



A solution based on fuzzy max-min approach to the bi-level programming model of energy and flexiramp procurement in day-ahead market

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Abstract. In this paper, we focus on solving the integrated energy and flexiramp procurement problem in the day-ahead market. The problem of energy and ramp procurement could be perfectly analyzed through Stackelberg concept because of its hierarchical nature of decision-making process. Such a circumstance was modeled via a bi-level programming in which suppliers acted as leaders and the Independent System Operator (ISO) was the follower. The ISO intended to minimize energy and spinning reserve procurement cost, and the suppliers aimed to maximize their profit. To solve the proposed model, a fuzzy max-min approach was applied to maximizing the utilities of players. The objectives and dynamic offers of suppliers, determined with regard to the market clearing prices, were reformulated through fuzzy utility functions. The proposed approach is an effective and simple alternative to the Karush Kuhn Tucker (KKT) method, especially for non-convex problems at lower levels.

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1. Introduction

1.1. Background and motivation

Efficient integration of renewable generation into the conventional power systems requires advanced market structures and analysis methods [1]. By increased penetration of the unreliable generation, it has been recognized that the conventional operating reserves are insufficient for responding to the net load variations. Therefore, the establishment of effective strategies for flexibility promoting is necessary [2–5]. Because of the inadequacy of the traditional reserves, California Independent System Operator (CAISO) declared

the need for more ramp rate capability in the next decade [2]. For additional up and down ramping capability, CAISO [6] and Midwest Independent System Operator (MISO) [7] have proposed establishment of the markets for flexible ramping products.

Wind and hydro generators, due to their high flexibility, are interesting strategic options. The strategic significance of wind generators in offering ramp rate arises from the market abnormality that emanates from the regional pricing structure of the electricity market [8].

An aspect of providing flexiramp through renewable power plants is the intentional curtailment of variable generation by the system operator or market-based mechanisms [9]. However, such curtailment is not common and rational with wind plants. In addition, even though power plants may have the technical capability of flexiramp providing, they usually offer a lower rate of the ramping capability to maintain their

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output in a stable level [10]. This issue is examined in [8]. Through an effective optimization model, the optimum contribution of each player based on its object was determined.

There are numerous researches which consider the bi-level programming for problem modeling [8,11–19]. In this section, a number of these studies are reviewed.

Moiseeva et al. [8] developed a bi-level programming model to capture the behavior in offering ramp rate. They considered a market operator who collected bids in the form of marginal costs, quantities, and ramp rates; and run a ramp-constrained economic dispatch in the real-time market. Therefore, the lower-level problem was the ramp-constrained economic dispatch and the higher level represented the profit maximization problems solved by strategic generators.

Riaz et al. [11] developed a Stackelberg game to capture the interaction between Independent System Operator (ISO) and Renewable Energy sources Aggregation (REA). In their game, the ISO attempted to minimize the generation cost, whereas REA maximized the revenue. Wu et al. [12] developed a day-ahead scheduling model through bi-level programming, in which the hourly demand response was considered to reduce the system operation cost and incremental changes in generation dispatch.

Tohidi et al. [17] developed a mathematical model to explore transmission network switching from an economic perspective in the context of the market power. They developed a model to capture the game between strategic Generation Companies (GenCos) based on the Cournot game as the lower-level problem in the bi-level model. At the upper level, the Transmission System Operator (TSO) as a leader minimized the system dispatch cost using transmission switching decisions.

Chen et al. [20] developed an inexact bi-level simulation-optimization model for conjunctive regional renewable energy planning and air pollution control for electric power systems. An improved interactive solution algorithm based on the satisfactory degree was introduced to make a tradeoff between meeting the constraints and attaining the optima for the objectives.

1.2. Gaps in the literature and our contribution

Despite the existence of several promotional strategies for the corporation of wind power plants in power markets, the attention has mostly been concentrated on some strategies such as investment subsidies, feed-in tariff schemes, fixed premium schemes, obligated quotas, and tendering and tradable green certificate systems. These instruments are discussed in more detail in [21]. In this paper, we examine a dynamic price offering method in which the wind units are allowed to propose bids with regard to the market clearing price in the previous period.

We concentrate on solving an integrated energy and flexiramp procurement problem in which the partners intend to attain their objectives. Procurement planning and order allocation to the suppliers are modeled via a bi-level programming in which the suppliers are the upper-level decision makers and the ISO, as the follower, acts in accordance with the decisions of the leaders. The reason for applying a bi-level programming model according to the Stackelberg game lies in the innate nature of the problem. In fact, the ISO chooses its actions after action selection by suppliers. Actions of the suppliers are their bids including the proposed prices and the amounts of available energy and ramp. Also, the leaders (suppliers) know that the follower (ISO) selects its actions after observing actions of the suppliers (bids).

In our proposed model, using a centralized model or KKT optimality conditions transforming the bi-level programming into an equivalent single-level problem [18] leads to wrongly eliminating the distributed nature of the problem and implies that information is completely available to the upper-level players [19]. To solve the bi-level model, a fuzzy max-min approach is applied to maximizing the utilities of the players. The fuzzy max-min approach is designed to obtain a satisfactory compromise between the objectives of the decision-makers [22].

For the case of a multiple-leader-multiple-follower equilibrium, Equilibrium Problem with Equilibrium Constraints (EPEC) optimization models are usually sufficient for representation of the interaction between the market participants. However, EPEC models are too hard to solve and very difficult to compute for large systems [23].

EPECs are a set of coupled Mathematical Program with Equilibrium Constraints (MPECs). Therefore, they inherit the bad properties of MPECs. They are non-convex and non-linear and finding a global solution to them is challenging [23].

Shih et al. [24] used the concepts of membership functions and multiple objective optimization to develop a fuzzy approach to solving the bi-level models. Their approach relied on changes of membership functions expressing satisfactory degrees of potential solutions for both upper- and lower-level decision makers. Such a satisfactory concept is more acceptable than optimality, because it is difficult to define a solid optimality in a multi-person game. Also, optimality is questionable due to restricting the potential solutions to the corners of the feasible region while potential satisfactory solutions are in the non-dominated region. Thus, the fuzzy approach is more efficient and will not increase the complexities of the original problems [24].

Therefore, a simple reason for using the fuzzy approach is its convenience. However, the more important reason is its capability for finding the satisfying

solution for all partners through transactions during the decision-making process instead of an optimal solution [19].

