Shape optimization of concrete arch dams considering stage construction

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Abstract. This paper describes a methodology to develop the interface between a finite element software and optimization algorithm for optimization of concrete high arch dams. The objective function is the volume of the dam. The numbers of design variables are 31 including the thickness and upstream profile of crown cantilever, thickness of the left and right abutments, radius of curvature of water and air faces left and right by use of polynomial curve fitting and cubic spline function. The constraint conditions are the geometric shape, stress, and the stability against sliding. Initially, a program is developed in MATLAB in order to generate the coordinates of nodes; then, finite element software ANSYS is taken for modeling the geometry of dam. Finally, the optimization technique is performed by Simultaneous Perturbation Stochastic Approximation algorithm. To include dead weight of dam body, stage construction is considered. The proposed method is applied successfully to an arch dam and good results are achieved. The results indicate that the concrete volume of the optimized dam is reduced by an average of 21%. Compared with the initial shape, the time of convergence in this method is very short and the method is fairly effective. It can be applied to practical engineering design.

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1. Introduction

The optimal shape is the best design for a structure subject to various constraints imposed by the restrictions placed on the design. Shape optimization is the key step in the design of an arch dam.

The geometrical shape defined during the initial design phase is not always the best one from technical and economical points of view. The best shape should be defined by means of optimization studies, which employ a set of structural safety and minimal cost criteria [1]. Introduction to Optimum Design started from the late 1960’s and several different researchers continued it [2-22]. In recent years, many methods of optimization are being developed rapidly and much attention has been paid by several authors to the fields of concrete arch dam.

Sun et al. [23] established an optimization model for the shape design of arch dam by the use of cubic spline arch for the shape parameters, such as the coordinates of nodes, semi-center angle, and the thickness of arch abutment. The results indicated that the concrete volume of the arch dam optimized by the proposed cubic spline was less than the original design scheme optimized using parabolic shape. Li et al. [24] used the modified complex method which can search for the optimal solution directly, has no special request on the condition of the objective function and constraint function, and does not need to derive during iteration pilot calculation. Fanelli [25] showed that the degrees of
freedom, which are strictly necessary to be considered in the shape optimization procedure of an arch dam, can be reduced by a judicious choice of basic model and design variables. Peng et al. [26] expressed that the optimization of arch dams is complex, because its objective function and constraint conditions appear non-linear and it applies a genetic algorithm with closure temperature field for shape optimization of arch dams.

Tajalli et al. [27] used Bofang formulation for parabolic arch dam. The finite element analysis and optimization procedure are implemented by commercial programs. The combination of Simultaneous Perturbation Stochastic Approximation (SPSA) and Particle Swarm Optimization (PSO) algorithm is introduced by Hamidian et al. [28] to find the optimal shapes of arch dams. Akbari et al. [29] employed a new algorithm for geometry modeling of arch dams using Hermit cubic splines and the optimization problem solved via the Sequential Quadratic Programming (SQP) method. Talakoccezah and Ghaemian [30] did the shape optimization of arch dams considering abutment stability with Particle Swarm Optimization (PSO) method.

The present study describes a method for shape optimization of double curvature concrete arch dam. In the optimization process, the objective function is volume of the concrete arch dam. Design variables of processing are the geometric shape parameters of the double curvature of arch dam and the constraint conditions are geometric shape, stress, and stability against sliding. The program basically consists of three parts that are described in the paper:

1. Generation of the coordinate of nodes by MATLAB code;
2. Call ANSYS batch file for analysis;
3. Optimization of the arch dams by SPSA algorithm according to the established load combination.

For optimization purposes, it is convenient to consider the effects of dam body dead weight and upstream hydrostatic pressures. To include dead weight of dam body, stage construction is considered. The proposed method is successfully applied to an arch dam, where good results are achieved. The results indicate that the concrete volume of the optimized arch dam is reduced by an average of 21%. Compared with the initial shape, the time of convergence in this method is very short and the method is fairly effective. It can be applied to practical engineering design.

2. Mathematical equation of arch dam design

2.1. Preliminary design

The main geometric parameters of the arch dam are shown in Table 1.

<table>
<thead>
<tr>
<th>K</th>
<th>Arch number</th>
<th>E.L.</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_c</td>
<td>Thickness of the crown cantilever</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_AL</td>
<td>Left abutment thickness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_AR</td>
<td>Right abutment thickness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USP</td>
<td>Crown cantilever upstream profile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DSP</td>
<td>Crown cantilever downstream profile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_LUS</td>
<td>Radius of curvature of left water face</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_RUS</td>
<td>Radius of curvature of right water face</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_LDS</td>
<td>Radius of curvature of left air face</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_RDS</td>
<td>Radius of curvature of right air face</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_eL</td>
<td>Left abutment curve</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_eR</td>
<td>Right abutment curve</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2. The geometric model of an arch dam

The shape of an arch dam is of paramount importance in its ultimate behavior and eventually settles all design criteria. Variable curvature arch dams evolved to be economical in shape optimization studies [31].

