Contractors' Partnership in Project Resource Management Application of Cooperative Game Theory Approach

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Abstract

It is accepted that project breakdown into several independent subprojects can help to have a successful and effective project management. On the other hand, it can lead to inefficiently use of some renewable resources, and increase the total project cost and time. This article studies the benefits of the horizontal partnering among contractors assigned to subprojects through the sharing renewable resources and proposes a model based on cooperative game theory to solve it. The improvement of the net present value of the project is considered as the benefit of the cooperation among contractors. Therefore, a mixed-integer non-linear programming (MINLP) model is developed for the resource constrained project scheduling with objective function of maximizing the net present value (NPV) of each coalition. Seven widely used cooperative game theory solution methods are used to solve the benefit (NPV) allocation problem and then the stability criteria are suggested to find the best allocation scheme. Finally, an example is represented to more comprehensively illustrate the problem.

Keywords: Transferable utility cooperative game; Partnering; Renewable resource allocation; Net present value (NPV); Stability analysis; Project scheduling problem.

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1. Introduction

Many Stakeholders influence in the implementation of construction projects and it is accepted that there is a strong link between project success and successful stakeholders' relationship [1]. The benefits of this relationship or collaboration will ultimately lead to greater satisfaction of them. Contractors at different levels of a construction projects are among these stakeholders. They carry out a large portion of the work done in construction projects and may account for up to 90% of the total value of the project [2-4].

In recent years, cooperation between the project stakeholders, including the main contractors and subcontractors, has been considered by various researchers [5-7]. It is evidently seen in the research of Phua and Rowlinson [8] entitled "How important is cooperation to construction project success? A grounded empirical quantification" that cooperation is perceived as vital to construction project success. Chan et al. [9] evaluate the critical success factors such as efficient cooperation, effective communication and mutual trust between contracting parties for running partnering construction projects. In the study of Hartmann et al. [4] about the subcontractor selection process of main contractors from Singapore, cooperation is one of the important selection criteria.

One of the problems that occur in the cooperation is how the benefits are divided among the parties, so the use of game theory in such situations that there is a conflict between rational parties is an efficient approach. In fact, using game theory, a win-win solution can be found for all parties or players.

A limited number of studies have conducted about the application of game theory in the problem of interaction between the project stakeholders. Perng et al. [10] studied the formwork subcontractors cooperation that hires open shop workers in a coalition, rather than union workers and earns more profit. They used Shapley value and nucleolus to divide profit among subcontractors.

Asgari and Afshar [11] used a cooperative game theory approach for modeling subcontractors cooperation in Time. They considered the real cost of the project based on time-cost and time-efficiency functions and showed subcontractors cooperation can improve total real cost. Finally, the benefits of coalition distributed using the shapely value and nucleolus methods. Barough et al. [12] used Prisoner dilemma and chicken game application in solving the construction project conflicts between the involved parties. Tsai and chi [13] showed the importance of cooperation learning for achieving win-win outcomes between two parties.

Joint resource management is one of the areas of cooperation among subcontractors in the construction projects. Asgari et al. [14] suggested cooperative game theory as an efficient tool for analyzing joint resource management in construction projects. In their study, first, for all possible subcontractors' coalitions, subcontractors' characteristic functions was built using a resource-leveling model. Then, cooperation benefits allocated among the subcontractors using various cooperative game theoretic solution methods. Finally, plurality rule and propensity to disrupt methods were used to select the most acceptable and stable allocation.

Samsami and Tavakolan [15] divided subcontractors' partnerships into two directions, horizontal and vertical. They defined a model to build and analyze the joint resource management as a horizontal partnership based on game theory. The objective function of their model is to minimize the net cost included the cost of hiring a fixed number of resources during the project and cost of repair/maintenance. They showed the overall payoff of coalition increases and used shapely value method to allocate benefits of joint resource management.

Previous research has shown that one of the main restrictions in construction projects affects project cost and time is the limited renewable resources such as labors and equipment [16-17]. So far, two different types of resource restricted problems have been considered:

resource smoothing (also known as resource leveling) problems and resource constrained (also known as resource allocation) problems. The project resource leveling problem was proposed to smooth resource usage and reduce resource fluctuation with determined project finish time, while the resource-constrained project scheduling problem focuses on optimizing the project duration with limited resources. In fact, in this study, the second problem is considered, and resource constrained project scheduling with the objective function of maximizing net present value (NPV) of the project will be developed.

The rest of this paper is organized as follows. Section 2 defines the problem of this study with tow sub-sections: "Resource constrained project scheduling problem maximizing the net present value (NPV)" formulates the effect of cooperation among contractors and "Cooperative Game theory" reviews the mechanisms for allocation of cooperative gains and the stability criteria of different allocation schemes. Section 3 is devoted to solving an example and the analysis of the results. Finally, the conclusions of this study are presented in Section 4.

2. Problem definition

This article studies the benefits of the cooperation among contractors assigned to subprojects through the sharing renewable resources. The improvement of the net present value (NPV) of the project is considered as the benefit of this cooperation and the problem is determining of the share of each contractors from this improvement. To solve the problem, first a coalitional based multi- mode resource-constrained project scheduling problem (RCPSP) with objective function of NPV is developed, which calculate the best NPV of each coalition among contractors and then various solutions of cooperative games with transferable utility (TU-cooperative games) will be used to the distribution of the NPV among contractors in the grand coalition.

2.1. Resource constrained project scheduling problem maximizing the net present value (NPV)

Over the past decades the resource-constrained project scheduling problem (RCPSP) has been extensively addressed in numerous studies [18-21]. Whereas the RCPSP attempts to minimize the total project duration or makespan, several alternative objectives exist such as the minimization of resource idle time, the minimization of earliness and tardiness, or the maximization of project net present value (NPV) [22].

