Optimal Production Inventory Decision with Learning and Fatigue Behavioral Effect in Labor Intensive Manufacturing

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Abstract

Behavioral economic has received much attention recently. Learning and fatigue are two typical behavioral phenomena in industrial production operation processes. The existence of learning and fatigue result in a dynamic change in productivity. In this paper, a classical economic production quantity model is extended to consider the behavioral economic value of learning and fatigue. Based on a real case study, each production cycle is divided into five phases. That is the learning phase, stable phase, fatigue phase, fatigue recovery (rest) phase, and the relearning phase. The new production inventory decision model is incorporated with dynamic productivity and learning-stable-fatigue-recovery effect. Numerical simulation and sensitivity analysis show that appropriate rest alleviates employees fatigue and increases productivity, resulting in a lower average production cost. On the other hand, when the rest time is too high, exceeding a certain value, it leads to the decline of the actual labor productivity, resulting in an increase in the average cost of the system.

Keywords: Behavioral economics, productivity, human factor, learning effect, fatigue effect, production inventory decision

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1. Introduction

In recent years, behavioral economic has received increasing attention [1] (Bendoly et al., 2006). In 2017, Richard Thaler was awarded the Nobel Prize in Economic Sciences for his excellent work in behavioral economics. In 2002, Daniel Kahneman was awarded Nobel Prize in Economic Science for his excellent work in behavioral economics. In management science, there is an increasing trend to consider human factors in operations management, leading to a new management field, the behavioral operations management [2]. Learning effect is a kind of human behavior which reflects the worker’s experience accumulation during production. Since Wright [3] first discovered the phenomenon of learning effect in airplane production and established the first learning effect mathematical model (experience curve), there are many learning behavioral effect studies in production operations management.

Since the cost of retaining skilled labor is expensive, effectively utilizing worker’s learning effect can reduce the workforce cost. Although more automatic equipment is being used nowadays, there are still many labor-intensive manufacturing companies. Therefore, studying how to improve the productivity of workforce is very important. This paper studies the behavioral value of learning and fatigue recovery in production operations decision, and provides managerial implications for practitioners.
The study is motivated by our connection with a garment factory (detail information of the case company background will be presented in section 3). From our observation, it is found that the phenomenon of learning with fatigue effect is obvious, and this inspires our research interest.

Jaber et al., [4] presented the “learning–forgetting–fatigue–recovery model” (LFFRM) that addressed the comprehensive effect of the learning, forgetting, and fatigue recovery by rest in a production process. However, they only established the productivity change curve using the LFFRM model; they did not derive the production inventory decision, or analyze how fatigue recovery impacts the inventory decision.

Although many scholars have extended EPQ (Economic Production Quantity) models with learning effect, they do not consider learning effect and fatigue effect simultaneously. We develop an EPQ model and establish the production inventory decision model for one production cycle time with five phases: learning phase, stable phase, fatigue phase, rest phase and relearning phase. Furtherly, we develop a production inventory optimization model to determine the optimal production lot size considering learning and fatigue effect, and study the impact of fatigue recovery policy on the production inventory decision. Through our study, we can answer the following questions,

(1) What is the optimal fatigue phase production time when the production fatigue during production is considered?

(2) In order to reduce fatigue in the production process, what is the optimal worker’s rest time for fatigue recovery?

(3) Compared with the traditional production-inventory decision, how fatigue effect influences the optimal production decision?
The contribution of our study is to consider multiple stage effects of learning, fatigue and recovery in the production inventory system decision. The significance of our study is the introduction of a new phase of productivity change based on a real case study of a trousers manufacturing factory, and analyze the impact of fatigue recovery on the production inventory decision. This improvement made by our model has shown some value in practical application. The rest of the paper is organized as follows: Section 2 conducts the literature review and compared the differences between previous studies and our work; Section 3 describes the case study background, problem assumptions, and formulates the problem; Section 4 describes the solution algorithm; Section 5 demonstrates the application of the model using numerical example and sensitivity analysis; Section 6 summarizes the conclusions and future research directions.

2. Literature Review

As one of the most important behavioral phenomena, learning effect has received much attention. Many researchers have studied the application of learning effect in production inventory decision. Some of them extended the EPQ models to consider learning effects [5-7]. Other authors have combined the human learning effect into the economic production quantity (EPQ) models to improve the application value of the traditional EPQ models [8-9]. Salameh et al. [10] developed an economic production quantity model with learning effect. Later, Jaber and Salameh [11] extended Salameh’s [10] model by taking into account of shortage and backorder. Balkhi [12] studied the learning effect on the production lot size for deteriorating and partially backordered items considering varying demand and deterioration rate. Boer and Zwart [13] studied dynamic pricing and learning for perishable products with finite initial inventory. Teng et al. [14] analyzed the effect of learning rate on the optimal credit payment period and the optimal batch size in the EPQ model.
Khan et al. [15] studied an integrated vendor–buyer inventory decision model with learning in production at the vendor’s end. Mahata [16] investigated the learning effect on the unit production time for an imperfect production system with partial backlogging and fuzzy environment. Khan [17] investigated the role of variable lead time, learning in production and screening errors in a vendor–buyer supply chain with defective items.

Besides the learning effect, another kind behavioral phenomenon, known as forgetting effect, also exists. During operations, experience and skills can be accumulated by learning effect, but they can also be forgotten with time. This is called forgetting effect. Forgetting effect can reduce production rate and output. Jaber and Bonney [18] explored both the learning effect and the forgetting effect in a production process. They developed a learning-forgetting curve to model the learning and forgetting effect with production break. Chiu and Chen [19] established a dynamic lot size model considering the effects of learning and forgetting during setup and production. Alamri and Balkhi [20] studied the learning and forgetting effect on the production lot size for deteriorating items with varying demand. Teyarachakul et al. [21] investigated the effect of learning and forgetting with respect to batch sizes. Their finding is that small batch production is better when there is learning and forgetting effect. Zanoni et al. [22] studied how learning and forgetting effects give flexibility to the supply chain stake holders under VMI (Vendor managed inventory) environment. Glock and Jaber [23] studied a multi-stage production-inventory decision with product rework; they considered the learning and forgetting effect in production and rework. Teyarachakul et al. [24] studied the long-term characteristics of workers’ skill levels under learning and forgetting.
However, the above literatures failed to consider fatigue during production [25-27]. Fatigue can lead to increasing error rates, quality problems, and reduction in efficiency and productivity. When fatigue becomes chronic or excessive, it can contribute to work-related disorders [28-30]. Many reasons may cause fatigue, such as overwork and sleeplessness [25], and physical discomfort [31]. A few authors have considered the fatigue phenomenon in production operation decision. Battini et al [32] developed an economic lot-sizing model that considered ergonomic issues in calculating optimal lot sizes, in which a rest allowance function is used to take account of recovery periods that help to maintain low levels of fatigue and ergonomic risks.