2. Stackelberg equilibrium

Unlike in the simultaneous games, players in Stackelberg game choose their strategies sequentially in a known order being aware of the strategies chosen by the precedent players. Also, the leader/leaders (suppliers in our problem) know that the follower (ISO in our problem) observes their actions [21,25]. Solution procedure for sequential games is backward, which means that we solve first the problem at the second level (follower's problem) and then, the problem at the first level (leader's problem). The simple algorithm for the bi-level problem is explained as follows [26]:

1. At the second level, the follower chooses their reaction given what the leader has chosen.
 - Outcome: reaction function of the follower.
2. At the first level, the leader chooses its action taking the reaction function of the follower into account (i.e., the leader optimizes its objective function subject to the reaction function of the follower)
 - Outcome: Stackelberg strategy.

In static games, if cost functions of the two players are $J_1(u_1, u_2)$ and $J_2(u_1, u_2)$, J_2 will be the leader and J_1 will be the follower. Also, u_2 will be decision variable of the leader and u_1 decision variable of the follower. The Stackelberg strategy is obtained by minimizing the objective of the leader (J_2) subject to the reaction function of the follower ($\frac{\partial J_1}{\partial u_1} = 0$) [26]. The general solution procedure is shown in Figure 1.

Unlike in the matrix games, the Stackelberg solution in static games is not always needed [25]. Thus, to find a near optimal solution, several methods have been adopted. In this paper, we use the fuzzy max-min method presented in [27].

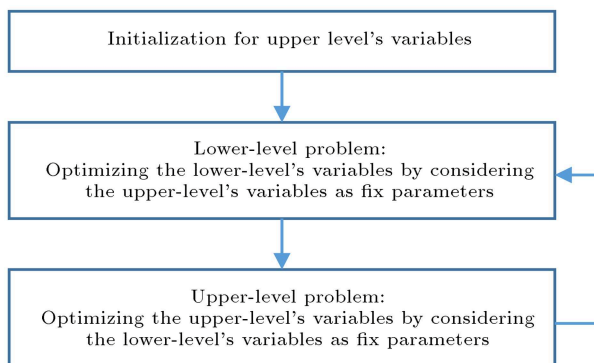


Figure 1. A simple solution process for the Stackelberg game in the bi-level form.

It should be noticed that the current auction-game ends in a period in which all players draw back from the game and a new game with a lower number of players is begun. However, in the new game, the principal player whose refusal was the end of the previous game interestingly becomes a follower or ISO, not the leader. To more clarify this, it is enough to remember the definitions for the leader and the follower. The leader is the player who determines its strategy first and knows that the other player (follower) observes its action. The proposed model is completely introduced in the next section.

3. Bi-level programming model

3.1. Bi-level programming

The bi-level programming problem is a special case of the multi-level programming, which is categorized as an NP-hard non-convex programming problem [28].

A general formulation of the bi-level programming problems is as follows:

$$\begin{aligned}
 & \max_{x,y} F_1(x, y), \\
 & \text{s.t.} \\
 & G_1(x, y,) \leq 0, \\
 & y \in \arg \min_{\hat{y}} f_1(x, \hat{y}), \\
 & \text{s.t.} \\
 & g_1(x, \hat{y}) \leq 0.
 \end{aligned}$$

Bi-level programming problems are complicated optimizations, because:

1. They are intrinsically NP-hard;
2. Their nested structure has inherent difficulties even for the notion of a solution;
3. For many methods, regularity conditions cannot be satisfied at any feasible point.

These features make bi-level programming problems very difficult to globally solve. In recent years, considerable research has been devoted to addressing the solution strategies for Bi-Level Programming Problems (BLPP) [29].

There are several methods for solving BLPP, like methods based on KKT conditions [30]; fuzzy approach [31]; metaheuristic algorithms like genetic [32], Particle Swarm Optimization (PSO) [33], and hybrid GA and PSO [34]; and evolutionary multi-agent system [35]. In [36], it is mentioned that in the hierarchical decision making, no one can achieve their individual optimum decision while the existing competitor has conflicting objectives and thus, a satisfactory decision

is rational for all players trying to maximize their individual objectives as much as possible.

3.2. The proposed bi-level programming

In this section, we explain our proposed mathematical model. Set $S_{i1} : \{C_{i,k}^t, R_{i,t}^{\text{up}}, R_{i,t}^{\text{dn}}\}$ is the set of variables which are controlled by each supplier independently at the upper level and sent to the lower level as fixed parameters. Also, set $S_2 : \{TG_{t,i}, \mu_{t,i}^A, \mu_{t,i}^{\text{up}}, \mu_{t,i}^{\text{Down}}, \text{OBJ}_L\}$ is the set of variables which are controlled by the ISO at the lower level and sent to the upper level as fixed parameters. Unlike in [37], the dispatched quantity of the wind power producer and the proposed prices in the second and third offers are dependent on market prices through a dynamic approach. For simplicity, the grid constraints are not considered in this model, but the inclusion of grid constraints is straightforward.

The upper-level model is a multi-objective programming, which intends to maximize the energy and ramp revenue, and minimize the ramp penalty and fuel cost for conventional generating units. The proposed model is as follows:

$$\begin{aligned} & \max_{S_{i1}} \left(\sum_{t,i,k} g_{t,i,k}^p \times C_{t,i,k} \right), \\ & \max_{S_{i1}} \left((R_{i,t}^{\text{up}} \times \mu_{t,i}^{\text{up}}) + (R_{i,t}^{\text{dn}} \times \mu_{t,i}^{\text{Down}}) \right), \\ & \min_{S_{i1}} \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right), \\ & \min_{S_{i1}} \left(\sum_{t,i} \alpha_{t,i} + (TG_{t,i}^p \times \beta_i) + ((TG_{t,i}^p)^2 \times \gamma_i) \right), \quad (1) \\ & R_{t,i}^{\text{up}} \leq MR_{t,i}^{\text{up}}, \\ & R_{t,i}^{\text{dn}} \leq MR_{t,i}^{\text{Down}}. \quad (2) \end{aligned}$$

Eqs. (1) and (2) are the upper-level problem, and the lower-level problem acts as constraints for the upper-level problem. Also, the objectives at both levels are transformed to the utility functions. Based on the max-min method [38], we must use the min operator to aggregate the satisfactory levels and then, maximize the aggregated utility. Therefore, the upper-level problem is not a separable problem.

At the lower level, ISO as a follower solves a multi-period economic dispatch problem. The objective of the ISO is to minimize the procurement cost of energy and reserve.

$$\min \left(\sum_{t,i,k} g_{t,i,k} \times C_{t,i,k} \right) + (SR_{t,i} \times \mu_{t-1,i}^{\text{reserve}}), \quad (3)$$

$$L_{t,i} \times z_{t,i}^1 \leq g_{t,i,1} \leq L_{t,i},$$

$$L_{t,i} \times z_{t,i}^2 \leq g_{t,i,2} \leq L_{t,i} \times z_{t,i}^1,$$

$$0 \leq g_{t,i,3} \leq L_{t,i} \times z_{t,i}^2. \quad (4)$$

The above equations are considered for the general nonlinear format and $L_{t,i}$ is the length of each piece of Heaviside function. However, if the ramp of the proposed Heaviside function is in an incremental order for the minimization problem, we can use the linear form.