Yang et al. [23], presented an integer programming algorithm for solving the limited-resource project scheduling problem with the objective of maximizing project net present value (NPV). Vanhoucke et al. [24] studied RCPSP with discounted cash flows. They assumed each activity of this RCPSP has certain resource requirements and a known deterministic cash flow. They developed a depth-first branch-and-bound algorithm which uses a new fast recursive search algorithm for the max-npv problem. Vanhoucke [25] developed a scatter search procedure for maximizing the net present value of a resource-constrained project with fixed activity cash flow. Khoshjahan et al. [26] considered the resource-constrained project scheduling problem with objective of minimizing the net present value of the earliness–tardiness penalty costs. They first modeled the problem, then, proposed two meta-heuristics, genetic algorithm and simulated annealing to solve it. Leyman and Vanhoucke [27] discussed the single- and multi-mode resource-constrained project scheduling problem with discounted cash flows (RCPSPDC and MRCPSPDC) and they solved the model with a proposed genetic algorithm metaheuristic.

In this study, maximizing the net present value (NPV) is taken into account as the objective function of scheduling problem to find an assignment of modes to activities as well as precedence and resource-feasible starting times for all activities. In other words, this study mathematically formulates the model for a multi-mode resource constrained project scheduling problem with discounted cash flows (MRCPSPDC). In this problem, it is assumed that

- Each activity can be performed in several modes and in each mode, it has a specific duration, cash flow (positive or negative), and amount of renewable resources.
- Cash flows are assumed to occur upon activity finish time.
- The project is breakdown into several subprojects with due dates under bonus–penalty policies. Each subproject assigned to a contractor wants to maximize their own NPV.
- Bonus (penalty) is allotted when the subproject is finished before (after) its pre-defined due date.
- Contractors can form a coalition, and scheduling problem of all subprojects performing in the coalition is an MRCPSPDC model.

The MRCPSPDC model of coalition $S \in 2^N \setminus \varnothing$ is developed as follows.

Summary of all notations used in the model is presented in Table 1.

By using the above notations, the proposed mathematical model can be formulated as:

MRCPSPDC Model:

$$Max\, NPV_s = \sum_{k \in C_s} \left[\sum_{i \in Act_s} \sum_{l=1}^{L_i} \sum_{t=1}^{T} cf_{il}\, x_{ilt} e^{-\alpha t} - w_k \left(FT_k - cd_k \right) y_k e^{-\alpha\, FT_k} + \vartheta_k \left(cd_k - FT_k \right) \left(1 - y_k \right) e^{-\alpha\, FT_k} \right]$$

subject to,

$$\sum_{l=1}^{L_i} \sum_{t=1}^{T} x_{ilt} = 1 \qquad \forall i \in Act_s$$

$$pred_{ij} \sum_{l=1}^{L_i} \sum_{t=1}^{T} t\, x_{ilt} \le \sum_{l=1}^{L_j} \sum_{t=1}^{T} \left(t - d_{jl} \right) x_{jlt} \qquad \forall i, j \in Act_s$$

$$\sum_{l=1}^{L_i} \sum_{t=1}^{T} t\, x_{ilt}\, A_{ik} \le FT_k \qquad \forall k \in C_s \;, \forall i \in Act_s$$

$$\sum_{i\in Act_s}\sum_{l=1}^{L_i} r_{pil}\sum_{b=t}^{t+d_{il}-1} x_{ilt} \le \sum_{k=1}^{N} res_{pkt} \qquad \forall t\in E\ ,\forall\, p\in R$$

$$FT_k\left(1-y_k\right)\le cd_k \qquad \forall\, k\in C_s$$

$$cd_k\, y_k \le FT_k \qquad \forall\, k\in C_s$$

$$FT_k \ge 0 \qquad \forall\, k\in C_s$$

$$x_{ilt}\in\{0,1\} \qquad \forall i\in Act_s, \quad \forall t\in E\ , \quad \forall l\in M_i$$

$$y_k\in\{0,1\} \qquad \forall\, k\in C_s$$

The objective function (1) maximizes the net present value of s-th coalition based on a discount rate α. It consists of three parts as: (a) the present values of the cash flow of the all activities performed by coalition s, (b) the present values of the tardiness penalties of all contractors included in coalition s, and (c) the present values of the earliness bonuses of all contractors included in coalition s, respectively. Eq. (2) states that every activity is assigned exactly one mode and exactly one finishing time. Constraint (3) ensures the precedence relations between activities. Inequality (4), ensures that the makespan of sub-project k, FT_k, is the maximum of its all activities finish times. Constraint (5) enforces the resource constraints at time interval t. Inequalities (6) and (7) determine the earliness or tardiness of sub-project k. Finally, constraints (8-10) denote the domain of the variables.

Based on above model, cooperation among contractors via joint resource management can increase their gain (NPV) by using the renewable resources efficiently. A TU-cooperative game (N,v) with the set of players N including all contractors and the characteristic function v equals to NPV can be used to determine the share of each contractor in the grand coalition.

2.2. Cooperative Game theory

Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" [28-29]. Game theory is divided into two branches: cooperative and non-cooperative. A game is cooperative (or coalitional) if the players are able to form binding commitments (or coalitions) externally enforced. A game is non-cooperative if players cannot form alliances or if all agreements need to be self-enforcing [29].

One of the problems in cooperative game theory is how to distribute the payoff of coalition among the members or players [28].

Cooperative games can be transferable utility games (TU-games) or non-transferable utility games (NTU-games). TU-cooperative games are used to model situations where the players in a coalition can compare and transfer part of their utility with each other. In the situations of NTU-cooperative games, it is not always possible that the players can compare or transfer their utility.