Other authors have also considered human errors and customer satisfaction in the production inventory decision. Kang et al., [33], incorporated human errors into the decision making process focusing on group technology inventory model. Cheng et al., [34] assumed the inspection process was carried out by the vendor and the defective items are disposed in multiple batches. They proposed an optimal integrated vendor-buyer inventory model with defective items, but did not consider inspection error. Customer satisfaction was considered by Besheli et al., [35] in a fuzzy dynamic muti-objective, muti-item model. Jaber and Glock [29] developed a new learning curve model that has cognitive and motor components.

Based on the above literature review, we summarize the major characteristics of the models and compare them with our work. The comparison is shown in Table 1. Unlike previous studies, we consider learning effect, fatigue effect in a production inventory environment.

Inserted Table 1

3. Formulation of the problem
In this section, we describe the case background, motivation of our study, data collection and develop a mathematical model.

3.1 Case background examination

This study is motivated by our involvement in a labor intensive manufacturing company. The X Company in our case study is a trousers production factory with 200 workers. There are nearly 50 processing procedures in the production line of trousers, and the processing flow can be classified into five stages.

(1) Production preparation. In this stage, the workers prepare the needed clothes; the main works include select and check cloths according to the sample size and specification.

(2) Tailoring cloth. In this stage, the workers cut cloths into different components and sizes according to the specification of trousers. This work is half automation by workers operating the cloth-cutting machines.

(3) Sewing. This is the core step of garment manufacturing. In this stage, the workers sew clothes into different trousers according to the production instruction. This is labor intensive work, and the work loading is very high.

(4) Ironing. When trousers are finished, in order to make smooth the faces of trousers, ironing is a necessary process for garment production. This work usually is completely manual, so the loading is also very high.

(5) Check. This is the last operation of garment manufacturing before package and storage. It is a necessary quality check done manually.

Inserted Fig.1
From the above production processes, most works in the garment factory are done manually. With the accumulation of experience, the production efficiency of the workers increases due to a learning effect phenomenon. However, fatigue and decreasing production rate may occur if working time is prolonged.

In order to understand the behavioral phenomena in production, on-site data collection is done for different production phases. The following tables illustrated some of the data we collected. Table 2 shows a worker’s sewing time for sewing the waist of a pair of trousers.

Inserted Table 2

In Table 2, some data points are abnormal, for example, the data point of 61.10 and 66.06 at the observation instant of 16:37 and 16:44. The reason of the results is because the worker just checked the trousers without actually working on the waist of the trousers. We then measure the worker’s productivity (unit of trousers /10 minutes), the result is depicted in the Fig 2.

Inserted Fig. 2

After deleting the abnormal data points, we depict the result in Fig3.

Inserted Fig. 3

From the productivity curve shown in Fig.3, we can see that a worker’s productivity change reveals a typical multiple phases change law, i.e., at the beginning, productivity increases with learning effect, it then enters a relative stable phase; with extended time, productivity decreases due to a fatigue.

Another example was shown in Table 2. This example was conducted in the process of ironing for an observed worker B. This work was conducted from the morning till afternoon. Based
on the original data in Table 2, we can calculate the productivity of worker B (trousers /10 minutes) in one day. Similar to the situation of worker A, work B did not very follow the work standard strictly; there are some interferences leading to abnormal data points. As expected, the productivity change curve was not consistent with our expectation.

Fig. 4 is the productivity curve for worker B based on the original data of Table 3. From Fig. 4, we can also see that worker B’s productivity also reveals the similar change law like worker A, i.e., productivity increases at the beginning, it then enters a relative stable phase; with extended time, productivity decreases due to a fatigue.

Inserted Table 3

In Fig. 4, the fatigue recovery effect also can be seen. Worker B works from the morning till afternoon with a break for lunch at noon. After a time of rest, his productivity has increase slightly as compared with the time before lunch break. In the afternoon, after an initial rises in productivity due to learning, it then maintains constant for a while before it decreases in the late afternoon.

Inserted Fig. 4

3.2 Problem description

Based on the examination of above case, we develop a theoretical production inventory decision model considering the dynamic productivity phenomena during production with learning and fatigue, recovery (rest) and relearning effect. Fig. 5 shows the production-inventory profile (shown in the upper part of Fig. 5) under the dynamic productivity process (shown in the lower part of Fig. 5).
Fig. 5, (2) shows the assumption that the production process includes learning phase, stable phase, fatigue phase, rest (fatigue recovery) phase. In learning phase, the productivity increases, in stable phase, the productivity keeps stable, in fatigue phase, the productivity decreases, in rest phase, the productivity is zero, in relearning phase, the productivity increases. This assumption fits the examination of above example. In our case background, for example, worker A production process can be divided into learning phase, stable phase and fatigue phase (Fig.3). For worker B, in one day, his production process can also be divided into learning phase, fatigue phase, rest phase, relearning phase and fatigue phase (Fig.4). The observed dynamic productivity changing process combines the phenomena of worker A and worker B. The production process includes learning phase, stable phase, fatigue phase, rest (fatigue recovery) phase, and relearning phase.

Based on the above productivity change characteristics, we extend the classic economic production quantity model considering the learning and fatigue effect in production. We then analyze how learning, fatigue and fatigue recovery impact the production decision. The study derives the optimal production schedule and the most effective rest time for each stage of production.

Inserted Fig. 5

3.3 Assumptions and notations

The following assumptions are used in this study:

(1) The manufacturer’s demand rate is continuous and constant. Shortage is not allowed.

(2) There are learning effect and fatigue effect during production.

