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Optimal production inventory decision with learning and fatigue behavioral effects in labor-intensive manufacturing

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Abstract. Behavioral economics has received much attention recently. Learning and fatigue are two typical behavioral phenomena in industrial production operation processes. The existence of learning and fatigue results in a dynamic change in productivity. In this paper, a classical Economic Production Quantity (EPQ) model is extended to consider the behavioral economic value of learning and fatigue. Based on a real case study, each production cycle was divided into five phases, namely learning phase, stable phase, fatigue phase, fatigue recovery (rest) phase, and relearning phase. The new production inventory decision model was formulated with dynamic productivity and learning-stable-fatigue-recovery effect. Numerical simulation and sensitivity analysis showed that appropriate rest would alleviate employees' fatigue and increase productivity, resulting in a lower average production cost. On the other hand, when the rest time was too long, exceeding a certain value, it led to the decline of the actual labor productivity, resulting in an increase in the average cost of the system.

1. Introduction

In recent years, behavioral economics has received increasing attention [1]. In 2002, Daniel Kahneman and in 2017, Richard Thaler were awarded the Nobel Prize in Economic Sciences for their excellent work in behavioral economics. In management science, there has been a strengthening inclination to consider human factors in operations management, leading to a new management field, namely behavioral operations management [2]. Learning effect is a kind of human behavior which reflects the accumluation of experience in a worker during production. Since Wright [3] discovered the phenomenon of learning efffect in airplane production for the first time and established the first learning effect mathematical model (experience curve), there have been many learning behavioral effect studies in production operations management.

Since the cost of retaining skilled labor is high, effectively utilizing learning effect of the workers can

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reduce the workforce cost. Although more automatic equipment is being used nowadays, there are still many labor-intensive manufacturing companies. Therefore, studying how to improve the productivity of workforce is very important. This paper studies the behavioral value of learning and fatigue recovery in production operations decision and provides managerial implications for practitioners.

The study is motivated by our connection with a garment factory (detailed information on the background of the case-study company will be presented in Section 3). In the observations, the effect of the phenomenon of learning with fatigue appeared obvious and this inspired carrying out this research.

Jaber et al. [4] presented the Learning-Forgetting-Fatigue-Recovery Model (LFFRM), which addressed the comprehensive effect of the learning, forgetting, and fatigue recovery by rest in a production process. However, they only established the productivity change curve using the LFFRM model and they did not derive the production inventory decision or analyze how fatigue recovery would impact the inventory decision.

Although many scholars have extended Economic Production Quantity (EPQ) models with learning effect, they do not consider learning and fatigue effects, simultaneously. We develop an EPQ model and establish the production inventory decision model for one production cycle time with five phases: learning phase, stable phase, fatigue phase, rest phase, and relearning phase. Furthermore, we develop a production inventory optimization model to determine the optimal production lot size considering learning and fatigue effects, and study the impact of fatigue recovery policy on the production inventory decision. Through this study, we can answer the following questions:

1. What is the optimal fatigue phase production time when the fatigue during production is considered?
2. In order to reduce fatigue in the production process, what is the optimal rest time for workers for fatigue recovery?
3. Compared with the traditional production inventory decision, how does fatigue effect influence the optimal production decision?

The contribution of our study is to consider multiple-stage effects of learning, fatigue, and recovery on the production inventory system decision and its significance is introducing a new phase of productivity change based on the real case study of a trousers manufacturing factory as well as analyzing the impact of fatigue recovery on the production inventory decision. The improvement suggested by this model is of significance in practical applications. The rest of the paper is organized as follows; Section 2 presents a review of the literature and compares the differences between previous studies and our work. Section 3 describes the background of the case study and problem assumptions, and formulates the problem. Section 4 describes the solution algorithm. Section 5 demonstrates applicability of the model using a numerical example and sensitivity analysis. Finally, Section 6 summarizes the conclusions and future research directions.

2. Literature review

As one of the most important behavioral phenomena, learning effect has received much attention during the recent years. Many researchers have studied the application of learning effect to production inventory decision. Some of them extended the EPQ models to consider learning effect [5–7] and others combined the human learning effect into the EPQ models to improve the applicability of the traditional EPQ models [8–9]. Salameh et al. [10] developed an EPQ model with learning effect. Afterwards, Jaber and Salameh [11] extended Salameh's [10] model by taking into account shortage and backorder. Balkhi [12] studied the learning effect on production lot size for deteriorating and partially backordered items considering varying demand and deterioration rate. Boer and Zwart [13] studied dynamic pricing and learning for perishable products with finite initial inventory. Teng et al. [14] analyzed the effect of learning rate on the optimal credit payment period and the optimal batch size in the EPQ model. Khan et al. [15] studied an integrated vendor-buyer inventory decision model with learning in production at the vendor's end. Mahata [16] investigated the learning effect on the unit production time for an imperfect production system with partial backlogging and fuzzy environment. Khan et al. [17] investigated the role of variable lead time, learning in production, and screening errors in a vendor-buyer supply chain with defective items.

Besides the learning effect, another kind of behavioral phenomenon, known as forgetting effect, also exists. During operations, experience and skills can be accumulated by learning effect, but they can also be forgotten with time. This is called forgetting effect. Forgetting effect can reduce production rate and output. Jaber and Bonney [18] explored both learning and forgetting effects in a production process. They developed a learning-forgetting curve to model the learning and forgetting effects with production break. Chiu and Chen [19] established a dynamic lot size model considering the effects of learning and forgetting during setup and production. Alamri and Balkhi [20] studied the learning and forgetting effects on the production lot size for deteriorating items with varying demand. Teyarachakul et al. [21] investigated the effects of learning and forgetting with respect to batch sizes. Their finding was that small batch production

Table 1. Major considerations of the models in the literature.

Literature	Inventory	Learning	Forgetting	Fatigue
Chen and Tsao [36]	Yes	Yes	No	No
Khan et al. [15]	Yes	Yes	No	No
Khan et al. [17]	Yes	Yes	No	No
Manna et al. [37]	Yes	Yes	No	No
Mahata [38]	Yes	Yes	No	No
Grosse and Glock [39]	Yes	Yes	Yes	No
Glock and Jaber [40]	Yes	Yes	Yes	No
Jaber and Glock [26]	Yes	Yes	Yes	No
Kumar and Goswami [41]	Yes	Yes	No	No
Sturm et al. [42]	Yes	Yes	No	No
Givi et al. [27]	No	Yes	No	Yes
This paper	Yes	Yes	No	Yes

was better when there were learning and forgetting effects. Zanoni et al. [22] studied how learning and forgetting effects provided supply chain stakeholders with flexibility in Vendor Managed Inventory (VMI) environment. Glock and Jaber [23] studied a multi-stage production-inventory decision with product rework. They considered the learning and forgetting effects on production and rework. Teyarachakul et al. [24] studied the long-term characteristics of skill levels of the workers under learning and forgetting.

However, the above studies in the literature failed to consider fatigue during production [25–27]. Fatigue can lead to increasing error rates, quality problems, and reduction in efficiency and productivity. When fatigue becomes chronic or excessive, it can contribute to work-related disorders [28–30]. Many reasons may cause fatigue, such as overwork and sleeplessness [25] as well as physical discomfort [31]. Some authors have considered the fatigue phenomenon in production operation decision. Battini et al. [32] developed an economic lot-sizing model that considered ergonomic issues in calculating optimal lot sizes and used a rest allowance function to take account of recovery periods for maintaining low levels of fatigue and ergonomic risks.

Other authors have also considered human errors and customer satisfaction in the production inventory decision. Kang et al. [33] incorporated human errors into the decision making process focusing on group technology inventory model. Cheng et al. [34] assumed that the inspection process was carried out by the vendor and the defective items were disposed in multiple batches. They proposed an optimal integrated vendor-buyer inventory model with defective items, but did not consider inspection error. Customer satisfaction was considered by Besheli et al. [35] in a fuzzy dynamic multi-objective, multi-item model. Jaber and Glock [26] developed a new learning curve model that comprised cognitive and motor components.

Based on the above literature review, the major characteristics of the models have been summarized and compared with our work. The comparison is shown in Table 1. Unlike previous studies, we consider learning and fatigue effects in the production inventory environment.

3. Formulation of the problem

In this section, we describe background of the case, motivation of our study, and the collected data and develop a mathematical model.

