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Partnership of contractors in cooperative game theory approach to project resource management

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Project scheduling problem.

Abstract. It is generally agreed that decomposing a project into several independent subprojects can help to have a successful and effective project management. On the other hand, this may lead to inefficient use of some renewable resources and increase in the total cost and time of project. This study deals with the benefits of horizontal partnering among contractors assigned to subprojects through sharing renewable resources and proposes a model based on cooperative game theory for the problem. The improvement of the Net Present Value (NPV) of the project was considered as the benefit of cooperation among contractors. Therefore, a Mixed-Integer Non-Linear Programming (MINLP) model was developed for the resource constrained project scheduling with the objective function of maximizing the NPV of each coalition. Seven widely used cooperative game theory solution methods were used to solve the NPV allocation problem and then, stability criteria were utilized to find the best allocation scheme. An example is presented in the paper to more comprehensively illustrate the problem.

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1. Introduction

Many stakeholders are involved in the implementation of construction projects and it is generally accepted that there is a strong link between project success and effective relationship between stakeholders [1]. The benefits of this relationship or collaboration will ultimately lead to greater satisfaction of the stakeholders. Contractors at different levels of a construction project are among these stakeholders. They carry out a large portion of the work done in a project and may account for up to 90% of its total value [2–4].

In recent years, cooperation between project stakeholders, including the main contractors and subcontractors, has been considered by various researchers [5–7]. Phua and Rowlinson [8] perceived

cooperation as a vital element of construction project success. Chan et al. [9] evaluated the critical success factors such as efficient cooperation, effective communication, and mutual trust between contracting parties for running partnering construction projects. Hartmann et al. [4], who dealt with the subcontractor selection process for the main contractors in Singapore, considered cooperation as one of the important selection criteria.

One of the problems that occur in cooperation is how the benefits are divided among the parties. Thus, the use of game theory in such situations that there is a conflict between rational parties is an efficient approach. In fact, using game theory, a win-win solution can be found for all parties or players.

A limited number of studies have been conducted about the application of game theory to the problem of interaction between project stakeholders. Perng et al. [10] studied the formwork subcontractors cooperation by hiring open shop workers in a coalition, rather than union workers, to earn more profit. They used

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Shapley value and nucleolus to divide profit among subcontractors.

Asgari and Afshar [11] used a cooperative game theory approach to modeling of subcontractors cooperation in time. They considered real cost of the project based on time-cost and time-efficiency functions and showed that cooperation of subcontractors could improve total real cost. Finally, the benefits of coalition were distributed using the Shapely value and nucleolus methods. Barough et al. [12] applied prisoner dilemma and chicken game to solving the construction project conflicts between the involved parties. Tsai and chi [13] showed the importance of cooperative learning for achieving win-win outcomes between two parties.

Joint resource management is one of the areas of cooperation among subcontractors in the construction projects. Asgari et al. [14] suggested cooperative game theory as an efficient tool for analyzing joint resource management in construction projects. In their study, first, characteristic functions of subcontractors for all possible coalitions were determined using a resource-leveling model. Then, cooperation benefits were allocated to the subcontractors using various cooperative game theoretic solution methods. Finally, plurality rule and propensity to disrupt methods were used to select the most acceptable and stable allocation.

Samsami and Tavakolan [15] divided partnership of subcontractors into two directions, namely horizontal and vertical. They defined a model to build and analyze joint resource management as a horizontal partnership based on game theory. The objective function of their model was to minimize the net cost consisting of the cost of hiring a fixed number of resources during the project and the cost of repair/maintenance. They showed the overall payoff of increase in coalition and used Shapely value method to allocate benefits of joint resource management.

Previous research has shown that one of the main restrictions on construction projects affecting project cost and time is limited renewable resources, such as labor and equipment [16–17]. So far, two different types of resource-restricted problems have been considered: resource-smoothing (also known as resource leveling) problems and resource-constrained (also known as resource allocation) problems. The project resource leveling problem has been proposed to smooth resource usage and reduce resource fluctuation with determined project finish time, while Resource-Constrained Project Scheduling Problem (RCPSP) focuses on optimizing project duration with limited resources. In this study, the latter is considered and resource-constrained scheduling is carried out with the objective function of maximizing Net Present Value (NPV) of the project.

The rest of this paper is organized as follows. Section 2 defines the problem in two sub-sections of which

the first formulates the effect of cooperation among contractors and the second reviews the mechanisms for allocation of cooperative gains and the stability criteria of different allocation schemes. Section 3 is devoted to solving an example and analysis of the results. Finally, the conclusions are presented in Section 4.

2. Problem definition

This study investigates the benefits of cooperation among contractors assigned to subprojects through sharing renewable resources. Improvement in the NPV of the project is considered as the benefit of cooperation and the problem is determining the share of each contractor from this improvement. To solve the problem, first, a coalition-based multi-mode RCPSP with the objective function of finding the best NPV for each coalition among contractors is developed and then, various solutions of cooperative games with Transferable Utility (TU-cooperative games) are used for the distribution of the NPV among contractors in the grand coalition.

