The Perils of Group Purchasing in a Competing Supply Chain

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Abstract

Group purchasing organizations (GPOs) are well-known intermediary firms that play an important role in some supply chains. An important question that arises regarding the GPOs, is whether a GPO that benefits from group buying discounts, always benefit the OEMs in the presence of market competition. In other words, does a GPO always lead to a win-win outcome for OEMs and the GPO? To answer this question, a bargaining framework has been used to investigate competing OEMs' procurement's strategies. The entrance of a GPO in a two tier supply chain that consists of two competing OEMs with a common supplier that has a quantity discount menu is analyzed. The result shows that low purchasing cost GPO may harm OEMs in a cost-benefit perspective. This unintuitive result can be explained by different impacts that a GPO has in purchasing process. Although, it can enlarge the size of trade surplus; but, it has an important influence on the size of the slice of the pie (profit sharing). Moreover, an OEM's procurement strategy in equilibrium not just only depends on his bargaining power; but also depends on his competitor OEM. Interestingly, a strong OEM may not prefer procuring through GPO, as well as a weak OEM does.

Keywords

Purchasing Strategy; Multiunit bilateral bargaining; market competition; quantity discount; wholesale price contract

Introduction

Because of economical evolutions in recent years, the emergence of complex international supply chains with worldwide manufacturers, mostly called original equipment manufacturers (OEMs) and also the strong role of intermediaries in supply chains because of their positive influence on supply chain efficiency have become very important. The notion of intermediary in economics literature refers to those economic agents that coordinate and arbitrate transactions between a group of supply chain firms' [1]. This importance led to proposing the intermediation theory of the firm by Spulber [2] in 1996. He believes that an intermediary acts as the fundamental building block of economic activities.
Group purchasing organizations (GPOs) are well-known intermediary firms that play an important role in some supply chains (especially in retail and healthcare supply chains).

There are lots of advantages in using GPOs for buyers. But, the fundamental rational for joining a GPO is that a buyer firm will incur a lower total purchasing cost by purchasing through a GPO rather than purchasing individually from a supplier (because of demand aggregation over a larger scale). Although the cost-reduction benefit of group purchasing is well understood, its effect on buyers' performance is not so transparent. On the one hand, GPO’s cost efficiency may bring savings to OEMs. On the other hand, the OEMs have to share a part of their profits with the GPO, and profit allocation naturally depends on the negotiated agreements.

This paper tries to shed some light on when competitors can cooperate in purchasing and also what are the perils of group purchasing for them. Our study offers another possible explanation of why some OEMs may not join a purchasing group.

In a stylized two-tier supply chain system, two competing OEMs are studied, who have the option to procure directly from a supplier or to use a GPO to delegate their procurement. To reach a delegating agreement, each OEM engages in bilateral negotiation with GPO. The OEMs subsequently compete in the product market by setting quantities and using an often-used wholesale-price contract form in negotiations.

The endogeneity of reservation profits in our problem highlights the importance of modeling firm negotiations under competition.

In this study, OEMs have four important characteristics, which represent real environment. First, each OEM seeks to maximize its total profit, not merely minimizing the purchasing costs. Second, each OEM's product demand, denoted as its purchasing requirement affects its product price in the market. Third, OEMs that agree to purchase through GPO, are not forced to purchase just only through GPO and they can purchase directly too. Fourth, OEMs determine their order quantities after deciding on the purchasing strategy that means competition for orders precedes procurement.

The final assumption may need some more clarification. In fact, firms compete and secure customer orders before negotiating contracts to procure inputs to fill those orders [3]. Indeed, preceding competition stage before vertical negotiation stage, would be applicable to environments, where input supply terms can be changed or renegotiated more frequently than customer orders (for instance, for durable goods or when customers are relatively patient) [3]. One well-known example is the supply chain management practices of Dell that has adopted a procurement strategy based around the principle: “Order from suppliers only when you
receive demand from customers”. Similar practices are common in some other industries like: building and architectural contracts, large scale services to governments, and electricity and gas retailing. This approach is usually relevant when in supply chain downstream firms find it more convenient to commit to contracts with customers than be locked into input supply contracts.

One important question to be addressed is whether a GPO that benefits from group buying discounts is always beneficial to OEMs (their purchasing process) in the presence of competition and negotiations. To answer this critical question, a game theoretic model is developed that includes a common supplier with a quantity discount schedule, a profit-maximizing GPO that negotiates with OEMs to derive a purchasing contract, and two profit-maximizing OEMs that compete in a common market. To uncover this question, a trade-off between two conflicting affects that drive an OEM’s purchasing decision is needed. On the one hand, an OEM can potentially enjoy cost savings because of more quantity discount obtained because of aggregated ordering by using a GPO. On the other hand, the OEM must share part of its profit with the GPO for the purchasing service. Naturally, profit sharing mechanisms depend on two main bargaining factors. One is the bargaining power of negotiators against each other and second is the bargaining position of the OEM and the GPO as well as their outside payoffs (i.e., their profits in the case of negotiation breakdown). The OEM’s outside payoff is its profit when procures directly from the supplier.

Analyzing of OEM–GPO negotiations highlights the strategic perils of group purchasing that using a GPO although is always preferable for symmetric OEMs; but may cause competing asymmetric OEMs worse off, leading to a win–lose outcome, in which the GPO gains and some OEM lose. In some conditions, OEM with lower bargaining power may obtain a lower profit and thus may be less likely to purchase through the GPO than with higher bargaining power. These results are driven by analyzing the entrance of a GPO in a two tier supply chain that consists of two competing OEMs with a common supplier. A bilateral bargaining framework is used to model negotiations of wholesale price contracts.

The sequence of events in our model is as follows. First, OEMs determine their purchasing strategy whether using a GPO or direct purchasing. The decision is known for all firms. Second, OEMs compete in the market and receive orders from consumers. Third, based on their known order quantities, OEMs procure their orders from supplier. If they agree on using a GPO, they make contracts with GPO in a bargaining framework. If not, they procure directly from the supplier. In the former case, the wholesale price is determined based on
bargaining and in the latter one, the wholesale price is determined based on a predefined quantity discount schedule.

The model is analyzed in two structures: one is symmetric case (OEMs with equal bargaining power) and the other one is asymmetric case (OEMs with different bargaining power).

The remainder of this paper is organized as follows. A review of the relevant literature is presented in section 2. The model is introduced in section 3. Extracting some insights and results based on analyzing the model is presented in section 4. Section 5 is conclusion and all proofs are relegated to the appendix.

2 Literature Review

This paper tries to integrate three main research streams in supply chain management literature (also called channel distribution literature in marketing) in just one river. Those different streams are: competition, contracting (bargaining), and quantity discount. This integrated river is procurement strategy.

Considering the competition stream, Ingene and Parry [4] are the first who introduce competing retailers to the quantity discount literature. Our model is based on deterministic demands and price competition; as Tsay and Agrawal [5] point out that, due to model complexity, typically deterministic formulations are found in most existing multi-echelon analyses that incorporate demand/price competition.

The second stream of related researches addresses the implication of bargaining on the allocation and level of supply chain profits. As is the case in our paper, some recent studies have considered bargaining contracts rather than take-it-or-leave-it price offers [6], [7], [8], [9], [10] and [11], which has been the case in previous models of channel interaction [12]. As Lovejoy [13] points out, a bargaining model would be more appropriate in many supply chain contexts because the solutions derived from the Stackelberg framework can be highly impractical because of various issues.