(3) The production process includes learning phase, stable phase, fatigue phase, rest (fatigue recovery) phase, and relearning phase.
(4) Productivity is greater than demand during the learning, stable and fatigue phases.

(5) The rest time for fatigue recovery depends on the production time.

(6) The productivity in the relearning phase depends on the rest time.

The assumption 3 is supported by the worker A’s and B’s productivity behavior in our case background.

All notation used in this paper is listed in Table 4.

Inserted Table 4

3.4 Model development

During the production phase, the inventory equation can be expressed as:

$$\frac{dI_i(t)}{dt} = P_i(t) - D$$

(1)

with the boundary conditions: $I_i(0)=0$, $t_0=0$, $I_{i-1}(t_{i-1})=I_i(t_{i-1})$, $i=2,3,4,5$.

Integrating of equation (1), the relation between the inventory level and time can be obtained:

$$I_i(t) = I_{i-1}(t_{i-1}) + \int_{t_{i-1}}^{t} [P_i(u) - D]du$$

(2)

The holding cost in different $i$ phase is:

$$C_{ih} = h \int_{t_{i-1}}^{t} I_i(t)dt = h \int_{t_{i-1}}^{t} \int_{t_{i-1}}^{u} [P_i(u) - D]du dt + h \int_{t_{i-1}}^{t} I_{i-1}(t_{i-1})du dt$$

(3)

In the following analysis, we model the total production cost for different production time

Case 1 When $t \leq t_i$, production is conducted during the learning phase only.

When the total production during the learning phase can meet the total demand, the manufacturer will not increase production. To minimize the average cost per unit time, it is necessary to derive the optimal production time $t^*$.
From the Wright learning curve theory (Wright, 1936), suppose \( T_0 \) is the production time of the first unit product, the production time for \( q \)th product is expressed as \( T_o q^{-b} \). The total time to produce \( Q \) quantity products is: \( t = \int_0^Q T_o q^{-b} \, dq = \frac{T_o Q^{1-b}}{1-b} \). The dynamic productivity (number of product per unit time) at instant \( t \) is: \( p(t) = \frac{dQ}{dt} \), one has:

\[
P_1(t) = \begin{cases} 
\frac{1}{1-b} \left( \frac{1-b}{T_o} \right)^{\frac{1}{1-b}} \frac{b}{t^{1-b}} & 0 < t \leq t_1 \\
\frac{1}{T_o} & t = 0 
\end{cases}
\]

(4)

Let \( \alpha = \frac{1}{1-b} \left( \frac{1-b}{T_o} \right)^{\frac{1}{1-b}} \), \( \beta = \frac{b}{1-b} \), since \( 0 < b < 1 \), and \( \beta > 1 \), Equation (4) can be rewritten as:

\[
P_1(t) = \begin{cases} 
\alpha t^\beta & 0 < t \leq t_1 \\
\frac{1}{T_o} & t = 0 
\end{cases}
\]

(5)

From equation (5), when \( 0 < t \leq t_1 \), the inventory level for any time \( t \) during the learning phase is:

\[
I_1(t) = \int_0^t (\alpha u^\beta - D) \, du = \left[ \frac{\alpha}{1+\beta} t^{1+\beta} - Dt \right]
\]

(6)

At the end of the learning phase, the inventory level from equation (6) becomes:

\[
S_1 = \frac{\alpha}{1+\beta} t^{1+\beta} - Dt. 
\]

The maximum output of the manufacturer is: \( Q_a = \frac{\alpha}{1+\beta} t^{1+\beta} \). The time available for the consumption is: \( T_{id} = \frac{\alpha t^{1+\beta}}{(1+\beta)D} - t \), where \( T_{id} \) is the non-production time. The total production cycle is: \( T_i = \frac{\alpha t^{1+\beta}}{(1+\beta)D} \). The holding cost during the learning phase is:

\[
C_{th} = h \int_0^t I_1(t) \, dt = h \left[ \frac{\alpha t^{2+\beta}}{(1+\beta)(2+\beta)} - \frac{1}{2} Dr^2 \right]
\]

(7)
The average cost within a cycle is:

\[
ATC(t) = \frac{1}{T_i} \left[ A + t h + \frac{h}{2D} \right]^{\frac{2}{1 + \beta}}
\]

Substitute \(T_i, C_{th}, S_1\) into (8), the average total cost is:

\[
ATC_1(t) = \frac{(1 + \beta)}{\alpha t^{1+\beta}} \left[ A + t h + \frac{h}{2D} \frac{1}{(1 + \beta)(2 + \beta)} \right]^{\frac{2}{1 + \beta}} + \frac{h}{2D} \left[ \frac{1}{(1 + \beta)} \right]^{\frac{2}{1 + \beta}}
\]

**Proposition 1.** \(ATC_1(t)\) is a convex function of \(t\), there exists \(t^*\) for optimal \(ATC_1(t)\).

**Proof.**

Taking the first and second derivatives of \(ATC_1(t)\), one has

\[
\frac{\partial ATC_1(t)}{\partial t} = \frac{[\alpha (2 + \beta)t^\beta - 2(1 + \beta)D]}{2(2 + \beta)} - \frac{t^{2-\beta}A(1 + \beta)^2 D + b D(1 + \beta)t^{1-\beta}}{\alpha}
\]

\[
\frac{\partial^2 ATC_1(t)}{\partial t^2} = \frac{\beta t^{-1} a h}{2} + \frac{(2 + \beta)t^{-3+\beta}A(1 + \beta)^2 D + b D(1 + \beta)^2 t^{-2-\beta}}{\alpha}
\]

Obviously, \(\frac{\partial^2 ATC_1(t)}{\partial t^2} > 0\), thus \(ATC_1(t)\) is a convex function of \(t\). Theoretically, we can derive the optimal solution by setting \(\frac{\partial ATC_1(t)}{\partial t} = 0\). However, because of the nonlinearity of the equation, the closed form solution \(t^*\) is difficult to obtain. Therefore, the optimal time \(t^*\) is obtained using the one-dimensional search method.

**Proposition 2** The optimal production time \(t^*\), the maximum output \(Q_1^*\), and the maximum inventory \(S_1^*\) have the same variation trend (increasing or decreasing) with respect to the learning coefficient \(b\).

