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Optimizing decisions on under- and out-of-warranty products in a finite planning horizon

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Abstract. In this paper, we consider a manufacturer that produces products in a finite horizon time and sells products with non-renewing Free Replacement Warranty (FRW) policy. The manufacturer is responsible to provide spare parts for failed products, whether the products are under or out of warranty. Previous research on warranty optimization has focused on maximizing manufacturer profit without considering the spare parts market for out-of-warranty products. This study proposes a novel nonlinear model that maximizes manufacturer profit by optimization of price, warranty length, and spare parts inventory for under- and out-of-warranty products in a manufacturing/remanufacturing system. Due to the unique structure of the model, we propose a new two-stage approach that combines metaheuristic and an exact method, in which the first stage is to determine prices and warranty length of product by the metaheuristic algorithm and in the second stage, the remaining inventory-related problem is transferred to a minimum cost network flow problem solved for spare parts inventory control. To illustrate effectiveness of the suggested method, the model is solved for the case study of Iranian SANAM electronic company with two different metaheuristic algorithms and a sensitivity analysis is conducted to study the effect of various parameters on the optimal solution.

1. Introduction

With tougher competition, technology advances and with shifts in customer preferences, it is more crucial than ever that companies use warranty as a competition advantage in order to increase their market share. Warranty signifies quality of the product in the customers' eyes, hence leading to growth in the satisfaction of customers and their willingness to buy the product. Warranty has two main functions of protection and promotion. Regarding the former, it protects the manufacturer from excessive claims and protects the customer from purchase risks. Regarding the latter, it is a competitive advantage to differentiate the manufacturer from its competitors [1]. Relatively long warranty period will increase willingness of the customers to buy. However, the manufacturers cannot propose long warranty, because they are responsible for the failure of products during the warranty period [2]. Also, they must take into account reliability of the products, because undesirable reliability may lead to high cost to them [3]. Therefore, they should determine warranty length in order to optimize their profit and customer satisfaction.

In the literature, price and warranty length are mentioned as two key factors affecting profit of the manufacturer [4–7]. Obviously, longer warranty period and lower price lead to increased sales, but they also tend to decrease marginal profit of the manufacturer.

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As a result, simultaneous decision about these two factors in order to optimize the profit is important and has widely been studied in the literature.

Glickman and Berger [8] proposed a model for maximizing profit of the manufacturer by determining price and warranty length, assuming that customers were homogeneous and their demand was an exponential function of price and warranty length. Nasrollahi and Asgharizadeh [9] solved a multi-objective problem by goal programming using demand function for estimating warranty length. The problem in the study conducted by Lin and Shue [10] was determining price and warranty length while demand was a function of price, warranty length, and cumulative sales and the objective was maximizing profit. They considered different product life distributions. In [6,7], the distribution of product lifetime was considered to be Normal and Gamma, and solution approaches were presented based on maximum principle. Huang et al. [11] took account of reliability in addition to price and warranty length in modeling, and studied the problem both in stable and dynamic market scenarios. Manna [12] considered the problem with price and warranty length decision variables and proposed a method to extend the model to two-dimensional warranty. The model proposed in [5] incorporated price, warranty length, and production rate as decision variables with the objective of maximizing profit of the manufacturer. Wu et al. [13] also developed a model with these decision variables considering Weibull product lifetime with the objective of maximizing profit. Zhou et al. [4] determined the optimal price and warranty length for repairable product and compared fixed-length warranty policy with dynamic policy. Fang and Huang [14] proposed a Bayesian decision model in which the integrated price of products, production quantity, and warranty length were determined for the situation in which the manufacturer did not have sufficient historical data. Shafiee and Chukova [15] developed a mathematical optimization model to determine price, warranty length, and upgrade strategy for the second-hand products. Faridimehr and Niaki [16] investigated optimal policies for price, warranty length, and production rate in both static and dynamic markets through the maximum principal approach. Mahmoudi and Shavandi [17] proposed a bi-objective model for maximizing profit of the manufacturer and minimizing the waiting time in queue. Also, they formulated the demand function as a fuzzy system. Tsao et al. [18] considered the problem of determining retail price and inventory level for hi-tech products when warranty length was predetermined. Wei et al. [19] proposed 5 decentralized models and determined equilibrium wholesale prices, retail prices, and warranty periods using game theoretical approaches. The problem in [20] was to determine pricing policy for returned used products along with their remanufacturing level and to identify pricing and warranty policy for the remanufactured products.