The other lower-level constraints are as follows:

$$\begin{aligned} & \sum_k g_{tik} = TG_{t,i}, \\ & \sum_i TG_{t,i} = D^t, \end{aligned} \quad (5)$$

$$\begin{aligned} & TG_{t,i} \geq p_{t,i}^{\text{min}}, \\ & TG_{t,i} \leq p_{t,i}^{\text{max}}, \end{aligned} \quad (6)$$

$$\begin{aligned} & TG_{t,i} - TG_{(t-1),i} \leq R_{t,i}^{\text{up}} \Leftrightarrow \mu_{t,i}^{\text{up}}, \\ & TG_{(t-1),i} - TG_{t,i} \leq R_{t,i}^{\text{dn}} \Leftrightarrow \mu_{t,i}^{\text{Down}}. \end{aligned} \quad (7)$$

The spinning reserve constraint is as follows:

$$SR_{t,i} \leq \min \{ SR_i^{\text{max}}, p_{t,i}^{\text{max}} - TG_{t-1,i} \}. \quad (8)$$

The above constraint is rewritten as follows:

$$y = \min \{ SR_i^{\text{max}}, p_{i,t}^{\text{max}} - TG_{i,t-1} \}, \quad (9)$$

$$\begin{aligned} & \text{if } SR_i^{\text{max}} \leq p_{i,t}^{\text{max}} - TG_{i,t-1} \rightarrow y = SR_i^{\text{max}}, \\ & \text{if } SR_i^{\text{max}} \geq p_{i,t}^{\text{max}} - TG_{i,t-1} \rightarrow y = p_{i,t}^{\text{max}} - TG_{i,t-1}. \end{aligned} \quad (10)$$

The linearized constraints are as follows:

$$\begin{aligned} & SR_i^{\text{max}} \leq (p_{i,t}^{\text{max}} - TG_{i,t-1}) + M\vartheta, \\ & (p_{i,t}^{\text{max}} - TG_{i,t-1}) \leq SR_i^{\text{max}} + M(1 - \vartheta), \\ & y = SR_i^{\text{max}} + M\vartheta, \\ & y = (p_{i,t}^{\text{max}} - TG_{i,t-1}) + M(1 - \vartheta). \end{aligned} \quad (11)$$

4. Model reformulating and solution procedure

4.1. Fuzzy solution procedure for bi-level models

Many heuristic and metaheuristic methods have been applied to solving the bi-level problems. The first step of many methods, e.g., the fuzzy method, is solving the problem at each level separately with the existing methods like CPLEX. If the optimal solution to the lower-level problem is also feasible for the upper level and vice versa, it is the optimal one for the BLP [39]. However, such a solution is usually unfeasible.

In the fuzzy max-min method, several linear or nonlinear utility functions should be considered for the objectives and upper-level variables. Some simple principles of the fuzzy method for bi-level problems are presented in the next section. In addition, we must use the min operator to aggregate the satisfactory levels and then, maximize the aggregated utility [38]. The general form of utility function for maximizing and minimizing objective functions is shown in Figure 2. Also, the general flowchart of the fuzzy max-min method is shown in Figure 3. To fully understand the fuzzy approach to the bi-level modeling, the interested reader can refer to [40].

It should be mentioned that the dimensions of various parts of the objective function are different. Therefore, either multi-objective solution procedures, like allocating weights, should be adopted for each objective or utility functions used and normalized. To this end, we introduced separate objectives and the associated normalized utility functions. This is an acceptable method to deal with the multi-objective problems.

4.1.1. Fuzzy utility functions for the day-ahead market
In this section, some simple linear utility functions for objectives are introduced. In the max-min method [27],

initially, the min operator should be used to aggregate the satisfactory levels and then, the aggregated utility is maximized. The general form of the utility function for maximizing and minimizing objective functions is shown in Figure 2. To fully understand the fuzzy approach to the bi-level modeling, the interested reader can refer to [22].

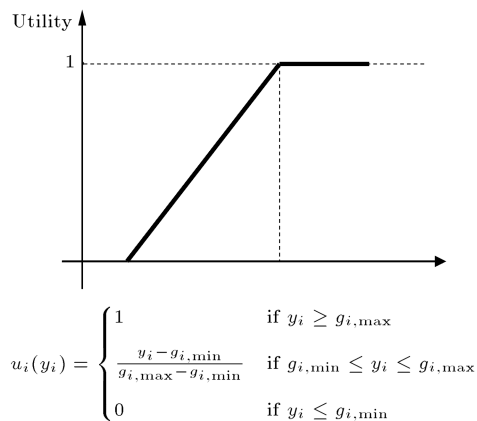
Some simple utility functions for the lower-level and upper-level objectives are introduced as follows:

1. An objective is the energy and ramp revenue maximization. Utility functions with respect to the energy revenue and ramp revenue are defined in Eqs. (12) and (13) as shown in Box I (Figure 4). In the fuzzy max-min method [28], the value of λ should be maximized subject to Constraints (14) and (15):

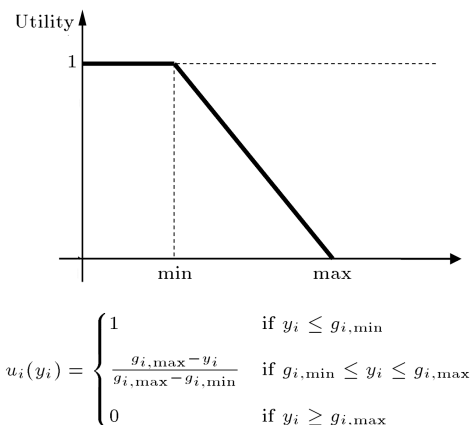
$$\frac{\left(\sum_{t,i,k} g_{t,i,k}^p \times C_{t,i,k} \right) - \text{OBJ}_L^P}{\left(\sum_{t,i,k} p_{t,i}^{\max} \times (3MC_{t,i} + \mu_{t,i}^A) \right) - \text{OBJ}_L^P} \geq \lambda, \quad (14)$$

$$\frac{\left(\sum_{t,i} (R_{t,i}^{\text{up}} \times \mu_{t,i}^{\text{up}}) + (R_{t,i}^{\text{Down}} \times \mu_{t,i}^{\text{Down}}) \right) - 0}{\left(\sum_{t,i,k} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{Down}}) \times (3MC_{t,i} + \mu_{t,i}^A) \right) - 0} \geq \lambda. \quad (15)$$

2. The other objective is minimization of the ramp cost for thermal units. The utility function is defined in Eq. (16) (Figure 5) and the value of λ should be maximized subject to Constraint (17); Eq. (16) and Constraint (17) are shown in Box II;
3. The other objective is minimization of the fuel cost. Its utility function is defined in Eq. (18) (Figure 6) and the value of λ must be maximized subject to



(a) Maximizing objective functions



(b) Minimizing objective functions

Figure 2. The general form of defining the utility function for the maximization and minimization objective functions.