These geometrical parameters can be defined as follows.

2.2.1. Crown cantilever shape

For definition of crown cantilever, two quadratic functions of vertical coordinates for water and air face are employed.

\[ USP(Z) = a_0 + a_1 Z + a_2 Z^2, \]
\[ DSP(Z) = b_0 + b_1 Z + b_2 Z^2, \]

in which \( Z \) is vertical coordinate; \( a_0, a_1, a_2, b_0, b_1, b_2 \) are the coefficients, and extrude is the curved upstream surface of the horizontal arch elements. The intrados is the curved downstream surface of horizontal arch elements; USP and DSP are the crown cantilever U/S (upstream) and D/S (downstream) profile, respectively. In other words, USP is the horizontal distance between the extrados and the axis on a line normal to the extrados and the DSP is the horizontal distance between the intrados and the axis on a line normal to the intrados. In Figure 1, the shape of crown cantilever and layers at control elevations are shown.

2.2.2. Thickness of arch dam

In this paper, the variations of thicknesses of the vertical crown cantilever and the horizontal arch sections are taken to be the third-degree polynomials of the vertical coordinate. The thicknesses of the arch horizontal sections are calculated using the following equations:

\[ t_c(Z) = c_0 + c_1 Z + c_2 Z^2 + c_3 Z^3, \]
\[ t_AL(Z) = d_0 + d_1 Z + d_2 Z^2 + d_3 Z^3, \]
Given the following list of points:

\[ a = x_0 < x_1 < \ldots < x_n = b \Rightarrow \ x \in [x_i, x_{i+1}] \]

\[(i = 0, 1, \ldots, n - 1)\]

\[y_0, y_1, \ldots, y_n.\]  \hspace{1cm} (7)

A cubic spline \( S(x) \) is a piecewise-defined function that satisfies the following conditions:

1. \( S(x) = S_i(x) \) is a cubic polynomial on each subinterval:

\[ [x_i, x_{i+1}] \quad (i = 0, 1, \ldots, n - 1), \]  \hspace{1cm} (8)

2. \( S(x_i) = y_i; \ i = 0, 1, \ldots, n, \)

\( S \) interpolates all the points.

3. \( S(x), S'(x), \) and \( S''(x) \) are continuous on \([a, b]\) (\( S \) is smooth).

So, the \( n \) cubic polynomial pieces can be written as:

\[ S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \]

\[ i = 0, 1, \ldots, n - 1, \]  \hspace{1cm} (10)

in which \( a_i, \ b_i, \ c_i, \) and \( d_i \) represent \( 4n \) unknown coefficients. The following conditions are held for interpolation and continuity:

a) \( S(x) \) is continuous at discrete points:

\[ S_i(x_i) = y_i, \quad i = 0, 1, \ldots, n - 1, \]  \hspace{1cm} (11)

\[ S_i(x_{i+1}) = y_{i+1}, \quad i = 0, 1, \ldots, n - 1. \]  \hspace{1cm} (12)

b) Derivatives of \( S(x) \) at discrete points are:

\[ S_i'(x_{i+1}) = S_{i+1}'(x_{i+1}) \quad i = 0, 1, \ldots, n - 2, \]  \hspace{1cm} (13)

\[ S_i''(x_{i+1}) = S_{i+1}''(x_{i+1}), \quad i = 0, 1, \ldots, n - 2. \]  \hspace{1cm} (14)

Based on conditions (a) and (b), the total number of equations is \( 4n - 2 \). The expressions for the derivatives of \( S_i \) can be written as:

\[ S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \]  \hspace{1cm} (15)

\[ S_i'(x) = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2, \]  \hspace{1cm} (16)

\[ S_i''(x) = 2c_i + 6d_i(x - x_i). \]  \hspace{1cm} (17)

If \( h_i = x_{i+1} - x_i \), then the spline conditions can be written as follow (substitute Eqs. (11) to (14) into Eqs. (15) to (17)):

\[ a_i = y_i, \]  \hspace{1cm} (18)

\[ a_i + b_i h_i + c_i h_i^2 + d_i h_i^3 = y_{i+1}, \]  \hspace{1cm} (19)
\begin{equation}
\begin{aligned}
    b_i + 2hc_i + 3h_i^2d_i - b_{i+1} &= 0, \\
    2c_i + 6h_id_i - 2c_{i+1} &= 0.
\end{aligned}
\end{equation}

The above equations can be written as a linear system for the 4m unknowns, i.e. \(a_0, b_0, c_0, d_0, a_1, b_1, c_1, d_1, \ldots, a_{n-1}, b_{n-1}, c_{n-1}, d_{n-1}\).