Besides the sale of the main products, aftermarket plays an important role in gaining profit by the manufacturer. Selling spare parts for out-of-warranty products can lead to substantial profit. However, it is very challenging to estimate the demand for spare parts due to its greater uncertainty than the uncertainty of the demand for products [21]. To the best of our knowledge, there is not any research that considers selling spare parts for out-of-warranty products, while in many industries, such as automobile and electronic devices, companies can increase their profits by up to 25 percent in the spare parts market [22].

Although pricing and warranty length are two key factors that affect inventory management of spare parts, only a few researchers have considered this interdependency in the literature. Kim and Park [23] proposed a two-stage optimal control model to jointly determine price, warranty length, and spare parts inventory for under-warranty products. They divided the planning horizon into two parts: life cycle of product and end of life period. Their study considered only under-warranty products, but a considerable portion of profit came from selling spare parts for out-of-warranty products. Also, they assumed that all spare parts were produced by the manufacturer, but in practice, components can be refurbished by remanufacturing with a lower amount of cost. Chari et al. [24] developed a mathematical optimization model to maximize total expected profit of the manufacturer by optimization of warranty length, sale price, age of reconditioned components, and the proportion of reconditioned components to be used. They assumed renewing Free Replacement Warranty (FRW) and static pricing strategy in their model. Also, they did not consider the role of out-of-warranty products in the profit for the manufacturer.

In order to present a concise review of the previous studies and demonstrate the characteristics of the proposed approach as compared to those in the literature, Table 1 illustrates a state-of-the-art survey of pricing and warranty inventory optimization. Although spare parts inventory decisions have a direct impact on warranty length and price decisions (as shown in Table 1), many of the proposed approaches seek to optimize warranty length and price without considering inventory of spare parts. To the best of the authors' knowledge, there is no research considering the effect of out-of-warranty products on the revenue of manufacturers.

The purpose of this paper is to develop a new mathematical model for optimizing product price (in different stages of the life cycle of a product), warranty length, and spare parts inventory control for under-

Table 1. Relevant previous research studies.

Reference	Pricing		WLO[1]				Inventory control		Planning horizon			Situation of product	
	Static	Dynamic	Renewing	Non-renewing	RO[2]	PRO[3]	Main product	Spare part	Selling period	EOL[4]	Infinite	Under-warranty	Out-of-warranty
Lin and Shue [10]	—	✓	✓	—	—	—	—	—	✓	—	—	✓	—
Wu et al. [7]	—	✓	✓	—	—	—	—	—	✓	—	—	✓	—
Huang et al. [11]	—	✓	—	✓	✓	—	—	—	✓	—	—	✓	—
Huang et al. [42]	—	—	—	—	—	—	✓	—	✓	—	✓	✓	—
Yeo et al. [43]	—	—	—	—	—		✓	✓	✓	—	—	✓	—
Khawam et al. [44]	—	—	—	—	—	—	✓	✓	✓	—	—	✓	—
Kim et al. [23]	—	✓	—	✓	—	—	—	✓	✓	✓	—	✓	—
Wu et al. [13]	✓	—	✓	—	—	✓	✓	—	✓	—	—	✓	—
Lin et al. [5]	—	✓	✓	—	—	✓	—	—	✓	—	—	✓	—
Faridimehr and Niaki [16]	✓	—	—	✓	—	—	—	—	✓	—	—	✓	—
Tsao et al. [18]	—	✓	—	—	—	—	—	✓	—	—	—	✓	—
Wei et al. [19]	✓	—	—	✓	—	✓	—	—	—	—	—	✓	—
Yazdian et al. [20]	✓	—	—	✓	—	✓	—	—	—	✓	—	✓	—
Chari et al. [24]	✓	—	✓	—	—	—	—	✓	✓	✓	—	✓	—
Darghouth et al. [45]	—	✓	—	✓	✓	—	—	—	✓	—	—	✓	—
This study	—	✓	—	✓	—	—	—	✓	✓	✓	—	✓	✓