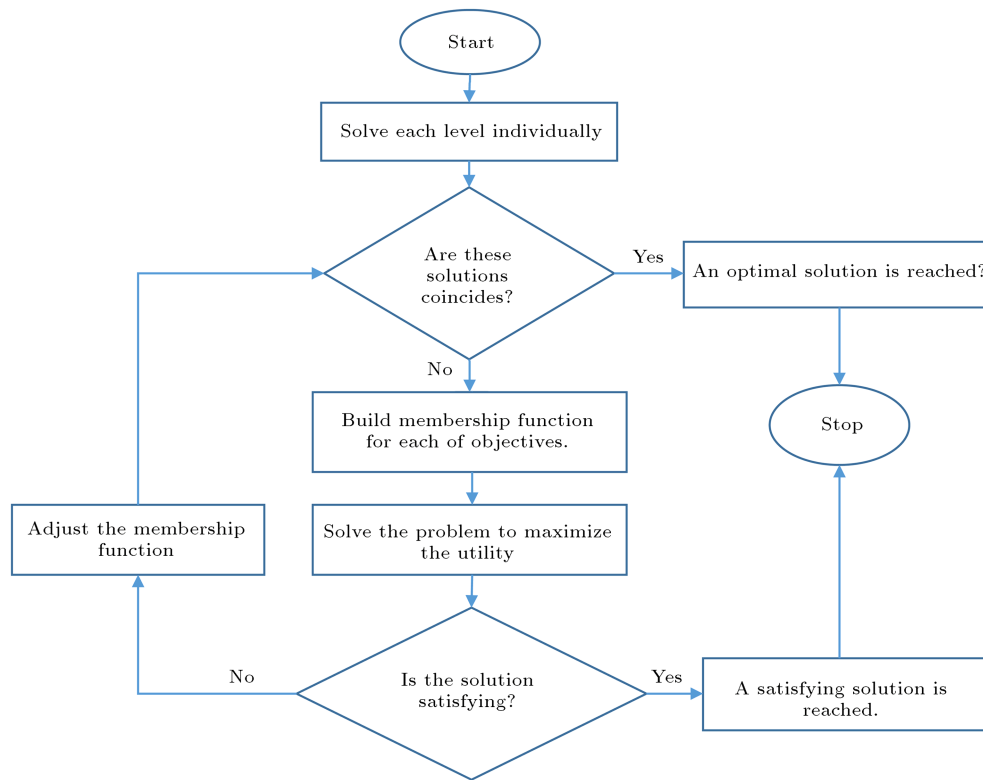


Figure 3. A simple solution process for the Stackelberg game in the bi-level form.

$$\mu_{\left(E_REV=\sum_{t,i,k} g_{t,i,k}^p \times C_{t,i,k}\right)} = \begin{cases} 1 & \text{if } (E_REV) > \left(\sum_{t,i,k} p_{t,i}^{\max} \times (3MC_{t,i} + \mu_{t,i}^A)\right) \\ \frac{(E_REV) - OBJ_L^P}{\left(\sum_{t,i,k} p_{t,i}^{\max} \times (3MC_{t,i} + \mu_{t,i}^A)\right) - OBJ_L^P} & \text{if } OBJ_L^P \leq (E_REV) \leq \left(\sum_{t,i,k} p_{t,i}^{\max} \times (3MC_{t,i} + \mu_{t,i}^A)\right) \\ 0 & \text{if } (E_REV) < OBJ_L^P \end{cases} \quad (12)$$

$$\mu_{\left(R_REV=\sum_{t,i} \left(R_{t,i}^{up} \times \mu_{t,i}^{up}\right) + \left(R_{t,i}^{Down} \times \mu_{t,i}^{Down}\right)\right)} = \begin{cases} 1 & \text{if } (R_REV) > \left(\sum_{t,i,k} p_{t,i}^{\max} \times (3MC_{t,i} + \mu_{t,i}^A)\right) \\ \frac{(R_REV) - 0}{\left(\sum_{t,i,k} \left(MR_{t,i}^{up} + MR_{t,i}^{Down}\right) \times (3MC_{t,i} + \mu_{t,i}^A)\right) - 0} & \text{if } 0 \leq (R_REV) \leq \left(\sum_{t,i,k} p_{t,i}^{\max} \times (3MC_{t,i} + \mu_{t,i}^A)\right) \end{cases} \quad (13)$$

$$\mu \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right) = \begin{cases} 0 & \text{if } \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right) > \left(\sum_{t,i} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{dn}}) \times RPN_i \right) \\ \frac{\left(\sum_{t,i} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{dn}}) \times RPN_i \right) - \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right)}{\left(\sum_{t,i} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{dn}}) \times RPN_i \right) - \text{OBJ}_U^P} & \text{if } \text{OBJ}_U^P \leq \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right) \leq \left(\sum_{t,i} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{dn}}) \times RPN_i \right) \\ 1 & \text{if } \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right) \leq \text{OBJ}_U^P \end{cases} \quad (16)$$

$$\frac{\left(\sum_{t,i} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{dn}}) \times RPN_i \right) - \left(\sum_{t,i} (R_{t,i}^{\text{up}} + R_{t,i}^{\text{dn}}) \times RPN_i \right)}{\left(\sum_{t,i} (MR_{t,i}^{\text{up}} + MR_{t,i}^{\text{dn}}) \times RPN_i \right) - \text{OBJ}_U^P} \geq \lambda. \quad (17)$$

Box II

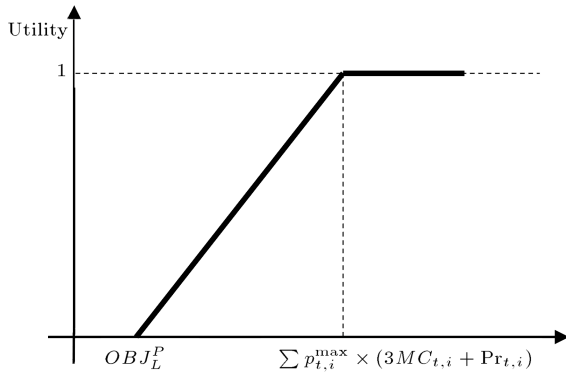


Figure 4. Utility function for the revenue of each supplier.