The definition of term \(m_i\) is given in Eq. (22):
\begin{equation}
S_i''(x_i) = 2c_i \quad \text{or} \quad c_i = m_i/2.
\end{equation}

Considering \(m_i\) as unknowns instead, we have (substitute Eqs. (22) and (14) into Eq. (21)):
\begin{equation}
d_i = (m_{i+1} - m_i)/(6h_i).
\end{equation}

Substitute Eqs. (11) and (12) into Eq. (19) in the following:
\begin{equation}
y_i + h_i b_i + h_i^2 c_i + h_i^3 d_i = y_{i+1}.
\end{equation}

Substituting \(c_i\) and \(d_i\) from Eqs. (22) and (23) into Eq. (21), the following is obtained:
\begin{equation}
b_i = \frac{y_{i+1} - y_i}{h_i} - \frac{h_i}{2} m_i - \frac{h_i}{6} (m_{i+1} - m_i).
\end{equation}

From Eq. (13) we have:
\begin{equation}
b_i + 2hc_i + 3h_i^2d_i = b_{i+1}.
\end{equation}

Substitute Eqs. (22), (23) and (25) into Eq. (21):
\begin{equation}
h_i m_i + 2(h_i + h_{i+1}) m_{i+1} + h_i + h_{i+1} m_{i+2} = 6 \left[ \frac{y_{i+2} - y_{i+1}}{h_{i+1}} - \frac{y_{i+1} - y_i}{h_i} \right].
\end{equation}

These are \((n - 1)\) linear equations for \((n + 1)\) unknowns, i.e. \(m_0, m_1, m_2, \ldots, m_n\), where \(m_i = d_i''(x_i)\).

For taken together gives an \((n+1)\times(n+1)\) system of equation as shown in Box I.

\begin{equation}
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & h_0 + h_1 & h_1 & \cdots & 0 \\
0 & h_1 & 2h_1 + h_2 & h_2 & \cdots \\
0 & 0 & h_2 & 2h_2 + h_3 & h_3 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & h_{n-2} & 2h_{n-2} + h_{n-1} & h_{n-1} & \cdots \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 1 \\
\end{bmatrix}
\begin{bmatrix}
m_0 \\
m_1 \\
m_2 \\
m_3 \\
\vdots \\
m_n \\
\end{bmatrix}
= \begin{bmatrix}
y_{i+2} - y_{i+1} \\
y_{i+1} - y_i \\
y_{i+1} - y_i \\
y_{i+1} - y_i \\
\vdots \\
y_{i+1} - y_{i+1} \\
\end{bmatrix} = 6
\end{equation}

If \(m_0 = 0, m_n = 0\), the above equation system will be reduced to \((n + 1)\times(n + 1)\) system.
RLUS and RRUS. In air face, \( y_h = DSP \), and the radius of curvature are RLDS and RRDS. Again, the radii of curvature at apex point are defined by spline. The axes of three parabolas are coincident and are positioned on dam reference plane.

To define the horizontal section, two parabolic curves are defined in the left and right sides of Figure 4. This is done in order to model an unsymmetrical arch dam. Each side is divided into two segments: constant thickness and variable thickness. The thickness of the dam in horizontal section is constant in the first segment and increases by parabolic function in the second one [32]. Coefficients \( k_r \) and \( k_l \) determine portion of the length of arch with constant thickness in the right and left banks. In this paper, \( k_r \) and \( k_l \) are equal to 2/3.

\[
\begin{align*}
T_{aR}(x) &= T_C + \frac{(x-x_{dR})^2}{(x_{dR}-x_{adj})^2} \quad x_{dR} < x < x_{adj} \\
T_{aL}(x) &= T_C + \frac{(x-x_{dL})^2}{(x_{dL}-x_{adj})^2} \quad x_{dL} < x < x_{adj}
\end{align*}
\]

(29)

[Diagram: A horizontal arch of the dam body.]