[1]WLO: Warranty Length Optimization; [2]RO: Reliability Optimization; [3]PRO: Production Rate Optimization;

[4]EOL: End Of Life.

warranty and out-of-warranty products in a manufacturing/remanufacturing system with the objective of maximizing the profit of the manufacturer. Planning horizon consists of three main parts:

1. Product life cycle;
2. End Of Life (EOL);
3. Guarantee period for spare part availability.

Demand for the product is considered as a function of price, time, and warranty length. The significant issue for the producer is to determine the price in each Pricing Period (PP) of the life cycle of the product in order to gain maximum profit. Another challenge is to determine warranty length, where longeer warranty length period tends to increase sales but, at the same time, increase warranty-relevant costs. Although failure of under- and out-of-warranty products changes stochastically in each period, the model can reach an acceptable estimation of failures in each Inventory Control Period (ICP) of planning horizon in order to effectively manage the spare parts inventory. In real word, a percentage of the failed items can be rectified by remanufacturing. Therefore, we assume that spare parts can be obtained in two ways:

1. Production by the original manufacturer;
2. Remanufacturing failed products.

Basically, this paper aims to perform the following tasks:

- Proposing a new model that considers out-of-warranty products as the main source of revenue for the manufacturer;
- Coordination between price, warranty length, and spare parts inventory decisions as an integrated model for under- and out-of-warranty products;
- Proposing a novel two-stage approach combining metaheuristic and exact method to solve the proposed model;
- Identifying how changes in life cycle of the product affect warranty length.

The rest of this paper is organized as follows: Section 2 presents problem definition. Section 3 explains the mathematical modeling. A solution method is introduced in Section 4. Section 5 demonstrates applicability of the presented mathematical model by a real-world numerical example taken from the Iranian SANAM electronic company along with sensitivity analyses. The paper is concluded in Section 6.

2. Problem definition

The problem in this paper is defined for maximizing the profit of the manufacturer, which consists of a set of revenues and cost elements. Revenues of the manufacturer include:

1. Sale of product in its life cycle;
2. Sale of spare parts for out-of-warranty products.

On the other hand, the costs comprise four main elements:

1. Production cost;
2. Inventory cost of spare parts;
3. Remanufacturing cost of spare parts;
4. Disposal cost.

Sale of the product and its production cost are directly related to the market demand in its life cycle. The demand is itself dependent upon time, sales price, and length of warranty. Sales price and warranty length are respectively inversely and directly proportional to the market demand. Therefore, simultaneous decision about sales price and length of warranty is of considerable significance in order to maximize profit.

Additionally, spare-part-related elements affect revenue of and cost to the manufacturer. Effective spare part inventory control decisions play an important role in reducing the cost to the manufacturer. A challenge here is to estimate the number of failures of a product in each ICP in order to optimize the inventory level.

Before presenting the model, the assumptions made for formulating the problem are given as follows:

1. All ICPs are equal and less than life cycle of the product;
2. All claims during the warranty period are valid;
3. Warranty policy for products is non-renewing FRW;
4. The original manufacturer is also responsible for remanufacturing of used products;
5. The refurbished components return to as-good-as-new state;
6. Production capacity is unlimited in product life cycle;
7. The firm is a monopolist and customers are myopic;
8. The amount of sale of the product is equal to the demand for the product;
9. Products have exponential failure distribution. This assumption was imposed by the product development division of SANAM electronic company;
10. Warranty length is a positive integer and multiple of the ICP;
11. The inventory delivery is assumed to be instantaneous (lead time is negligible);
12. Shortage is not allowed to avoid lost sales.

Assumptions 1 to 10 are common in reality for the problem (especially for electronic device manufacturers). However, assumptions 11 and 12 are set in order to make the problem technically more tractable. The notation used to formulate the problem is presented in Table 2.