Feng and Lu have some novel researches [8], [9] and [10] related to production outsourcing literature. They are the first researchers have investigated market competition and contract bargaining, concurrently, on an outsourcing problem. They have investigated market competition and vertical contracting in a two tier supply chain consisting of two manufacturers and one/two suppliers. Although, their researches have some similarities with this study, but there are some significant differences. First, our main focus is on the role of an intermediary (like GPO) in procurement strategy; while, their researches focus on
outsourcing strategy. Second, the production (procurement) cost is constant in their studies; but, it is related to order quantity in our model (using quantity discount). Third and more importantly, in their problem setting, downstream firms compete in the market after determining the wholesale prices under contract bargaining. In contrast, early ordering (competition before negotiation) is used in our problem definition. So, although our results confirm some of their results; but, the results of this paper shows some more new findings.

The most relative research in outsourcing literature with our study is from Feng and Lu [10] considering both negotiation and competition in a two tier supply chain. They have investigated whether low cost outsourcing always beneficial to downstream manufacturers. They consider a two tier supply chain with one common supplier and two competing manufacturers that have different fixed producing costs. They show that low cost outsourcing may lead to a win-lose outcome in which the suppliers gain and the manufacturers lose. Because of bargaining externality (comes from different bargaining positions), they conclude that as a manufacturer’s bargaining power decreases, her profit under outsourcing may increase and it may be more likely for her to outsource. In some cases, manufacturers with larger bargaining power may obtain a lower outsourcing profit and thus may be less likely to outsource than those with smaller bargaining power (in contrast with our results). Our research has some important differences with Feng and Lu [10]:

- They do not use any quantity discount in their model and all costs are fixed and predefined.
- In their problem setting, negotiation stage happens before competition stage. But, in our study, market quantities and prices are determined before negotiation.
- They use a revenue-sharing contract format in negotiations (assuming fixed centralized profit); but, a wholesale price contract format have been implemented in this paper that is more general in real world but more complicated.

There is a vast operations and supply chain management literature on contracting; but existing literature on GPOs and other contracting intermediaries is sparse and still developing. Assuming a manufacturer that offers a linear quantity discount to competing retailers, Chen and Roma [14] identify conditions under which a GPO would be formed. They are the first who study group buying in a competing distribution channel. They study a supply chain with a monopoly manufacturer that offers a linear quantity discount to two competing retailers. They identify conditions under which a GPO will form when the retailers’ demands are functions of retail prices in the market. Unlike our paper, they do not consider bargaining in
their model. The same as this paper but in different conditions, they show that downstream firms may get hurt from group purchasing. In their paper, group buying is always preferable for symmetric retailers in the market. However, under asymmetric case, it can be detrimental to the larger retailer. But in our paper, downstream firms are always symmetric in the market but they may differ in their bargaining power with a GPO. In this study, group purchasing can be detrimental to the weaker OEM that is in contrast with Chen and Roma [14].

The existing literature on group purchasing apparently has not modeled firm negotiations in a competitive setting, thereby unable to provide insights into this important trade-off in competing OEMs’ purchasing decisions. Specifically, most of studies on implementing a GPO in the supply chain assume that GPO's interests are aligned with the buyers’ and thus the GPO seeks to minimize the buyers’ total purchasing costs [15], [16] and [17]. All of these papers use a take-it-or-leave-it negotiation format if necessary. More recently Hu and Schwarz [18] have studied the role of a GPO as an independent entity in a healthcare supply chain that tries to maximize his own profit. Their study along with their previous study [16] provides the first theoretical analyses of healthcare GPOs. They have studied the impact of GPOs on healthcare-product supply chains [18]. They investigate how the presence of a GPO affects the buyers' total purchasing costs in a supply chain with one common upstream supplier and two heterogeneous buyers (and n identical buyers). Although there are some similarities in the structure of the supply chain in our paper and that paper but there are noticeable different among these two researches. The most important different is that there is no competition between buyers in that paper and all demands are fixed before. But the demands are determined in a competing market in our paper. As well as, they do not have any negotiation between firms and each firm tries to maximize (minimize) his profit (cost) individually.

Early ordering is another characteristic of our model that differentiates it from previous studies. There are few studies in the literature, modeling competition for orders prior to procurement negotiations. Stahl [19] seems to be the first study that assumes bidding for inputs takes place after downstream competition for forward contracts. He shows this leads to competitive outcomes across the entire vertical chain. Our study is different from his model, as he sets a single input price (there is no price discrimination in the wholesale negotiation) and downstream firms have all the power in that market. In contrast, our model presumes that the single upstream firm (GPO) bargains bilaterally with each of the downstream firms.

In a more relevant study, Gans [20] has studied vertical contracting when competition for orders precedes procurement. He shows that when competition for orders precedes
negotiations for component procurement, the outcome is the oligopolistic competitive
goal (i.e., Cournot competition outcome). He also studies the impact of vertical
integration on this outcome. Although, he considers a non-decreasing supplying cost with
respect to order quantities but we use a more realistic and often-used non-increasing
supplying cost called quantity discount model. Gans has considered a two tier supply chain
but our main focus is on a three tier supply chain.

More recently, Guo and Iyer [11] have studied a multiunit bilateral bargaining and
downstream competition in a two tier supply chain that market competing takes place before
ex post negotiations. Their main focus is on bargaining timing and they determine the firms'
equilibrium decision whether bargaining concurrently or sequentially.

Briefly, our study generalizes the existing knowledge on GPO by considering a number of
structure features: (i) quantity discount model for supplying the product, (ii) product
substitutability, (iii) firms’ competition, and (iv) bargaining powers (which highly have been
investigated in literature) in one framework, simultaneously. Why most previous papers have
considered two or three of these four features in one research framework.

3 The proposed model

A well-known supply chain framework is considered, consisting of one common supplier
who sells a homogeneous good based on a quantity discount menu to two OEMs
(indexed by i, j = 1, 2) who compete in a common market by selling to end consumers. A
quantity discount menu, in facts, represents a competitive supply market that there are many
of suppliers who can produce a general item. OEMs’ products are substitutable in the market
and they consist of a main component procured from a supplier market. Without loss of
generality OEMs’ production cost are assumed equal to zero. So, the procurement cost makes
the main part of the market price.

In the initial market equilibrium, each OEM acts individually in procuring from the supplier.
The OEMs then consider the option of procuring through a GPO to obtain a lower wholesale
price. This option is possible only when both OEMs agree to cooperate.

3.1 Market Competition

The demand quantity for each product in the market is elastic. For tractability, a Cournot
model is used known as an often-used linear price model.

\[ p_i = a - q_i - \gamma q_j \]  

Eq. (1)
Where \( i \in \{1, 2\} \) and \( j = 3 - i \). \( p_i \) and \( q_i \) show the market price and quantity, respectively. \( \alpha \) is the initial market price and \( \gamma \in (0, 1) \) measures the degree of product substitutability. When \( \gamma \) approaches 0, the two products become independent and there is no competition in the market. When \( \gamma \) approaches 1, they become perfect substitutes. As what understood from relation (1), both OEMs are symmetric in the market.

3.2 Procurement Process

Assuming that OEMs first compete in the market for orders and then they procure their needs based on their procurement strategy. Here, they have two alternatives for their procurement strategy: whether to procure directly from a common supplier (abbreviated by DP) or procuring through a GPO (abbreviated by GPO). According to the Robinson–Patman act, a common price menu must be offered to each buyer to preclude sellers "from giving different terms to different resellers in the same reseller class." So, the same price menu is offered under either individual or group purchase.