The rest of this section introduces basic notation, definitions and notions from TU-cooperative game theory.

2.2.1. Basic definitions and concepts

Definition 1- A *TU-game* is an ordered pair (N,v) consisting of the player set N (with n players) and the characteristic function $v:2^N \to R$ with $v(\varnothing)=0$. For each coalition $S \subseteq N$, The real number $v(S)$ denotes the maximal worth or cost savings that the members in S can obtain if they cooperate [30-31].

Definition 2- Let $x \in R^n$ be a payoff vector, where x_i represents the value allocated to player $i \in N$, in the grand coalition. A payoff vector $x \in R^n$ is called an imputation for the game (N,v) if it satisfies efficiency an individual rationality conditions, *i.e.*

$$(i)\sum_{i\in N} x_i = \nu(N) \tag{11}$$

$$(ii)\, x_i \geq \nu(i) \tag{12}$$

The set of imputations of the game (N,ν) is denoted by $I(\nu)$ [31].

Definition 3- The core $C(\nu)$ of the game (N,ν) is the set

$$C(\nu) = \{x \in I(\nu) | \sum_{i\in S} x_i \geq \nu(S),\ \forall S \in 2^N \setminus \varnothing\} \tag{13}$$

Or the Core of the game (N,ν) , $C(\nu)$, is a set of imputations that satisfy

$$(i)\sum_{i\in N} x_i = \nu(N), \tag{14}$$

$$(ii)\sum_{i\in S} x_i \geq \nu(S),\ \forall S \in 2^N \setminus \varnothing. \tag{15}$$

If $x \in C(\nu)$, no player has an incentive to deviate to form a different coalition [31].

2.2.2. Solution concepts for cooperative TU-games

2.2.2.1. The Shapely Value

Lloyd Shapley introduced the solution concept of the Shapely Value in 1953 [32].

Definition 4- Given a cooperative game (N,ν), the Shapley value $\phi_i(\upsilon)$ which is the expected payoff of player $i \in N$,is defined by

$$\phi_i(\upsilon) = \sum_{S:i\notin S} \frac{|S|!(n-1-|S|)!}{n!}\ \nu(S\cup\{i\}) - \nu(S)). \tag{16}$$

2.2.2.2. The τ –Value

Tijs defined the solution concept of the τ-value for each quasi-balanced game in 1981 [33].

Definition 5- For a quasi-balanced game $(N,\nu)$the τ-value, $\tau(\upsilon)$ is defined by

$$\tau(\upsilon) := \alpha m(N,\upsilon) + (1-\alpha) M(N,\upsilon) \tag{17}$$

Where $m(N,\upsilon)$ and $M(N,\upsilon)$ are the lower vector and upper vector of the game (N,ν), respectively and $\alpha \in [0,1]$ is uniquely obtained from $\sum_{i\in N} \tau_i(\upsilon) = \nu(N)$ [31,33].

2.2.2.3. The Average Lexicographic Value

The average lexicographic value or *AL*-value is defined for balanced cooperative games, which are games with a non-empty core [31,34].

Given a balanced game (N,ν) and an ordering σ of the players in N, the lexicographic maximum of the core $C(\nu)$ of ν with respect to σ is denoted by $L^{\sigma}(\upsilon)$. It is the unique point in $C(\nu)$ with the properties:

$$\begin{aligned}
&\left(L^{\sigma}(\upsilon)\right)_{\sigma(1)} = \max\left\{x_{\sigma(1)} | x \in C(\nu)\right\},\\
&\left(L^{\sigma}(\upsilon)\right)_{\sigma(2)} = \max\left\{x_{\sigma(2)} | x \in C(\nu) \ with \ x_{\sigma(1)} = \left(L^{\sigma}(\upsilon)\right)_{\sigma(1)}\right\},\\
&\ldots\\
&\left(L^{\sigma}(\upsilon)\right)_{\sigma(n)} = \max\left\{x_{\sigma(n)} | x \in C(\nu) \ with \ x_{\sigma(i)} = \left(L^{\sigma}(\upsilon)\right)_{\sigma(i)}, \ i = 1,2,\ldots,n-1\right\},
\end{aligned} \tag{18}$$

Note that $L^{\sigma}(\upsilon)$ is an extreme point of $C(\nu)$ for each $\sigma \in \pi(N)$.

Definition 6- For a balanced game (N,ν) the average lexicographic value $AL(\upsilon)$ is defined by the average of all lexicographically maximal vectors of the core $C(\nu)$, i.e.,

$$AL(\upsilon) = \frac{1}{n!} \sum_{\sigma \in \pi(N)} L^{\sigma}(\upsilon). \tag{19}$$

2.2.2.4. The Equal Split-Off Set

The equal split-off set is introduced as a solution concept for cooperative games based on egalitarian considerations [35].

Given a game (N,ν), in the first step one of the coalitions with maximal average worth, say T_1, forms, i.e.,

$$T_1 \in \arg \max_{S\in 2^N} \frac{\upsilon_k(S)}{|S|} \tag{20}$$

And the worth $\upsilon(T_1)$ is divided among the players in T_1 equally.

In step 2 one of the coalitions in $N \setminus T_1$ with maximal average marginal worth w.r.t. T_1, say T_2, forms, and joins costless T_1, and the value $\upsilon(T_1 \cup T_2) - \upsilon(T_1)$ is divided among the players in T_2 equally.