**Proof.**

From proposition 1, take the first order derivative of \(ATC_1(t)\) with respect to \(t\); letting:

\[
F(t^*, \beta) = \frac{[\alpha (2 + \beta)(t^*)^\beta - 2(1 + \beta)D] ah}{2\alpha (2 + \beta)} - \frac{(t^*)^{2-\beta}A(1 + \beta)^2 D + b D(1 + \beta) (t^*)^{1-\beta}}{\alpha}
\]

Taking the first order derivatives of \(F(t^*, \beta)\) with respect to \(t^*\) and \(\beta\), we get
\[ F_r = \frac{\beta(t^*)^{\beta-1} ah}{2} + \frac{(2+\beta)(t^*)^{3-\beta} A(1+\beta)^2 D + bDl(1+\beta)^2 (t^*)^{-2-\beta}}{\alpha} > 0 \]

\[ F_\beta = \frac{h\left[ \alpha \ln(t^*)(2 + \beta)^2 (t^*)^{\beta} - 2D \right]}{2(2+\beta)^3} + \frac{DA(1+\beta)\left[-2 + (1+\beta) \ln(t^*)\right](t^*)^{-2-\beta}}{\alpha} + \frac{DL\left[\beta(1+\beta) \ln(t^*) - 2\beta - 1\right](t^*)^{-1-\beta}}{\alpha}. \]

The relationship of \( t^* \) and \( \beta \) is expressed as \( \frac{\partial t^*}{\partial \beta} = -\frac{F_\beta}{F_r}. \) From \( \beta = \frac{b}{1-b}, \) one has:

\[ \frac{\partial \beta}{\partial b} = \frac{1}{(1-b)^2} > 0. \] From \( Q_1^* = \frac{\alpha}{1+\beta} (t^*)^{\beta}, \) one has: \( \frac{\partial Q_1^*}{\partial t^*} = \alpha(t^*)^{\beta} > 0. \) Based on our assumption, we have:

\[ \frac{\partial S_1^*}{\partial t^*} = \alpha(t^*)^{\beta} - D > 0. \]

If \( F_\beta > 0, \) then \( \frac{\partial t^*}{\partial b} < 0, \frac{\partial Q_1^*}{\partial b} < 0, \frac{\partial S_1^*}{\partial b} < 0. \) If \( F_\beta < 0, \) then \( \frac{\partial t^*}{\partial b} > 0, \frac{\partial Q_1^*}{\partial b} > 0, \frac{\partial S_1^*}{\partial b} > 0. \) This means that the optimal \( t^*, \) \( Q_1^* \) and \( S_1^* \) have the same change law. □

Proposition 3 The optimal production time \( t^*, \) the maximum output \( Q_1^*, \) and the maximum inventory \( S_1^* \) increase with respect to the initial unit production time \( T_o. \)

Proof.

Set

\[ F(t^*, \alpha) = \frac{\left[\alpha(2+\beta)(t^*)^{\beta} - 2(1+\beta)D\right] ah}{2\alpha(2+\beta)} - \frac{(t^*)^{-2-\beta} A(1+\beta)^2 D + bDl(1+\beta)(t^*)^{-1-\beta}}{\alpha} \]

Taking the first order derivatives of \( F(t^*, \alpha) \) with respect to \( t^* \) and \( \alpha, \) one has:

\[ F_{t^*} = \frac{\beta(t^*)^{\beta-1} ah}{2} + \frac{(2+\beta)(t^*)^{-3-\beta} A(1+\beta)^2 D + bDl(1+\beta)^2 (t^*)^{-2-\beta}}{\alpha} > 0 \]
\[ F_a = \frac{(t^*)^\beta h}{2} + \frac{(t^*)^{-2+\beta} A(1+\beta)^2 D + bDl(1+\beta)(t^*)^{-1-\beta}}{\alpha^2} > 0 \]

Since \( F_a > 0 \), \( F_i > 0 \), thus \( \frac{\partial t^*}{\partial \alpha} = -\frac{F_a}{F_i} < 0 \).

From \( \alpha = \frac{1}{1-b} \left( 1-b \right)^{1-\beta} \), we have \( \frac{\partial \alpha}{\partial t} = -(1-b)^{1-\beta} T_o^{-2-b} < 0 \).

\[ \frac{\partial t^*}{\partial t_o} = \frac{\partial t^*}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial t_o} = \frac{F_a}{F_i} \cdot (1-b)^{1-\beta} T_o^{-2-b} > 0. \]

\[ \frac{\partial Q}{\partial t} = \frac{\partial t^*}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial t} = \alpha(t^*)^\beta \cdot \frac{F_a}{F_i} \cdot (1-b)^{1-\beta} T_o^{-2-b} > 0. \]

\[ \frac{\partial S}{\partial t_o} = \frac{\partial t^*}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial t_o} = \left[ \alpha(t^*)^\beta - D \right] \frac{F_a}{F_i} \cdot (1-b)^{1-\beta} T_o^{-2-b} > 0. \]

\[ \Box \]

**Remark:** When the production is carried out in the learning phase only, when \( b \to 0 \), thus, \( \beta = \frac{b}{1-b} \to 0 \) one has \( P_1 \to \frac{1}{T_o} \), where the productivity is constant. That is a classical EPQ model.