2.2.5. Programming and implementation of 3D model and loadings

According to the above formula, a MATLAB program for geometrical design of arch dams was written so that Finite element model was developed in the APDL programming language of the ANSYS code. In finite element modeling of the arch dam, geometry is considered as double curvature arch dam. In the finite element model of an arch dam, 1580 eight-node elements in the foundation and 180 twenty-node elements in the dam body are used. Each node has three degrees of freedom: translations in the nodal X, Y and Z directions. Two layers of elements were set along thickness of the dam. The finite element model of the dam is developed so that it includes the foundation. As it is shown in Figure 4, the length and width of the foundation along the global X and Y axes are taken to be 1560 m. For the 3D arch dam analysis, mass concrete and rock were assumed to be homogeneous with linear elastic materials. The modulus of elasticity of mass concrete was taken as 28 GPa and that of the foundation rock as 9 GPa. The Poisson’s ratios of mass concrete and rock were taken as 0.18 and 0.25, respectively. Mass density of the concrete was chosen as 2400 kg/m\(^3\) and no gravity load was applied on the foundation rock. Concrete and rock were assumed to be homogeneous and isotropic materials. As the foundation is assumed as massless, only the effects of foundation flexibility are considered in the analysis. For the boundary conditions in the finite element model of the dam, all degrees of freedom are fixed at the outside surfaces of the foundation.

Figure 5 illustrates the dam, foundation, and reservoir finite element model. The usual static cases include the effects of silt and tail water pressures and temperature (either summer or winter), while for optimization purposes, it is convenient to exclude all these effects and merely consider the effects of dam body dead weight and upstream hydrostatic pressures. In this research, two basic loading cases, as follows, have been considered for the optimization procedure:

1. SU1 (the first usual static load combination) or self-weight;
2. SUN1 (the first unusual static unusual load com-

[Diagram: Three-dimensional shape of an arch dam with foundation.]
3. The optimization method of arch dam shape

Shape optimization aims to minimize consumed concrete volume while enhancing safety criteria. The shape optimization problem is to find the design variable $X$ while minimizing the objective function $F(x)$ under the constraint functions $h_j(X)$ and $g_k(X)$ that can be stated mathematically as:

Find $X = [X_1, X_2, ..., X_n]^T$,

$a_i \leq X \leq b_i$ ($i = 1, 2, ..., n$).

To minimize $F(x)$

$h_j(X) = 0$ ($j = 1, 2, ..., p$).

$g_k(X) \leq 0$ ($k = 1, 2, ..., m$). \hspace{1cm} (31)

The subscripts $p$, $m$, and $n$ denote the number of equality constraints, behavioral constraints, and design variables, respectively, where $a_i$ and $b_i$ are allowable lower and upper limits of the design variables, which are introduced to deal with various requirements.

3.1. Design variables

Shape optimization can be improved by increasing the number of design variables, but it raises the cost of calculations. According to the geometrical model of arch dams described before in the paper, the design variables can be selected as: 31 design variables, which will be used in the process of optimization, as shown in Table 3.

Crown cantilever design variables are shown in Figure 7.

3.2. Objective function

The purpose of optimization is to choose proper geometric shape of arch dam to make the project cost minimal on the premise of meeting the needs of

Table 2. Load combinations used for presenting the analyses results.

<table>
<thead>
<tr>
<th>Load combination number</th>
<th>Single load parts</th>
<th>Factor of safety</th>
<th>Load combination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead weight</td>
<td>Normal water</td>
<td>Tension</td>
</tr>
<tr>
<td>SU1</td>
<td>√</td>
<td>√</td>
<td>2</td>
</tr>
<tr>
<td>SUN1</td>
<td>√</td>
<td>-</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 3. Design variables.

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Radius</th>
<th>USP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{c1}$</td>
<td>$R_{LUS1}$</td>
<td>$R_{US1}$</td>
</tr>
<tr>
<td>$t_{c7}$</td>
<td>$R_{LUS7}$</td>
<td>$R_{US7}$</td>
</tr>
<tr>
<td>$t_{c11}$</td>
<td>$R_{LUS11}$</td>
<td>$R_{US11}$</td>
</tr>
<tr>
<td>$t_{c16}$</td>
<td>$R_{LUS16}$</td>
<td>$R_{US16}$</td>
</tr>
</tbody>
</table>
strength and stability. Generally, the cost of arch dam is mainly dependent upon the volume of dam body concrete. So, the objective function is the dam body volume.

3.3. Constraint functions

In shape optimization of concrete arch dams, the following three types of constraint sets should be satisfied, as required by the demands of design and construction:

1. Geometrical constraints;
2. Stress constraints;

3.3.1. Geometrical constraints set

Thickness of horizontal arch: The thickness of crown cantilever decreases from base to the dam crest.

\[ T_{C_{i+1}} < T_{C_{i}} = \frac{T_{C_{i+1}}}{T_{C_{i}}} - 1 \leq 0 \quad (i = 0, 1, ..., n). \]  

(32)

For different elevations, the crown cantilever thickness is lower than abutment thickness.

\[ T_{C_{i}} < T_{AR_{i}} = \frac{T_{C_{i}}}{T_{AR_{i}}} - 1 \leq 0 \quad (i = 0, 1, ..., n). \]  

(33)

Slope of overhang in upstream and downstream of arch dam: To facilitate construction, the maximum slope of overhang at the upstream and downstream faces should be controlled as follows (Figure 8).