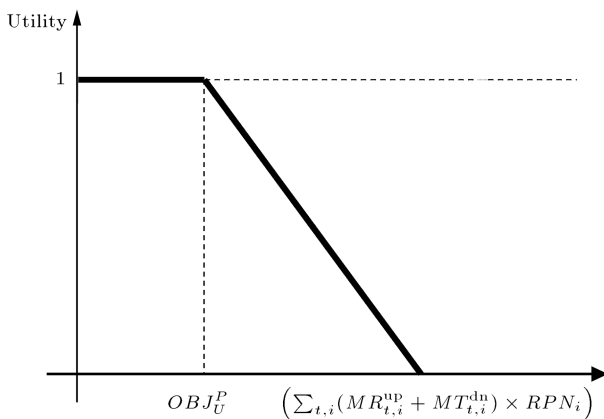


Figure 5. Utility function for ramping cost.

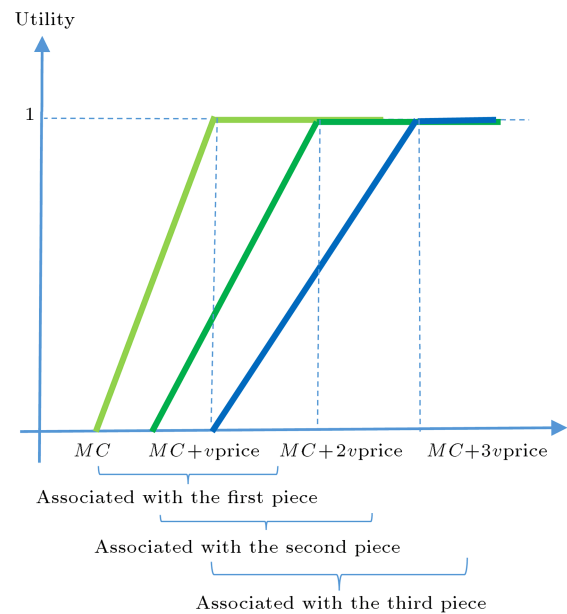


Figure 6. Utility function for the proposed prices.

Constraint (19); Eq. (18) and Constraint (19) are shown in Box III.

4. If the market is cleared/pay-as-bid, suppliers will be interested in taking their proposed prices closer to the market balancing price, to increase their revenue and improve the chance of winning the bids. Thus, the objective could be as follows:

$$\mu_{(C_i^{\text{fuel}})} = \begin{cases} 0 & \text{if } C_i^{\text{fuel}} > \left(\sum_{t,i} \alpha_{t,i} + (p_{t,i}^{\text{max}} \times \beta_i) + ((p_{t,i}^{\text{max}})^2 \times \gamma_i) \right) \\ \frac{\left(\sum_{t,i} \alpha_{t,i} + (p_{t,i}^{\text{max}} \times \beta_i) + ((p_{t,i}^{\text{max}})^2 \times \gamma_i) \right) - (C_i^{\text{fuel}})}{(C_i^{\text{fuel}}) - \text{OBJ}_{\text{help}2}^P} & \text{if } \text{OBJ}_{\text{help}2}^P \leq C_i^{\text{fuel}} \leq \left(\sum_{t,i} \alpha_{t,i} + (p_{t,i}^{\text{max}} \times \beta_i) + ((p_{t,i}^{\text{max}})^2 \times \gamma_i) \right) \\ 1 & \text{if } C_i^{\text{fuel}} \leq \text{OBJ}_{\text{help}2}^P \end{cases} \quad (18)$$

$$\frac{\left(\sum_{t,i} \alpha_{t,i} + (p_{t,i}^{\text{max}} \times \beta_i) + ((p_{t,i}^{\text{max}})^2 \times \gamma_i) \right) - \left(\sum_{t,i} \alpha_{t,i} + (TG_{t,i}^p \times \beta_i) + ((TG_{t,i}^p)^2 \times \gamma_i) \right)}{\left(\sum_{t,i} \alpha_{t,i} + (p_{t,i}^{\text{max}} \times \beta_i) + ((p_{t,i}^{\text{max}})^2 \times \gamma_i) \right) - \text{OBJ}_{\text{help}2}^P} \geq \lambda. \quad (19)$$

Box III

$$\min \sum_{t,k} |\mu_{i,t}^A - C_{i,t,k}|, \quad k = \{1, \dots, K\}. \quad (20)$$

This objective through goal programming could be rewritten as follows:

$$\min (d1_{t,i,k}^+ - d1_{t,i,k}^-)$$

$$(C_{t,i,k} - \mu_{t,i}^A) - o = d1_{t,i,k}^+ - d1_{t,i,k}^-. \quad (21)$$

Nevertheless, suppliers do not tend to confine their proposed prices to the value of $\mu_{t,i}^A$, which is an upper-level parameter. Therefore, fuzzy programming is much more suitable for modeling this objective. The price bidding is dynamic and related to the market clearing prices. Also, suppliers tend to propose prices at least equal to their marginal costs. Two types of utility functions (a, b) associated with the approach of suppliers to proposing prices are as follows.

The utility functions associated with the first, second, and third blocks of the proposed cost function are brought in Eqs. (22)–(24) as shown in Box IV. In the fuzzy max-min method, the value of the λ must be maximized subject to constraint 25.

$$\begin{aligned} \frac{C_{t,i,1'} - (MC_{t,i})}{\mu_{t,i}^A} &\geq \lambda, \\ \frac{C_{t,i,2'} - (MC_{t,i} + \mu_{t,i}^A)}{\mu_{t,i}^A} &\geq \lambda, \\ \frac{C_{t,i,3'} - (MC_{t,i} + 2\mu_{t,i}^A)}{\mu_{t,i}^A} &\geq \lambda. \end{aligned} \quad (25)$$

Also, the lower-level utility function is given in

Eq. (26) and its associated constraint is brought in Eq. (27); Eqs. (26) and (27) are shown in Box V. The flowchart of the auction-based bi-level programming is brought in Figure 7 and the computational analyses are illustrated in the next section.

5. Computational analyses

In this section, we illuminate the validity of our approach by solving the proposed model for a simple example. The model was implemented on GAMS platform 24.7.4 and solved through BARON (Branch and Reduce Optimization Navigator) solver. The GAMS code was run on 4 GHz Intel Processor core i7 and 16 GB of RAM. The problem parameters are shown in Tables 1, 2, and 3. The market clearing prices in each period in the 10th iteration of the game

Table 1. Demand in each period.