In our problem setting for the sake of analytical simplicity and tractability, the supplier (indexed by \( s \)) offers a linear quantity discount schedule, relation (2), which satisfies two important practical situation conditions: non-increasing volume discount and non-decreasing total purchasing cost.

\[
w_{si} = \lambda - \mu q_i ; \quad \text{Eq. (2)}
\]

Where \( i \in \{1, 2, c\} \) and \( w_{si} \) shows the wholesale price offered from the supplier to OEM (i) or GPO (indexed by c). \( \lambda \) is the basic wholesale price (when quantity is zero) that without loss of generality, it is normalized to 1; and \( \mu \) stands for discount rate. Although theoretically, \( \mu \) can vary in \([0, 1]\) interval; but, we do not have discount rates higher than \( \frac{1}{2} \) in practice. Since \( \mu \leq \frac{1}{2} \) simplifies the analytic process in this paper, we assume \( \mu \) less than \( \frac{1}{2} \).

3.3 Bargaining and Contracts

To model negotiations in the procurement through the GPO, a multi-unit bilateral bargaining framework is used to model contract negotiations. Each OEM negotiates bilaterally with a common GPO over a wholesale price. This bilateral formulation of negotiations reflects the fact that competing OEMs typically negotiate independently. Under competition, the bargaining outcome of one channel naturally depends on that of the other. When procuring through the GPO, OEMs negotiate with the GPO to determine the wholesale
price that they must pay to the GPO. To model these bilateral negotiations, the asymmetric Nash bargaining solution is used.

\[
\max_{w_i} \phi_i = (\pi_i (w_i) - \delta_i)^\theta (\pi_c (w_i) - \delta_c)^{(1-\theta)}
\]

Eq. (3)

Where \( \phi_i \) is the Nash bargaining product, \( w_i \) is the wholesale price offered by the GPO to the OEM \( i \), and \( \pi_i \) is the firm \( i \)'s profit. When the OEM \( i \) and the GPO negotiate, \( \delta_i \) and \( \delta_c \) stand for the OEM \( i \)'s and the GPO's reservation profits (that is also called disagreement point or outside option, in the literature i.e., the firm's profit when no agreement is achieved), respectively. \( \theta_i \in [0,1] \) denotes OEM \( i \)'s bargaining power vis-à-vis the GPO.

Consider the negotiation between the OEM \( i \) and the GPO over the wholesale price of product \( i \). Two possible outcomes arise from the negotiation: (1) the trade agreement fails and the OEM procures directly from the supplier, or (2) an agreement is reached and the GPO procures the product for the OEM. To define the bargaining problem, the trading parties’ disagreement points should be specified, i.e., their profits when the negotiation breaks down (\( \delta_i \) and \( \delta_c \)).

The value of a negotiator’s outside option is referred as its “bargaining position.” This is distinguished from “bargaining power,” which refers to the relative skill of negotiators.

The negotiation outcomes of the two competing supply chains form Nash equilibrium. This solution concept for our multiunit bilateral bargaining problem is known as the Nash–Nash solution. This approach is viewed as a direct extension of the single-unit Nash bargaining solution to multiple bargaining units [10].

### 3.4 Sequence of Events

At first, the OEMs determine their procurement strategy; whether to procure directly from a component supplier or to procuring through the GPO. If they decide to procure directly, they determine their order quantities in the market, and then they procure their orders from the supply market independently. In this strategy, the wholesale prices are determined based on a pre-defined quantity discount schedule. If they decide to use a GPO for component procurement, then:

1. The OEMs compete in the product market to determine their order quantities.
2. A contract is negotiated bilaterally between each of the OEMs and the GPO to determine the wholesale price of the component. All negotiations occur in parallel.
Upon reaching an agreement, the OEM contracts with the GPO to procure her demand; otherwise, the OEM procures directly from the supplier.

3. The OEMs order to the GPO (or directly procure from the supply market). The GPO procures from the supply market and charges the OEMs the negotiated prices.

3.5 Assumptions

- To focus on the competition and negotiation’s effects on optimal procurement structure, all model parameters are assumed to be deterministic and common knowledge.
- Zero coordination cost under group purchasing.
- $\mu \leq \frac{1}{2}$. This is a rational condition that happens in reality, most of the times and helps us in analytical proofs.
- OEMs are similar in the market and they may differ only in their bargaining power;
- Without loss of generality: $\theta_1 \leq \theta_2$.
- The supplier offers a quantity discount schedule that is nonincreasing in quantity and ensures nondecreasing revenue.
- For simplicity, the OEMs' production cost is zero. This is true when equal production costs are considered for both OEMs.
- All firms have common knowledge under complete information.
- The OEMs, GPO and supplier are risk neutral: each seeks to maximize its own expected profit.

3.6 Notations

Tables 1 and 2 show a snapshot of parameters and variables, respectively.

4 Analysis

In this section the behavior of firms under different procurement strategies is investigated analytically. Since there are numerous parameters that make it difficult to study the equilibrium behaviors, the problem is investigated in two symmetric and asymmetric cases with respect to equal or different bargaining powers, respectively. As these two cases differ only on OEMs' bargaining powers situation, so they would differ only under GPO procurement strategy and DP strategy is alike for both cases.
4.1 Symmetric Case

Symmetric case is when both OEMs have the same bargaining powers vis-à-vis a GPO ($\theta = \theta_1 = \theta_2$). In this case, both OEMs have similar behavior under GPO strategy; like they have under DP strategy. As a matter of fact, there is just one bilateral monopoly problem when both OEMs are identical. As shown in proposition 1, GPO is the dominant strategy in symmetric case.

**Proposition 1.** (Equilibrium Strategy) when OEMs have equal bargaining power, then: GPO strategy is the best decision for all supply chain players in equilibrium.

The intuition behind this proposition is straightforward and it has mentioned as the most important motive for group purchasing ([10], [14]). It is clear that the GPO always benefits from group purchasing strategy because he obtains zero profit otherwise. For analyzing the behavior of OEMs in the equilibrium it should be mentioned that determining the order quantities (accordingly, market prices) happen before bargaining over wholesale prices. When procuring through GPO, OEMs benefit from larger order quantities by more discounts. This enlarges the size of the pie (trade surplus) to be shared between OEMs and GPO. Since OEMs have larger order quantities under GPO, so their market prices become lower than under DP. Consequently, lower wholesale price that GPO have to pay the supplier (because of higher order quantity) and lower market price for OEMs, both of them force the negotiated wholesale price to decrease when bargaining. It can be simply shown that each OEM’s margin ($p_i - w_i$) is larger under GPO compared to DP and it is obvious that OEMs have larger order quantities under GPO too. As a result, there is a higher profit for each OEM under GPO compared to DP. So, GPO strategy would be a win-win strategy for both GPO and OEMs.

Since, GPO procurement strategy is the equilibrium decision in this game, in the following propositions; the role of different parameters on decision variables under GPO strategy is further investigated.

**Proposition 2.** (Effect of bargaining power) under GPO strategy, a higher bargaining power (higher $\theta$) leads to: i) higher market demand, ii) lower market price, iii) lower wholesale price, iv) higher profits for OEMs, and v) lower profit for GPO.