Below tangent points, the angles of tangents are negative and above it, they are positive. The plotting steps should be increased to avoid gap in curves of the upper and lower parts.

\[ \theta_{\text{max}}^{U} \leq \theta_{\text{max}}^{D}. \]  

(35)

\[ \theta_{\text{max}}^{U} \leq \theta_{\text{max}}^{D}. \]  

(36)

where \( \theta_{\text{max}}^{U} \) and \( \theta_{\text{max}}^{D} \) are the allowable maximal overhang slopes of the upstream and downstream surfaces, and \( \theta_{\text{max}}^{U} \) and \( \theta_{\text{max}}^{D} \) are the allowable maximal overhang slopes, respectively.

Crown cantilever profile: Below tangent points, the angles of tangents are negative and above it, they are positive. The plotting steps should be increased to avoid gap in curves of the upper and lower parts.

Crown cantilever upstream profile:

\[ \text{USP}_{i+1} < \text{USP}_{i} \Rightarrow \frac{\text{USP}_{i+1}}{\text{USP}_{i}} - 1 \leq 0 \quad (i = 0, 1, ..., n). \]  

(37)

Crown cantilever downstream profile:

\[ \text{DSP}_{i+1} < \text{DSP}_{i} \Rightarrow \frac{\text{DSP}_{i+1}}{\text{DSP}_{i}} - 1 \leq 0 \quad (i = 0, 1, ..., n). \]  

(39)

Location of the tangent point: As shown in Figure 9, the maximum distance between crest and tangent point is 0.6 H [33].

\[ H_{\text{Tangent point}} = 0.6H. \]  

(41)
3.3.2. Stress constraints
Stress constraints are used to control stress distribution in the structure. Under different loads imposed on arch dam, the maximum stress is less than the allowable stress. In this study, the behavior constraints are defined to prevent failure of each element (i) of arch dam under specified safety factor (sf). For this purpose, the failure criterion of concrete of Willam and Warneke [32] due to multiaxial stress state is employed as follows:

\[
\frac{f}{f_c} - \frac{s}{f_c} - \frac{s}{f_{ic}} \leq 0 \quad (i = 0, 1, \ldots, n_c), \tag{48}
\]

where \((f)\) is a function of the principal stress state \((\sigma_1 \geq \sigma_2 \geq \sigma_3)\) and \((s)\) is failure surface expressed in terms of principal stresses, uniaxial compressive strength of concrete \((f_c)\), uniaxial tensile strength of concrete \((f_{tc})\), and biaxial compressive strength of concrete \((f_{bc})\). Table 4 shows the four principal stress states by which the failure of concrete is categorized into four domains. In each domain, independent functions describe \((f)\) and the failure surface \((s)\). The details of failure criterion can be found in Willam and Warneke and the theory reference of ANSYS.

The angle of similarity \((\eta)\) describes the relative magnitudes of the principal stresses as:

\[
\cos \eta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{\sqrt{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 + \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}}. \tag{49}
\]

The parameters \((r_1)\) and \((r_2)\) represent the failure surface of all stress states with \((\eta = 0^\circ)\) and \((\eta = 60^\circ)\), respectively, and they are functions of principal stresses and concrete strengths. The parameters \((r_1)\) and \((r_2)\) and the angle \((\eta)\) are shown in Figure 10.

Therefore, Eq. (40) must be checked for the center of all dam elements \((n_c)\) with safety factor that is chosen as \(sf = 1\). If it is satisfied, there is no crack or crush. Otherwise, the material will crack if any principal stress is tensile, while crushing will occur if all principal stresses are compressive.

**Figure 9.** Location of the tangent point.

\[
H_{\text{Tangent point}} = H_{\text{Tangent point max}} - 1 \leq 0. \tag{42}
\]

**Radius of curvature.** The most important geometric constraints are those that prevent intersection of upstream and downstream faces as:

\[
R_{LDS} < R_{LUS}, \quad \frac{R_{LDS}}{R_{LUS}} - 1 \leq 0
\]

\((i = 0, 1, \ldots, n)\). \tag{43}

\[
R_{RDS} < R_{RUS}, \quad \frac{R_{RDS}}{R_{RUS}} - 1 \leq 0
\]

\((i = 0, 1, \ldots, n)\). \tag{44}

The variation of radius at crown cantilever along with height of dam should be satisfactory to some kind of nonlinear variation rule.

\[
R_{LUS} < R_{LUS+1}, \quad \frac{R_{LUS}}{R_{LUS+1}} - 1 \leq 0
\]

\((i = 0, 1, \ldots, n)\). \tag{45}

\[
R_{RUS} < R_{RUS+1}, \quad \frac{R_{RUS}}{R_{RUS+1}} - 1 \leq 0
\]

\((i = 0, 1, \ldots, n)\). \tag{46}

\[
R_{RDS} < R_{RDS+1}, \quad \frac{R_{RDS}}{R_{RDS+1}} - 1 \leq 0
\]

\((i = 0, 1, \ldots, n)\). \tag{47}

where \(R_{RUS}\) and \(R_{RDS}\) are radius of curvatures at the upstream and downstream faces of the dam in the \(i\)th layer in \(z\) direction.