2.1. RCPSP of maximizing the NPV

Over the past decades, RCPSP has been extensively addressed in numerous studies [18–21]. Whereas the RCPSP attempts to minimize the total project duration or makespan, several alternative objectives exist, such as minimization of the resource idle time, minimization of earliness and tardiness, or maximization of project NPV [22].

Yang et al. [23] presented an integer programming algorithm for solving the limited-resource project scheduling problem with the objective of maximizing project NPV. Vanhoucke et al. [24] studied RCPSP with discounted cash flows. They assumed that each activity of the RCPSP had certain resource requirements and a known deterministic cash flow. They developed a depth-first branch-and-bound algorithm, which used a new fast recursive search algorithm for the max-npv problem. Vanhoucke [25] developed a scatter search procedure for maximizing the NPV of a resource-constrained project with fixed activity cash flow. Khoshjahan et al. [26] considered the RCPSP with the objective of minimizing the NPV of the earliness-tardiness penalty costs. They first modeled the problem and then, proposed two meta-heuristics, namely genetic algorithm and simulated annealing, to solve it. Leyman and Vanhoucke [27] discussed the single- and multi-mode Resource-Constrained Project Scheduling Problems with Discounted Cash flows (RCPSPDC and MRCPSDC) and solved the model with a proposed genetic algorithm metaheuristic.

In this study, maximizing the NPV is taken into account as the objective function of the scheduling problem to achieve an assignment of modes to activities

as well as precedence and resource-feasible starting times for all activities. In other words, this study mathematically formulates the model for a Multi-mode Resource Constrained Project Scheduling Problem with Discounted Cash flows (MRCPSPDC). In this problem, it is assumed that:

- Each activity can be performed in several modes and in each mode, it has a specific duration, cash flow (positive or negative), and amount of renewable resources;
- Cash flows are assumed to occur upon activity finish time;
- The project is decomposed into several subprojects

with due dates under bonus-penalty policies. Each subproject assigned to a contractor is aimed at maximizing their own NPV;

- Bonus (penalty) is allotted when the subproject is finished before (after) its pre-defined due date;
- Contractors can form a coalition and the scheduling problem of all subprojects performed in the coalition is an MRCPSPDC model.

The MRCPSPDC model of coalition $S \in 2^N \setminus \phi$ is developed as follows.

The notations used in the model are summarized in Table 1. Using the notations, the proposed mathematical model is formulated as:

Table 1. Summary of notations

Indices:	
i, j	Activity
t, b	Time interval
p	Resource
$l \in \{1, 2, \dots, L_i\}$	Mode
$s \in \{1, 2, \dots, 2^n - 1\}$	Coalition
k	Contractor or sub-project
Sets:	
I	Set of all activities $s \in \{0, 1, \dots, m\}$
R	Set of renewable resources $= \{1, 2, \dots, P\}$
E	Set of all intervals $= \{1, \dots, T\}$
N	Grand coalition that includes all contractors $= \{1, 2, \dots, n\}$
M_i	Set of all modes of activity $i = \{1, \dots, L_i\}$
Act_s	Set of activities performed by coalition $s = \{i A_{ik} = 1, k \in C_s\}$
C_s	s -th coalition of contractors (s -th subset of grand coalition N)
Parameters:	
m	Number of activities
L_i	Number of modes of activity i
T	Time horizon of the project
n	Number of contractors
$A_{ik} = \begin{cases} 1 \\ 0 \end{cases}$	If activity i is performed by contractor k otherwise
P	Number of renewable resources
d_{il}	Duration of activity i executed in mode l
$pred_{ij} = \begin{cases} 1 \\ 0 \end{cases}$	If activity i is the predecessor of activity j otherwise
res_{pkt}	Number of units of renewable resource p available to contractor k in time t
r_{pil}	Number of units of renewable resource p required by activity i executed in mode l
cf_{il}	Net cash flow associated with activity i in mode l
w_k	Penalty per time unit of delay of sub-project k
ϑ_k	Bonus per time unit for early completion of sub-project k
cd_k	Due date of sub-project k
α	Discount rate
Binary variables:	
$x_{ilt} = \begin{cases} 1 \\ 0 \end{cases}$	If activity i is performed in mode l and finished at time t , $\forall i \in I$, $\forall l \in M_i$, and $\forall t \in E$ otherwise
$y_k = \begin{cases} 1 \\ 0 \end{cases}$	If sub-project k is finished after its due date (cd_k) otherwise
Continuous variables:	
FT_k	Makespan or finish time of sub-project k , $\forall k \in N$
NPV_s	NPV of coalition s , $\forall s \subset N$