Following an early ordering mechanism, the OEM knows that her share of the pie (trade surplus) increases in her bargaining power. So, she tries to enlarge the pie when her bargaining power increases. For this, she makes a larger order quantity. Based on relation (1), larger order quantity causes lower market price and lower wholesale price, consequently.
Most importantly, the OEM's bargaining power has positive effect on OEM's profit and negative effect on GPO and contract profits, intuitively. To further understand the effect of OEM's bargaining power on firms' profits, the firms' and the contract profits (total profit of both sides of the negotiation) are plotted as a function of the OEM's bargaining power in figure 1. It supports findings (iv) and (v) of proposition 2. The profit behaviors shown in figure 1 are similar for different $\alpha$ and $\gamma$.

It is also interesting in managerial viewpoint to investigate the impact of product competition intensity on decision variables; that is shown in the next proposition.

**Proposition 3.** *(Effect of competition intensity)* a higher competition level ($\gamma$) leads to: i) lower market demand, ii) higher/lower market price, iii) higher wholesale price, iv) lower profits for OEMs, and v) lower profit for GPO.

The effect of competition intensity in the market is not intuitive and needs more attention. Based on most of previous studies [8] and [14], competition intensity ($\gamma$) should have a negative impact on market price and positive impact on market demand. But here it has an unusual impact and assuming all other parameters fixed, higher competition intensity ($\gamma$) always causes lower market demand.

Another interesting result is that although higher market competition causes higher wholesale prices that OEMs pay to GPO but it decreases the GPO's profit. This unintuitive result is because that higher competition intensity has a negative impact on market demand and consequently the size of the pie decreases. Smaller size of the pie reduces the profits of OEMs and GPO.

As well as, the effect of competition intensity on wholesale price is positive; but, its effect on market price is dependent on other parameters. Higher competition intensity ($\gamma$) usually decreases market price (intuitive); but for $\frac{1}{4} < \mu < \frac{1}{2}$, there is a threshold for $\theta$ called $\tau_i(\mu)$ that market price would increase in $\gamma$ (unintuitive).

Focusing on more managerial decision variables (profits), the market competition has negative effects on both OEM's and GPO's profits as well as on contract profit. For more perception, the firms' and the contract profits (total profit of both sides of the negotiation) are plotted as a function of competition intensity in figure 2. The profit behaviors shown in figure 2 are similar for different $\alpha$ and $\mu$.

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1 This threshold has been defined in the appendix.
Another important aspect of this study is quantity discount schedule that supplier offers. The impact of quantity discount rate on decision variables are investigated in the following proposition.

**Proposition 4.** *(Effect of quantity discount)* a higher quantity discount rate ($\mu$) leads to: i) higher market demand, ii) lower market price, iii) lower wholesale price, iv) higher profits for OEMs, and v) higher profit for GPO.

Assuming all other parameters fixed, higher quantity discount rate ($\mu$) always causes lower wholesale price for GPO and consequently lower wholesale price for OEMs. This lower wholesale price for OEMs leads to lower market price. Lower market price softens the competition in the market and causes higher market demand. So, the size of the pie increases and the firms’ profit would increase too. All of the results in proposition 4 are intuitive.

Most importantly, the quantity discount rate has positive effect on both OEM's and GPO's profits as well as on the contract profit. This is because of the higher the quantity discount rate, the larger the size of the pie.

To further understand the effect of quantity discount rate on firms’ profits, the firms’ and the contract profits are plotted as a function of the quantity discount rate in figure 3. It supports findings (iv) and (v) of proposition 4. The profit behaviors shown in figure 3 are similar for different $\alpha$ and $\gamma$.

Consequently, at the end of this subsection, the impact of important parameters on different decision variables (results of propositions 2 to 4) are summarized in table 3. In a consumer viewpoint, consumers always benefit when bargaining power of OEMs or quantity discount rate (or both of them) increases.

Although most of the results in symmetric case are intuitive; but, there are more challenging outcomes when the bargaining power of OEMs are different in the next subsection.

### 4.2 Asymmetric Case

In this case, the model is analyzed when retailers have different bargaining powers. Without loss of generality it is assumed that OEM1 has less bargaining power compared to OEM2, when it negotiates with the GPO (i.e., $\theta_1 \leq \theta_2$). Comparison of the two OEMs’ profits indicates that both of them are identical when they directly procure from the supply market. It is also shown that OEM2 is better-off when OEMs procure through the GPO.

**Proposition 5.** *(OEMs’ profits comparison)* Under DP strategy, both OEMs are the same but, when they procure through the GPO, the stronger OEM always
i) gets better-off;

ii) offers lower prices in the market;

iii) pays lower wholesale price;

iv) receives larger order quantity in the market.

This proposition confirms the intuition that a stronger OEM benefits from her higher bargaining power receiving lower prices from the GPO. Higher bargaining power of one of the OEMs might originate from different sources; for example, the OEM might have more expertise in procurement negotiations than the other OEM. This allows the OEM to get a better price in her contract from the GPO. As a result, it would be able to offer more competitive prices in the market. In the first stage of the game, when OEMs compete over their order quantities, OEM2 who has higher bargaining power, knowing her higher negotiation skills and bargaining powers, decides to be more aggressive in terms of increasing its order quantity and reducing its price.

Proposition 6. (Equilibrium Decisions) With different bargaining powers, in equilibrium,

i) The stronger OEM (OEM2) prefers GPO to DP.

ii) The preferred procurement strategy of the weaker OEM depends on both OEMs' bargaining power.

iii) GPO always prefers GPO procurement strategy.

iv) The centralized profit in GPO procurement is greater than DP.

It is clear that the GPO always benefits from group purchasing strategy because he obtains no profit, otherwise. The stronger OEM (OEM2) also prefers GPO to DP since GPO strategy enlarges the trade surplus (size of the pie) and it also increases her profit share (slice of the pie). The weaker OEM (OEM1) prefers GPO to DP only when a threshold called

\[
\tau_2 = 2(1 - \mu) \left( \frac{1 - \theta_2}{1 - \theta_1} \right) \frac{\theta_2}{\theta_1} + 2\theta_1 - \gamma \frac{1 - \theta_1 \theta_2}{1 - \theta_2} \]

is positive (this threshold can be translated to \( \Delta\theta \) is small enough); even if her bargaining power is either low or high. The interesting point to OEM1’s behavior is that her tendency to GPO strategy is more dependent on her competitor bargaining power rather than her own bargaining power. If \( \mu \) and \( \gamma \) are considered to be fixed, the OEM1’s tendency to GPO increases when both OEMs' bargaining power approaches to \( \frac{1}{2} \). In the other words, the most tendency of OEM1 to alter GPO strategy is when all participants (both OEMs and GPO) have similar bargaining power. This tendency decreases in product competition (\( \gamma \)) while discount rate (\( \mu \)) has very small effect on it. When
Δθ becomes considerable, the weaker one would prefer GPO to DP only when product competition (γ) is very low.

To make a better sense of OEM1's behavior in equilibrium, it is demonstrated in figure 4. It can be concluded from τ₂ that discount rate (µ) has very small effect on it. This is shown in figure 5.

Because of higher trade surplus under GPO strategy, so the size of the pie that have to be shared between firms enlarges. This causes a higher centralized profit. Therefore, the GPO's cost advantage, while always benefiting himself, can be a double-edged sword for the competing OEMs (specifically, the weaker one).

Based on this proposition, the stronger OEM has to mention that although she achieves more profit under GPO strategy but her competitor may deviate from GPO when threshold τ₂ is not satisfied. Since the stronger OEM achieves less profit when she procures directly after deciding to use a GPO (because of deviation of her competitor) rather than her profit when she knows from the first that she should procure directly from the supplier; she has to make sure that the threshold is satisfied to implement the GPO strategy.