**Figure 10.** Failure surface in the compression-compression-compression Regime.
Table 4. The equation of Wilam and Warnke.

<table>
<thead>
<tr>
<th>Domain</th>
<th>(f, s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
<td>(f = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} )</td>
</tr>
<tr>
<td>Compression</td>
<td>(s = \frac{2\gamma [v_1^2 + \gamma]}{4r_1^2 \gamma \gamma_1} \left( 4r_1 + r_2 \right)^{1/2} )</td>
</tr>
<tr>
<td>Tension</td>
<td>(f = \frac{1}{\sqrt{2}} \left[ (\sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3)^2 \right]^{1/2} )</td>
</tr>
<tr>
<td>Compression</td>
<td>(s = \left( 1 - \frac{\sigma_1}{\sigma_1} \right) \frac{2p_2 [y_2 + \gamma]}{4p_2^2 \gamma^2 + 4p_2 \gamma + 4p_2} \left( 4p_2 + \gamma \right)^{1/2} )</td>
</tr>
<tr>
<td>Tension</td>
<td>(f = \sigma_i, \ i = 1, 2)</td>
</tr>
<tr>
<td>Tension</td>
<td>(s = \frac{k}{J} (1 + \frac{\gamma}{\gamma_1}) )</td>
</tr>
<tr>
<td>Tension</td>
<td>(f = \sigma_i, \ i = 1, 2, 3)</td>
</tr>
<tr>
<td>Tension</td>
<td>(s = \frac{k}{J} )</td>
</tr>
</tbody>
</table>

3.3.3. Stability constraints

**Central angle of the arch:** In this paper, constraints ensuring the sliding stability of the dam may be expressed by central angle of the arch.

\[
1 - \frac{\phi_U}{90} \leq 0, \quad \frac{\phi_U}{110} - 1 \leq 0, \quad (50)
\]

\[
1 - \frac{\phi_D}{90} \leq 0, \quad \frac{\phi_D}{110} - 1 \leq 0, \quad (51)
\]

where \((\phi)\) is the sum of central angles at the right and the left arches, and \(90 \leq \phi_U \leq 110, \quad 90 \leq \phi_D \leq 110\).

**Overturning:** To verify the overall overturning stability of crown cantilever arch dam monoliths:

\[
USP_1 < Y_{bar} \Rightarrow \frac{USP_1}{Y_{bar}} - 1 \leq 0. \quad (52)
\]

4. Optimization algorithm

The Simultaneous Perturbation Stochastic Approximation (SPSA) has recently attracted considerable attention in areas, such as statistical parameter estimation, feedback control, simulation-based optimization, signal and image processing, and experimental design. However, the SPSA has not been tested yet for structural optimization and this is the first study employed for the purpose. The promising feature of the SPSA optimization algorithm is that it requires only two structural analyses in each cycle of optimization process, regardless of the optimization problem dimensions. This attribute can drastically reduce the computational cost of the optimization, particularly in problems with a number of variables to be optimized.

The following step-by-step summary shows the process of SPSA in arch dam optimization:

**Step 1. Initialization and coefficient selection.** Set counter index \(k = 0\). Pick initial guess and non-negative coefficients \(a, c, A, \alpha, \gamma\) in the SPSA gain sequences \(a_k = a / (A + k + 1)\) and \(c_k = c / (k + 1)\). The choice of gain sequences \((a_k\) and \(c_k)\) is critical to performance of SPSA. Spall provides some guidance on picking these coefficients in a practically effective manner;

**Step 2. Generation of the simultaneous perturbation vector.** Generate an \(n_x\) dimensional random perturbation vector \(\Delta_k\) by Monte Carlo, where each of the \(n_x\) components of \(\Delta_k\) is independently generated from a zero mean probability distribution satisfying some conditions. A simple and theoretically valid choice for each component of \(\Delta_k\) is using a Bernoulli ±1 distribution with probability of 1/2 for each ±1 outcome. Note that uniform and normal random variables are not allowed for the element in \(\Delta_k\) by the SPSA regularity conditions;