Period (hour)	Demand	Period (hour)	Demand
1	700	13	860
2	700	14	860
3	700	15	900
4	700	16	900
5	700	17	900
6	700	18	900
7	800	19	970
8	830	20	990
9	830	21	990
10	830	22	990
11	830	23	950
12	830	24	800

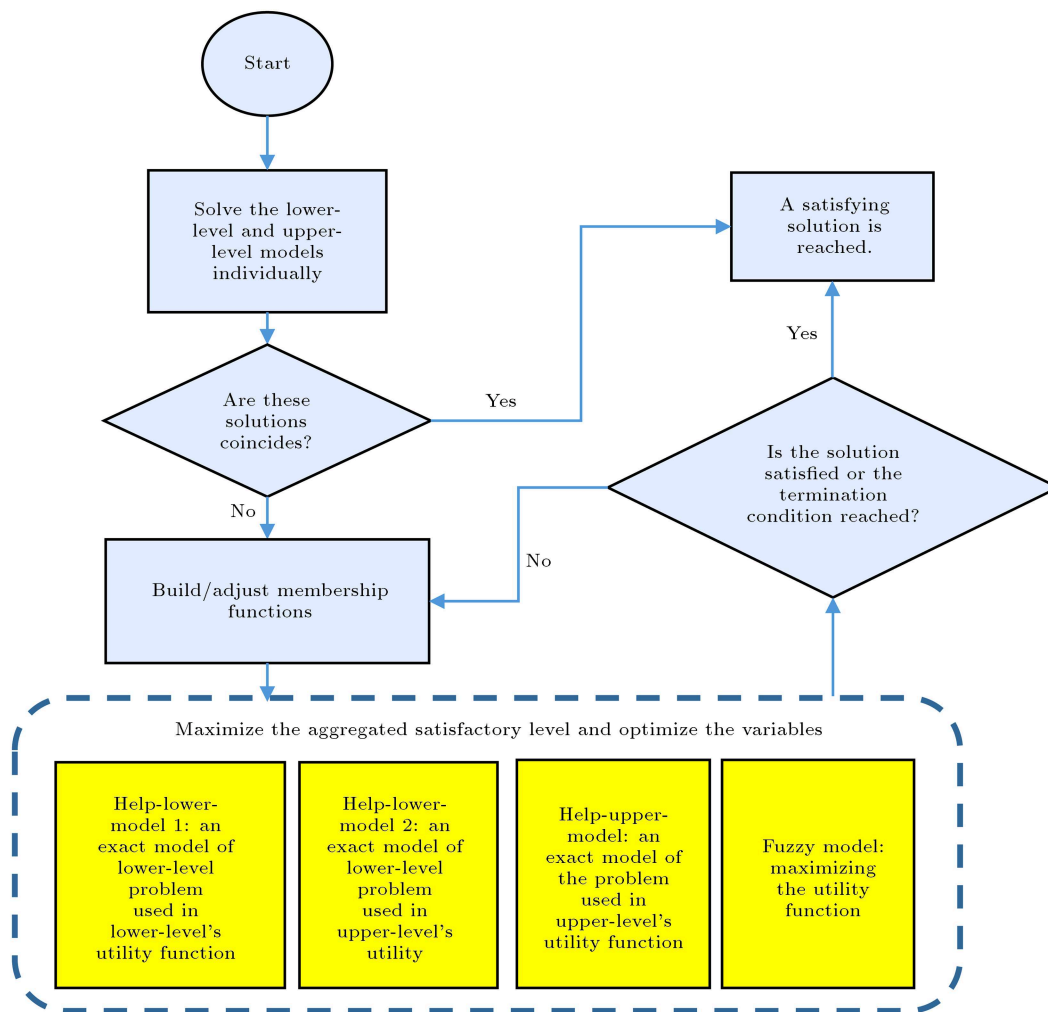


Figure 7. Flowchart of the auction-based bi-level programming.

are shown in Table 4. Table 5 illustrates the proposed prices for units in each proposed interval for the 10th iteration of the game. Table 6 illuminates the ramp up

of the units in the 10th iteration of the game. Table 7 demonstrates the ramp down of the units in the 10th iteration of the game. Table 8 elucidates the allocated

$$\mu_{C_{t,i},1'} = \begin{cases} \frac{C_{t,i},1' - (MC_{t,i})}{\mu_{t,i}^A} & \text{if } MC \leq c_{t,i},1' \leq MC + \mu_{t,i}^A \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

$$\mu_{C_{t,i},2'} = \begin{cases} \frac{C_{t,i},2' - (MC_{t,i} + \mu_{t,i}^A)}{\mu_{t,i}^A} & \text{if } MC_{t,i} + \mu_{t,i}^A \leq c_{t,i},2' \leq MC_{t,i} + 2\mu_{t,i}^A \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

$$\mu_{C_{t,i},3'} = \begin{cases} \frac{C_{t,i},3' - (MC_{t,i} + 2\mu_{t,i}^A)}{\mu_{t,i}^A} & \text{if } MC_{t,i} + 2\mu_{t,i}^A \leq c_{t,i},3' \leq MC_{t,i} + 3\mu_{t,i}^A \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

$$\mu(\text{ISO_Energy Cost}) = \begin{cases} 0 & \text{if } \left(\left(\sum_{t,i,k} g_{t,i,k} \times C_{t,i,k} \right) + \left(\sum_{t,i} SR_{t,i} \times \mu_{t-1,i}^{\text{reserve}} \right) \right) > (\text{OBJ}_{\text{Max Cost}}^P) \\ \frac{(\text{OBJ}_{\text{Max Cost}}^P) - \left(\left(\sum_{t,i,k} g_{t,i,k} \times C_{t,i,k} \right) + \left(\sum_{t,i} SR_{t,i} \times \mu_{t-1,i}^{\text{reserve}} \right) \right)}{(\text{OBJ}_{\text{Max Cost}}^P) - (\text{OBJ}_{\text{Min Cost}}^P)} & \text{if } (\text{OBJ}_{\text{Min Cost}}^P) \leq \left(\left(\sum_{t,i,k} g_{t,i,k} \times C_{t,i,k} \right) + \left(\sum_{t,i} SR_{t,i} \times \mu_{t-1,i}^{\text{reserve}} \right) \right) \leq (\text{OBJ}_{\text{Max Cost}}^P) \\ 1 & \text{if } \left(\left(\sum_{t,i,k} g_{t,i,k} \times C_{t,i,k} \right) + \left(\sum_{t,i} SR_{t,i} \times \mu_{t-1,i}^{\text{reserve}} \right) \right) \leq (\text{OBJ}_{\text{Min Cost}}^P) \end{cases} \quad (26)$$

$$\frac{(\text{OBJ}_{\text{Max Cost}}^P) - \left(\left(\sum_{t,i,k} g_{t,i,k} \times C_{t,i,k} \right) + \left(\sum_{t,i} SR_{t,i} \times \mu_{t-1,i}^{\text{reserve}} \right) \right)}{(\text{OBJ}_{\text{Max Cost}}^P) - (\text{OBJ}_{\text{Min Cost}}^P)} \geq \lambda. \quad (27)$$

Box V

Table 2. Type of each generating unit, fuel cost coefficient, and the capacity limits.