In a Stackelberg setting, the GPO is the one that offers wholesale prices to the OEMs. This is the same as θ₂ = θ₁ = 0 in the paper. According to Figures 4 and 5 in the paper (and also substituting θ₂ = θ₁ = 0 in the appendix), it is straightforward to check that both the GPO and DP strategies are equivalent. The intuition for this finding is that under the GPO strategy, the GPO charges the OEMs the same price as they get through the DP strategy from the supply market. This leaves the OEMs with the same profit as they get under the DP strategy.

5 Conclusion

In this paper, a general model was constructed of competing OEMs with two possible strategies: DP or GPO, given a quantity discount schedule from the manufacturer. Based on linear demand curves, a number of insights were offered on OEMs’ group buying decisions when they compete with each other in a common market and have an opportunity to bargain with a GPO to determine the wholesale price. Intuitively, GPO is always advantageous in the absence of competition. Under competition, symmetric OEMs are always better off using GPO.

Although group purchasing organizations (GPOs) has been widely used as an intermediary for group purchasing in supply chain and a variety of buyers has been benefitted by purchasing cost reductions using GPOs; but this paper shows that buyers should not use GPO
in their purchasing process with closed eyes. Our analysis of OEM–GPO negotiations highlights the strategic perils of group purchasing and shows that low purchasing cost GPO may harm OEMs in their profits. This unintuitive result can be explained by different impacts that a GPO have in purchasing process. Although, he can enlarge the size of the pie (trade surplus) causing more quantity discount; but, he can effect on the slice of the pie (profit sharing) by his bargaining power.

In symmetric case, using GPO strategy is always the best decision for all supply chain players in equilibrium; but, it is not the case in asymmetric case. When OEMs are asymmetric, procuring through GPO is not always the OEMs' dominant strategy in equilibrium. Moreover, it was shown that an OEM's procurement strategy in equilibrium does not just depend on her bargaining power; also depends on her competitor OEM. Interestingly, a strong OEM (with respect to GPO) may not prefer procuring through GPO as well as a weak OEM does. This paper shed some lights on vague aspects of purchasing strategy. To expand more, investigators may generalize quantity discount function using a general form introduced by Schotanus [21] that describes the underlying function of different discount schedules. Based on this general form, the unit wholesale price $w(q)$ is

$$w(q) = \lambda + \frac{\mu}{q^e}$$  \hspace{1cm} (4)

Where $\lambda$ is the base price, $\mu$ is the discount scale, and $e$ is the steepness. They show that this QDF fits well with 66 discount schedules found in practice, with $e$ varying from -1.00 to +1.60. Also, researchers may use nonlinear demand functions. In many competitive equilibrium analyses, linear demand functions are often used because of their tractability in providing analytical results. Yet it is expected to have demand nonlinearity in many real problems. However, this makes it very difficult to analyze this structure analytically. Considering two different OEMs in share of the market (different $\alpha$'s) or more than two competing OEMs may be other extensions on this research.
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>the basic price of product in the market</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>the substitution effect of the products</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>the basic price of key component (with no discount)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>quantity discount rate offered from supplier</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>OEM i’s bargaining power vis-a-vis GPO</td>
</tr>
</tbody>
</table>

Table 5-Summary of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i$</td>
<td>the demand for OEM i’s product in the market</td>
</tr>
<tr>
<td>$p_i$</td>
<td>the price for OEM i’s product in the market</td>
</tr>
<tr>
<td>$w_{si}$</td>
<td>the wholesale price of key component offered from supplier to OEM i</td>
</tr>
<tr>
<td>$w_{ci}$</td>
<td>the wholesale price of key component offered from GPO to OEM i</td>
</tr>
<tr>
<td>$w_{sc}$</td>
<td>the wholesale price of key component offered from supplier to GPO</td>
</tr>
<tr>
<td>$\pi_i$</td>
<td>the profit of firm i</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>the reservation profit of firm i</td>
</tr>
</tbody>
</table>

Figure 6- Effect of bargaining power on firms’ profits under GPO strategy; Solid thick line: Contract Profit; Solid line: OEM’s Profit; Dashed line: GPO’s Profit. $\alpha = 2$ and $\gamma = 0.5$
Figure 1a - Low Discount ($\mu = 0.05$)

Figure 1b - Moderate Discount ($\mu = 0.25$)
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Figure 7 - Effect of competition intensity on firms' profits under GPO strategy; Solid thick line: Contract Profit; Solid line: GPO's Profit; Dashed line: OEM's Profit. α = 2 and μ = 0.25

Figure 2a - Low Bargaining Power (θ = 0.1)
Figure 2b- Moderate Bargaining Power ($\theta = 0.5$)

Figure 2c- High Bargaining Power ($\theta = 0.9$)

Figure 8- Effect of quantity discount on firms' profits under GPO strategy; Solid thick line: Contract Profit; Solid line: GPO's Profit; Dashed line: OEM's Profit. $\alpha = 2$ and $\gamma = 0.5$
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Figure 3b- Moderate Bargaining Power ($\theta = 0.5$)
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<table>
<thead>
<tr>
<th></th>
<th>bargaining power (θ)</th>
<th>competition level (γ)</th>
<th>quantity discount rate (μ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>market demand</td>
<td>higher</td>
<td>lower</td>
<td>higher</td>
</tr>
<tr>
<td>market price</td>
<td>lower</td>
<td>higher/lower</td>
<td>lower</td>
</tr>
<tr>
<td>wholesale price</td>
<td>lower</td>
<td>higher</td>
<td>lower</td>
</tr>
<tr>
<td>OEMs' profit</td>
<td>higher</td>
<td>lower</td>
<td>higher</td>
</tr>
<tr>
<td>GPO's profit</td>
<td>lower</td>
<td>lower</td>
<td>higher</td>
</tr>
<tr>
<td>Contract profit</td>
<td>lower</td>
<td>lower</td>
<td>higher</td>
</tr>
</tbody>
</table>

Figure 3c—High Bargaining Power (θ = 0.9)

Figure 9—Equilibrium Strategy of OEM1 for α = 2
Figure 10-Equilibrium Strategy of OEM1. $\alpha = 2$ and $\gamma = 0.5$
6 Technical biography of authors

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He is a PhD candidate in department of industrial engineering at Iran University of Science and Technology. His main research focus is on logistics and supply chain management. He has published 9 books and more than 15 journal and conference papers in this area, also doing 12 research projects mostly for the “ministry of industry, mine and trade” of Iran during his 13 years research experience. Also he is the editor in chief of “logistics and supply chain” journal published monthly in Iran.

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6.3 Ehsan Bolandifar

Professor Ehsan Bolandifar is an assistant professor at the Chinese University of Hong kong. His research mainly focuses on the operations management interfaces with finance and marketing. He is also interested in component procurement in supply chains and his research has been featured in top-tier journals in operations management like Manufacturing and Service Operations Management and Production and Operations Management. Professor Bolandifar has received his Ph.D. in Business Administration from Washington University in St Louis and his M.Sc. from Sharif University of Technology.
Appendix: proofs

Lemma 1. Using direct procurement, OEMs are the same in equilibrium and GPO is inactive with no profit.