MRCPSPDC model:

$$\max NPV_s = \sum_{k \in C_s} \left[\sum_{i \in Act_s} \sum_{l=1}^{L_i} \sum_{t=1}^T cf_{il} x_{ilt} e^{-\alpha t} - w_k (FT_k - cd_k) y_k e^{-\alpha FT_k} + \vartheta_k (cd_k - FT_k) (1 - y_k) e^{-\alpha FT_k} \right], \quad (1)$$

subject to,

$$\sum_{l=1}^{L_i} \sum_{t=1}^T x_{ilt} = 1 \quad \forall i \in Act_s, \quad (2)$$

$$pred_{ij} \sum_{l=1}^{L_i} \sum_{t=1}^T tx_{ilt} \leq \sum_{l=1}^{L_j} \sum_{t=1}^T (t - d_{jl}) x_{jlt} \quad \forall i, j \in Act_s, \quad (3)$$

$$\sum_{l=1}^{L_i} \sum_{t=1}^T tx_{ilt} A_{ik} \leq FT_k \quad \forall k \in C_s, \quad \forall i \in Act_s, \quad (4)$$

$$\sum_{i \in Act_s} \sum_{l=1}^{L_i} r_{pil} \sum_{b=t}^{t+d_{il}-1} x_{ilt} \leq \sum_{k=1}^N res_{pkt} \quad \forall t \in E, \quad \forall p \in R, \quad (5)$$

$$FT_k (1 - y_k) \leq cd_k \quad \forall k \in C_s, \quad (6)$$

$$cd_k y_k \leq FT_k \quad \forall k \in C_s, \quad (7)$$

$$FT_k \geq 0 \quad \forall k \in C_s, \quad (8)$$

$$x_{ilt} \in \{0, 1\} \quad \forall i \in Act_s, \quad \forall t \in E, \quad \forall l \in M_i, \quad (9)$$

$$y_k \in \{0, 1\} \quad \forall k \in C_s. \quad (10)$$

Objective function (1) maximizes the NPV of the s -th coalition based on a discount rate α . It consists of three parts:

- The present values of cash flow for all the activities performed by coalition s ;
- The present values of tardiness penalties for all contractors included in coalition s ;
- The present values of earliness bonuses for all contractors included in coalition s .

Eq. (2) states that every activity is assigned exactly one mode and exactly one finishing time. Constraint (3) ensures the precedence relations between activities. Inequality (4) ensures that the makespan of sub-project k , FT_k , is the maximum finish time of all of its activities. Constraint (5) enforces the resource

constraints at time interval t . Inequalities (6) and (7) determine the earliness and tardiness of sub-project k , respectively. Finally, Constraints (8)–(10) denote the domain of the variables.

Based on above model, cooperation among contractors via joint resource management can increase their gain (NPV) by using the renewable resources efficiently. A TU-cooperative game (N, ν) with the set of players N including all contractors and the characteristic function ν equal to NPV can be used to determine the share of each contractor in the grand coalition.

2.2. Cooperative game theory

Game theory is “the study of mathematical models of conflict and cooperation between intelligent rational decision-makers” [28,29] and is divided into two branches: cooperative and non-cooperative. A game is cooperative (or coalitional) if the players are able to form binding commitments (or coalitions) externally enforced. A game is non-cooperative if players cannot form alliances or if all agreements need to be self-enforcing [29].

One of the problems in cooperative game theory is how to distribute the payoff of coalition among the members or players [28].

Cooperative games can be Transferable-Utility (TU) games or Non-Transferable-Utility (NTU) games. TU-cooperative games are used to model situations in which the players in a coalition can compare and transfer part of their utilities with each other. In the situations of NTU-cooperative games, it is not always possible for the players to compare or transfer utilities.

The rest of this section introduces the basic notation, definitions, and notions of the TU-cooperative game theory.

2.2.1. Basic definitions and concepts

Definition 1. A TU-game is an ordered pair (N, ν) consisting of the player set N (with n players) and the characteristic function $\nu : 2^N \rightarrow R$ with $\nu(\emptyset) = 0$. For each coalition $S \subseteq N$, The real number $\nu(S)$ denotes the maximal worth the members in S can obtain or the cost savings they can make if they cooperate [30,31].

Definition 2. Let $x \in R^n$ be a payoff vector, where x_i represents the value allocated to player $i \in N$ in the grand coalition. A payoff vector $x \in R^n$ is called an imputation for the game (N, ν) if it satisfies efficiency and individual rationality conditions, i.e.:

$$(i) \quad \sum_{i \in N} x_i = \nu(N), \quad (11)$$

$$(ii) \quad x_i \geq \nu(i). \quad (12)$$

The set of imputations of the game (N, ν) is denoted by $I(\nu)$ [31].

Definition 3. The core $C(\nu)$ of the game (N, ν) is the set:

$$C(\nu) = \{x \in I(\nu) \mid \sum_{i \in S} x_i \geq \nu(S), \quad \forall S \in 2^N \setminus \emptyset\}, \quad (13)$$

or the Core of the game (N, ν) , $C(\nu)$, is a set of imputations that satisfy:

$$(i) \quad \sum_{i \in N} x_i = \nu(N), \quad (14)$$

$$(ii) \quad \sum_{i \in S} x_i \geq \nu(S), \quad \forall S \in 2^N \setminus \emptyset. \quad (15)$$

If $x \in C(\nu)$, no player has an incentive for deviation to form a different coalition [31].

2.2.2. Solution concepts for cooperative TU-games

Shapely value

The solution concept of Shapely value was introduced by Shapley in 1953 [32].