3.1. Background of the case

This study is motivated by our involvement in a labor-intensive manufacturing company. Company X in our case study is a trousers production factory with 200 workers. There are nearly 50 processing procedures in the production line of trousers and the processing flow can be classified into five stages (Figure 1):

1. ***Preparation for production***: At this stage, the workers prepare the needed clothes. The main tasks include selecting and checking cloths according to sample sizes and specifications;
2. ***Tailoring clothes***: At this stage, the workers cut cloths into different components and sizes according to the specifications of trousers. This work is half automated by workers operating the cloth-cutting machines;
3. ***Sewing***: This is the core step of garment manufacturing. At this stage, the workers sew clothes into different trousers according to the production instructions. This stage demands labor-intensive work and work load is very high in it;
4. ***Ironing***: When trousers are finished, in order to make the faces of trousers smooth, ironing is a necessary process for garment production. This work

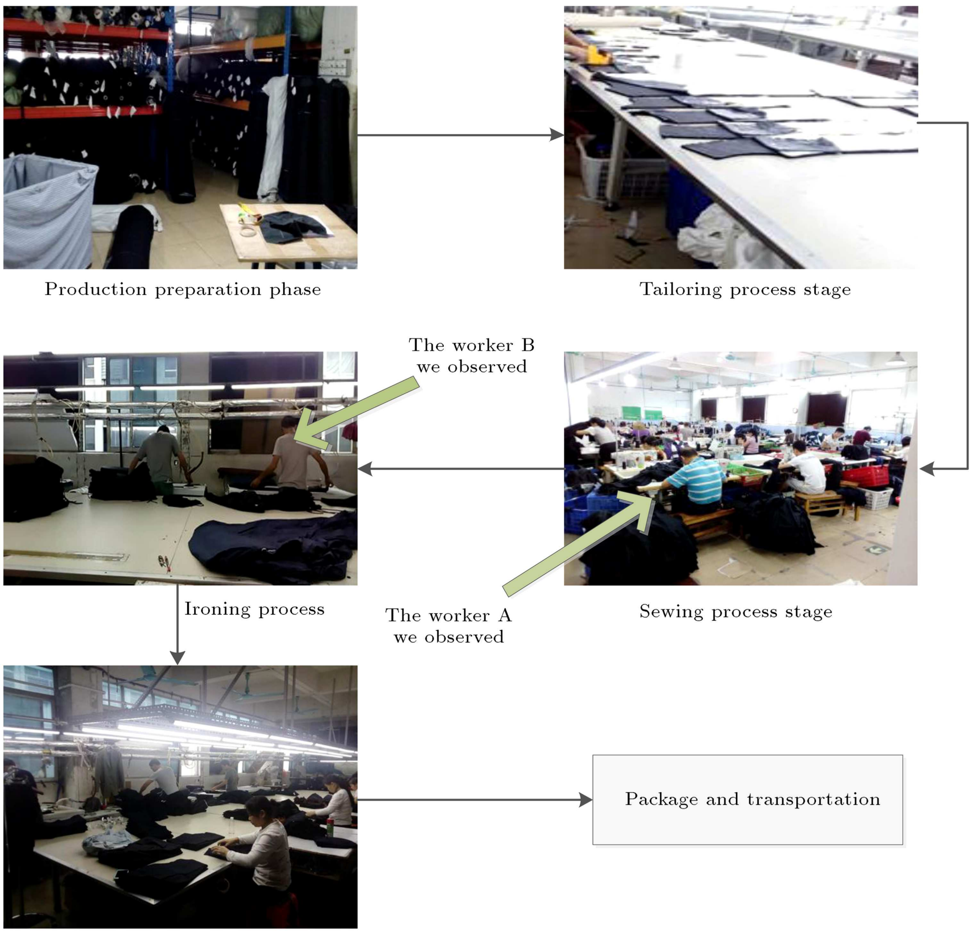

Figure 1. The main production processes for trousers in Company X.

usually is completely manual, hence the loading is also very high;

5. ***Checking***: This is the last operation of garment manufacturing before package and storage. It is a necessary quality check done manually.

In the above production process, most of the work in the garment factory is done manually. With the accumulation of experience, the production efficiency of the workers increases due to the learning effect phenomenon. However, fatigue and decreasing production rate may occur if working time is prolonged.

In order to understand the behavioral phenomena in production, on-site data collection was done for different production phases. The following tables illustrate some of the collected data.

Table 2 shows the sewing time of a worker for sewing the waist of a pair of trousers.

In Table 2, some data points are abnormal, e.g., the data points of 61.10 and 66.06 at the observation instants of 16:37 and 16:44. The reason for these results is that the worker A has checked the trousers without actually working on the waist of the trousers. We measured productivity of the worker (unit of trousers/10 minutes) and the result is depicted in the Figure 2.

Table 2. Time for sewing the waist of a pair of trousers.

Observation instant	**Time per trousers (sec)**	**Observation instant**	**Time per trousers (sec)**
14:05	94.42	16:32	105.89
14:15	96.73	16:37	114.17
14:40	87.89	16:43	61.10
14:42	84.42	16:44	66.06
15:01	81.24	17:28	129.13
15:32	85.91	17:32	114.49
15:42	92.81	17:40	127.34
15:55	101.48	17:45	157.22

The results are depicted in Figure 3 after deleting the abnormal data points.

From the productivity curve shown in Figure 3, we can see that a change in the productivity of a worker leads to a typical multiple-phase change law. At the

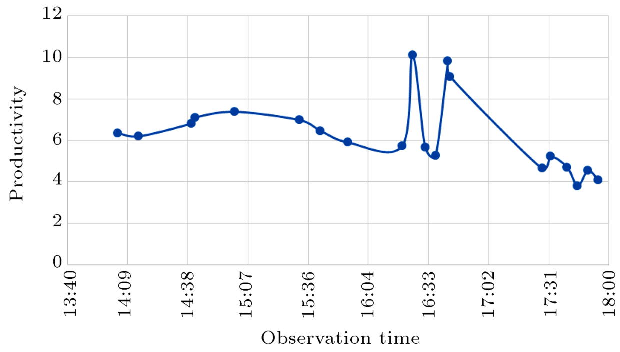

Figure 2. Productivity curve for worker A in the sewing process (trousers/10 minutes).

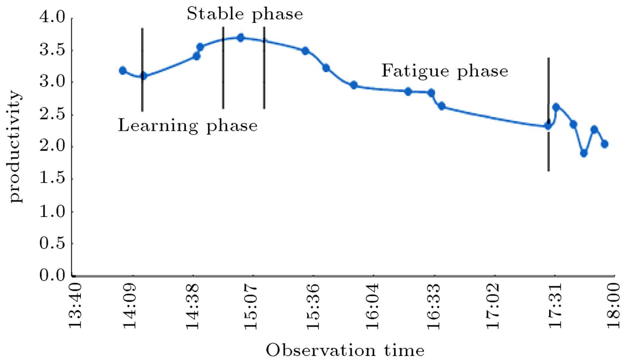

Figure 3. Productivity curve for a worker in the sewing process (after deleting abnormal data).

beginning, productivity increases with learning effect; then, it enters a relatively stable phase; finally, with the progression of time, productivity decreases due to the fatigue.

Another example is shown in Table 2. This example was surveyed in the process of ironing for an observed worker B. The work was carried out from the morning until afternoon. Based on the data in Table 2, we could calculate the productivity of worker B (trousers/10 minutes) in one day. Similar to the situation for worker A, worker B did not follow the work standard very strictly and there were some interferences leading to abnormal data points. As predicted, the productivity change curve was not consistent with the expectation of classical models.

Figure 4 illustrates the productivity curve for worker B based on the data in Table 3. In Figure 4, we can observe that productivity of worker B also follows a change law similar to that for worker A, i.e., productivity increases at the beginning; then, it enters a relatively stable phase; and finally, with the progression of time, productivity decreases due to the fatigue.

In Figure 4, the fatigue recovery effect can be observed. Worker B works from the morning until afternoon with a break for lunch at noon. After a time of rest, his productivity increases slightly as compared

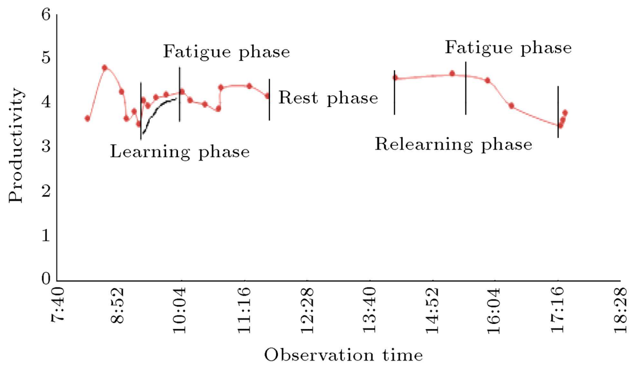

Figure 4. Productivity curve for worker B in the ironing process (trousers/10 minutes).

Table 3. Observed time needed to iron a pair of trousers by worker B.