**Case 2** When \( t_1 < t \leq t_2 \), production goes beyond the learning phase and ends in the stable phase.

At the end of the learning phase, the productivity level reaches, \( P_2(t = t_1) = P_1(t = t_1) = \alpha t_1^\beta \), the manufacturer continue to produce at a steady level of productivity. From the boundary condition \( I_1(t_1) = I_2(t_1) \), the inventory change with time during the stable phase is:

\[ I_2(t) = (\alpha t_1^\beta - D)t - \frac{\alpha \beta}{1+\beta} t_1^{1+\beta}, \quad t_1 < t \leq t_2. \quad (10) \]

The holding cost during stable phase is:

\[ C_{th} = h \int_0^t I_2'(t) dt = h \left[ \frac{1}{2} (\alpha t_1^\beta - D)(t^2 - t_1^2) - \frac{\alpha \beta}{1+\beta} t_1^{1+\beta}(t - t_1) \right]. \quad (11) \]

From equation (10), the inventory level at \( t \) is \( S_2 = (\alpha t_1^\beta - D)t - \frac{\alpha \beta}{1+\beta} t_1^{1+\beta} \). The maximum output is \( Q_2 = \frac{\alpha t_1^\beta (1+\beta)t - \alpha \beta t_1^{1+\beta}}{(1+\beta)}. \)
The total time of a production cycle is 
\[ T_2 = \frac{\alpha t_1^\beta (1 + \beta)t - \alpha \beta t_1^{1+\beta}}{(1 + \beta)D}. \]

The average cost of the production system within a cycle, including costs of learning and stable period, is:

\[ ATC_2(t) = \frac{D}{S_2 + Dt} \left( A + lt + \sum_{i=1}^{2} C_{ih} + \frac{h}{2D} S_2^2 \right) \tag{12} \]

where, inside the bracket of the first term is the setup cost, the second term is the labor cost, the third term is the holding cost of inventory during production of leaning and stable phases, \( C_{ih} \) and \( C_{2h} \) are from (7) and (11) respectively. The forth term is the holding cost of the inventory during non-production.

Case 3  
When \( t > t_2 \), the production time includes the learning, and the stable and the end of the fatigue phase.

We assume that the productivity function in the fatigue phase is \( P(t) = ae^{-ct} + dt^{-f} + g \). This assumption is based on two factors that affect human fatigue behavior. The two factors are about physiological and labor intensity. In the natural state, the productivity is getting smaller and smaller as the body is able to bear less and less load. Different people have different physiological load. If the labor intensity is large, the employees will be fatigue quickly. In this formula, the first term \( ae^{-ct} \) is the productivity under normal physiological load; it diminishes with time \( t \), and \( c \) is the physiological load index. The second term \( dt^{-f} \) is the productivity under normal labor load; it diminishes with time \( t \), and \( f \) is the labor load index. The third term is the initial productivity when becoming fatigue.

The reasonability of this assumption can be explained theoretically and practically. A laboratory experiment was designed by Okogbaa (1983) to examine mental work output with and without rest. Okogbaa (1983) observed hyperbolic as well as exponential decay function. Bechtold
(1988) found out that when workers had fatigue process during production, instant productivity is an exponential function with passage of time. Lindstrom et al. (1997) observed that the relationship between the work load and maximum endurance is exponential function. Practically, from our examination case, from worker A’s productivity curve in Fig.3 and Worker B’s productivity curve in Fig.4, we think the productivity curve in fatigue phase is a function combining of hyperbolic function and exponential function. Consequently, the assumption is reasonable.

From the boundary condition, \( P_2(t_2) = P_3(t_f) = 0 \), since \( P_2(t_2) = P_1(t_1) = \alpha t_1^\beta \), one has: \( g = \alpha t_1^\beta - ae^{-ct_2} - dt_2^{-f} \). Thus, during fatigue phase, the productivity can be expressed as:

\[
P_1(t) = a(e^{-at} - e^{-ct_2}) + d(t^{-f} - t_2^{-f}) + \alpha t_1^\beta.
\] (13)

Based on the boundary condition \( I_2(t_2) = I_3(t_3) \), the inventory level changes with time is:

\[
I_3(t) = -\frac{a}{c}(e^{-at} - e^{-ct_2}) + \frac{d}{1-f}(t^{1-f} - t_2^{1-f}) + (\alpha t_1^\beta - ae^{-ct_2} - dt_2^{-f} - D)(t - t_2) +
\]

\[
(\alpha t_1^\beta - D)t_2 - \frac{a\beta}{1+\beta} t_1^{1+\beta}, \quad t_2 < t \leq t_3
\] (14)

When the production ends at \( t \) during fatigue production phase, the inventory level is \( S_3 = I_3(t) \).

The holding cost during fatigue phase is:

\[
C_{3h} = h \frac{a}{c^2}(e^{-ct} - e^{-ct_2}) + h \frac{d}{(1-f)(2-f)}(t^{2-f} - t_2^{2-f}) + \frac{1}{2} h(\alpha t_1^\beta - ae^{-ct_2} - dt_2^{-f} - D)(t^2 - t_2^2) + h(t
\]

\[-t_2) \left[ \frac{a}{c} e^{-ct_2} - \frac{d}{1-f} t_2^{1-f} + (ae^{-ct_2} + dt_2^{-f})t_2 \right] - h(t - t_2) \frac{a\beta}{1+\beta} t_1^{1+\beta}
\] (15)

The average cost of a production cycle is:

\[
ATC_3(t) = \frac{D}{Dt} + S_3 \left( A + Lt + \sum_{i=1}^{3} C_{ih} \frac{hS_i^2}{2D} \right).
\] (16)
The problem is to minimize $ATC_j(t)$, which is an unconstrained nonlinear programming equation. Due to the nonlinearity of equation (16), it is difficult to derive the closed form optimal solution. Therefore, Newton iteration is used to solve the problem.

Case 4 When $t > t_4$, production includes the learning, stable, fatigue, rest and relearning phases.

When production goes to the rest phase, production stops, the productivity rate is zero during rest time $t_3 \leq t \leq t_4$. Though there is no production during rest time, demand still exists; the inventory level at time $t$ in the rest phase is:

$$I_4(t) = -\frac{a}{c}(e^{-ct_3} - e^{-ct_2}) + \frac{d}{1-f}(t_3^{i-f} - t_2^{i-f}) + (\alpha t_1^\beta - a e^{-ct_2} - dt_2^{i-f} - D)(t_3 - t_2) + (\alpha t_1^\beta - D)t_2 - \frac{\alpha \beta}{1 + \beta} t_1^{i+\beta} - D(t - t_3) \quad t_3 < t \leq t_4.$$

The holding cost during rest phase is:

$$C_{ch} = h\left(-\frac{a}{c}(e^{-ct_3} - e^{-ct_2}) + \frac{d}{1-f}(t_3^{i-f} - t_2^{i-f}) + (\alpha t_1^\beta - a e^{-ct_2} - dt_2^{i-f} - D)(t_3 - t_2) + (\alpha t_1^\beta - D)t_2 - \frac{\alpha \beta}{1 + \beta} t_1^{i+\beta} + D t_2\right) (t - t_3) - \frac{1}{2} h D(t^2 - t_3^2)$$