Definition 4. Given a cooperative game (N, ν) , the Shapley value $\phi_i(v)$, which is the expected payoff of player $i \in N$, is defined by:

$$\phi_i(v) = \sum_{S: i \notin S} \frac{|S|!(n-1-|S|)!}{n!} \nu(S \cup \{i\}) - \nu(S). \quad (16)$$

τ -value

Tijs defined the solution concept of τ -value for each quasi-balanced game in 1981 [33].

Definition 5. For a quasi-balanced game (N, ν) , the τ -value $\tau(\nu)$ is defined by:

$$\tau(v) := \alpha m(N, \nu) + (1 - \alpha) M(N, \nu), \quad (17)$$

where $m(N, \nu)$ and $M(N, \nu)$ are the lower vector and upper vector of the game (N, ν) , respectively, and $\alpha \in [0, 1]$ is uniquely obtained from [31,33]:

$$\sum_{i \in N} \tau_i(v) = \nu(N).$$

Average lexicographic value

The Average Lexicographic value or AL -value is defined for balanced cooperative games, which are games with a non-empty core [31,34].

Given a balanced game (N, ν) and an ordering σ of the players in N , the lexicographic maximum of the core $C(\nu)$ of ν with respect to σ is denoted by $L^\sigma(v)$. It is the unique point in $C(\nu)$ with the following properties:

$$(L^\sigma(v))_{\sigma(1)} = \max\{x_{\sigma(1)} \mid x \in C(\nu)\},$$

$$(L^\sigma(v))_{\sigma(2)} = \max\{x_{\sigma(2)} \mid x \in C(\nu) \text{ with}$$

$$x_{\sigma(1)} = (L^\sigma(v))_{\sigma(1)}\},$$

...

$$(L^\sigma(v))_{\sigma(n)} = \max\{x_{\sigma(n)} \mid x \in C(\nu)$$

$$\text{with } x_{\sigma(i)} = (L^\sigma(v))_{\sigma(i)},$$

$$i = 1, 2, \dots, n-1\}. \quad (18)$$

Note that $L^\sigma(v)$ is an extreme point of $C(\nu)$ for each $\sigma \in \pi(N)$.

Definition 6. For a balanced game (N, ν) , the average lexicographic value $AL(v)$ is defined by the average of all the lexicographically maximal vectors of the core $C(\nu)$, i.e.:

$$AL(v) = \frac{1}{n!} \sum_{\sigma \in \pi(N)} L^\sigma(v). \quad (19)$$

The equal split-off set

Equal split-off set is introduced as a solution concept for cooperative games based on egalitarian considerations [35].

Given a game (N, ν) , in the first step, one of the coalitions with maximal average worth, say T_1 , is formed, i.e.:

$$T_1 \in \arg \max_{S \in 2^N} \frac{v(S)}{|S|}, \quad (20)$$

and the worth of $\nu(T_1)$ is equally divided among the players in T_1 .

In step 2, one of the coalitions in $N \setminus T_1$ with maximal average marginal, say T_2 , is formed, and the value $v(T_1 \cup T_2) - v(T_1)$ is equally divided among the players in T_2 .

Similarly, in step k , T_k is formed, i.e.:

$$T_k \in \arg \max_{S \in 2^{N \setminus \bigcup_{i=1}^{k-1} T_i}}} \frac{v\left(\bigcup_{i=1}^{k-1} T_i \cup S\right) - v\left(\bigcup_{i=1}^{k-1} T_i\right)}{|S|}, \quad (21)$$

and the value of $v\left(\bigcup_{i=1}^{k-1} T_i \cup S\right) - v\left(\bigcup_{i=1}^{k-1} T_i\right)$ is equally divided among the players in T_k . This process continues until a partition of N of the form $\langle T_1, \dots, T_k \rangle$ for some $1 \leq k \leq n$ is reached [31].

Nucleolus

The nucleolus was first introduced by Schmeidler (1969) as a solution concept in cooperative game theory [36].

Consider a game (N, ν) and a payoff vector $x \in R^n$. The excess of the coalition $S \in 2^N \setminus \phi$ associated with x is defined as $e(S, x) = v(S) - \sum_{i \in S} x_i$, which is the gain that players in coalition S can obtain if they withdraw the grand coalition N under payoff x and instead, take the payoff $v(S)$. In other words, $e(S, x)$ represents a measure of dissatisfaction of coalition S in the grand coalition.

The nucleolus tries to find an imputation inside the core, $x \in C(\nu)$, that lexicographically minimizes the vector of non-increasing ordered excesses of coalitions $e(S, x)$, $S \in 2^N \setminus \phi$. The nucleolus of the game (N, ν) can be reached by solving a sequence of Linear Programs (LPs) defined recursively as follows:

$$(LP_1) \begin{cases} \min \varepsilon \\ \text{subject to} \begin{cases} v(S) - \sum_{i \in S} x_i \leq \varepsilon & \forall S \in 2^N \setminus \phi \\ \sum_{i \in N} x_i = v(N) \\ \varepsilon, x_i \in R, i \in N \end{cases} \end{cases} \quad (22)$$