Observation instant	Time per trousers (sec)	Observation instant	Time per trousers (sec)
8:18	165.90	10:33	151.37
8:37	125.86	10:48	156.30
8:57	141.70	10:51	138.58
9:02	165.15	11:23	137.16
9:11	158.75	11:44	145.37
9:16	170.41	14:11	131.88
9:22	148.70	15:16	129.48
9:27	152.64	15:57	133.76
9:36	145.96	16:24	152.90
9:48	143.86	17:20	172.19
10:06	141.69	17:23	167.30
10:16	148.03	17:26	159.01

with the time before lunch break. In the afternoon, after an initial rise due to learning, productivity remains constant for a while before it decreases in the late afternoon.

3.2. Problem description

Based on examination of the above case, we develop a theoretical production inventory decision model considering the dynamic productivity phenomena during production with learning and fatigue, recovery (rest), and relearning effects. Figure 5 shows the production-inventory profile (shown in the upper part of the figure) under the dynamic productivity process (shown in the lower part of the figure).

Figure 5(b) shows the assumption that the production process includes learning phase, stable phase, fatigue phase, and rest (fatigue recovery) phase. In the learning phase, productivity increases; in the stable phase, it remains stable; in the fatigue phase, it decreases; in the rest phase, it is zero; and in the relearning phase, it increases. This assumption fits

Table 4. Notations and descriptions.

i	Production phase	$I_i(t)$	Inventory level of t in phase i
$i=1$	Learning phase	S_i	Maximum inventory when phase i ends
$i=2$	Stable phase	A	Setup cost
$i=3$	Fatigue phase	b	Learning coefficient
$i=4$	Rest phase	d	Productivity under normal labor load
$i=5$	Relearning phase	f	Labor load index
D	Demand rate	T_0	First-piece production time
t_i	Time instant	C_{ih}	Holding cost in phase i
a	Productivity under normal physiological load	$P_i(t)$	Productivity of t in phase i
c	Physiological load index	Q_i	Maximum production quantity when phase i ends
h	Holding cost per unit per unit time	l	Labor cost per unit time

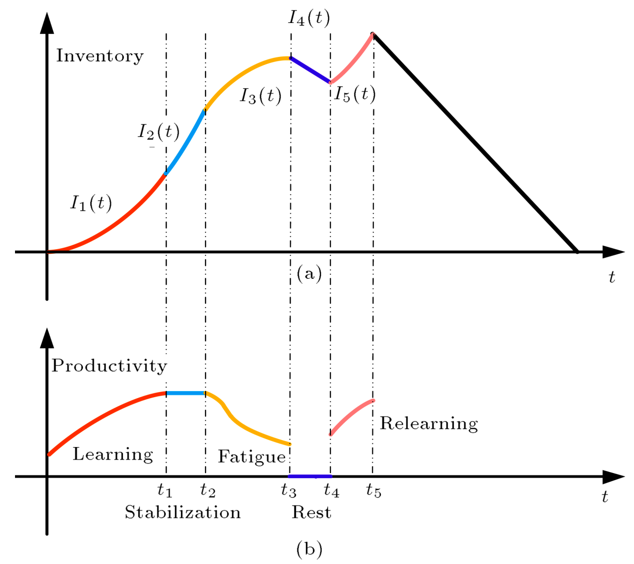

Figure 5. Production inventory change with dynamic productivity over time.

the above examination of examples. In this case, production process of worker A can be divided into the learning phase, stable phase, and fatigue phase (Figure 3). For worker B, in one day, the production process can also be divided into the learning phase, fatigue phase, rest phase, relearning phase, and fatigue phase (Figure 4). The observed dynamic productivity changing process combines the observations for worker A and worker B. The production process includes learning phase, stable phase, fatigue phase, rest (fatigue recovery) phase, and relearning phase.

Based on the above productivity change characteristics, we extend the classic EPQ model considering the learning and fatigue effects on production. Then, we analyze how learning, fatigue, and fatigue recovery impact the production decision. The study derives the optimal production schedule and the most effective rest time for each stage of production.

3.3. Assumptions and notation

The following assumptions are made in this study:

1. The demand rate of the manufacturer is continuous and constant. Shortage is not allowed;
2. There are learning effect and fatigue effect during production;
3. The production process includes learning phase, stable phase, fatigue phase, rest (fatigue recovery) phase, and relearning phase;
4. Productivity is greater than demand during the learning, stable, and fatigue phases;
5. The rest time for fatigue recovery depends on the production time;
6. The productivity in the relearning phase depends on the rest time.

Assumption 3 is supported by the productivity behavior of workers A and B in our case study.

The notation used in this paper is listed in Table 4.

3.4. Model development

During the production phase, the inventory equation can be expressed as:

$$\frac{\mathrm{d}I_i(t)}{\mathrm{d}t} = P_i(t) - D, \tag{1}$$

with the boundary conditions:

$$I_1(0)=0 \quad t_0=0,$$

$$I_{i-1}(t_{i-1}) = I_i(t_{i-1}), \qquad i=2,3,4,5.$$

From Eq. (1), the relation between the inventory level and time can be obtained:

$$I_i(t)=I_{i-1}(t_{i-1})+\int_{t_{i-1}}^{t}[P_i(u)-D]\mathrm{d}u, \quad t_{i-1}\le t\le t_i. \tag{2}$$

The holding cost in phase i is:

$$C_{ih} = h\int_{t_{i-1}}^{t_i} I_i(t)\mathrm{d}t = h\int_{t_{i-1}}^{t_i}\int_{t_{i-1}}^{t}\left[P_i(u)-D\right]\mathrm{d}u\mathrm{d}t$$

$$+h\int_{t_{i-1}}^{t_i} I_{i-1}(t_{i-1})\mathrm{d}u\mathrm{d}t. \quad (3)$$

In the following analysis, we model the total production cost for different production times.

Case 1. *When $t \le t_1$, production is carried out only during the learning phase.*

When the total production during the learning phase can meet the total demand, the manufacturer will not increase production. To minimize the average cost per unit time, it is necessary to derive the optimal production time t^*.

Based on the Wright learning curve theory [3], suppose that T_0 is the production time of the first unit product; the production time for the qth product is expressed as $T_0 q^{-b}$. The total time to produce the quantity Q of products is:

$$t \approx \int_0^Q T_0 q^{-b}\mathrm{d}q = \frac{T_0 Q^{1-b}}{1-b}.$$

The dynamic productivity (number of products per unit time) at instant t is: $p(t) = \frac{\mathrm{d}Q}{\mathrm{d}t}$. We have:

$$P_1(t) = \begin{cases} \frac{1}{1-b}\left(\frac{1-b}{T_0}\right)^{\frac{1}{1-b}} t^{\frac{b}{1-b}} & 0 < t \le t_1 \\ \frac{1}{T_0} & t = 0 \end{cases} \quad (4)$$

Let $\alpha = \frac{1}{1-b}\left(\frac{1-b}{T_0}\right)^{\frac{1}{1-b}}$ and $\beta = \frac{b}{1-b}$. Since $0 < b < 1$ and $\beta > 1$, Eq. (4) can be rewritten as:

$$P_1(t) = \begin{cases} \alpha t^{\beta} & 0 < t \le t_1 \\ \frac{1}{T_0} & t = 0 \end{cases} \quad (5)$$

From Eq. (5), when $0 < t \le t_1$, the inventory level for any time t during the learning phase is:

$$I_1(t) = \int_0^t \left(\alpha u^{\beta} - D\right)\mathrm{d}u = \left(\frac{\alpha}{1+\beta}t^{1+\beta} - Dt\right). \quad (6)$$

At the end of the learning phase, the inventory level in Eq. (6) becomes $S_1 = \frac{\alpha}{1+\beta}t^{1+\beta} - Dt$. The maximum output of the manufacturer is $Q_1 = \frac{\alpha}{1+\beta}t^{1+\beta}$. The time available for the consumption is $T_{1d} = \frac{\alpha t^{1+\beta}}{(1+\beta)D} - t$, where T_{1d} is the non-production time. The total production cycle is $T_1 = \frac{\alpha t^{1+\beta}}{(1+\beta)D}$. The holding cost during the learning phase is:

$$C_{1h} = h\int_0^t I_1(t)\mathrm{d}t = h\left[\frac{\alpha t^{2+\beta}}{(1+\beta)(2+\beta)} - \frac{1}{2}Dt^2\right]. \quad (7)$$

The average cost within a cycle is:

$$ATC_1(t)\frac{1}{T_1}\left[A + lt + C_{1h} + \frac{h}{2D}S_1^2\right]. \quad (8)$$

By substituting T_1, C_{1h}, and S_1 into Eq. (8), the average total cost is:

$$ATC_1(t) = \frac{(1+\beta)D}{\alpha t^{1+\beta}}\Bigg\{A + lt + h\left[\frac{\alpha t^{2+\beta}}{(1+\beta)(2+\beta)} - \frac{1}{2}Dt^2\right] + \frac{h}{2D}\left[\frac{\alpha t^{1+\beta} - D(1+\beta)t}{(1+\beta)}\right]^2\Bigg\}. \quad (9)$$