The longer the time of fatigue production, the more time is needed to relieve fatigue. The rest time depends on the production time of the fatigue phase, and the relationship between the rest time and fatigue production time is $t_4 - t_3 = \delta(t_3 - t_2)$, where $\delta > 0$ is a scale factor. After some time of rest, the workers’ physical energy recovers to a higher level and the workers’ productivity is higher than the productivity at the end of the fatigue phase. This phenomenon is called fatigue recovery. From equation (4) and after some derivation, the productivity in the relearning phase is:
\[ P_s(t) = \frac{1}{1-b} \left[ \frac{\delta(t_3-t_2)}{T_o} \right]^{\frac{1}{1-b}} (t-t_2)^{\frac{b}{1-b}}, \quad t_4 < t \]

Let \( \gamma = \frac{1}{1-b} \left[ \frac{\delta(t_3-t_2)}{T_o} \right]^{\frac{1}{1-b}} \), \( \beta = \frac{b}{1-b} \), then \( P_s(t) = \gamma(t-t_4)^\beta \).

Based on the boundary condition \( I_4(t_4) = I_5(t_4) \), the inventory level changes with time is:

\[ I_5(t) = \frac{\gamma}{1+\beta}(t-t_4)^{1+\beta} - D(t-t_4) - \frac{\alpha\beta}{1+\beta} t_1^{1+\beta} - \frac{a}{c} \left( e^{-ct_3} - e^{-ct_2} \right) + \frac{d}{1-f} (t_3^{1-f} - t_2^{1-f}) + \]

\[(\alpha t_1^\beta - ae^{-ct_2} - dt_2^{1-f} - D)(t_3-t_2) - D(t_4-t_3) + (\alpha t_1^\beta - D)t_2 \quad , \quad t_4 < t \]  \hspace{1cm} (20)

The holding cost during relearning phase is:

\[ C_{sh} = \frac{h\gamma(t-t_4)^{2+\beta}}{(1+\beta)(2+\beta)} - \frac{1}{2} hD(t^2-t_4^2) + h \left[ Dt_3 - \frac{a}{c} \left( e^{-ct_3} - e^{-ct_2} \right) + \frac{d}{1-f} (t_3^{1-f} - t_2^{1-f}) \right] + \]

\[(\alpha t_1^\beta - D)t_2 - \frac{\alpha\beta}{1+\beta} t_1^{1+\beta} + (\alpha t_1^\beta - ae^{-ct_2} - dt_2^{1-f} - D)(t_3-t_2) \]

\[(t-t_4) . \]  \hspace{1cm} (21)

The production time of relearning phase depends on the rest time; the longer the rest time, the longer is the production time in the relearning phase. The relearning production time \( t-t_4 \) is equal to \( \kappa \delta(t_3-t_2) \), where, \( \kappa \) is a scale factor. From \( t_4-t_3 = \delta(t_3-t_2) \), we derive \( t_3 = \frac{t+(1+\kappa)\delta t_2}{1+\delta+\kappa\delta} \), and \( t_4 = \frac{t+\delta t-\kappa\delta t_2-2\kappa^2\delta^2 t_2}{1+\delta+\kappa\delta} \).

After the end of the relearning phase, the maximum inventory level \( S_4 \) from equation (20) is:

\[ S_4 = \frac{\gamma}{1+\beta}(t-t_4)^{1+\beta} - Dt + D t_3 - \frac{a}{c} \left( e^{-ct_3} - e^{-ct_2} \right) + \frac{d}{1-f} (t_3^{1-f} - t_2^{1-f}) + \]

\[(\alpha t_1^\beta - ae^{-ct_2} - dt_2^{1-f} - D)(t_3-t_2) + (\alpha t_1^\beta - D)t_2 - \frac{\alpha\beta}{1+\beta} t_1^{1+\beta} . \]  \hspace{1cm} (22)
When the production stops, the time to consume the maximum inventory is $S_x / D$. The inventory cost during the depletion period is $hS_x^2 / 2D$. The production time and consumption time in a cycle (also known as the production cycle) is $T = t + S_x / D$.

The average cost of a production cycle is:

$$ATC_x(t) = \frac{D}{D} + S_x \left( A + lt + \sum_{j=1}^{\infty} C_{ij} + \frac{hS_x^2}{2D} \right).$$

(23)

Due to the high nonlinearity of equation (23), a closed form solution is not possible. Therefore, the Newton iteration method is used to solve the optimal solutions.

4. Solution algorithm

For the complex nonlinear models developed in last section, we adopt the Newton Raphson iteration method in solving the problem. In the Newton Raphson iteration method, two Taylor expansions of the objective function to minimize the function. The first few terms of the Taylor series are used to derive the roots of the equations. It is an approximation method for linearizing nonlinear equations. That will enable us to derive a square convergence near the single root of the equation. The method is to obtain $t^*_3$ in Case 4.

Set $f(X) = ATC(t_3)$,

The solution steps are as follows:

**Step 1** Given the initial value $X_0$, set the allowable error $\varepsilon$ ($\varepsilon = 0.0001$ in this case);

**Step 2** Calculate $X_n = X_{n-1} - \frac{f(X_{n-1})}{f'(X_{n-1})}$;

**Step 3** if $|X_n - X_{n-1}| < \varepsilon$, then go to Step 4, if $|X_n - X_{n-1}| \geq \varepsilon$, then go to Step 2;

**Step 4** $t^* = X^* = X_n$.
In Case 4, the manufacturer should stop production at the moment \( t^*_3 \), where
\[
t^*_3 = \frac{t^* + (1 + \kappa)\delta t_2}{1 + \delta + \kappa\delta}.
\]
The rest time is \( \delta(t^*_3 - t_2) \). The reproduction time is \( t^*_4 \), where
\[
t^*_4 = \frac{t^* + \delta t^* - \kappa \delta t_2 - 2\kappa \delta^2 t_2 - 2\kappa^2 \delta^3 t_2}{1 + \delta + \kappa\delta}.
\]
The relearning production time is \( \kappa \delta(t^*_4 - t_2) \). The same procedure is used to derive \( t^* \) in Case 3.