$$(LP_k) \begin{cases} \min \varepsilon \\ \text{subject to} \begin{cases} v(S) - \sum_{i \in S} x_i = \varepsilon_0 \\ \forall S \in S_1 \\ \vdots \\ v(S) - \sum_{i \in S} x_i = \varepsilon_{k-1} \\ \forall S \in S_{k-1} \setminus S_{k-2} \\ v(S) - \sum_{i \in S} x_i \leq \varepsilon \\ \forall S \in 2^N \setminus S_{k-1} \\ \sum_{i \in N} x_i = v(N) \\ \varepsilon, x_i \in R, i \in N \end{cases} \end{cases} \quad (23)$$

where ε_{k-1} is the optimal objective value for LP_{k-1} and S_{k-1} is the set of coalitions for which the excess has been fixed in a previous LP in the sequence [37–40]. Fromen [41] introduced an algorithm for solving this sequence of LPs.

Per-capita excess

The per-capita nucleolus represents a measure of dissatisfaction per capita of a coalition. It is determined by replacing:

$$e(S, x) = v(S) - \sum_{i \in S} x_i,$$

with:

$$\bar{e}(S, x) = \frac{v(S) - \sum_{i \in S} x_i}{|S|},$$

in the optimization programs of the nucleolus [30].

Nash-Harsanyi (N-H) solution

The Nash-Harsanyi (N-H) solution concept maximizes the product of the difference between the allocated utilities (income or NPV in this paper) from cooperation in

grand coalition and the non-cooperation case, subject to core conditions, by equating the utility gains of all players [36,39]. Given a cooperative game (N, ν) , the optimization model of the N-H solution is as follows:

$$\max \prod_{i \in N} (x_i - v(i)),$$

subject to,

$$v(S) - \sum_{i \in S} x_i \leq 0 \quad \forall S \in 2^N \setminus \phi,$$

$$\sum_{i \in N} x_i = v(N),$$

$$x_i \in R, i \in N. \quad (24)$$

2.2.3. Solution stability criteria

Based on the mathematical calculations of the core, all the solutions in it are potentially acceptable for all players in the grand coalition. However, in practice, many of these allocations may seem unfair from the viewpoint of some of the players and they have incentive for leaving the grand coalition and forming partial coalitions or act individually. This makes grand coalition unstable. Therefore, for the sake of stability, the concept of “fairness” in allocation should be considered.

On the other hand, players, knowing that they can gain more in the core, may bargain or threaten the grand coalition to leave. Thus, an additional concept of stability, namely “propensity to disrupt,” should be considered.

Fairness index

The Shapley-Shubik power index was suggested by Shapley and Shubik [42] to measure power in voting game. Loehman et al. [43] used an index similar to the Shapley-Shubik power index to measure power in a cooperative game. This index was used in several later studies to evaluate the fairness of a given allocation among all players [37,40,44].

The power index (α_i) compares the gain of player $i \in N$ with the gains of coalition. The power index (α_i) is:

$$\alpha_i = \frac{x_i - v(i)}{\sum_{i \in N} (x_i - v(i))}, \quad i \in N; \quad \sum_{i \in N} \alpha_i = 1, \quad (25)$$

where x_i is the solution allocation for player $i \in N$ and $v(i)$ is the worth of player i .

The power index of each player is calculated separately. If power is distributed almost evenly among the players, then the coalition is more likely to be stable. Based on the concept of power index, the Fairness Index (FI) can be defined as:

$$FI = \frac{\sigma_\alpha}{\bar{\alpha}}, \quad 0 \leq FI \leq 1, \quad (26)$$

where FI indicates the Fairness Index. σ_α is the standard deviation and $\bar{\alpha}$ represents the average value. The greater the value of FI, the larger the instability of the solution [37,40,44].

Propensity to Disrupt (DP)

Propensity to Disrupt (DP), as another measure of solution stability, was introduced by Gately in 1974 [45]. DP_i is defined as the ratio of how much the other players would lose if player $i \in N$ does not cooperate in the grand coalition to how much the player would lose in this situation, i.e.:

$$DP_i = \frac{\sum_{j \in N \setminus \{i\}} x_j - v(N \setminus \{i\})}{x_i - v(i)}, \quad i \in N, \quad (27)$$

where x_i is the solution allocation for player $i \in N$; $v(N \setminus \{i\})$ is the worth of coalition $N \setminus \{i\}$; and $v(i)$ is the worth of player i . DP can be defined for the solution as:

$$DP = \max_{i \in N} (DP_i). \quad (28)$$

The greater the value of DP, the larger the instability of the solution. Decision makers must determine an acceptance upper limit for DP and eliminate any imputation not inside the limit [37,45].

3. Case study

3.1. Description

The proposed approach is illustrated with an example. Consider a construction project including three similar sub-projects with three contractors in charge. The due date of all sub-projects is at the end of time unit 25 with delay penalty and earliness bonus of 30 and 20\$ per time unit, respectively. The discount rate is assumed 2%. Each activity has two possible execution modes. Table 2 presents the list of activities, corresponding precedence relations between them and their durations, required resources, and cash flow with respect to each mode.

Number of units of each renewable resource available to each contractor at different time intervals is shown in Figure 1 and 2.