According to the above-mentioned assumptions, the planning horizon of the problem is divided into three segments (see Figure 1):

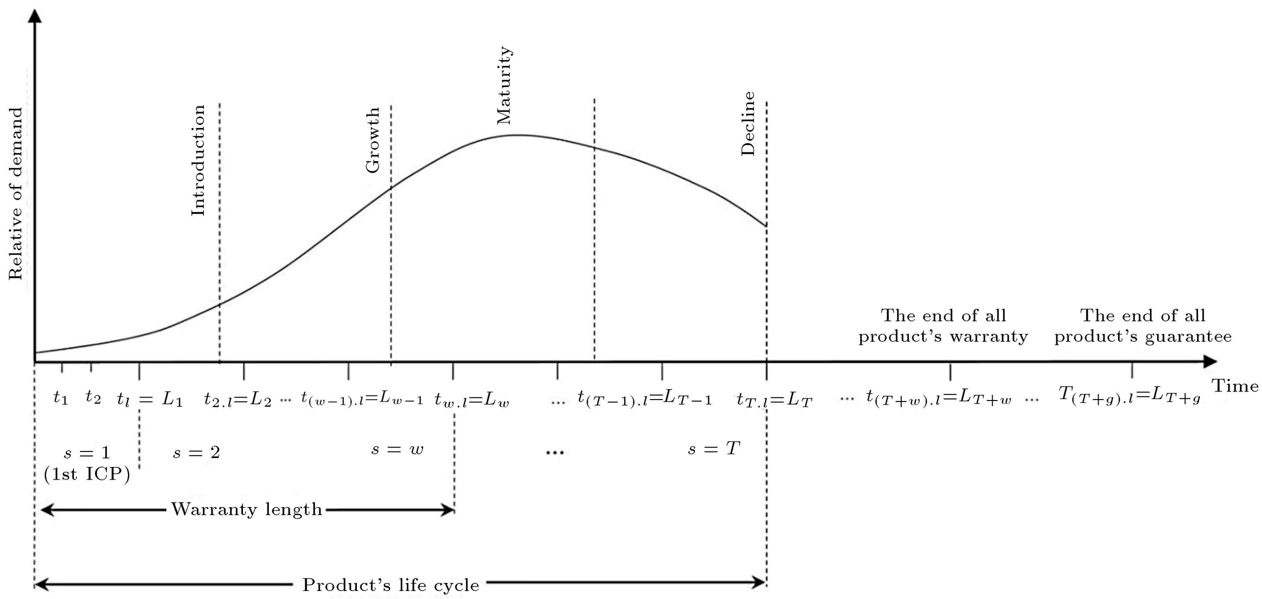


Figure 1. Demand in product life cycle.

Table 2. Indices, parameters, and decision variables.

	Description
Index:	
i	Counter of key components for each product, $i \in \{1, 2, \cdots, I\}$
s	Counter of Inventory Control Period (ICP), $s \in \{1, 2, \cdots, T+g\}$
j	Counter of Pricing Periods (PPS), $j \in \{1, \cdots, l.(T+g)\}$
Parameter:	
I	The number of key components in each product
l	The number of PPs in each ICP, $L_s - L_{s-1} = l \times t_1$
t_j	The end of the jth PP, $t_{j+1} - t_j = t_1 \; \forall \; j \in \{1, \cdots, l.(T+g)\}$
T	The number of ICPs in the life cycle of the product
g	The number of ICPs that guarantee spare part availability after termination of the life cycle of the product
L_s	Calendar time for the end of the sth ICP
$W_{\min}$	Lower bound for warranty length
$W_{\max}$	Upper bound for warranty length
$P_{\min}$	Lower bound for prices
$P_{\max}$	Upper bound for prices
c	Per-unit procurement cost of product
pc_i	Unit selling price for component i
co_i	Procurement cost per unit for component i
cr_i	Unit refurbishing cost for component i
v_i	Unit disposing cost for component i
h_i	Holding cost per unit per time unit for component i
α_i	Percentage of the failure of the product for component i
δ_i	Percentage of component i to be sent to the refurbishing center
κ_i	Percentage of successful refurbishing for component i
λ	Rate of failure of the product
ρ_w	The service level parameter for products under warranty
ρ_{pw}	The service level parameter for products out of warranty
U	Maximum market demand
η	Time of peak demand
D_0	Initial demand
λ_d	A given positive coefficient
Dependent variable:	
y_{js}^k	Random variable for the number of failures of products in situation k (situations will be described) that are produced in the jth PP and fail in the sth ICP
n_{js}^k	Maximum number of failures for products in situation k that are produced in the jth PP and fail in the sth ICP
$S(j, P_j, w)$	Selling function for the jth PP with price P_j and warranty length w
$D_w(s)$	Maximum number of failures for products under warranty in the sth ICP
$D_{pw}(s)$	Maximum number of failures for products out of warranty in the sth ICP
$F_i(s)$	All components i that may undergo refurbishing in the sth ICP
$E_i(s)$	All components i that have gone successful refurbishing in the sth ICP
$V_i(s)$	All components i that may be disposed in the sth ICP
$X_i(s)$	The number of in-hand inventory at the end of the sth ICP