Proposition 1. *$ATC_1(t)$ is a convex function of t. There exists t^* for optimal $ATC_1(t)$.*

Proof. Taking the first and second derivatives of $ATC_1(t)$, we have:

$$\frac{\partial ATC_1(t)}{\partial t} = \frac{\left[\alpha(2+\beta)t^{\beta} - 2(1+\beta)D\right]h}{2(2+\beta)} - \frac{t^{-2-\beta}A(1+\beta)^2 D + bDl(1+\beta)t^{-1-\beta}}{\alpha}$$

$$\frac{\partial^2 ATC_1(t)}{\partial t^2} = \frac{\beta t^{\beta-1}\alpha h}{2} + \frac{(2+\beta)t^{-3-\beta}A(1+\beta)^2 D + bDl(1+\beta)^2 t^{-2-\beta}}{\alpha}.$$

Obviously, $\frac{\partial^2 ATC_1(t)}{\partial t^2} > 0$. Thus, $ATC_1(t)$ is a convex function of t. Theoretically, we can derive the optimal solution by setting $\frac{\partial ATC_1(t)}{\partial t} = 0$. However, because of the nonlinearity of the equation, the closed-form solution t^* is difficult to obtain. Therefore, the optimal time t^* is obtained using the one-dimensional search method.

Proposition 2. *The optimal production time t^*, the maximum output Q_1^*, and the maximum inventory S_1^* have the same variation trend (increasing or decreasing) with respect to the learning coefficient b.*

Proof. Taking the first-order derivative of $ATC_1(t)$ with respect to t from Proposition 1, let:

$$F(t^*, \beta) = \frac{\left[\alpha(2+\beta)(t^*)^{\beta} - 2(1+\beta)D\right]\alpha h}{2\alpha(2+\beta)} - \frac{(t^*)^{-2-\beta}A(1+\beta)^2 D + bDl(1+\beta)(t^*)^{-1-\beta}}{\alpha}.$$

Taking the first-order derivatives of $F(t^*,\beta)$ with respect to t^* and β, we get:

$$F_{t^*}=\frac{\beta(t^*)^{\beta-1}\alpha h}{2}+\frac{(2+\beta)(t^*)^{-3-\beta}A(1+\beta)^2D+bDl(1+\beta)^2(t^*)^{-2-\beta}}{\alpha}>0,$$

$$F_\beta=\frac{h\left[\alpha\ln(t^*)(2+\beta)^2(t^*)^\beta-2D\right]}{2(2+\beta)^2}+\frac{DA(1+\beta)\left[-2+(1+\beta)\ln(t^*)\right](t^*)^{-2-\beta}}{\alpha}+\frac{Dl\left[\beta(1+\beta)\ln(t^*)-2\beta-1\right](t^*)^{-1-\beta}}{\alpha}.$$

The relationship between t^* and β is expressed as $\frac{\partial t^*}{\partial\beta}=-\frac{F_\beta}{F_{t^*}}$. From $\beta=\frac{b}{1-b}$, we have $\frac{\partial\beta}{\partial b}=\frac{1}{(1-b)^2}>0$. From $Q_1^*=\frac{\alpha}{1+\beta}(t^*)^{1+\beta}$, we have $\frac{\partial Q_1^*}{\partial t^*}=\alpha(t^*)^\beta>0$. Based on our assumption, we have $\frac{\partial S_1^*}{\partial t^*}=\alpha(t^*)^\beta-D>0$.

$$\frac{\partial t^*}{\partial b}=\frac{\partial t^*}{\partial\beta}\cdot\frac{\partial\beta}{\partial b}=-\frac{F_\beta}{F_{t^*}}\cdot\frac{\partial\beta}{\partial b},$$

$$\frac{\partial Q_1^*}{\partial b}=\frac{\partial Q_1^*}{\partial t^*}\cdot\frac{\partial t^*}{\partial\beta}\cdot\frac{\partial\beta}{\partial b}=-\alpha(t^*)^\beta\cdot\frac{F_\beta}{F_{t^*}}\cdot\frac{\partial\beta}{\partial b},$$

$$\frac{\partial S_1^*}{\partial b}=\frac{\partial S_1^*}{\partial t^*}\cdot\frac{\partial t^*}{\partial\beta}\cdot\frac{\partial\beta}{\partial b}=-\left[\alpha(t^*)^\beta-D\right]\cdot\frac{F_\beta}{F_{t^*}}\cdot\frac{\partial\beta}{\partial b}.$$

If $F_\beta>0$, then $\frac{\partial t^*}{\partial b}<0$, $\frac{\partial Q_1^*}{\partial b}<0$, and $\frac{\partial S_1^*}{\partial b}<0$. If $F_\beta<0$, then $\frac{\partial t^*}{\partial b}>0$, $\frac{\partial Q_1^*}{\partial b}>0$, and $\frac{\partial S_1^*}{\partial b}>0$. This means that the optimal t^*, Q_1^*, and S_1^* have the same change law. □

Proposition 3. *The optimal production time t^*, the maximum output Q_1^*, and the maximum inventory S_1^* increase with respect to the initial unit production time T_0.*

Proof. Set:

$$F(t^*,\alpha)=\frac{\left[\alpha(2+\beta)(t^*)^\beta-2(1+\beta)D\right]\alpha h}{2\alpha(2+\beta)}-\frac{(t^*)^{-2-\beta}A(1+\beta)^2D+bDl(1+\beta)(t^*)^{-1-\beta}}{\alpha}.$$

Taking the first-order derivatives of $F(t^*,\alpha)$ with respect to t^* and α, we have:

$$F_{t^*}=\frac{\beta(t^*)^{\beta-1}\alpha h}{2}+\frac{(2+\beta)(t^*)^{-3-\beta}A(1+\beta)^2D+bDl(1+\beta)^2(t^*)^{-2-\beta}}{\alpha}>0,$$

$$F_\alpha=\frac{(t^*)^\beta h}{2}+\frac{(t^*)^{-2-\beta}A(1+\beta)^2D+bDl(1+\beta)(t^*)^{-1-\beta}}{\alpha^2}>0.$$

Since $F_{t^*}>0$ and $F_\alpha>0$, $\frac{\partial t^*}{\partial\alpha}=-\frac{F_\alpha}{F_{t^*}}<0$. From $\alpha=\frac{1}{1-b}\left(\frac{1-b}{T_0}\right)^{\frac{1}{1-b}}$, we have $\frac{\partial\alpha}{\partial T_0}=-(1-b)^{\frac{1}{1-b}}T_0^{-\frac{2-b}{1-b}}<0$.

$$\frac{\partial t^*}{\partial T_0}=\frac{\partial t^*}{\partial\alpha}\cdot\frac{\partial\alpha}{\partial T_0}=\frac{F_\alpha}{F_{t^*}}\cdot(1-b)^{\frac{1}{1-b}}T_0^{-\frac{2-b}{1-b}}>0,$$

$$\frac{\partial Q_1^*}{\partial T_0}=\frac{\partial Q_1^*}{\partial t^*}\cdot\frac{\partial t^*}{\partial\alpha}\cdot\frac{\partial\alpha}{\partial T_0}=\alpha(t^*)^\beta\cdot\frac{F_\alpha}{F_{t^*}}\cdot(1-b)^{\frac{1}{1-b}}T_0^{-\frac{2-b}{1-b}}>0,$$

$$\frac{\partial S_1^*}{\partial T_0}=\frac{\partial S_1^*}{\partial t^*}\cdot\frac{\partial t^*}{\partial\alpha}\cdot\frac{\partial\alpha}{\partial T_0}=\left[\alpha(t^*)^\beta-D\right]\frac{F_\alpha}{F_{t^*}}\cdot(1-b)^{\frac{1}{1-b}}T_0^{-\frac{2-b}{1-b}}>0.\ \square$$

Remark. When production is carried out only in the learning phase, with $b\to0$, $\beta=\frac{b}{1-b}\to0$ and we have $P_1\to\frac{1}{T_0}$, where productivity is constant. This is a classical EPQ model.