Similarly, in step k, T_k forms, i.e.,

$$T_k \in \arg \max_{S\in 2^{N\setminus \bigcup_{i=1}^{k-1} T_i}} \frac{\upsilon\left(\bigcup_{i=1}^{k-1} T_i \bigcup s\right) - \upsilon\left(\bigcup_{i=1}^{k-1} T_i\right)}{|S|} \tag{21}$$

And the value $\upsilon\left(\bigcup_{i=1}^{k-1} T_i \bigcup s\right) - \upsilon\left(\bigcup_{i=1}^{k-1} T_i\right)$ is divided among the players in T_{k} equally. This process continues until a partition of N of the form $T_1, \ldots, T_k$ for some $1 \leq K \leq n$ is reached [31].

2.2.2.5. The Nucleolus

The nucleolus was first introduced by Schmeidler (1969) as a solution concept in cooperative game theory [36].

Let a game (N,ν), and a payoff vector $x \in R^n$. The excess of the coalition $S \in 2^N \setminus \varnothing$ associated with x is defined as $e(S,x) = \upsilon(S) - \sum_{i\in S} x_i$, which is, the gain that players in coalition S can obtain if they withdraw from the grand coalition, N, under payoff x and instead take the payoff $\upsilon(S)$. In other words, The $e(S,x)$ represents a measure of dissatisfaction of coalition S in the grand coalition.

The nucleolus tries to find an imputation inside the core, $x \in C(\nu)$, that lexicographically minimizes the vector of non-increasing ordered excesses of coalitions $e(S,x)$, $S \in 2^N \setminus \varnothing$.

The nucleolus of the game (N,ν) can be reached by solving a sequence of linear programs (LPs) defined recursively as follows:

$$(LP_1)\begin{cases}\text{Min } \varepsilon \\ \text{subject to}\begin{cases}\upsilon(S)-\sum_{i\in S}x_i \le \varepsilon & \forall S \in 2^N \setminus \varnothing \\ \sum_{i\in N}x_i = \upsilon(N) \\ \varepsilon, x_i \in \mathrm{R}, i \in N\end{cases}\end{cases} \tag{22}$$

$$(LP_k)\begin{cases}\text{Min } \varepsilon \\ \text{subject to}\begin{cases}\upsilon(S)-\sum_{i\in S}x_i = \varepsilon_0 & \forall S \in \mathrm{S}_1 \\ . \\ . \\ . \\ \upsilon(S)-\sum_{i\in S}x_i = \varepsilon_{k-1} & \forall S \in \mathrm{S}_{k-1} \setminus \mathrm{S}_{k-2} \\ \upsilon(S)-\sum_{i\in S}x_i \le \varepsilon & \forall S \in 2^N \setminus \mathrm{S}_{k-1} \\ \sum_{i\in N}x_i = \upsilon(N) \\ \varepsilon, x_i \in \mathrm{R}, i \in N\end{cases}\end{cases} \tag{23}$$

Where ε_{k-1} is the optimal objective value to LP_{k-1} and S_{k-1} is the set of coalitions for which its excess has been fixed in a previous LP in the sequence [37-40]. Fromen [41] introduced an algorithm for solving this sequence of linear programs (LPs).

2.2.2.6. The per capita excess

The per capita nucleolus represents a measure of dissatisfaction per capita of such a coalition. It is found by replacing $e(S,x) = \upsilon(S) - \sum_{i \in S} x_i$ with $\bar{e}(S,x) = \frac{\upsilon(S) - \sum_{i \in S} x_i}{|S|}$ in the optimization programs of the nucleolus [30].

2.2.2.7. The Nash–Harsanyi (N–H) solution

The Nash–Harsanyi (N–H) solution concept maximizes the product of the difference between the allocated utilities (income, or NPV in this paper) from cooperation in grand coalition and the non-cooperation case, subject to core conditions, by equating the utility gains of all players [36,39]. Given a cooperative game (N, ν), The optimization model of the N–H solution is as follows:

$$\text{Max} \prod_{\forall i \in N} \left(x_i - \upsilon(i) \right), \tag{24}$$

subject to

$$\begin{aligned} \upsilon(S) - \sum_{i \in S} x_i \le 0 \quad & \forall S \in 2^N \setminus \varnothing, \\ \sum_{i \in N} x_i = \upsilon(N) & \\ x_i \in \mathrm{R}, i \in N & \end{aligned}$$

2.2.3. Solution Stability Criteria

Based on the mathematical calculations of the core, all the solutions in it are potentially acceptable to all players in the grand coalition, however in practice many of these allocations may seem unfair in the view of some of the players and they have incentives for leaving the grand coalition and forming partial coalitions or act individually. So it makes grand coalition unstable. Therefore, for the sake of stability, the concept of "fairness" in allocation should be considered.

On the other hand, players, with the knowledge that they can gain more in the core, may bargain or threaten the grand coalition to leave. So an additional concept of stability, "propensity to disrupt" should be considered.

2.2.3.1. Fairness Index

The Shapley-Shubik Power Index was suggested by Shapley and Shubik [42] to measure power in voting game. Loehman et al. [43] used an index similar to the Shapley–Shubik Power Index to measure power in a cooperative game. This index then used in several studies to evaluate the fairness of a given allocation among all players [37,40,44].

The power index (α_i) compares the gains to player $i \in N$ with the gains to the coalition. The power index (α_i) is

$$\alpha_i = \frac{x_i - \upsilon(i)}{\sum_{i \in N}(x_i - \upsilon(i))}, \ i \in N; \sum_{i \in N} \alpha_i = 1. \tag{25}$$

where x_i is the solution allocation for player $i \in N$ and $\upsilon(i)$ is the worth of player i.

The power index of each player is calculated separately. If the power is distributed almost evenly among the players, then the coalition is more likely to be stable. Based on the concept of Power Index, the Fairness Index (FI) can be defined as:

$$FI = \frac{\sigma_\alpha}{\bar{\alpha}}, 0 \le FI \le 1 \tag{26}$$

where FI indicates the Fairness Index. σ_α is the standard deviation and $\bar{\alpha}$ represents the average value. The greater the value of FI the larger the instability of the solution [37,40,44].