Table 2. Indices, parameters, and decision variables (continued).

	Description
First-stage decision variable:	
P_j	Selling price during the jth PP
w	Warranty length
Second-stage decision variable:	
$Q_i(s)$	Production amount of component i in the sth ICP

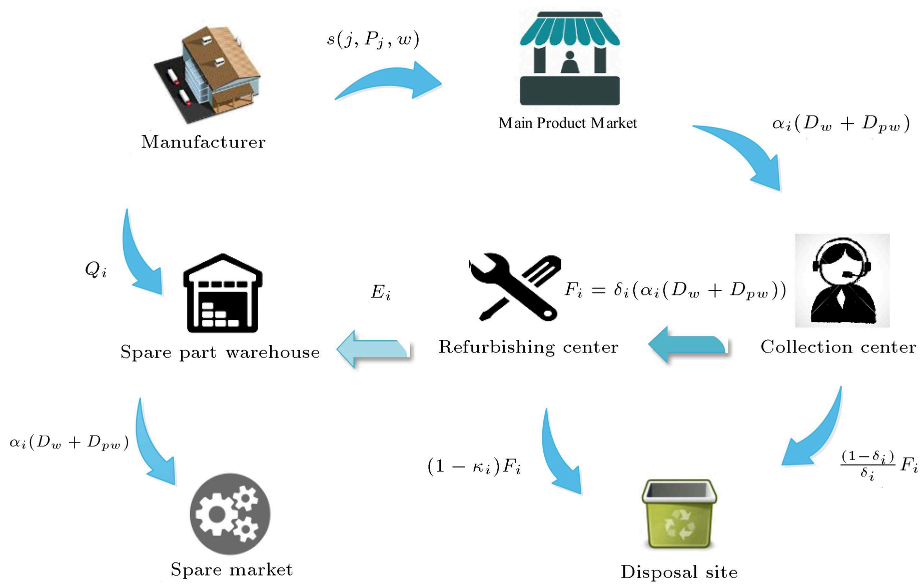


Figure 2. Cycle of production, marketing, and spare part inventory control.

1. Life cycle of product $(0, L_T)$;
2. EOL (L_T, L_{T+w});
3. Guarantee period for spare part availability (L_{T+w}, L_{T+g}).

In each of the ICPs, an estimation of the number of products under warranty should be made. It is a challenging task because, on the one hand, these estimation conditions differ in each of the above-mentioned segments and, on the other hand, in each ICP, a number of manufactured products are added to the products under warranty and some products become out of warranty. After calculating the number of under-warranty and out-of-warranty products in each ICP, the estimated number of failures can be calculated using failure behavior of the product, which is employed to estimate the demand for spare parts.

As can be seen in Figure 1, planning horizon is divided into $T + g$ ICPs, in which the spare part order size is determined based on the demand for the spare parts minus the remaining inventory from the previous period and the amount of remanufactured spare parts. In addition, each ICP is divided into l equal sub-periods in which the amount of production is a function of time, price, and warranty length. Since price of the product is determined in each of these sub-periods, they are named PPs. If warranty length is equal to w ICPs, we may have products under warranty in the market until at most L_{T+w}. Therefore, from L_T to L_{T+w}, the decision is limited to spare parts inventory control for under-warranty and out-of-warranty products. Finally, from L_{T+w} to L_{T+g}, spare parts inventory control for out-of-warranty products is the only decision to be made. This period is for ensuring the customers of the availability of spare parts for a fixed period of time after the whole warranty of the product.