Case 2. *When $t_1<t\le t_2$, production goes beyond the learning phase and ends in the stable phase.* At the end of the learning phase, the productivity level reaches $P_2(t=t_1)=P_1(t=t_1)=\alpha t_1^\beta$ and the manufacturer continues to produce at a steady level of productivity. From the boundary condition $I_1(t_1)=I_2(t_1)$, the inventory change with time during the stable phase is:

$$I_2(t)=(\alpha t_1^\beta-D)t-\frac{\alpha\beta}{1+\beta}t_1^{1+\beta},\quad t_1<t\le t_2.\quad(10)$$

The holding cost during the stable phase is:

$$C_{2h}=h\int_{t_1}^{t}I_2(t)\mathrm{d}t=h\left[\frac{1}{2}(\alpha t_1^\beta-D)(t^2-t_1^2)-\frac{\alpha\beta}{1+\beta}t_1^{1+\beta}(t-t_1)\right].\quad(11)$$

From Eq. (10), the inventory level at t is:

$$S_2 = (\alpha t_1^\beta - D)t - \frac{\alpha\beta}{1+\beta} t_1^{1+\beta},$$

and the maximum output is:

$$Q_2 = \frac{\alpha t_1^\beta (1+\beta)t - \alpha\beta t_1^{1+\beta}}{(1+\beta)}.$$

The total time of a production cycle is:

$$T_2 = \frac{\alpha t_1^\beta (1+\beta)t - \alpha\beta t_1^{1+\beta}}{(1+\beta)D}.$$

The average cost of the production system within a cycle, including the costs in learning and stable periods, is:

$$ATC_2(t) = \frac{D}{S_2 + Dt}\left(A + lt + \sum_{i=1}^{2} C_{ih} + \frac{h}{2D} S_2^2\right), \quad (12)$$

where, inside the bracket, the first term is the setup cost; the second term is the labor cost; the third term is the holding cost of inventory during production in leaning and stable phases (C_{1h} and C_{2h} are taken from Eqs. (7) and (11), respectively); and finally, the forth term is the holding cost of the inventory during non-production.

Case 3. *When $t > t_2$, the production time includes the learning and the stable phases in addition to the fatigue phase up to the end.* We assume that the productivity function in the fatigue phase is $P(t) = a\mathrm{e}^{-ct} + dt^{-f} + g$. This assumption is based on two factors that affect human fatigue behavior, namely physiological and labor intensities. In the natural state, productivity gets lower and lower as the body becomes weaker and weaker in bearing the load. Different people have different physiological loads. If labor intensity is large, the employees will be fatigued quickly. In this formula, the first term, $a\mathrm{e}^{-ct}$, is productivity under normal physiological load, which diminishes with time t; c is the physiological load index. The second term, dt^{-f}, is productivity under normal labor load, which diminishes with time t; f is the labor load index. The third term is the initial productivity when becoming fatigue.

The reasonability of this assumption can be explained theoretically and practically. A laboratory experiment was designed by Okogbaa (1983) [43] to examine mental work output with and without rest. Okogbaa (1983) observed hyperbolic as well as exponential decay function. Bechtold (1988) [44] found out that when workers underwent fatigue process during production, instant productivity was an exponential function of passage of time. Lindstrom et al. (1997) [45] observed that the relationship between the work load and maximum endurance was an exponential function. Practically, based on our examinations, referring to the productivity curves for worker A in Figure 3 and worker B in Figure 4, we believe that the productivity curve in the fatigue phase is a function of the combination of the hyperbolic and exponential functions. Consequently, the assumption is reasonable.

From the boundary condition $P_2(t_2) = P_3(t_f = 0)$, since $P_2(t_2) = P_1(t_1) = \alpha t_1^\beta$, we have $g = \alpha t_1^\beta - ae^{-ct_2} - dt_2^{-f}$. Thus, during the fatigue phase, productivity can be expressed as:

$$P_3(t) = a(\mathrm{e}^{-ct} - \mathrm{e}^{-ct_2}) + d(t^{-f} - t_2^{-f}) + \alpha t_1^\beta. \quad (13)$$

Based on the boundary condition $I_2(t_2) = I_2(t_3)$, the inventory level changes with time as:

$$\begin{aligned} I_3(t) = &-\frac{a}{c}\left(\mathrm{e}^{-ct} - \mathrm{e}^{-ct_2}\right) + \frac{d}{1-f}(t^{1-f} - t_2^{1-f}) \\ &+ (\alpha t_1^\beta - a\mathrm{e}^{-ct_2} - dt_2^{-f} - D)(t - t_2) \\ &+ (\alpha t_1^\beta - D)t_2 - \frac{\alpha\beta}{1+\beta} t_1^{1+\beta}, \quad t_2 < t \leq t_3, \quad (14) \end{aligned}$$

when the production ends at t during the fatigue production phase, the inventory level is: $S_3 = I_3(t)$.

The holding cost during the fatigue phase is:

$$\begin{aligned} C_{3h} = &h\frac{a}{c^2}(e^{-ct} - e^{-ct_2}) + h\frac{d(t^{2-f} - t_2^{2-f})}{(1-f)(2-f)} \\ &+ \frac{1}{2}h(\alpha t_1^\beta - ae^{-ct_2} - dt_2^{-f} - D)(t^2 - t_2^2) \\ &+ h(t - t_2)\Big[\frac{a}{c}\mathrm{e}^{-ct_2} - \frac{d}{1-f} t_2^{1-f} \\ &+ (ae^{-ct_2} + dt_2^{-f})t_2\Big] - h(t - t_2)\frac{\alpha\beta}{1+\beta} t_1^{1+\beta}. \quad (15) \end{aligned}$$

The average cost of a production cycle is:

$$ATC_3(t) = \frac{D}{Dt + S_3}\left(A + lt + \sum_{i=1}^{3} C_{ih} + \frac{hS_3^2}{2D}\right). \quad (16)$$

The problem is to minimize $ATC_3(t)$, which is an unconstrained nonlinear programming equation. Due to the nonlinearity of Eq. (16), it is difficult to derive the closed-form optimal solution. Therefore, Newton iteration is used to solve the problem.

Case 4. *When $T > T_4$, production includes the learning, stable, fatigue, rest, and relearning phases.* When production goes to the rest phase, it stops. Productivity rate is zero during the rest time ($t_3 \leq$

$t \leq t_4$). Although there is no production during the rest time, demand still exists. The inventory level at time t in the rest phase is:

$$I_4(t) = -\frac{a}{c}\left(\mathrm{e}^{-ct_3} - \mathrm{e}^{-ct_2}\right) + \frac{d}{1-f}(t_3^{1-f} - t_2^{1-f}) + (\alpha t_1^{\beta} - a\mathrm{e}^{-ct_2} - dt_2^{-f} - D)(t_3 - t_2) + (\alpha t_1^{\beta} - D)t_2 - \frac{\alpha\beta}{1+\beta}t_1^{1+\beta} - D(t - t_3), \quad t_3 < t \leq t_4. \tag{17}$$

The holding cost during the rest phase is:

$$C_{4h} = h\Big\{ -\frac{a}{c}(e^{-ct_3} - e^{-ct_2}) + \frac{d}{1-f}(t_3^{1-f} - t_2^{1-f}) + (\alpha t_1^{\beta} - ae^{-ct_2} - dt_2^{-f} - D)(t_3 - t_2) + (\alpha t_1^{\beta} - D)t_2 - \frac{\alpha\beta}{1+\beta}t_1^{1+\beta} + Dt_3\Big\}(t - t_3) - \frac{1}{2}hD(t^2 - t_3^2). \tag{18}$$

The longer the time of fatigue production, the more time is needed to relieve the sfatigue. The rest time depends on the production time of the fatigue phase and the relationship between the rest time and fatigue production time is $t_4 - t_3 = \delta(t_3 - t_2)$, where $\delta > 0$ is a scale factor. After some time of rest, physical energy of the workers recovers to a higher level and the their productivity gats higher than that at the end of the fatigue phase. This phenomenon is called fatigue recovery. From Eq. (4), after some derivation, the productivity in the relearning phase is:

$$P_5(t) = \frac{1}{1-b}\left\{\frac{[\delta(t_3 - t_2)]^{\varepsilon}(1-b)}{T_0}\right\}^{\frac{1}{1-b}}(t - t_4)^{\frac{b}{1-b}}, \quad t_4 < t. \tag{19}$$

Let :

$$\gamma = \frac{1}{1-b}\left\{\frac{[\delta(t_3 - t_2)]^{\varepsilon}(1-b)}{T_0}\right\}^{\frac{1}{1-b}},$$

and:

$$\beta = \frac{b}{1-b}.$$

Then, $P_5(t) = \gamma(t - t_4)^{\beta}$.