2.2.3.2. Propensity to Disrupt (DP)

Propensity to disrupt (DP), another measure of the solution stability, is introduced by Gately in 1974 [45]. DP_i is defined as the ratio of how much the other players would lose if player

$i \in N$ doesn't cooperate in the grand coalition to how much he would lose in this situation, i.e.,

$$\mathrm{DP}_i = \frac{\sum_{j \in N \setminus \{i\}} x_j - \upsilon\left(N \setminus \{i\}\right)}{x_i - \upsilon\left(i\right)},\ i \in N \tag{27}$$

where x_i is the solution allocation for player $i \in N$, $\upsilon\left(N \setminus \{i\}\right)$ is the worth of coalition $N \setminus \{i\}$ and $\upsilon\left(i\right)$ is the worth of player i. then, DP can be defined for solution as

$$DP = \max_{i \in N}(DP_i). \tag{28}$$

The greater the value of DP, the larger the instability of the solution. Decision makers must determine an acceptance upper limit for DP, and eliminate any imputation not inside the limit [37,45].

3. Case study

3.1. Description

The proposed approach is illustrated with an example. Consider a construction project including three similar sub-projects with three in charge contractors. The due date of all sub-projects is at the end of time unit 25 with delay penalty and earliness bonus of 30 and 20 $ per time unit respectively. The discount rate is assumed 2%. Each activity has two possible execution modes. Table 2 presents the list of activities, corresponding precedence relations between them and their durations, required resources and cash flows with respect to each mode.

Number of units of each renewable resource available to each contractor at different time intervals is shown in Fig. 1 and Fig. 2.

3.2. NPVs for various degrees of cooperation

Based on the above information, the proposed mathematical models of all possible coalitions including non-cooperation (act alone), partial cooperation (subset coalition) and the grand coalition are solved by Branch-And-Reduce Optimization Navigator (BARON) under GAMS and the results consist of the best NPV found for each coalition, the finish time of each activity, and the makespan of each sub-project are found (Table 3).

As shown in Table 3, if no coalition created or the contractors perform the subprojects separately, the NPV of the project will be 3266.41 $, if the subset coalitions of (1,2), (1,3) and (2,3) are formed the NPV of the project will be 3423.014$, 3381.122$ and 3400.969$ respectively and if project is performed by grand coalition, the NPV will be 3553.879$.

The resulting gant charts for all five possible combinations of coalitions are shown in Fig. 3. In this figure the color of the bars of all activities of one coalition is same.

3.3. The core of the cooperation game

The allocation of the NPV of all three contractors cannot migrate outside of the Core, $x \in C(v)$. The core of the game $C(v)$ is shown in Figure 4, representing all possible payoff allocation.

3.4. NPV allocation schemes based on seven solutions

By applying the introduced seven methods of cooperative game theory to the case study, the NPV assignment strategy of each method can be deduced and summarized in Table 4. Apparently, the sum of the allocated value of each contractor is equal to the total NPV (3553.879$) through the grand coalition. Meanwhile, all of the allocation schemes satisfy the Core requirements as shown in Table 4.

3.5. Stability for different allocation schemes

In the following, in order to investigate the stability of different allocation schemes, as mentioned in Section 2-2-3, DP and Shapley-Shubik Power Index value are calculated.

Table 5 shows the DP value for each contractor using the seven allocation methods. It can be found that the calculated DP value for the 3-th contractor employing the equal split-off set method is 1.504 referring Eq.(28) and as mentioned in Section 2-2-3, the smaller the DP value is, the greater incentive the player will have to join the coalition and vice versa. So, one player will disrupt the coalition only when the DP value is less than 1. Therefore, the 3-th contractor will refuse to accept the allocation strategy based on the equal split-off set method. Therefore, it can be concluded that the equal split-off set method is unstable.

Table 6 shows the Power Index within each scheme by utilizing Eq. (25), and based on which the Fairness Index can be calculated through Eq. (26). As mentioned before, the greater the value of Fairness Index, the lower the fairness of the allocation strategy. Thus, it can be concluded that, although all of the values are within a reasonable range ($0 \leq FI_{\alpha} \leq 1$) referring Eq. (26), the most fairness scheme is the N-H solution due to the lowest Fairness Index.

Therefore, in summary, the best allocation scheme can be deduced through the N-H solution method which can meet both of the stability criteria simultaneously.

Based on the N-H solution method, the utilities or NPVs 1159.946$, 1165.749$ and 1228.184$ are allocated to contractors 1, 2 and 3 respectively. On other words, the shares of contractors 1, 2 and 3 in the grand coalition are 32.6%, 32.8%, and 34.6% respectively.

4. Conclusions

One of the constraints on construction projects which leads to increase the time and cost of the projects is the limitation of renewable resources (e.g., labor or equipment). Sharing this resources is one of the areas of cooperation among the project's contractors, which can lead to decrease total time and cost of the projects and improve net present value (NPV). In this study, various solution methods of the cooperative game theory were used to solve the problem of determining the share of the participating contractors from this improvement.

In order to model this problem, a multi-mode resource constrained project scheduling problem with discounted cash flows (MRCPSPDC) for each coalition is taken into account and a mixed integer nonlinear programming (MINLP) model is developed. The suggested model can be used by partnering contractors to manage renewable resources more efficiently on a cooperative basis. On other words, partnering makes the feasible solution space of the problem larger and it can improve the solution.