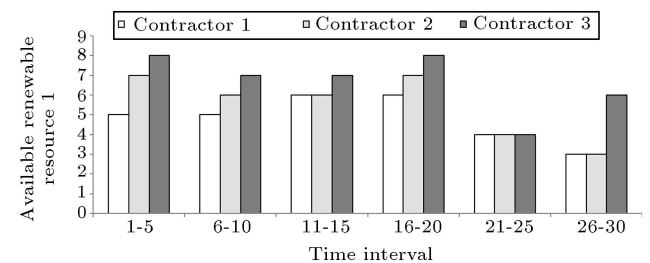


Figure 1. Number of units of renewable resource 1 available to each contractor at each time interval.

Table 2. The list of activities, corresponding precedence relations between them and their durations, required resources, and cash flows with respect to each mode.

Subcontractor	Activity	Precedence	Mode							
			1				2			
			Duration	Required resource		Cash flow	Duration	Required resource		Cash flow
				1	2			1	2	
1	1	–	5	1	3	–320	3	2	4	–306
	2	1	4	2	4	–508	3	4	6	–564
	3	1	10	0	1	–330	6	0	0	–346
	4	2,3	6	3	0	–280	4	5	3	–300
	5	3	10	5	2	–780	8	6	5	–792
	6	4,5	6	1	3	4596	4	2	4	4550
2	7	–	5	1	3	–320	3	2	4	–306
	8	7	4	2	4	–508	3	4	6	–564
	9	7	10	0	1	–330	6	0	0	–346
	10	8,9	6	3	0	–280	4	5	3	–300
	11	9	10	5	2	–780	8	6	5	–792
	12	10,11	6	1	3	4596	4	2	4	4550
3	13	–	5	1	3	–320	3	2	4	–306
	14	13	4	2	4	–508	3	4	6	–564
	15	13	10	0	1	–330	6	0	0	–346
	16	14,15	6	3	0	–280	4	5	3	–300
	17	15	10	5	2	–780	8	6	5	–792
	18	16,17	6	1	3	4596	4	2	4	4550

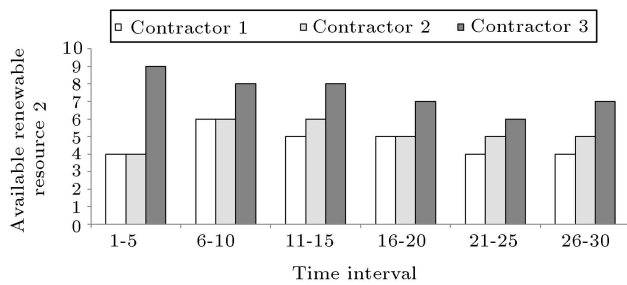


Figure 2. Number of units of renewable resource 2 available to each contractor at each time interval.

3.2. NPVs for various degrees of cooperation

Based on the above information, the proposed mathematical models of all possible coalitions including non-cooperation (act alone), partial cooperation (subset coalition), and the grand coalition are solved by branch-and-reduce optimization navigator (BARON) under GAMS. The results consist of the best NPV found for each coalition, the finish time of each activity, and the makespan of each sub-project (Table 3).

As shown in Table 3, if no coalition is created or the contractors perform the subprojects separately, the NPV of the project will be 3266.41\$; if the subset coalitions of (1,2), (1,3), and (2,3) are formed, the NPV of the project will be 3423.014\$, 3381.122\$, and

3400.969\$, respectively; and if the project is performed by grand coalition, the NPV will be 3553.879\$.

The resulting Gantt charts for all 5 possible combinations of coalitions are shown in Figure 3. In this figure, the color of the bars for all activities of one coalition is the same.

3.3. The core of the cooperation game

The allocation of the NPV to all the three contractors cannot migrate outside the core, $x \in C(\nu)$. The core of the game $C(\nu)$ is shown in Figure 4, representing all possible payoff allocations.

3.4. NPV allocation schemes based on the solutions

By applying the introduced 7 methods of cooperative game theory to the case study, the NPV assignment strategy for each method is deduced and summarized in Table 4. Apparently, the sum of the allocated values for each contractor is equal to the total NPV (3553.879\$) through grand coalition. Meanwhile, all of the allocation schemes satisfy the core requirements as shown in Table 4.

3.5. Stability for different allocation schemes

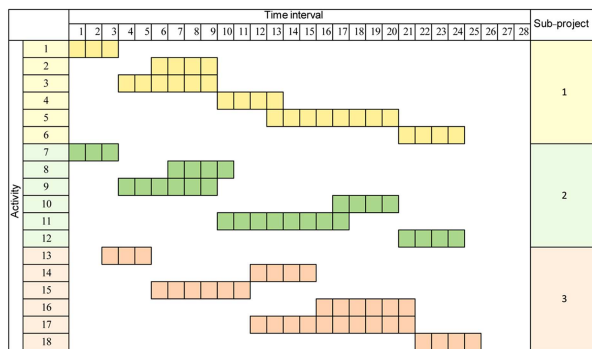
In the following, in order to investigate the stability of different allocation schemes presented in Section 2.2.3,

Table 3. Net Present Value (NPV) and makespan of each sub-project for various degrees of cooperation.