**Step 3. Fitness function evaluations.** Obtain two measurements of the fitness function \(f(0)\) based on the simultaneous perturbation around the current design vector \(\hat{x}_k\): \(f(\hat{x}_k + c_k \Delta_k)\) and \(f(\hat{x}_k - c_k \Delta_k)\) with the \(c_k\) and \(\Delta_k\) from Steps 1 and 2;

**Step 4. Gradient approximation.** Generate the simultaneous perturbation approximation with the unknown accurate gradient \(G(\hat{x}_k)\):

\[
G_k(\hat{x}_k) \cong \frac{f(\hat{x}_k + c_k \Delta_k) - f(\hat{x}_k - c_k \Delta_k)}{2c_k} \quad (\Delta_k)_{i=1}^{n_x} \]

where \(\Delta_{ki}\) is the \(i^{th}\) component of \(\Delta_k\) vector;
Step 5. Updating $\hat{x}$ estimate. Use the standard Stochastic Approximation (SA) to update $\hat{x}_k$ to a new value $\hat{x}_{k+1}$:

$$\hat{x}_{k+1} = \hat{x}_k - a_k \tilde{G}_k(\hat{x}_k);$$  \hspace{1cm} (54)

Step 6. Iteration or termination. Return to Step 2 with $k + 1$ replacing $k$. Terminate the algorithm if the Maximum Number of Iterations (MNI) has been reached [28]. The flow chart of SPSA algorithm for the arch dam optimization problem can be shown in Figure 11.

5. Result

The optimization process of arch dam according to the above methodology converged after 1000 iterations.

Figure 12 shows the evolution of crown cantilever shape. The initial and optimum values of shape design variables are given in Table 5 (all dimensions are in meters). As can be seen, the volume of the dam body defined by the present optimization is 1057550 m$^3$ less than the initial volume, i.e. 21% less.

| Table 5. Initial and optimum values of shape design variables. |
| --- | --- | --- | --- | --- |
| | 840 | 725 | 625 | 515 |
| TC | Initial design | 10 | 33.22 | 44.52 | 50 |
| | Optimum design (iteration:1000) | 8.48 | 24.67 | 34.60 | 45.80 |
| $T_{AL}$ | Initial design | 10 | 48.87 | 60.1 | 50 |
| | Optimum design | 10.69 | 40.67 | 70.24 | 60.84 |
| $T_{AR}$ | Initial design | 10 | 48.87 | 60.1 | 50 |
| | Optimum design | 9.49 | 41.66 | 67.34 | 52.65 |
| USP | Initial design | 63.12 | 18.65 | 6.83 | 20.00 |
| | Optimum design | 57.36 | 14.30 | 5.42 | 26.35 |
| RLUS | Initial design | 200.00 | 144.56 | 115.07 | 100.00 |
| | Optimum design | 194.54 | 128.88 | 108.32 | 100.72 |
| RRUS | Initial design | 200.00 | 144.56 | 115.07 | 100.00 |
| | Optimum design | 203.23 | 130.45 | 108.67 | 91.77 |
| RLDS | Initial design | 192.03 | 95.32 | 60.81 | 50.83 |
| | Optimum design | 192.37 | 105.56 | 70.47 | 49.99 |
| RRDS | Initial design | 192.03 | 94.98 | 61.48 | 50.83 |
| | Optimum design | 199.03 | 105.76 | 70.50 | 50.52 |

Figure 11. The flowchart of SPSA algorithm.
The difference between the initial and optimum design shapes can be seen in Figure 13. It is observed that the optimal design is thinner than the initial design and slope of overhang in upstream and downstream of surfaces in the optimum design are smaller than those of the initial design, which are benefits for construction [34].

The boldness coefficient in Table 6 is calculated by Lombardi’s formula, as follows:

$$\text{Boldness coefficient} = \frac{\text{Mid surface area}^2}{(\text{Height of dam}) \times \text{volume}} \quad (55)$$

The boldness coefficients for the initial and optimum designs are shown in Table 6. The boldness coefficient for the optimum design is higher than that for the initial design.

The principal stresses for two load cases are shown in Figure 14 and Table 7. The position of maximum tensile stress is found close to the one third of dam height. The maximum tensile stresses are obtained for the U/S face in two cases SUN1 and SU1. For the case of SUN1, the maximum compression stress is observed in the U/S face and for the case the SU1, it is obtained in the D/S case.
<table>
<thead>
<tr>
<th>SUL, σ₁</th>
<th>Initial design</th>
<th>Optimum design</th>
</tr>
</thead>
</table>
| -0.113E+07  
-814717  
-494948  
-174278  
145941  
466161  
786380  
0.111E+07  
0.143E+07  
0.175E+07 | -0.129E+07  
-940018  
-500072  
-240126  
109819  
459705  
809711  
0.116E+07  
0.151E+07  
0.186E+07 |