Therefore, in the solution procedure, first the suggested model is solved for all coalitions and then the benefit (NPV) allocation problem is solved by seven widely used cooperative game theory solution methods: The Shapely value, The τ–Value, The Average Lexicographic Value, The Equal Split-Off Set, The Nucleolus, The per capita excess, The N-H solution. At last, the best allocation scheme based on the stability criteria: the Shapley-Shubik Power Index and DP value will be chosen.

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Figure captions

Figure 1- Number of units of renewable resource 1 available to each contractor at each time interval

Figure 2- Number of units of renewable resource 2 available to each contractor at each time interval

Figure 3-Gantt charts of all possible combinations of coalitions

Figure 4-The Core space

Table captions

Table 1. Summary of notation.

Table 2. The list of activities, corresponding precedence relations between them and their durations, required resources and cash flows with respect to each mode.

Table 3. NPV and makespan of each sub-project for various degrees of cooperation.

Table 4. NPV assignment using different schemes.

Table 5. Propensity to Disrupt (DP) of different schemes.

Table 6. The fairness evaluation of each scheme.

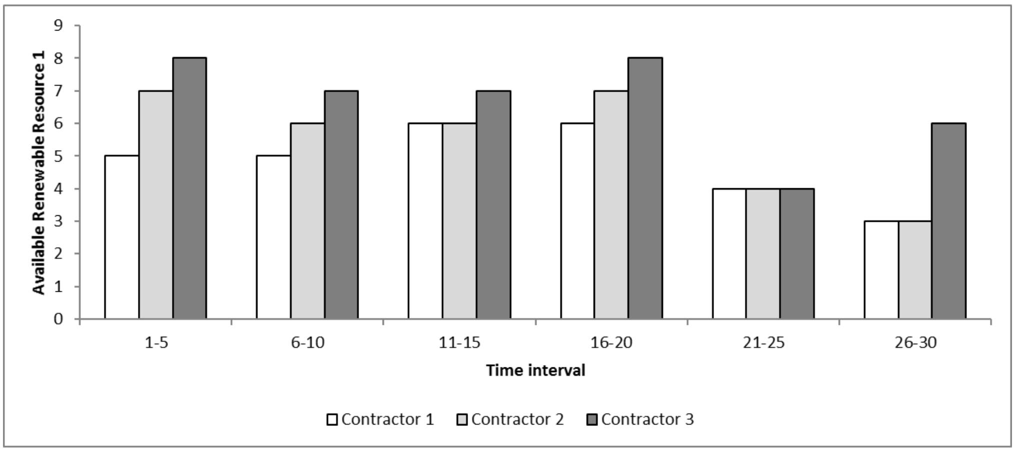


Figure 1

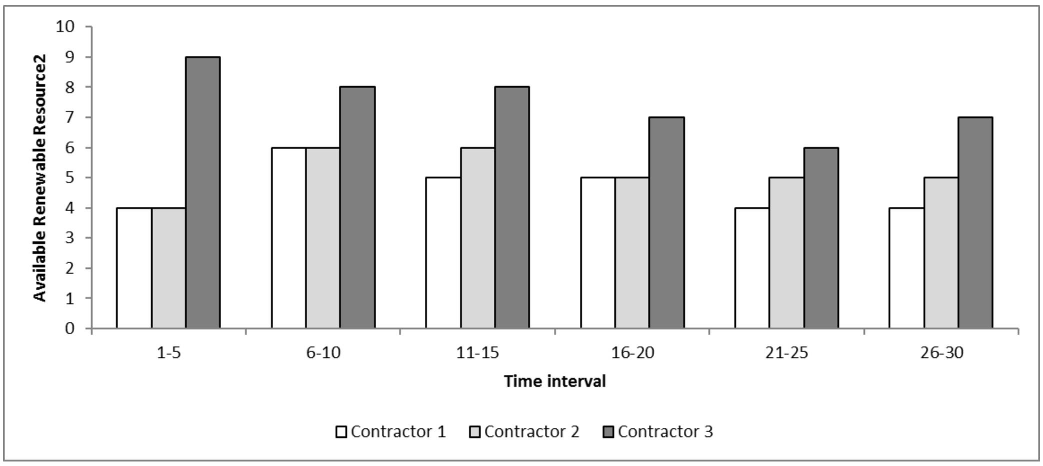
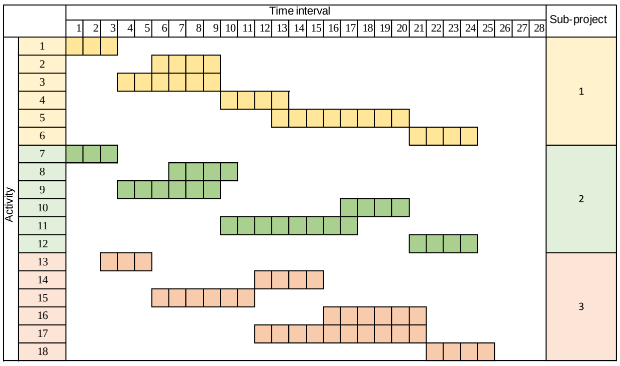
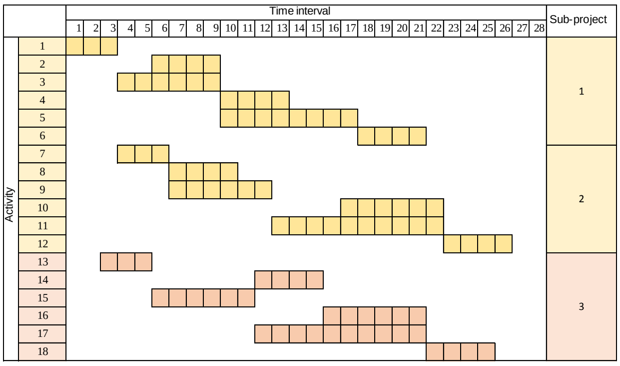