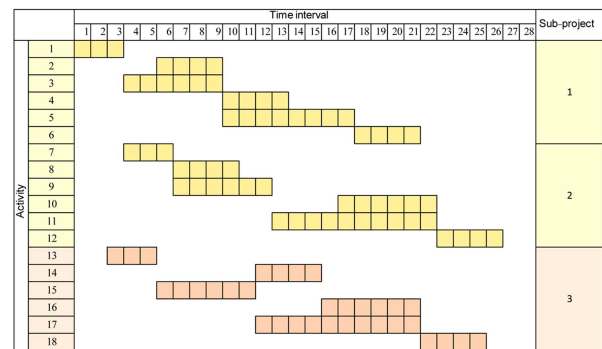
Scenario	Coalition	NPV(\$) of coalition	Combinations	NPV(\$) of project	Makespan		
					Sub-project 1	Sub-project 2	Sub-project 3
Act alone	1	1064.13	{(1),(2),(3)}	3266.41	24	–	–
	2	1069.923			–	24	–
	3	1132.357			–	–	25
Subset coalition	1,2	2290.657	{(1,2),(3)}	3423.014	21	26	–
	1,2	2290.657	{(1,2),(3)}	3423.014	21	26	–
	1,3	2311.199	{(1,3),(2)}	3381.122	27	–	21
	2,3	2336.839	{(1),(2,3)}	3400.969	–	25	22
Grand coalition	1,2,3	3553.879	{(1,3, 2)}	3553.879	28	21	21

Table 4. Net Present Value (NPV) assignment using different schemes.

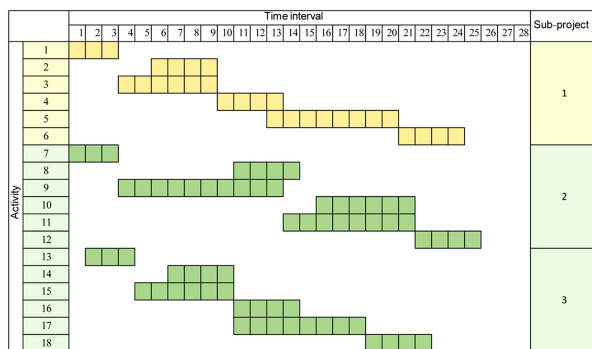
Solution scheme	NPV allocation (\$)			In core
	x_1	x_2	x_3	
Shapely value	1160.319	1176.036	1217.524	Yes
τ -Value	1159.952	1179.562	1214.365	Yes
Average lexicographic value	1174.702	1177.999	1201.179	Yes
Equal split-off set	1184.626	1184.626	1184.626	Yes
Nucleolus	1,160.686	1,186.326	1,206.868	Yes
Per-capita excess	1,160.686	1,186.326	1,206.868	Yes
N-H solution	1159.946	1165.749	1228.184	Yes



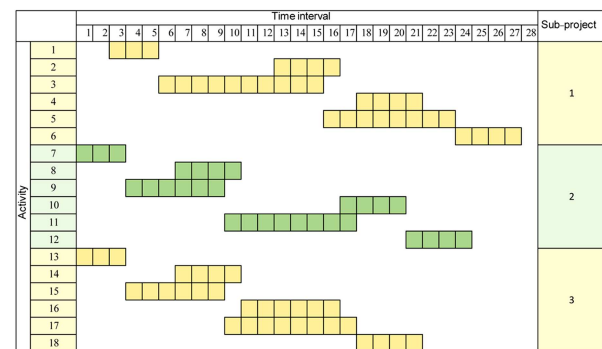
(a) The project gantt chart in the non-cooperative mode (singleton coalitions) or $\{(1),(2),(3)\}$



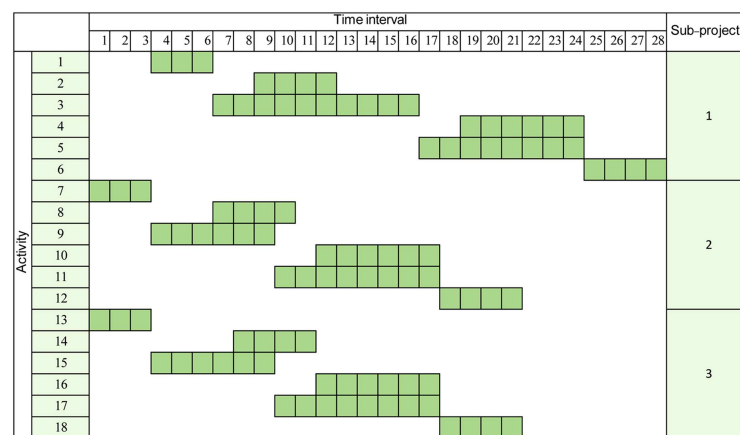
(b) The project gantt chart in the situation of Contractors 1 and 2 cooperate and Contractor 3 acts alone or $\{(1,2),(3)\}$.



(c) The project gantt chart in the situation of Contractors 2 and 3 cooperate and Contractor 1 acts alone or $\{(1),(2,3)\}$.