Based on the boundary condition $I_4(t_4)=I_5(t_4)$, the inventory level changes with time as:

$$I_5(t) = \frac{\gamma}{1+\beta}(t - t_4)^{1+\beta} - D(t - t_4) - \frac{\alpha\beta}{1+\beta}t_1^{1+\beta} - \frac{a}{c}\left(e^{-ct_3} - e^{-ct_2}\right) + \frac{d}{1-f}(t_3^{1-f} - t_2^{1-f}) + (\alpha t_1^{\beta} - ae^{-ct_2} - dt_2^{-f} - D)(t_3 - t_2) - D(t_4 - t_3) + (\alpha t_1^{\beta} - D)t_2, \quad t_4 < t. \tag{20}$$

The holding cost during the relearning phase is:

$$C_{5h} = \frac{h\gamma(t - t_4)^{2+\beta}}{(1+\beta)(2+\beta)} - \frac{1}{2}hD(t^2 - t_4^2) + h\Big[Dt_3 - \frac{a}{c}\left(e^{-ct_3} - e^{-ct_2}\right) + \frac{d}{1-f}(t_3^{1-f} - t_2^{1-f}) + (\alpha t_1^{\beta} - D)t_2 - \frac{\alpha\beta}{1+\beta}t_1^{1+\beta} + (\alpha t_1^{\beta} - ae^{-ct_2} - dt_2^{-f} - D)(t_3 - t_2)\Big](t - t_4). \tag{21}$$

The production time of the relearning phase depends on the rest time; the longer the rest time, the longer is the production time in the relearning phase. The relearning production time $t - t_4$ is equal to $\kappa\delta(t_3 - t_2)$, where κ is a scale factor. From $t_4 - t_3 = \delta(t_3 - t_2)$, we derive $t_3 = \frac{t+(1+\kappa)\delta t_2}{1+\delta+\kappa\delta}$ and $t_4 = \frac{t+\delta t-\kappa\delta t_2-2\kappa\delta^2 t_2-2\kappa^2\delta^2 t_2}{1+\delta+\kappa\delta}$.

After the end of the relearning phase, the maximum inventory level S_4 from Eq. (20) is:

$$S_4 = \frac{\gamma}{1+\beta}(t - t_4)^{1+\beta} - Dt + Dt_3 - \frac{a}{c}\left(\mathrm{e}^{-ct_3} - \mathrm{e}^{-ct_2}\right) + \frac{d}{1-f}(t_3^{1-f} - t_2^{1-f}) + (\alpha t_1^{\beta} - a\mathrm{e}^{-ct_2} - dt_2^{-f} - D)(t_3 - t_2) + (\alpha t_1^{\beta} - D)t_2 - \frac{\alpha\beta}{1+\beta}t_1^{1+\beta}, \tag{22}$$

when production stops, the time to consume the maximum inventory is S_4/D. The inventory cost during the depletion period is $hS_4^2/2D$. The production time and consumption time in a cycle (also known as the production cycle) are $T = t + S_4/D$.

The average cost of a production cycle is:

$$ATC_4(t) = \frac{D}{Dt + S_4}\left(A + lt + \sum_{i=1}^{5} C_{ih} + \frac{hS_4^2}{2D}\right). \tag{23}$$

Due to the high nonlinearity of Eq. (23), achieving a closed-form solution is not possible. Therefore, the Newton iteration method is used to solve the optimal solutions.

4. Solution algorithm

For the complex nonlinear models developed in the previous section, we adopt the Newton Raphson iteration method in solving the problem. In the Newton Raphson iteration method, two Taylor expansions of the objective function are utilized to minimize the function. The first few terms of the Taylor series are used to derive the roots of equations. It is an approximation method for linearizing nonlinear equations. This will enable us to derive a square convergence near the single root of the equation. The method is to obtain t_3^* in Case 4.

Set : $f(X) = ATC(t_3)$,

The solution steps are as follows:

Step 1. Given the initial value X_0, set the allowable error ε ($\varepsilon = 0.0001$ in this case);

Step 2. Calculate $X_n = X_{n-1} - \frac{f(X_{n-1})}{f'(X_{n-1})}$;

Step 3. If $|X_n - X_{n-1}| < \varepsilon$, then go to Step 4; If $|X_n - X_{n-1}| \geq \varepsilon$, then go to Step 2;

Step 4. $t^* = X^* = X_n$.

In Case 4, the manufacturer should stop production at moment t_3^*, where:

$$t_3^* = \frac{t^* + (1+\kappa)\delta t_2}{1+\delta+\kappa\delta}.$$

The rest time is $\delta(t_3^* - t_2)$; the reproduction time is t_4^*, where:

$$t_4^* = \frac{t^* + \delta t^* - \kappa\delta t_2 - 2\kappa\delta^2 t_2 - 2\kappa^2\delta^2 t_2}{1+\delta+\kappa\delta},$$

and the relearning Production time is $\kappa\delta(t_3^* - t_2)$. The same procedure is used to derive t^* in Case 3.

5. Numerical example and sensitivity analysis

5.1. Setting parameters

A numerical example is provided to validate the proposed model. The following basic data are used: $A = 100$, $D = 12$, $l = 10$, $h = 0.2$, $T_0 = 0.04$, $b = 0.54$, $a = 50$, $c = 1.3$, $d = 180$, $f = 1.28$, $\varepsilon = 0.04$, $\delta = 2.04$, and $\kappa = 0.9$.

5.2. Optimal solution

Case 1. From Proposition 1, we have $t^* = 0.7705$; the optimal average cost is $ATC_1(t^*) = 21.47$; the maximum inventory is $S_1^* = 105$; the maximum output is $Q_1^* = 115$; the non-production time for a period is 8.7921; and the total time per cycle is 9.5626;

Case 2. When production time in the learning phase is $t_1 = 0.50$, the optimal production time is $t^* = 0.8592$; production time in the stable phase is 0.3592; and the optimal average cost is 21.52.

Case 3. Given $t_1 = 0.50$ and $t_2 = 0.75$, through the algorithm solving procedure, the optimal total production time is obtained as $t^* = 0.8708$; the fatigue phase production time is 0.1208; the maximum yield is $Q_3^* = 114$; the maximum inventory is $S_3^* = 104$; and the optimal average cost is 21.53. Because of the fatigue effect, the average cost in Case 3 is more than that in Case 2.

Case 4. Given $t_1 = 0.50$ and $t_2 = 0.75$, by the Newton Raphson method, we derive $t_3^* = 0.8491$. It means that production should stop at the moment 0.8491, followed by a rest. The rest time is 0.2022. After the end of the rest time, production with relearning restarts. The relearning production time is 0.182. The optimal average cost of the system is 21.62. Production time during the fatigue phase is 0.0991, which is less than 0.1208 in Case 3.

5.3. Sensitivity analysis and managerial implications

5.3.1. The influence of the learning factor b and the initial production time T_0 in Case 1

Figure 6 shows that when the learning coefficient is less than a critical value, it increases as the optimal production time decreases. When the learning coefficient is greater than a critical value, it increases as the optimal production time increases. The influence of the learning coefficient on the maximum yield is consistent with the optimal production time (Proposition 2). The greater value of T_0 means the lower initial productivity; thus, it takes a longer time to produce a certain product with the required quantity. With increase in the initial

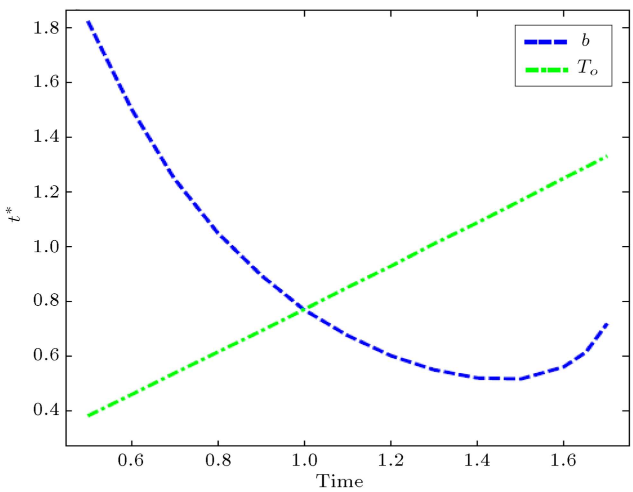

Figure 6. Impact of b and T_0 on the production time.

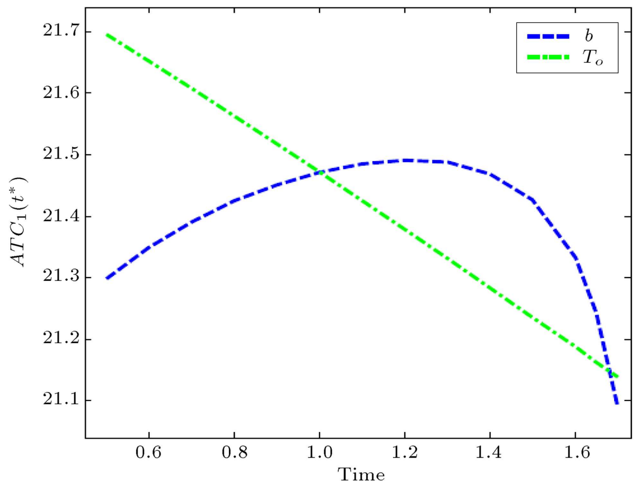

Figure 7. Impact of b and T_0 on the optimal cost.

production time, the optimal production time increases linearly (see Figure 6).