<table>
<thead>
<tr>
<th>SUL, σ₂</th>
<th>Initial design</th>
<th>Optimum design</th>
</tr>
</thead>
</table>
| -0.122E+08  
-0.108E+08  
-0.94E+08  
-0.815E+07  
-0.681E+07  
-0.548E+07  
-0.414E+07  
-0.280E+07  
-0.146E+07 | -0.127E+08  
-0.113E+08  
-0.990E+07  
-0.850E+07  
-0.710E+07  
-0.570E+07  
-0.430E+07  
-0.290E+07  
-0.151E+07 |

<table>
<thead>
<tr>
<th>SUL, σ₃</th>
<th>Initial design</th>
<th>Optimum design</th>
</tr>
</thead>
</table>
| -0.223E+07  
-0.150E+07  
-0.769623  
-37261  
695101  
0.143E+07  
-0.216E+07  
0.286E+07  
0.362E+07  
0.436E+07 | -0.224E+07  
-0.133E+07  
-414294  
498110  
0.141E+07  
0.232E+07  
0.324E+07  
0.415E+07  
0.506E+07  
0.597E+07 |

<table>
<thead>
<tr>
<th>SUL, σ₃</th>
<th>Initial design</th>
<th>Optimum design</th>
</tr>
</thead>
</table>
| -0.888E+07  
-0.812E+07  
-0.735E+07  
-0.658E+07  
-0.581E+07  
-0.504E+07  
-0.427E+07  
-0.349E+07  
-0.272E+07 | -0.108E+08  
-0.980E+07  
-0.883E+07  
-0.786E+07  
-0.690E+07  
-0.593E+07  
-0.496E+07  
-0.400E+07  
-0.303E+07 |

**Figure 14.** Principal stresses σ₁ and σ₃ in the initial and optimum design shapes for usual load combination (SU) and unusual load combination (SUN).
Table 6. Boldness coefficients for the initial and optimum designs.

<table>
<thead>
<tr>
<th>Design</th>
<th>Height (m)</th>
<th>Volume (m³)</th>
<th>Mid surface area (m²)</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial design</td>
<td>325</td>
<td>4.924.850</td>
<td>137314</td>
<td>11.78</td>
</tr>
<tr>
<td>Optimum design</td>
<td>325</td>
<td>3.863.840</td>
<td>137972</td>
<td>15.16</td>
</tr>
</tbody>
</table>

Table 7. Summary of the results of the models.

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th></th>
<th>DS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Maximum</td>
<td>Maximum</td>
<td>Maximum</td>
</tr>
<tr>
<td></td>
<td>tension</td>
<td>compression</td>
<td>tension</td>
<td>compression</td>
</tr>
<tr>
<td>Initial design</td>
<td>SUN1 1.72</td>
<td>-12.5</td>
<td>0.77</td>
<td>-4.20</td>
</tr>
<tr>
<td></td>
<td>SU1  4.3</td>
<td>-7.43</td>
<td>2.12</td>
<td>-8.90</td>
</tr>
<tr>
<td>Optimum design</td>
<td>SUN1 1.85</td>
<td>-13</td>
<td>1.5</td>
<td>-4.4</td>
</tr>
<tr>
<td></td>
<td>SU1  6.12</td>
<td>-9.9</td>
<td>3.3</td>
<td>-10.9</td>
</tr>
</tbody>
</table>

Figure 15. Convergence rate of the dam body volume.

The values of stresses for the optimum design are bigger than those for the initial design.

Convergence rate of the objective function in the optimization process is shown in Figure 15.

After performing the optimization process, the dam volume decreased by 21% in comparison with that in the initial design.

6. Conclusion

This paper employs a methodology to develop the interface between a finite element method and optimization algorithm for shape optimization of double curvature concrete arch dam.

In order to create the geometry of arch dams, a new algorithm is proposed in MATLAB. This algorithm is able to model the different shapes of an arch dam. The finite element of software ANSYS is taken for modeling the geometry of an arch dam used to consider the effects of dam body dead weight and upstream hydrostatic pressures. The following conclusions, some of which are important, are drawn from the present work:

- It is concluded that SPSA can be effectively used in the shape optimization of arch dams;
- Arch dam is a massive structure and therefore, its construction is staged into a step by step procedure. If the dead load is applied to the dam all at once, without taking into account the fact that horizontal load transfer cannot occur before the dam is complete, fictitious stresses will be indicated. By considering stage construction, there have been longer optimization process and lower optimum volume;
- The maximum tensile stresses are obtained for the U/S face in two cases SUN1 and SU1. For the case of SUN1, the maximum compression stress is observed in the U/S face and for the case of SU1, it is obtained in the D/S case. After the shape optimization of the arch dam, the dam body volume is reduced by 21%.

References


**Biographies**

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