Proof of lemma 1:
The market price is determined based on relation \( (1) \) depending on order quantity \( (q_i) \):
\[ p_i = \alpha - q_i - \gamma q_j; \]
Under DP strategy, the wholesale price is determined as follows:
\[ w_i = w_i = \lambda - \mu q_i; \]
So, the final profit of each OEM is determined based on relation \( (A.1) \):
\[ \pi_i = q_i (p_i - w_i); \quad \text{Eq. (A.1)} \]
Since, each OEM seeks to maximize its own expected profit; so, they compete in the market based on profit maximization. Each OEM maximizes her gross profit showed in relation \( (A.1) \) by choosing order quantities \( (\max \pi_i) \) which leads to the following first-order conditions:
\[ \frac{\partial \pi_i}{\partial q_i} = (\alpha -1) - 2(1 - \mu)q_i - \gamma q_j = 0; \quad (i = 1, 2 \& j = 3 - i) \quad \text{Eq. (A.2)} \]
Solving Eq. (A.2) simultaneously for \( i = 1 \) and 2, gives order quantities:
\[ q_i = q_j = \frac{\alpha - 1}{2 + \gamma - 2\mu}; \quad \text{Eq. (A.3)} \]
As order quantities of both OEMs are the same; so, other variables would be the same too:
\[ p_i = p_j = \frac{1 + \alpha + \gamma - 2\alpha \mu}{2 + \gamma - 2\mu}; \quad \text{Eq. (A.4)} \]
\[ w_i = w_j = \frac{1 - \mu(\alpha -1)}{2 + \gamma - 2\mu}; \quad \text{Eq. (A.5)} \]
\[ \pi_i = \pi_j = \frac{(\alpha - 1)^2 (1 - \mu)}{(2 + \gamma - 2\mu)^2}; \quad \text{Eq. (A.6)} \]
Since, GPO has no role in DP strategy, it is clear that he would not get any profit.

□

Lemma 2. Using GPO strategy, the most important variables (order quantity and wholesale price) are as follows:
\[ q_i = \frac{\xi (1 - \alpha)(\xi (2 - \gamma) - 2\mu \nu)}{\eta}; \quad \text{Eq. (A.7)} \]
\( w_{ci} = \frac{1}{\eta} \left( \left( \gamma^2 - 4 + \mu(2 + 6\alpha - 4\alpha \mu - \gamma - 3\gamma) \right) \xi^2 + 2\mu(2 - 2\alpha + \alpha \mu + \alpha \gamma - \mu) \kappa_i \xi + 2\mu \kappa_i \xi + 4\alpha \mu \kappa \kappa_j \right) \); Eq. (A.8)

**Proof of lemma 2:**

We define \( \theta_i \) and \( \kappa_i \) as the bargaining power of OEM (i) and GPO in a bilateral negotiation, respectively and we have \( \theta_i = 1 - \kappa_i \). For simplicity, we define parameters \( \xi \) and \( \eta \) as follows:

\[
\xi = \kappa_i + \kappa_j - \kappa_i \kappa_j ; \quad \text{Eq. (A.9)}
\]

\[
\eta = \xi^2 \left( \gamma^2 - 4\left( \mu - 1 \right)^2 - 2\mu \gamma \right) + 2\mu \kappa_i \kappa_j \left( \gamma \xi + 2\mu \left( 1 - \xi \right) \right) ; \quad \text{Eq. (A.10)}
\]

Since we apply an "early ordering" approach in procurement strategy; so, OEMs compete in the market firstly to determine their order quantities and then bargain bilaterally with the GPO to determine the wholesale price they should pay based on Nash bargaining solution.

Here, we have a sub-game perfect equilibrium and we can use a backward induction approach to determine two main variables for each OEM.

At the first step, we consider order quantities as given and try to determine "wholesale prices" in bargaining stage. For this and according to the Nash-bargaining solution approach, the wholesale prices are determined by simultaneously maximizing Nash-product relation pre-defined in relation (3) for \( i = 1, 2 \) & \( j = 3 - i \):

\[
w_{ci} = \frac{1 - \mu q_i - \theta_j \left( \theta_j + 2\mu q_j - \mu \theta_j q_i - 2\mu \theta_j q_j \right)}{1 - \theta_i \theta_j} \quad \text{Eq. (A.11)}
\]

It is worth commenting that the profit of GPO (\( \pi_c \)) is defined as follows:

\[
\pi_c = q_1 \left( w_{c1} - w_{sc} \right) + q_2 \left( w_{c2} - w_{sc} \right) ; \quad \text{Eq. (A.12)}
\]

Where \( q_1 \) and \( q_2 \) are given. \( w_{ci} \) is going to be determined and \( w_{sc} \) is the wholesale price that GPO pays to supplier based on quantity discount schedule:

\[
w_{sc} = \lambda - \mu \left( q_1 + q_2 \right) ; \quad \text{Eq. (A.13)}
\]

Also, the reservation profits in relation (3) are determined as follows:

\[
\delta_i = q_i \left( p_i - w_{si} \right) ; \quad \text{Eq. (A.14)}
\]

\[
\delta_{ci} = 0 ; \quad \text{Eq. (A.15)}
\]

Relation (A.14) states that OEM (i) can directly procure her committed orders from the supplier when bilateral negotiation with GPO fails (disagreement point). Also, if there is no agreement between OEM (i) and GPO, then GPO can just contract with OEM (i)'s competitor. But, since there is no advantage for OEM (j) in using GPO (\( q_c = q_j \Rightarrow w_{cj} = w_{sc} \)); so, GPO's reservation profit becomes zero.
At the second step, OEMs compete in the market to determine their order quantities based on
maximizing their gross profits showed in relation (A.1). For this, we should solve the
following first-order conditions simultaneously:
\[
\left( \frac{\partial \pi_i}{\partial q_i} = 0; \frac{\partial \pi_j}{\partial q_j} = 0 \right) \quad (i = 1, 2 \& j = 3 - i) \quad \text{Eq. (A.16)}
\]
This leads to the following results:
\[
q_i = \frac{((1 - \alpha)(1 - \theta)\theta_i)((2 - \gamma)(1 - \theta)\theta_j - 2(1 - \theta)\mu)}{(-4 + \gamma^2)(1 - \theta)\theta_j + 2(1 - \theta)\theta_i(4 - 4\theta_i\theta_j - \gamma(\theta_j + \theta_i(1 - 2\theta_j)))\mu - 4(1 - \theta)\theta_i(3 - \theta_i - \theta_j))\mu^2} \quad \text{Eq. (A.17)}
\]
Using \( \kappa_i, \xi \) and \( \eta \) (as defined above) relation (A.17) is simplified to relation (A.7). After
substituting \( q_i \) and \( q_j \) in relation (A.11) and some simplifications, we arrive to relation (A.8)
and the proof is completed.

According to relations (1) and (A.7), we can get the market prices.
\[
p_i = \frac{2\mu \xi(\gamma(\alpha \kappa_i + \kappa_j) - (\alpha - 1)\kappa_j) - \xi^2((2 - \gamma)(1 + \alpha + \gamma - 4\alpha \mu) + 4\alpha \mu^2) + 4\alpha \mu^2 \kappa_i \kappa_j(1 - \xi))}{\eta} \quad \text{Eq. (A.18)}
\]
When we put relations (A.9), (A.17), and (A.18) in relation (A.1) then firms' profits are
derived:
\[
\pi_i = \frac{\xi^2(\alpha - 1)^2(1 - \mu)(2\mu \kappa_i - (2 - \gamma)\xi)^2}{\eta^2}; (i = 1, 2 \& j = 3 - i) \quad \text{Eq. (A.19)}
\]
\[
\pi_e = \frac{2\kappa_i \kappa_j \xi(\alpha - 1)^2 \mu((2 - \gamma)^2 \xi^2 - 2\mu(2 - \gamma)\xi^2 + \kappa_i \kappa_j \left(\xi - \frac{2\mu}{2 - \gamma}\right))}{\eta^2} \quad \text{Eq. (A.20)}
\]

Lemma 3. (Feasible Conditions): Besides some general conditions for parameters, there is
always a unique upper bound for "\( \alpha \)" under different procurement strategies. The
feasible area under DP is always larger than it under GPO.