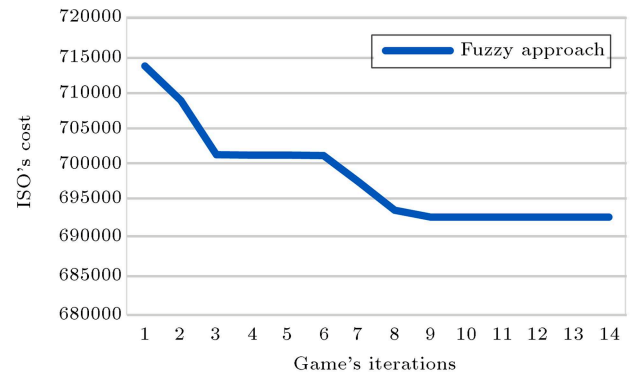
Unit	Type	α	β	γ	p_i^{\min}	p_i^{\max}
1	Thermal	240	7	0.007	50	500
2	Hydro	0	0	0	0	140
3	Hydro	0	0	0	0	400
4	Wind	0	0	0	0	300
5	Wind	0	0	0	0	200
6	Thermal	190	12	0.0075	50	500

Table 3. Maximum spinning reserve, ramp up, and ramp down of the units.

Unit	SR_i^{\max}	$MR_{t,i}^{\text{up}}$	$MR_{t,i}^{\text{Down}}$
1	80	100	100
2	40	70	70
3	80	200	200
4	80	300	300
5	80	200	200
6	80	100	100

energy by each offer in each period in the 10th iteration of the game.

We compare the results of the fuzzy method with those of the EPEC reformulation method in terms of the cost to the ISO. The cost to the ISO calculated

**Figure 8.** Cost of the Independent System Operator (ISO) in the fuzzy method.

by the fuzzy max-min method is shown in Figure 8. According to Figure 8, in comparison with the cost achieved by the EPEC method (\$694328.739), it is decreased during the iterations of the game and it converges to a value less than the cost to the ISO in the EPEC method.

The proposed approach decreases the cost to the ISO without decreasing revenues of the units through introducing multiple utility functions and considering ramp penalties for conventional units.

The ramp prices (Lagrange multipliers $\mu_{t,i}^{\text{up}}$ and $\mu_{t,i}^{\text{Down}}$) in all iterations of the game are equal to zero, implying that the flexiramp price in the day-ahead market is equal to zero. According to the results, we should not rely only on the analysis of the day-ahead market to study the ramp procurement. The

Table 4. Market clearing price in each period in the 10th iteration of the game (with scheme 1 for proposing the prices).

Period	Energy price	Period	Energy price	Period	Energy price	Period	Energy price
1	35	7	112.195	13	143.443	19	193.911
2	35	8	143.117	14	144.210	20	199.493
3	36.934	9	115.977	15	185.505	21	214.282
4	36.151	10	142.909	16	190.265	22	221.359
5	35.907	11	133.404	17	191.031	23	210.595
6	35	12	158.706	18	175.269	24	181.034

price of flexiramp is determined regarding the possible contingencies in the real-time market.

6. Conclusion and further research

In this paper, a bi-level programming model based on the Stackelberg game for integrated energy and ramp procurement problem in the day-ahead market has been developed. In addition, to solve the proposed bi-level programming model, a fuzzy max-min approach was applied to maximizing utilities of the players. The proposed approach is an effective and simple alternative to the KKT method for problems with numerous constraints in the lower level or the non-convex lower-level ones. Also, it simulates the iterative hierarchical game.

According to the results of the comparison in this study, the cost to the Independent System Operator (ISO) determined by the fuzzy method decreases during the iterations of the game and converges to a value lower than the cost in the EPEC method. The proposed fuzzy method is very simple and does not have the complexities of the EPEC.

Since the calculated ramp prices were equal to zero, it is concluded that we should not rely only on the day-ahead market analysis to study the ramp procurement. In fact, the price of flexiramp has to be determined with regard to the possible contingencies in the real-time market and by taking the accepted bids and commitments in the day-ahead market into account.

We aim to improve this paper from several aspects. The most important aspect is the improvement of the utility functions to include more complex and nonlinear ones.

Nomenclature

Indices

i Index of the generating units,
 $i \in \{1, \dots, I\}$

k Index of the blocks of the balancing energy offers, $k : \{1, \dots, K\}$

t Index of time periods, $t \in \{1, \dots, T\}$

p Index of the parameters which are variable at the other level

Decision variables at the upper level

$C_{t,i,k}$ Proposed price in the k th bid (the ramp of the k th block of the offer) by unit i in period t

$R_{i,t}^{\text{up}}$ Proposed ramp-up by unit i in period t

$R_{i,t}^{\text{dn}}$ Proposed ramp-down by unit i in period t

ϑ Binary variable for linearization

Decision variables at the lower level

$g_{t,i,k}$ The allocated energy in each offer to each unit in each period

TG_i^t Total amount of allocated energy to generator i in period t

$\mu_{t,i}^A$ LM of unit i energy balance in period t

$\mu_{t,i}^{\text{up}}$ LM of unit i ramp-up constraint in period t

$\mu_{t,i}^{\text{Down}}$ LM of unit i ramp-down constraint in period t

$z_{t,i}$ Binary variables for the general non-linear form of the Heaviside function

Parameters

D^t Demand in period t

p_i^{max} Maximum capacity of unit i

p_i^{min} Minimum capacity of unit i

$SR_{t,i}^{\text{max}}$ Spinning reserve of unit i in period t

$MR_{t,i}^{\text{up}}$ Maximum ramp-up rate capability for unit i in period t

Table 5. Proposed prices for the units in each interval in the 10th iteration of the game.