As illustrated in Figure 7, with increase in learning coefficient, the optimal total average cost shows an inverted U character type feature, i.e., the cost first increases and then, decreases. Only when the learning coefficient is bigger than a critical value, the effect of learning on cost reduction is obvious. On the other hand, with increase in the initial production time T_0, the optimal average total cost decreases linearly.

In Figures 6 and 7, we can see that the impact of learning coefficient b on the production time and cost is reverse. Similar condition exists with the initial production time T_0.

Observation 1. *The role of learning coefficient in reducing the production time and the total cost is reversed. It is necessary to choose the right learning coefficient to balance the optimal production time and optimal cost.*

5.3.2. The impact of t_1 on the total production time and the average cost in Case 2

Figures 8 and 9 show the impact of t_1 on total production time and the average cost in Case 2.

In Figure 8, the blue colored line is the optimal total production time (t^*) and red colored line is the stable production time ($t^* - t_1$). It is shown that with increase in learning production time (t_1), both the total optimal production time and the stable production time decrease. When the learning production time approaches 0.7705, the stable production time tends to be zero in the stable phase and the total production time approaches 0.7705.

Figure 9 shows the influence of the learning production time on the optimal average cost. It is shown that with increase in learning production time, the cost curve becomes an S type curve. When $t_1 <$ 0.32, with increase in learning production time, the

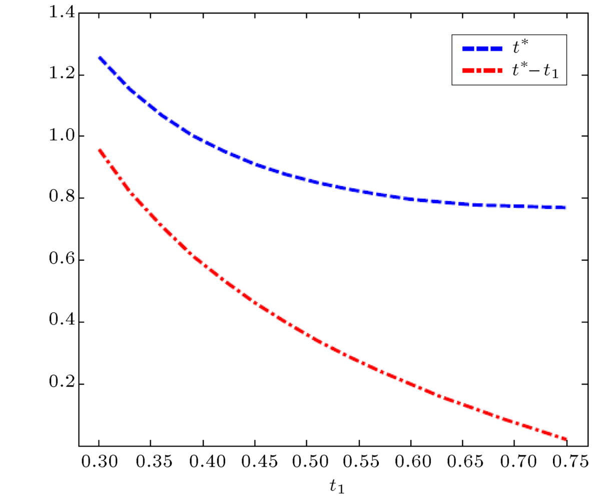

Figure 8. Impact of t_1 on production time.

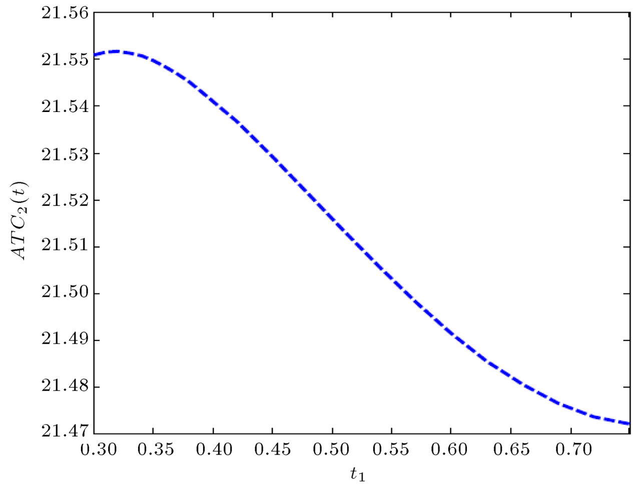

Figure 9. Impact of t_1 on $ATC_2(t)$.

maximum inventory increases; thus, optimal average cost of the system increases. When $t_1 > 0.32$, with increase in the learning production time, the optimal average cost decreases.

Obsevation 2: *If production process involves only learning and stable phases, the effect of prolonging learning production time on the reduction in cost is obvious; thus, prolonging production time is beneficial.*

5.3.3. The influence of t_1, t_2 on the total production time and the average cost in Case 3

The impact of t_1, t_2 on the total production time and average cost is illustrated in Figures 10 and 11 for the situation in which production goes beyond the production and stable phases and ends in the fatigue phase in Case 3.

In Figure 10, it is shown that when $t_1 < 0.36$, the increase in t_1 prolongs fatigue production time; thus, the total production time increase. When $t_1 >$

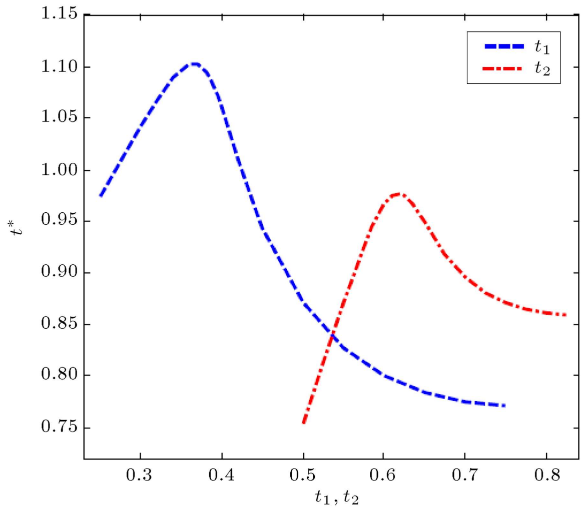

Figure 10. Impact of t_1, and t_2 on the total production time.

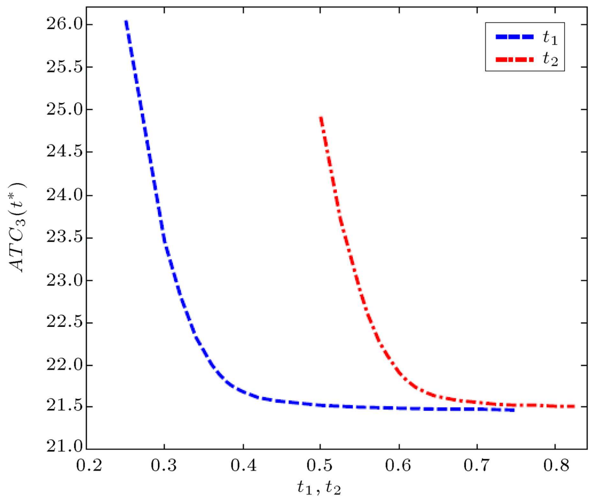

Figure 11. Impact of t_1, and t_2 on $ATC_3(t^*)$.

0.36, the increase in t_1 shortens the fatigue production time; thus, the total production time decreases. When the learning production time is given (in the case of $t_1 = 0.50$), the effect of t_2 on total production time is similar to that of t_1. When $t_2 < 0.62$, the increase in stable production time prolongs fatigue production time; thus, the total production time increases. When $t_2 > 0.62$, the increase in stable production time shortens the fatigue production time; thus, the total production time decreases.

In Figure 11, we can observe that the impacts of t_1, and t_2 are similar. Prolonging the stable production time results in higher productivity in a longer time, hence the average productivity will be higher. As a result, the average cost decreases quickly. Based on Figure 11, when $t_1 > 0.4$ or $t_2 > 0.7$, with increase in the learning production time or the stable production time, the average cost remains almost unchanged. This means that the longer the learning time and the stable production time, the higher the productivity and the lower the production cost.

Based on the above analysis, Observation 3 holds.

Observation 3. *When production goes beyond the learning and stable phases, prolonging of the learning or stable production time can reduce the average cost.*

5.3.4. Influence of productivity coefficients a, c, d, and f on the fatigue phase in Case 3

In Case 3, since production goes through the learning, stable, and fatigue phases, the productivity in the fatigue phase also affects the result. The impact of the productivity coefficient on the fatigue phase is discussed in the following. First, the effects of the fatigue related parameters a, d, and f on the total production time and cost are analyzed. In Figures 12 and 13, we can see that the total production time and the average cost are not sensitive to a. With increase

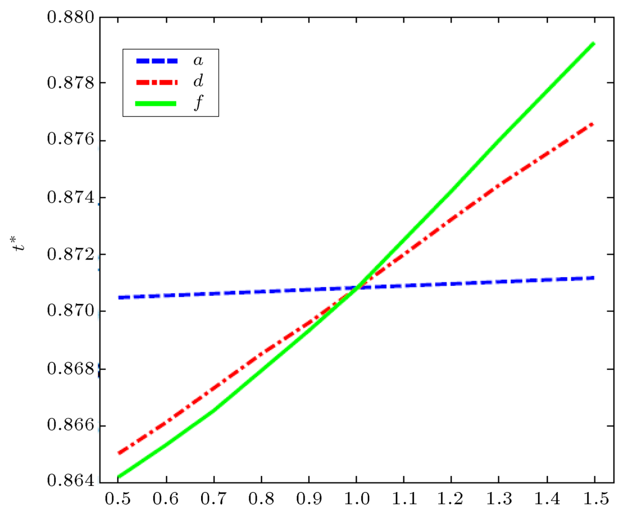

Figure 12. Impact of a, d, and f on t^*.