5. Numerical example and sensitivity analysis

5.1 Parameters setting

A numerical example is provided to validate the proposed model. The following basic data are used: \( A = 100 \), \( D = 12 \), \( l = 10 \), \( h = 0.2 \), \( T_o = 0.04 \), \( b = 0.54 \), \( a = 50 \), \( c = 1.3 \), \( d = 180 \), \( f = 1.28 \), \( \varepsilon = 0.04 \), \( \delta = 2.04 \), \( \kappa = 0.9 \).

5.2 Optimal solution

(1) Case 1. From proposition 1, we have \( t^* = 0.7705 \), the optimal average cost \( ATC(t^*) = 21.47 \). The maximum inventory \( S^*_1 = 105 \), the maximum output \( Q^*_1 = 115 \), the non-production time for a period is \( 8.7921 \), and the total time per cycle is \( 9.5626 \).

(2) Case 2. When the learning phase production time is \( t_1 = 0.50 \), the optimal production time is \( t^* = 0.8592 \), the stable phase production time is \( 0.3592 \) and the optimal average cost is \( 21.52 \).

(3) Case 3. Given \( t_1 = 0.50 \), and \( t_2 = 0.75 \), from the algorithm solving procedure, the optimal total production time is \( t^* = 0.8708 \). The fatigue phase production time is \( 0.1208 \). The maximum yield is \( Q^*_3 = 114 \), the maximum inventory is \( S^*_3 = 104 \), and the optimal average cost is \( 21.53 \). Because of fatigue effect, the average cost in Case 3 is more than that in Case 2.

(4) Case 4. Given \( t_1 = 0.50 \), and \( t_2 = 0.75 \), with the Newton Raphson method, we derive \( t^*_3 = 0.8491 \). That means the production should stop at the moment 0.8491, it should be followed by a rest, the rest time is 0.2022. After the end of the rest time, production with relearning restarts. The
relearning production time is 0.182. The optimal average cost of the system is 21.62. Production time during fatigue phase is 0.0991, which is less than 0.1208 in Case 3.

5.3 Sensitivity analysis and managerial implications

(1) The influence of the learning factor $b$ and the initial production time $T_o$ in Case 1

Fig. 6 shows that when the learning coefficient is less than a critical value, it increases as the optimal production time decreases. When the learning coefficient is greater than a critical value, it increases as the optimal production time increases. The influence of the learning coefficient on the maximum yield is consistent with the optimal production time (Proposition 2). The greater value of $T_o$ means the lower initial productivity, thus it takes a longer time to produce a certain product that meets the required quantity. With the increase in the initial production time, the optimal production time increases linearly (see Fig. 6).

Inserted Fig. 6 and Fig. 7

As illustrated in Fig. 7, with the increase in learning coefficient, the optimal total average cost shows an inverted U character type feature, i.e., the cost increases first and then decreases. Only when the learning coefficient is bigger than a critical value, the effect of learning on cost reduction is obvious. On the other hand, with the increase of initial production time $T_o$, the optimal average total cost decrease linearly.

From Fig. 6 and Fig. 7, we can see that the impact of learning coefficient $b$ on the production time and cost are reverse. Similar condition exists with the initial production time $T_o$.

Observation 1. The roles of learning coefficient in reducing the production time and the total cost are reversed. It is necessary to choose the right learning coefficient to balance the optimal production time and optimal cost.
(2) **The impact of** \( t_1 \) **on the total production time and the average cost in Case 2.**

Fig 8 and 9 show the impacts of \( t_1 \) on the total production time and the average cost in Case 2.

Inserted Fig.8 and Fig.9

In Fig.8, the blue colored line is the optimal total production time \( (t^*) \), and red colored line is the stable production time \( (t^*-t_1) \). It is shown that with the increase in learning production time \( (t_1) \), both the total optimal production time and the stable production time decreases. When the learning production time approaches 0.7705, the stable production time tends to be zero in the stable phase, and the total production time approaches 0.7705.

Fig.9 shows the influence of the learning production time on the optimal average cost. It is shown that with the increase of learning production time, the cost curve is an S type curve. When \( t_1 < 0.32 \), with the increase of learning production time, the maximum inventory increases, thus the system's optimal average cost increases. When \( t_1 > 0.32 \), with the increase of the learning production time, the optimal average cost decreases.

**Observation2:** *If production process only involves learning phase and stable phase, the effect of prolonging learning production time on the reduction of cost is obvious; thus, prolonging production time is beneficial.*

(3) **The influence of** \( t_1,t_2 \) **on the total production time and the average cost in Case 3**

When the production goes beyond the production and stable phases, and ends in the fatigue phase in Case 3, the impacts of \( t_1,t_2 \) on the total production time and average cost are illustrated in Fig. 10 and 11.
From Fig.10, it is shown that when \( t_1 < 0.36 \), the increase of \( t_1 \) prolongs fatigue production time, thus the total production time increases. When \( t_1 > 0.36 \), the increase of \( t_1 \) shortens the fatigue production time, thus the total production time decreases. When the learning production time is given (in the case, \( t_1 = 0.50 \)), the effect of \( t_2 \) on total production time is similar to the effect of \( t_1 \) on the total production time. When \( t_2 < 0.62 \), the increase of stable production time prolongs fatigue production time, thus the total production time increases. When \( t_2 > 0.62 \), the increase of stable production time shortens the fatigue production time, thus the total production time decreases.

From the Fig.11, we can see that the impact of \( t_1, t_2 \) are similar. Prolonging of the stable production time results in a higher productivity for a longer time, so the average productivity will be higher. Thus, the average cost decreases quickly. From Fig.11, we know that when \( t_1 > 0.4 \) or \( t_2 > 0.7 \), with the increase of the learning production time or the stable production time, the average cost almost remains unchanged. This means that the longer the learning time and stable production time, the higher the productivity and the lower the production cost.

Inserted Fig.10 and Fig.11

From the above analysis, observation 3 can be summarized as followings:

**Observation 3.** When production goes beyond the learning and stable phases, prolonging of the learning or stable production time can reduce the average cost.