Spare parts inventory can be provided from two sources:

1. Remanufactured failed components;
2. Manufactured components.

Because the planning horizon is finite and the holding and production costs of the component can change with time, it is essential for the manufacturer to optimize the amount of production of the component in each ICP.

The relationship between production, market, and spare part inventory is depicted in Figure 2. Manufacturer supplies the product according to market demand, which is a function of time, price, and warranty length. The product includes I critical components. The amount of failed products that enter the collection center in each ICP is dependent upon the situation of the product (to be defined later) and failure rate. Based on historical data, we can determine the proportion of times of failure of the product for each component. Then, based on conditions of the remanufacturing process, components that can be refurbished are sent to the refurbishing center and irreparable ones are disposed. With the assumption that refurbishing is perfect, refurbished components are as good as new. Thus, they can meet a proportion of the demand for spare parts. Finally, the manufacturer satisfies the demand by the spare market (demands with under- and out-of-warranty products) with spare parts that are refurbished and produced in the factory. Since the key elements of reverse logistics are considered in the literature to be failed item remanufacturing, disposition, spare parts inventory management, after-sale service, and product pricing [25,26], we can say that the proposed framework belongs to reverse logistics. Therefore, the manufacturer can overcome reverse logistics challenges related to under-warranty and out-of-warranty products by using the proposed model.

3. Mathematical model

In this section, we describe the mathematical model for the problem, which is constructed based on the mentioned notation and assumptions.

3.1. Objective function

The objective function maximizes manufacturer profit gained by selling products and spare parts. The manufacturer can sell spare parts only for out-of-warranty product.

$$\max\ z=\sum_{j=1}^{l.T}(P_j-c).S(j,P_j,w)+\sum_{i=1}^{I}pc_i.\sum_{s=w+1}^{T+g}(\alpha_i D_{pw}(s))-\sum_{i=1}^{I}cr_i.\sum_{s=1}^{T+g}E_i(s)-\sum_{i=1}^{I}v_i.\sum_{s=1}^{T+g}V_i(s)-\sum_{i=1}^{I}h_i.\sum_{s=1}^{T+g}X_i(s)-\sum_{i=1}^{I}co_i.\sum_{s=1}^{T+g}Q_i(s). \quad (1)$$

In Eq. (1), the first term is the profit obtained by the sale of the product, which is calculated by multiplying the net profit of the product by demand in each PP. The revenue earned by the sale of spare parts for out-of-warranty products is calculated in the second term. The third to fifth terms are the remanufacturing, disposal, and holding costs, respectively. The final term calculates the cost of production of spare parts in each ICP.

3.1.1. Market demand for the product or number of products sold in each period

Eq. (2) shows the demand behavior according to time, which is increasing up to η and decreasing afterwards. This is congruent with the life cycle of the product in which demand is rising until the maturity and falling afterwards. Interested reader can find more information in [27].

$$S(j)=\begin{cases}\frac{U}{\left(1+\Psi e^{-\lambda_d U j}\right)}, & 0\le j\le\eta\\ \frac{U}{(\lambda_d U(j-\eta)+\theta)}, & \eta\le j\le T.l\end{cases}$$

$$\Psi=\frac{U}{D_0}-1,$$

$$\theta=1+\varphi e^{-\lambda_d U\eta}. \quad (2)$$

Eq. (3) calculates the demand for the main product based on the assumption that the amount of sale of the product is equal to the demand for it. Therefore, the sale of the manufacturer is a function of time (j), price (p_j), and warranty length (w). We assume that $S(j)$ is the potential market demand in each PP. It is obvious that sale is inversely proportional to price and directly proportional to warranty length. The element $-k_1(p_j-P_{\min})+k_2w$ demonstrates this type of dependence. In Eq. (3), k_1 is price coefficient and k_2 is warranty coefficient in the demand function. Constraint (4) shows the lower bound and upper bound for warranty length.

$$S(j,p_j,w)=S(j)-k_1(p_j-P_{\min})+k_2w \quad \text{for} \quad k_1>0,\quad k_2>0, \quad (3)$$

$$W_{\min}\le w\le W_{\max}. \quad (4)$$

Because customers are considered myopic, the price of the product is decreasing in time. Constraint (5) takes account of this fact, which is called markdown pricing. Constraint (6) represents the lower bound and upper bound for the price of the product in each PP.