Proof of lemma 3:
We have some general conditions for parameters that are held in real world (as explained in
§3, remember that forcing \( \theta_i \leq \theta_2 \) is just an assumption we made in this paper):
\[
0 < \gamma < 1 \& 0 < \mu < \frac{1}{2} \& 1 < \alpha \& 0 < \theta_i \leq \theta_2 < 1 \quad \text{Eq. (A.21)}
\]
Besides these general conditions, all variables of problem (order quantities, market prices,
wholesale prices, and firms' profits) have to be positive too.
When OEMs procure directly from the supplier, according to general conditions (A.21), relations (A.3), (A.4), and (A.6) in lemma 1 are always positive. But, being positive of wholesale prices based on relation (A.5) needs an extra condition:

\[ 1 < \alpha \leq \frac{2 + \gamma - \mu}{\mu} \quad \text{Eq. (A.22)} \]

\[ \alpha_{DP}^G = \frac{2 + \gamma - \mu}{\mu} \] is an upper bound for \( \alpha \) under DP strategy. This unique upper bound increases in \( \gamma \) and decreases in \( \mu \).

When OEMs procure through GPO:

According to general conditions (A.21) and comparing \( P_i^{GPO} \) with \( w_{ci}^{GPO} \) (relations (A.11) and (A.18)), we understand that \( P_i^{GPO} > 0 \) is dominated by \( w_{ci}^{GPO} > 0 \). So we just need to consider \( w_{ci}^{GPO} > 0 \). \( \pi_{ci}^{GPO} \) is always positive for both OEMs and \( \pi_c^{GPO} > 0 \) is dominated by \( q_i^{GPO} > 0 \).

So, under GPO strategy, \( q_i^{GPO} > 0 \) and \( w_{ci}^{GPO} > 0 \) are sufficient conditions. \( q_i^{GPO} > 0 \) is always positive. Considering \( w_{ci}^{GPO} > 0 \) we would have two different upper bounds for \( \alpha \) called \( \alpha_{1GPO}^{GPO} \) and \( \alpha_{2GPO}^{GPO} \):

\[ \alpha_{1GPO}^{GPO} = \frac{(\gamma^2 - 4 + 2\mu - \gamma\mu)\xi^2 + 2\mu(2 - \mu)\kappa_1\xi + 2\gamma\mu\kappa_2\xi}{3\mu(\gamma - 2)\xi^2 + 2\mu(\mu - \gamma + 2)\kappa_2\xi + 4\mu^2\kappa_1(\xi - \kappa_1)} \quad \text{Eq. (A.23)} \]

\[ \alpha_{2GPO}^{GPO} = \frac{(\gamma^2 - 4 + 2\mu - \gamma\mu)\xi^2 + 2\mu(2 - \mu)\kappa_2\xi + 2\gamma\mu\kappa_1\xi}{3\mu(\gamma - 2)\xi^2 + 2\mu(\mu - \gamma + 2)\kappa_1\xi + 4\mu^2\kappa_2(\xi - \kappa_2)} \quad \text{Eq. (A.24)} \]

As \( \theta_1 \leq \theta_2 \) and \( \kappa_2 \leq \kappa_1 \), accordingly. Then, it is obvious that \( \alpha_{2GPO}^{GPO} \leq \alpha_{1GPO}^{GPO} \). So, we need just consider \( \alpha_{2GPO}^{GPO} \) as the upper bound for \( \alpha \) and \( \alpha^{GPO} = \alpha_{2GPO}^{GPO} \) is the unique upper bound for \( \alpha \) under GPO strategy.

By comparing upper bounds for \( \alpha \) under two different procurement strategies, we get

\[ \alpha^{GPO} \leq \alpha^{DP}. \] That means the feasible area under DP is always larger than it under GPO.

\[ \square \]

**Proof of proposition 1:**
The proof for GPO is straightforward. The GPO is out of the trade under DP strategy; so, he receives no profit. But, he has positive profit under GPO strategy. We define \( \rho = 2 + \gamma - 2\mu \); so, based on relation (A.20) the gross profit for GPO in symmetric case is as follows:

\[
\pi_{c_{GPO}} = \frac{2\mu(\alpha - 1)^2(1 - \theta^2)}{(\rho + (\rho - 2\mu)\theta)^2}
\]

Eq. (A.25)

The above term is always positive (under general conditions described in lemma 3); except when \( \theta = 1 \) that means OEMs have all bargaining power and get the whole trade surplus under negotiation (we know that GPO's reservation profit is zero) with no profit for GPO.

Simplifying relation (A.19) for symmetric case, we got OEMs' profit as follows:

\[
\pi_{i_{GPO}} = \frac{(1 - \mu)(\alpha - 1)^2(1 + \theta)^2}{(\rho + (\rho - 2\mu)\theta)^2}
\]

Eq. (A.26)

Comparing relations (A.6) and (A.26), it is simply proved that always \( \pi_{i_{GPO}} \geq \pi_{i_{DP}} \).

\( \square \)

**Proof of proposition 2:**

According to relations (A.7), (A.8), (A.18), (A.19), and (A.20) if we differentiate them with respect to \( \theta \), we have: \( \frac{\partial p_i}{\partial \theta} \leq 0; \frac{\partial q_i}{\partial \theta} \geq 0; \frac{\partial \pi_i}{\partial \theta} \geq 0; and \frac{\partial \pi_{c}}{\partial \theta} \leq 0 \). They show the results in proposition 2.

\( \square \)

**Proof of proposition 3:**

According to relations (A.7), (A.8), (A.19), and (A.20) if we differentiate them with respect to \( \gamma \), we have: \( \frac{\partial q_i}{\partial \gamma} \leq 0; \frac{\partial w_i}{\partial \gamma} \geq 0; \frac{\partial \pi_i}{\partial \gamma} \leq 0; and \frac{\partial \pi_{c}}{\partial \gamma} \leq 0 \). They show the results in proposition 2.

But the behavior of market price with respect to \( \gamma \) is different and depends on other parameters. \( \frac{\partial p_i}{\partial \gamma} \) is negative most of the time; so, the market price usually decreases in \( \gamma \). In the case of \( \frac{1}{4} < \mu < \frac{1}{2} \), we have a threshold \( \theta < \frac{1 - 2\mu}{4\mu - 1} \) (called \( \tau_1 \)). It means that if \( \theta > \tau_1 \) then market price would increase in \( \gamma \).

\( \square \)

**Proof of proposition 4:**
According to relations (A.7), (A.8), (A.18), (A.19), and (A.20) if we differentiate them with respect to \( \mu \), we have: 
\[
\frac{\partial p_i}{\partial \mu} \leq 0; \quad \frac{\partial q_i}{\partial \mu} \geq 0; \quad \frac{\partial w_{ci}}{\partial \mu} \leq 0; \quad \frac{\partial \pi_c}{\partial \mu} \geq 0; \text{ and } \frac{\partial \pi_c}{\partial \mu} \geq 0.
\]
They show the results in proposition 4.

\[\square\]

**Proof of proposition 5:**

When procuring directly from supplier, OEMs are equal in all parameters; why the only difference between them appears when procuring through GPO. So, it is clear that they have identical behavior under DP strategy and all variables are derived according to lemma 1.