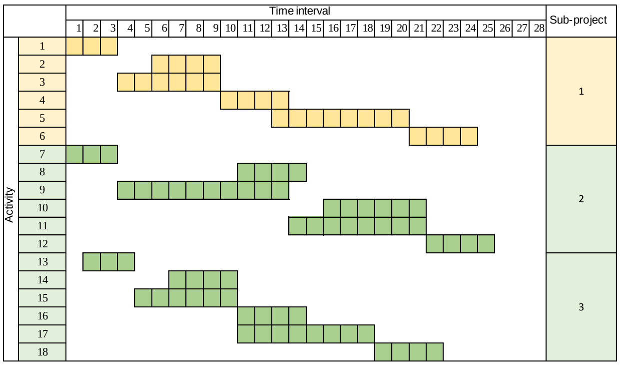
Figure 2



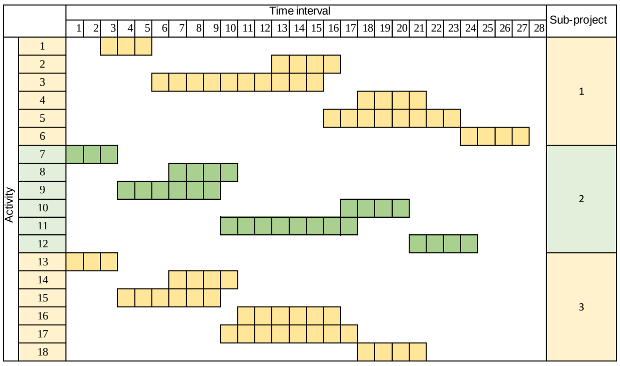
a- The project gant chart in the non-cooperative mode (singleton coalitions) or {(1),(2),(3)}



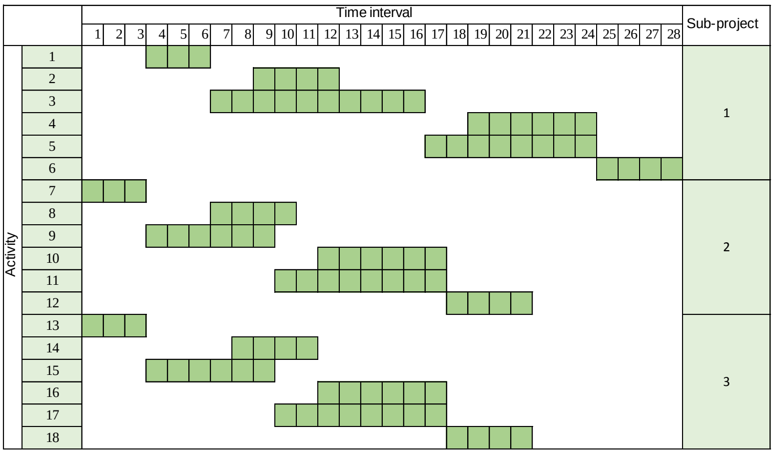
b- The project gant chart in the situation of contractors 1 and 2 cooperate and contractor 3 acts alone or {(1, 2),(3)}.



c- The project gant chart in the situation of contractors 2 and 3 cooperate and contractor 1 acts alone or {(1),(2,3)}.

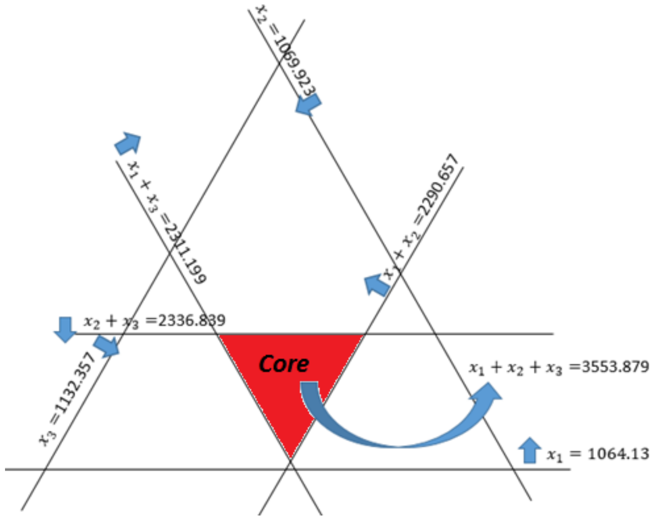


d- The project gant chart in the situation of contractors 1 and 3 cooperate and contractor 2 acts alone or {(1,3),(2)}.



e-The project gant chart in the situation of grand coalition or {(1,3, 2)}.

Figure 3



x_2 =1069.923
$x_1 + x_3$ =2311.199
$x_1 + x_2$ =2290.657
$x_2 + x_3$ =2336.839
Core
$x_1 + x_2 + x_3$ =3553.879
x_3 =1132.357
x_1 = 1064.13

Figure 4

Table 1.