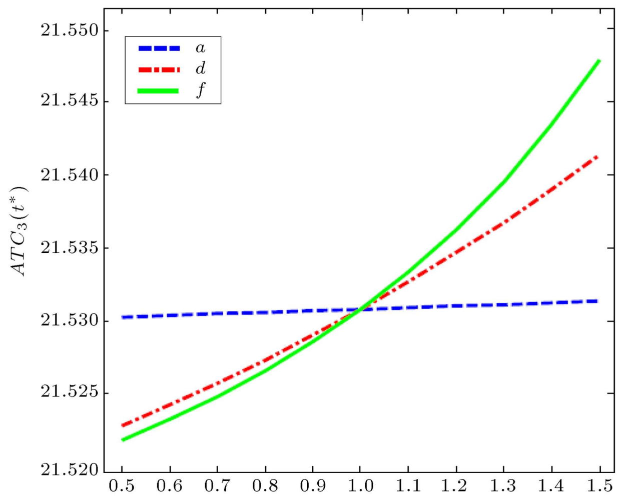

Figure 13. Impact of a, d, and f on $ATC_3(t^*)$.

in d or f, the production time tends to increase linearly and the average cost increases almost exponentially.

The larger values of f mean higher fatigue level, leading to a faster decrease in productivity. In order to meet the demand, we may need to prolong the production time. As a result, this further increases fatigue level and cost.

Observation 4. *When production goes beyond fatigue phase, fatigue leads to faster decrease in productivity, resulting in prolonged production time and decrease in productivity. This productivity degeneration will cause increase in total cost.*

From this observation, we can deduce that when fatigue of the workers reaches a certain level, the labor productivity decreases rapidly. Therefore, it becomes unwise to continue production and proper rest for fatigue recovery is necessary.

In addition, the influence of the fatigue related parameter c on the total production time and cost is analyzed. Figure 14 shows the effect of c on the production time (t^*) and Figure 15 shows the effect of

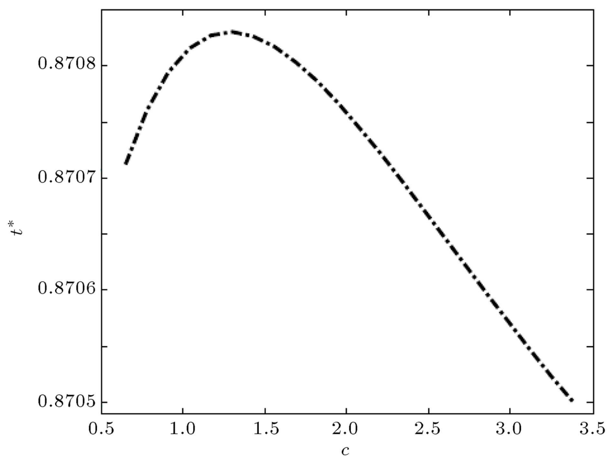

Figure 14. Impact of c on t^*.

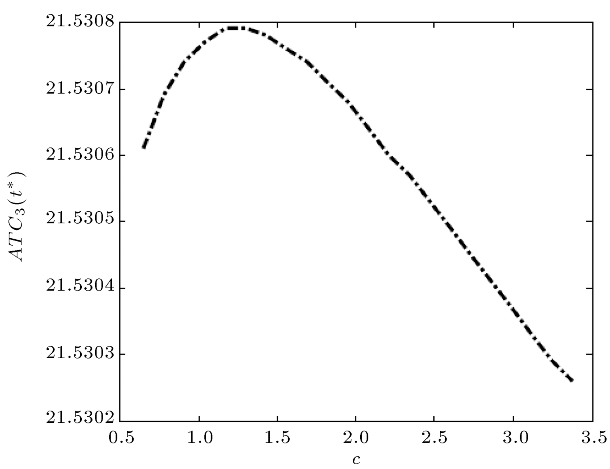

Figure 15. Impact of c on $ATC_3(t^*)$.

c on the average cost ($ATC_3(t^*)$). Both of them have an inverted shape. When c is smaller, production time and the average cost increase with increase in c. When c is bigger than a certain value, both the production time and the average cost decrease.

5.3.5. The influence of δ on fatigue production time and average cost in Case 4

In Case 4, production goes through five phases: learning phase, stable phase, fatigue phase, rest phase, and relearning phase. The scale factor δ reflects the relationship between rest time and fatigue production time, which also impacts the result. We analyze the effect of δ on the fatigue production time and the average cost. The result of the analysis is given in Figures 16 and 17.

Figure 16 illustrates the relationship between δ and the fatigue production time ($t_3^* - t_2$), and Figure 17 illustrates the impact of δ on the average cost $ATC_4(t^*)$ when $\varepsilon = 0.02, 0.04$, and 0.06.

According to Figure 16, regardless of the value of

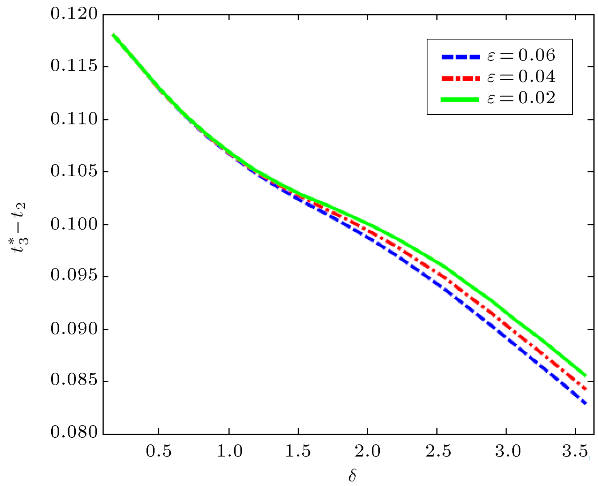

Figure 16. Impact of δ on $t^* - t_2$.

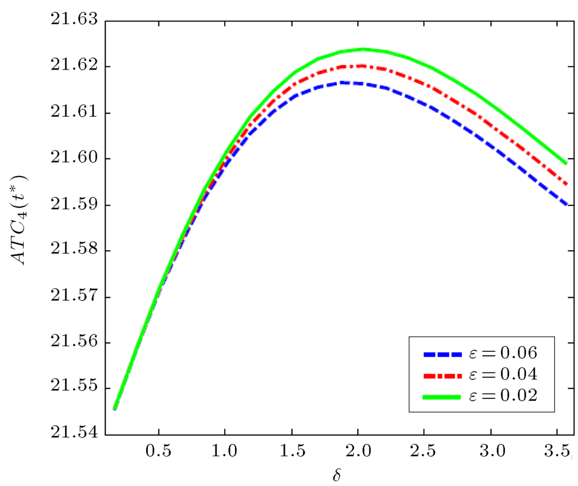

Figure 17. Impact of δ on $ATC_4(t^*)$.

ε, the fatigue production time decreases as δ increases. This is because increasing means an increased fatigue level, which requires a longer rest time. As illustration in Figure 17, with increase in δ, the optimal average cost initially increases and then, decreases. This implies that a short rest time is not beneficial for reducing cost. One should have sufficient rest time in order to reduce cost and increase economic value. In the results, we also observe that for larger ε values, all curves in Figures 16 and 17 are low. This is because larger δ means a better recovery after rest, leading to higher productivity after the relearning. As a result, the fatigue production time becomes shorter and the average cost gets lower.

Based on this observation, an appropriate fatigue recovery time is very important to improve the efficiency of production.

6. Conclusions

In a labor-intensive manufacturing industry, human factor plays an important role in operations decisions. This study dealt with the problem of behavioral economic value of leaning and fatigue recovery in a production inventory decision. Based on a real case study, we developed a new Economic Production Quantity (EPQ) model considering learning, fatigue, and fatigue recovery effects. The optimal production time and fatigue recovery time were determined to minimize the average cost of the production system. Newton Raphson method was used to derive the optimal solutions. The main conclusions of our study are as follows:

1. Learning effect plays an important role in the early phase of production. The presence of fatigue will decrease the efficiency of the production system by lowering productivity;
2. An appropriate fatigue recovery is necessary for reducing cost and increase productivity. When the rest time is shorter than required, fatigue cannot be alleviated. With increase in the optimal rest time, the average total cost will decrease;
3. It is not always appropriate to cut the fatigue production time. Proper prolonging of the fatigue phase production time can reduce the cost.

Although human factor in operations management has been researched for a long time, modeling the behavioral value of learning and fatigue effects on production inventory decision is lacking. Our study has investigated the behavioral value of learning and fatigue effects on production inventory decision using a real-life example. For further research, we can consider multiple products and multi-stage production systems in the models.

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