Period	Unit	Proposed prices in each interval			Period	Unit	Proposed prices in each interval		
		1	2	3			1	2	3
1	1	31.712	66.712	101.712	7	1	19.049	131.244	243.439
	2	33.751	68.751	103.751		2	19.609	131.804	243.999
	3	30.401	65.401	100.401		3	26.168	138.364	250.559
	4	—	35.000	70.000		4	—	112.195	224.390
	5	—	35.000	70.000		5	—	112.195	224.390
	6	33.077	68.077	103.077		6	32.509	144.704	256.899
2	1	29.155	64.155	99.155	8	1	33.256	176.372	319.489
	2	18.816	53.816	88.816		2	38.559	181.676	324.793
	3	20.704	55.704	90.704		3	31.230	174.347	317.464
	4	—	35.000	70.000		4	—	143.117	286.233
	5	—	35.000	70.000		5	—	143.117	286.233
	6	32.665	67.665	102.665		6	26.779	169.896	313.013
3	1	32.817	69.751	106.686	9	1	38.887	154.864	270.841
	2	29.627	66.561	103.495		2	36.231	152.208	268.186
	3	31.044	67.978	104.912		3	30.016	145.994	261.971
	4	—	36.934	73.868		4	—	115.977	231.955
	5	—	36.934	73.868		5	—	115.977	231.955
	6	36.934	73.868	110.803		6	19.994	135.972	251.949
4	1	20.452	56.603	92.754	10	1	29.894	172.803	315.712
	2	24.373	60.524	96.675		2	31.787	174.696	317.605
	3	33.197	69.348	105.500		3	35.112	178.021	320.931
	4	—	36.151	72.302		4	—	142.909	285.819
	5	—	36.151	72.302		5	—	142.909	285.819
	6	36.151	72.302	108.453		6	26.727	169.637	312.546
5	1	34.988	70.894	106.801	11	1	28.548	161.953	295.357
	2	34.899	70.805	106.712		2	38.020	171.424	304.828
	3	26.808	62.714	98.621		3	24.351	157.755	291.160
	4	—	35.907	71.813		4	—	133.404	266.808
	5	—	35.907	71.813		5	—	133.404	266.808
	6	35.907	71.813	107.720		6	32.646	166.051	299.455
6	1	29.755	64.755	99.755	12	1	30.676	189.382	348.087
	2	20.860	55.860	90.860		2	37.690	196.396	355.101
	3	28.180	63.180	98.180		3	35.652	194.357	353.063
	4	—	35.000	70.000		4	—	158.706	317.411
	5	—	35.000	70.000		5	—	158.706	317.411
	6	27.011	62.011	97.011		6	37.058	195.763	354.469

Table 5. Proposed prices for the units in each interval in the 10th iteration of the game (continued).

Period	Unit	Proposed prices in each interval			Period	Unit	Proposed prices in each interval		
		1	2	3			1	2	3
13	1	26.861	170.303	313.746	19	1	36.482	230.393	424.304
	2	37.706	181.149	324.592		2	41.907	235.818	429.728
	3	28.285	171.728	315.170		3	48.842	242.753	436.664
	4	—	143.443	286.886		4	—	193.911	387.821
	5	—	143.443	286.886		5	—	193.911	387.821
	6	31.612	175.055	318.498		6	40.476	234.387	428.297
14	1	27.052	171.262	315.472	20	1	38.145	237.637	437.130
	2	32.580	176.790	321.000		2	44.353	243.846	443.339
	3	42.477	186.687	330.896		3	47.309	246.802	446.295
	4	—	144.210	288.420		4	—	199.493	398.986
	5	—	144.210	288.420		5	—	199.493	398.986
	6	45.104	189.314	333.524		6	41.783	241.276	440.768
15	1	35.793	221.299	406.804	21	1	44.011	258.294	472.576
	2	37.835	223.341	408.846		2	46.180	260.463	474.745
	3	42.565	228.070	413.576		3	46.617	260.899	475.181
	4	—	185.505	371.011		4	—	214.282	428.565
	5	—	185.505	371.011		5	—	214.282	428.565
	6	43.170	228.676	414.181		6	44.757	259.039	473.322
16	1	39.422	229.686	419.951	22	1	44.193	265.552	486.911
	2	42.024	232.289	422.553		2	45.203	266.561	487.920
	3	43.361	233.625	423.890		3	47.055	268.414	489.773
	4	—	190.265	380.529		4	—	221.359	442.718
	5	—	190.265	380.529		5	—	221.359	442.718
	6	44.454	234.718	424.983		6	48.008	269.367	490.726
17	1	38.362	229.393	420.424	23	1	39.918	250.512	461.107
	2	42.308	233.339	424.370		2	40.864	251.459	462.053
	3	47.338	238.369	429.400		3	44.892	255.487	466.081
	4	—	191.031	382.062		4	—	210.595	421.189
	5	—	191.031	382.062		5	—	210.595	421.189
	6	44.117	235.148	426.180		6	46.100	256.695	467.289
18	1	25.299	200.568	375.837	24	1	37.359	218.393	399.427
	2	34.817	210.087	385.356		2	35.618	216.652	397.686
	3	41.463	216.733	392.002		3	38.097	219.131	400.164
	4	—	175.269	350.539		4	—	181.034	362.068
	5	—	175.269	350.539		5	—	181.034	362.068
	6	47.005	222.275	397.544		6	36.345	217.379	398.412

Table 6. Ramp up of the units in the 10th iteration of the game.

Period	Scheme 1					
	Unit					
	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	20	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	20
7	80	0	0	0	20	0
8	0	0	0	0	0	100
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	110	0	0	0	0	0
12	100	0	0	0	0	0
13	30	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	100	80	0
16	0	0	0	0	0	0
17	0	0	0	20	0	0
18	100	0	0	0	0	0
19	0	0	0	120	80	0
20	20	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0

Table 7. Ramp down of the units in the 10th iteration of the game.

Period	Scheme 1					
	Unit					
	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	20
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	20	0
7	0	0	0	0	20	0
8	80	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	100
12	0	0	100	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	100	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	20	0
18	0	0	0	120	60	0
19	100	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	40	0	0	0	0	0
24	30	0	0	120	0	0

Table 8. Allocated energy by each offer in each period in the 10th iteration of the game.

Period	The allocated energy to each unit					
	1	2	3	4	5	6
1	130	50	120	180	100	120
2	130	50	120	180	100	120
3	130	50	120	180	120	100
4	130	50	120	180	120	100
5	130	50	120	180	100	120
6	130	50	120	160	120	120
7	210	50	120	180	120	120
8	130	50	120	180	120	230
9	130	50	120	180	120	230
10	130	50	120	180	120	230
11	130	50	230	180	120	120
12	240	50	120	180	120	120
13	270	50	120	180	120	120
14	270	50	120	180	120	120
15	310	50	120	180	120	120
16	310	50	120	180	120	120
17	310	50	120	180	120	120
18	310	50	120	180	120	120
19	310	50	120	180	120	190
20	310	50	120	180	120	210
21	310	50	120	180	120	210
22	310	100	160	180	120	120
23	310	100	120	180	120	120
24	130	100	120	180	120	150

$MR_{t,i}^{\text{Down}}$	Maximum ramp-down rate capability for unit i in period t
$L_{t,i,k}$	The length of the k th block of energy offered by generating unit i
$MC_{t,i}$	Marginal cost of generator i in period t
$g_{t,i,k}^p$	The allocated energy in each offer to each unit in each period determined at the lower level; it is a fixed parameter at the upper level
$TG_{t,i}^p$	Total amount of allocated energy to generator i in period t determined at the lower level; it is a fixed parameter at the upper level

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