests include supply chain management, nonlinear optimization, and scheduling.