Indices	
i, j	Activity
t, b	Time interval
p	Resource
$l \in \{1, 2, \ldots, L_i\}$	Mode
$s \in \{1, 2, \ldots, 2^n - 1\}$	Coalition
k	Contractor or sub-project
Sets	
I	Set of all activities $= \{0, 1, \ldots, m\}$
R	Set of renewable resources $= \{1, 2, \ldots, P\}$
E	Set of all intervals $= \{1, \ldots, T\}$
N	Grand coalition that includes all Contractors $= \{1, 2, \ldots, n\}$
M_i	Set of all modes of activity i $= \{1, \ldots, L_i\}$
Act_s	Set of activities is performed by coalition s $= \{i \mid A_{ik} = 1, k \in C_s\}$
C_s	s -th Coalition of Contractors (s -th subset of the grand coalition N)
Parameters	
m	Number of activities
L_i	Number of modes of activity i
T	Time horizon of the project
n	Number of contractor

$A_{ik} = \begin{cases} 1 & \text{If activity } i \text{ is performed by contractor } k \\ 0 & \text{otherwise} \end{cases}$

P Number of renewable resources

d_{il} Duration of activity i executed in mode l

$pred_{ij} = \begin{cases} 1 & \text{if activity } i \text{ is the predecessor of activity } j \\ 0 & \text{otherwise} \end{cases}$

res_{pkt} Number of units of renewable resource p available to the contractor k in the time t

r_{pil} Number of units of renewable resource p required by activity i executed in mode l

cf_{il} Net cash flow associated with activity i in mode l

w_k Penalty per time unit of delay of sub-project k

ϑ_k Bonus per time unit for early completion of sub-project k

cd_k Due date of sub-project k

α Discount rate

<u>Binary variables</u>

$$x_{ilt} = \begin{cases} 1 & \text{if activity } i \text{ is performed in mode } l \text{ and finished at time } t,\ \forall i \in I,\ \forall l \in M_i \text{ and } \forall t \in E \\ 0 & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if sub-project } k \text{ is finished after its due date } (cd_k) \\ 0 & \text{otherwise} \end{cases}$$

<u>Continuous variables</u>

FT_k	Makespan or finish time of sub-project k , $\forall k \in N$
NPV_s	NPV of coalition s , $\forall s \subset N$

Table 2

Subcontractor	Activity	Precedence	Mode 1				Mode 2			
			Duration	Required resource 1	Required resource 2	Cash flow	Duration	Required resource 1	Required resource 2	Cash flow
1	1	-	5	1	3	-320	3	2	4	-306
	2	1	4	2	4	-508	3	4	6	-564
	3	1	10	0	1	-330	6	0	0	-346
	4	2,3	6	3	0	-280	4	5	3	-300
	5	3	10	5	2	-780	8	6	5	-792
	6	4,5	6	1	3	4596	4	2	4	4550
2	7	-	5	1	3	-320	3	2	4	-306
	8	7	4	2	4	-508	3	4	6	-564
	9	7	10	0	1	-330	6	0	0	-346
	10	8,9	6	3	0	-280	4	5	3	-300
	11	9	10	5	2	-780	8	6	5	-792
	12	10,11	6	1	3	4596	4	2	4	4550
3	13	-	5	1	3	-320	3	2	4	-306
	14	13	4	2	4	-508	3	4	6	-564
	15	13	10	0	1	-330	6	0	0	-346
	16	14,15	6	3	0	-280	4	5	3	-300
	17	15	10	5	2	-780	8	6	5	-792
	18	16,17	6	1	3	4596	4	2	4	4550

Table 3

Scenario	Coalition	NPV($) of coalition	Combinations	NPV($) of project	Makespan		
					sub-project 1	sub-project 2	sub-project 3
Act alone	1	1064.13			24	-	-
	2	1069.923	{(1),(2),(3)}	3266.41	-	24	-
	3	1132.357			-	-	25
Subset coalition	1,2	2290.657	{(1, 2),(3)}	3423.014	21	26	-
	1,3	2311.199	{(1,3),(2)}	3381.122	27	-	21
	2,3	2336.839	{(1),(2,3)}	3400.969	-	25	22
Grand coalition	1,2,3	3553.879	{(1,3, 2)}	3553.879	28	21	21

Table 4

Solution scheme	NPV allocation ($)			In core
	x_1	x_2	x_3	
The Shapely value	1160.319	1176.036	1217.524	Yes
The τ –Value	1159.952	1179.562	1214.365	Yes
The Average Lexicographic Value	1174.702	1177.999	1201.179	Yes
The Equal Split-Off Set	1184.626	1184.626	1184.626	Yes
The Nucleolus	1,160.686	1,186.326	1,206.868	Yes
The per capita excess	1,160.686	1,186.326	1,206.868	Yes
The N-H solution	1159.946	1165.749	1228.184	Yes

Table 5

Solution scheme	NPV allocation ($)			DP
	DP_1	DP_2	DP_3	
The Shapely value	0.590	0.628	0.537	0.628
The τ –Value	0.596	0.576	0.596	0.596
The Average Lexicographic Value	0.383	0.598	0.902	0.902
The Equal Split-Off Set	0.269	0.506	1.504	1.504
The Nucleolus	0.584	0.484	0.756	0.756
The per capita excess	0.584	0.484	0.756	0.756
The N-H solution	0.596	0.803	0.366	0.803

Table 6

Solution scheme	The Shapley-Shubik power index			Fairness index (FI_{α})
	α_1	α_2	α_3	
The Shapely value	0.321	0.377	0.302	0.095
The τ –Value	0.333	0.381	0.285	0.118
The Average Lexicographic Value	0.385	0.376	0.239	0.200
The Nucleolus	0.336	0.405	0.259	0.179
The per capita excess	0.336	0.405	0.259	0.179
The N-H solution	0.333	0.333	0.333	0.000

Biography

Mahdieh Akhbari received her BS degree in Industrial Engineering from Khajeh Nasir Toosi University of Technology in 2004, an MS degree in Industrial Engineering from Isfahan University of Technology , in 2007, and a PhD degree in Industrial Engineering, in 2014, from Science And Research Branch of Islamic Azad University . She is currently Assistant Professor in the Industrial Engineering Department of the Electronic Branch of the Islamic Azad University. Her research interests include soft computing, game theory, and mathematical modeling.