(d) The project gantt chart in the situation of Contractors 1 and 3 cooperate and contractor 2 acts alone or $\{(1,3),(2)\}$.



(e) The project gantt chart in the situation of grand coalition or $\{(1,3),(2)\}$.

Figure 3. Gantt charts for all possible combinations of coalitions.

the values of DP and Shapley-Shubik Power Index are calculated.

Table 5 shows the DP value for each contractor using the 7 allocation methods. It can be observed that the calculated DP value for the third contractor employing the equal split-off set method is 1.504. According to Eq. (28) and as mentioned in Section 2.2.3, the smaller the value of DP , the greater the incentive of the player to join the coalition and vice versa. Therefore, one player will disrupt the coalition only

when the DP value is less than 1. Therefore, the third contractor will refuse to accept the allocation strategy based on the equal split-off set method. Therefore, it can be concluded that the equal split-off set method is unstable.

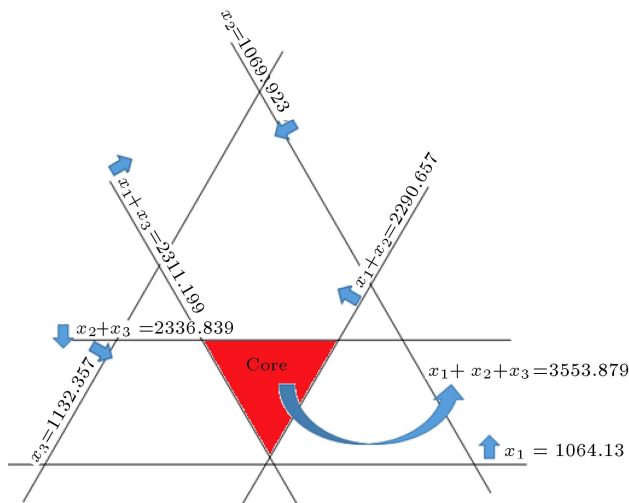
Table 6 shows the power index within each scheme by utilizing Eq. (25), based on which the FI can be calculated by Eq. (26). As mentioned before, the greater the value of FI, the lower the fairness of the allocation strategy. Thus, it can be concluded that,

Table 5. Propensity to disrupt (DP) of different schemes.

Solution scheme	NPV allocation (\$)			
	DP_1	DP_2	DP_3	DP
Shapely value	0.590	0.628	0.537	0.628
τ -value	0.596	0.576	0.596	0.596
Average lexicographic value	0.383	0.598	0.902	0.902
Equal split-off set	0.269	0.506	1.504	1.504
Nucleolus	0.584	0.484	0.756	0.756
Per-capita excess	0.584	0.484	0.756	0.756
N-H solution	0.596	0.803	0.366	0.803

Table 6. Fairness evaluation of each scheme.

Solution scheme	The Shapley-Shubik power index			Fairness index (FI_α)
	α_1	α_2	α_3	
Shapely value	0.321	0.377	0.302	0.095
τ -value	0.333	0.381	0.285	0.118
Average lexicographic value	0.385	0.376	0.239	0.200
Nucleolus	0.336	0.405	0.259	0.179
Per-capita excess	0.336	0.405	0.259	0.179
N-H solution	0.333	0.333	0.333	0.000

**Figure 4.** The core space of the game representing all possible payoff allocations.

although all of the values are within a reasonable range ($0 \leq FI_\alpha \leq 1$), based on Eq. (26), the scheme with the highest fairness is the N-H solution with the lowest FI.

Therefore, in summary, the best allocation scheme can be deduced by the N-H solution method, which can meet both of the stability criteria, simultaneously.

With the N-H solution method, the utilities or NPVs 1159.946\$, 1165.749\$, and 1228.184\$ are allocated to contractors 1, 2, and 3, respectively. Accordingly, the share of contractors 1, 2, and 3 in the grand coalition is 32.6%, 32.8%, and 34.6%, respectively.

4. Conclusions

One of the constraints on construction projects, which leads to increase in time and cost, is the limitation of renewable resources (e.g., labor or equipment). Sharing this resources is one of the areas of cooperation among the contractors of the project, which can lead to decrease in total time and cost and improve NPV. In this study, various solution methods of the cooperative game theory were used to solve the problem of determining the share of the participating contractors from this improvement.

In order to model this problem, a Multi-mode Resource-Constrained Project Scheduling Problem with Discounted Cash flows (MRCPSDC) for each coalition was taken into account and a Mixed-Integer Nonlinear Programming (MINLP) model was developed. The suggested model can be used by partnering contractors to manage renewable resources more efficiently on a cooperative basis. In other words, partnering makes the feasible solution space of the problem larger and it can improve the solution.

Therefore, in the solution procedure, first, the suggested model was solved for all coalitions and then, the benefit (NPV) allocation problem was solved by 7 widely used cooperative game theory solution methods, namely Shapely value, τ -value, average lexicographic value, equal split-off set, nucleolus, per-capita excess, and N-H solution. Eventually, the best allocation scheme based on the stability criteria of Shapley-Shubik power index and DP value was identified.

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Biography

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