**4. Influence of productivity coefficient** \( a, c, d, f \) **in the fatigue phase in Case 3**

In Case 3, since production goes through the learning, stable and fatigue phases, the productivity in the fatigue phase also affects the result. The impact of the productivity coefficient in the fatigue phase is illustrated as follows: Firstly, the effects of the fatigue related parameters: \( a, d, f \) on the total production time and cost are analyzed. From Fig.12 and 13,
we can see that the total production time and the average cost are not sensitive to parameter $a$. With the increase of $d$ or $f$, the production time tends to increase linearly, and the average cost increases almost exponentially.

Inserted Fig.12 and Fig.13

The larger value of $f$ means the higher fatigue level leading to a faster decrease in productivity. In order to meet demand, we may need to prolong the production time. As a result, this further increases fatigue level and cost.

**Observation 4.** When production goes beyond fatigue phase, fatigue leads to faster decrease in productivity, which leads to prolong production time, and decrease in productivity. This productivity degeneration will result in total cost increases.

From this observation, we can imply that when workers’ fatigue reaches a certain level, the labor productivity decrease rapidly; it is then unwise to continue production; a proper rest for fatigue recovery is necessary.

Inserted Fig.14 and Fig.15

Secondly, the influence of fatigue related parameter $c$ on the total production time and cost is analyzed. Fig.14 shows that the effects of $c$ on the production time ($t^*$) and Fig.15 shows that the effects of $c$ on the average cost ($ATC_s(t^*)$). Both of them have an inverted shape. When $c$ is smaller, the production time as well as the average cost increase with increasing $c$. When $c$ is bigger than a certain value, both the production time and the average cost decrease.

(5) The influence of $\delta$ on fatigue production time and average cost in case 4
In Case 4, production goes through the five phases: learning phase, stable phase, fatigue phase, rest phase and relearning phase. The scale factor \( \delta \) reflects the relationship between rest time and fatigue production time, which also impacts the result. We analyze the effect of \( \delta \) on the fatigue production time and the average cost. The analysis results are shown in Fig.16 and 17.

Inserted Fig.16 and Fig.17

Fig.16 illustrates the relationship between the \( \delta \) and the fatigue production time \( (t_3^* - t_2^*) \), and Fig.17 illustrates the impact of \( \delta \) on the average cost \( ATC_4(t^*) \), when \( \varepsilon = 0.02, 0.04, \) and \( 0.06 \) respectively.

From the Fig. 16, regardless of the value \( \varepsilon \), the fatigue production time decreases as \( \delta \) increases. This is because increasing \( \delta \) means an increased fatigue level which requires a longer rest time. As illustration in Fig.17, it can be seen that with the increase of \( \delta \), the optimal average cost increases initially and then decreases. Therefore, this implies that a short rest time is not beneficial for reducing cost. One should have sufficient rest time in order to reduce cost and increase economic value. From the results, we also see that for larger \( \varepsilon \) values, all curves in Fig 16 and 17 are low. This is because larger \( \varepsilon \) mean a better recovery after rest. It results in higher productivity after the relearning. As a result, the fatigue production time is shorter and the average cost is lower.

From this observation, we can see that an appropriate fatigue recovery time is very important to improve the production efficiency.

6. Conclusions
In a labor intensive manufacturing industry, human factor plays an important role in operations decisions. This study deals with the problem of behavioral economic value of leaning and fatigue recovery in a production inventory decision. Based on our real case study, we develop a new economic production quantity model considering learning, fatigue and fatigue recovery effects. The optimal production time and fatigue recovery time are determined to minimize the average cost of the production system. Newton Raphson method is used to derive the optimal solutions. The main conclusions of our study are as follows:

(1) The learning effect plays an important role in the early phase of production; the presence of fatigue will decrease the efficiency of the production system by lowering the productivity.

(2) An appropriate fatigue recovery is necessary for reducing cost and increase productivity. When the rest time is shorter than required, fatigue cannot be alleviated. With optimal rest time increases, the average total cost will decrease.

(3) It is not always appropriate to cut the fatigue production time, proper prolonging the fatigue phase production time can reduce the cost.

Although human factor in operations management has been researched for a long time, modeling the behavioral value of learning and fatigue effect in production inventory decision is lacking. Our study investigates the behavioral value of learning and fatigue effect in a production inventory decision using a real life example. For further research, we can consider multiple products and multi-stage production system in our models.

Acknowledgments
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References


Table 1 Major consideration of the models

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<th>Learning</th>
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<th>Fatigue</th>
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<td>Manna et al. [37]</td>
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Table 2 Time for sewing the waist of a pair of trousers

Table 3 Observed time needed to iron a pair of trousers by worker B

Table 4. Notation and descriptions
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Figure captions

Fig.1 The main production processes of trousers in X Company

Fig. 2 Worker A’s productivity curve in the sewing process (trousers/10 minute)

Fig.3 A worker’s productivity curve in the sewing process (after deleting abnormal data)

Fig.4 Worker B’s productivity curve in the ironing process (trousers/10 minutes)

Fig. 5 Production inventory change with dynamic productivity over time

Fig.6 Impact of $b, T_o$ on the production time

Fig.7 Impact of $b, T_o$ on the optimal cost

Fig.8 Impact of $t_1$ on production time

Fig. 9 Impact of $t_1$ on $ATC_{\lambda}(t)$

Fig.10 Impact of $t_1, t_2$ on the total production time

Fig. 11 Impact of $t_1, t_2$ and $ATC_{\lambda}(t^*)$
Fig. 12 Impact of $a, d, f$ on $t^*$

Fig. 13 Impact of $a, d, f$ on $ATC_3(t^*)$

Fig. 14 Impact of $c$ on $t^*$

Fig. 15 Impact of $c$ on $ATC_3(t^*)$

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Fig. 11 Impact of $t_1, t_2$ and $ATC(t^*)$

Fig. 12 Impact of $a, d, f$ on $t^*$

Fig. 13 Impact of $a, d, f$ on $ATC(t^*)$
Fig. 14 Impact of $c$ on $t^*$ 

Fig. 15 Impact of $c$ on $ATC_3(t^*)$

Fig. 16 Impact of $\delta$ on $t_3^* - t_2$

Fig. 17 Impact of $\delta$ on $ATC_4(t^*)$

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