When procuring through GPO, all variables are derived according to lemma 2. All results presented in proposition 5 are simply proved based on general conditions presented in relation (A.21).

\[\square\]

**Proof of proposition 6:**

**GPO:**

It is clear that the GPO always benefits from group purchasing strategy because he obtains zero profit under DP strategy.

**OEM2:**

For OEM2 we have to prove that \( \pi_2^{\text{GPO}} > \pi_2^{\text{DP}} \). From (A.6) and (A.19) we have: 
\[
\pi_2^{\text{GPO}} > \pi_2^{\text{DP}}
\]

\[
\Rightarrow \frac{\xi^2 (\alpha - 1)^2 (1 - \mu)(2 \mu k_2 - (2 - \gamma) \xi)^2}{\eta^2} > \frac{(\alpha - 1)^2 (1 - \mu)}{(2 + \gamma - 2 \mu)^2} \Rightarrow
\]

\[
\frac{\xi^2 (2 \mu k_2 - (2 - \gamma) \xi)^2}{\eta^2} > \frac{1}{(2 + \gamma - 2 \mu)^2} \Rightarrow \frac{\xi^2 (2 \mu k_2 - (2 - \gamma) \xi)^2}{\eta^2} (2 + \gamma - 2 \mu)^2 - \eta^2 > 0 \Rightarrow
\]

\[
\xi^2 (2 \mu k_2 - (2 - \gamma) \xi)^2 (2 + \gamma - 2 \mu)^2 - \eta^2 > 0 \Rightarrow
\]

\[
(\xi (2 \mu k_2 - (2 - \gamma) \xi) (2 + \gamma - 2 \mu) - \eta) (\xi (2 \mu k_2 - (2 - \gamma) \xi) (2 + \gamma - 2 \mu) + \eta) > 0; \quad \text{Eq. (A.27)}
\]

R.H.S of relation (A.27) consists of two parts:

a) \( \xi (2 \mu k_2 - (2 - \gamma) \xi) (2 + \gamma - 2 \mu) + \eta \) \quad \text{Eq. (A.28)}

b) \( \xi (2 \mu k_2 - (2 - \gamma) \xi) (2 + \gamma - 2 \mu) - \eta \) \quad \text{Eq. (A.29)}
As we know, $\eta < 0$ under conditions prescribed in lemma 3. We can simply show that $(2\mu \kappa_2 - (2 - \gamma) \xi) < 0$. As $\xi$ and $(2 + \gamma - 2\mu)$ are always positive; so, part (a) is always negative.

If we prove that part (b) is negative too; then it is proved that $\pi_2^\text{GPO} > \pi_2^\text{DP}$. Now, we focus on part (b):

$$\left(\xi(2\mu \kappa_2 - (2 - \gamma) \xi)(2 + \gamma - 2\mu) - \eta\right) < 0 \Rightarrow \xi(2\mu \kappa_2 - (2 - \gamma) \xi)(2 + \gamma - 2\mu) < \eta.$$ It can simply be shown that under feasible conditions (lemma 3), it is true. So, the proposition for OEM2 is proved.

**OEM1:**

We do a similar procedure for OEM1. To prove that $\pi_1^\text{GPO} > \pi_1^\text{DP}$, we have a relation similar to Eq. (A.27):

$$\left(\xi(2\mu \kappa_1 - (2 - \gamma) \xi)(2 + \gamma - 2\mu) - \eta\right) < 0 \Rightarrow \xi(2\mu \kappa_1 - (2 - \gamma) \xi)(2 + \gamma - 2\mu) < \eta.$$ Eq. (A.30)

L.H.S of relation Eq. (A.30) consists of two parts:

a) $$\left(\xi(2\mu \kappa_1 - (2 - \gamma) \xi)(2 + \gamma - 2\mu) + \eta\right)$$ Eq. (A.31)

b) $$\left(\xi(2\mu \kappa_1 - (2 - \gamma) \xi)(2 + \gamma - 2\mu) - \eta\right)$$ Eq. (A.32)

As mentioned before (proof of OEM2), we can simply show that part (a) is always negative. So, OEM1 prefers GPO to DP strategy if and only if part (b) is negative. Based on the definition of $\eta$ (lemma 2), part (b) is shown as:

$$\xi(2 + \gamma - 2\mu)\left(\xi(-2 + \gamma) + 2\kappa_1\mu\right) - \left(\xi^2 - 4(-1 + \mu)\gamma - 2\gamma\mu + 2\kappa_1\kappa_2\mu\right) < 0.$$ Eq. (A.33)

Being negative of part (b), as $\xi > 0$ then:

$$\xi\kappa_1\left(2 + \gamma - \gamma\kappa_2 - 2\mu\right) + 2\xi^2\left(-1 + \mu\right) + 2\kappa_1\kappa_2\left(-1 + \xi\right)\mu < 0 \Rightarrow$$

$$\left(2\xi(-1 + \mu) + \kappa_1\left(2 + \gamma - \gamma\kappa_2 + 2(-1 + \kappa_2)\mu\right)\right) < 2 \frac{\kappa_1\kappa_2}{\xi}\mu \Rightarrow$$

$$\left(2 - \frac{\xi}{\kappa_1\kappa_2}(-1 + \mu) + (2 + \gamma - 2\mu)\frac{1}{\kappa_2} - (\gamma - 2\mu)\right) < 2 \frac{\mu}{\xi} \Rightarrow \left(2 - \gamma\right) - 2\left(1 - \mu\right)\frac{1}{\kappa_1} + \gamma\frac{1}{\kappa_2} < 2 \frac{\mu}{\xi} \Rightarrow$$

$$\left(2 - \gamma\right)\xi - 2\left(1 - \mu\right)\left(1 + \frac{\kappa_2}{\kappa_1} - \kappa_2\right) + \gamma\left(1 + \frac{\kappa_1}{\kappa_2} - \kappa_1\right) < 2\mu \Rightarrow$$

$$\left((2 - \gamma)(\xi - 1) + \gamma\left(\frac{\kappa_1}{\kappa_2} - \kappa_1\right) - 2\left(1 - \mu\right)\left(\frac{\kappa_2}{\kappa_1} - \kappa_2\right)\right) < 0 \Rightarrow$$
1 \left( \gamma \frac{\kappa_1}{\kappa_2} - 2(1-\mu) \frac{\kappa_3}{\kappa_1} \frac{1-\kappa_1}{1-\kappa_2} \right) - (2-\gamma)(1-\kappa_1) < 0 \quad \Rightarrow \\
2 \gamma \frac{1-\theta_2}{1-\theta_1} - 2(1-\mu) \frac{1-\theta_2}{1-\theta_1} \frac{\theta_1}{\theta_2} - 2\theta_1 < 0.

3 We define a threshold: \( r_2 = 2(1-\mu) \frac{1-\theta_2}{1-\theta_1} \frac{\theta_1}{\theta_2} + 2\theta_1 - \gamma \frac{1-\theta_2}{1-\theta_1} \). \quad \text{Eq. (A.33)}

4 Whenever \( r_2 > 0 \) then \( \pi_{i}^{GPO} > \pi_{i}^{DP} \) and vice versa.

5 □