$$P_{j-1}\ge P_j \qquad \forall\ j\in\{1,\cdots,(T+g)\times l\}, \quad (5)$$

$$P_{\min}\le p_j\le P_{\max} \qquad \forall\ j\in\{1,\cdots,(T+g)\times l\}. \quad (6)$$

3.2. Situations of the product during each inventory planning period

Each ICP, based on its position in the planning horizon, may include several types of situations for each product. Each situation will demonstrate how much time a product is under warranty or/and out of warranty in each ICP. Each product may have one or two of the five situations in each ICP. Situations 1, 2, and 3 are for calculating the number of failed products under warranty and situations 4 and 5 are for computing the number of failed out-of-warranty products. These situations are as follows:

- **Situation 1 (for under-warranty products)** For a product manufactured in an ICP in Situation 1, the probability of failure in that ICP is proportional to the amount of time it lies in that ICP. It is obvious that all ICPs in the life cycle of the products have products in Situation 1, because production is carried out in these ICPs;
- **Situation 2 (for under-warranty products)** This situation covers a case in which the product is under warranty throughout the ICP;
- **Situation 3 (for under-warranty products)** A product in Situation 3 falls in an ICP in which warranty expires;
- **Situation 4 (for out-of-warranty products)** Situation 4 is the complement to Situation 3 as it enables us to compute the probability of failure when the product becomes out of warranty;
- **Situation 5 (for out-of-warranty product)** In this situation, the warranty has been expired in one of the prior ICPs, hence the probability of failure is proportional to the whole ICP.

Figure 3 summarizes Situations 1 to 5 along with their features for products that are produced in t_1. In this figure, we have assumed that warranty length is equal to four ICPs. Therefore, the amount of products produced in t_1 is $S(1, P_1, 4)$. The products that are in $s = 1$ are in Situation 1 and the failure probability is proportional to $(L_1 - t_1)$. In $s = \{2, 3, 4\}$, products are in Situation 2; thus, their probability of failure is proportional to the whole length of ICP. Since warranty will expire in $s = 5$, as long as products are under warranty, they are in Situation 3. After they become out of warranty, they get in Situation 4 until the end of the 5th ICP. Finally, the products will be in Situation 5 in $s = \{6, \cdots, T + g\}$. Therefore, it can be concluded that the products are in Situation 1 when they are produced and they pass through Situations 2 to 5 sequentially in the whole planning horizon.

3.3. The number of failures of products under warranty

In order to calculate the number of failures of under-warranty products in each ICP, we must first compute how much time the product is under warranty in that ICP. As defined above, under-warranty products can be in at most three situations, i.e., 1, 2, or 3.

According to assumption 1, the probability of failure for the product in Situation 1 in the sth ICP, which is produced in the jth PP ($L_{s-1} < t_j < L_s$), is equal to $P^1_{js} = P(f \le L_s | f \ge t_j) = 1 - e^{-\lambda.(L_s - t_j)}$. On the other hand, the number of products produced in the jth PP is denoted by $S(j, p_j, w)$, hence the number of failures of products in Situation 1 is binomially distributed as $y^1_{js} \sim b(S(j, p_j, w), P^1_{js} = 1 - e^{-\lambda.(L_s - t_j)})$. All the products produced in the first ICP are in Situation 1. Therefore, using continuity correction, Eq. (7) calculates the maximum number of failures (n^1_{j1}) in the jth PP with the confidence ρ_w in the first ICP. Eq. (8) computes the total number of failures in the first ICP.

$$P\left(y^1_{j1} \le n^1_{j1}\right) \ge \rho_w$$

$$\Rightarrow P\left(z < \frac{n^1_{j1} + 0.5 - S(j, P_j, w).P^1_{j1}}{\sqrt{S(j, P_j, w).P^1_{j1}.(1 - P^1_{j1})}}\right) \ge \rho_w$$

$$\Rightarrow n^1_{j1} = \varphi^{-1}(\rho_w).\sqrt{S(j, P_j, w).P^1_{j1}.(1 - P^1_{j1})} + S(j, P_j, w).P^1_{j1} - 0.5 \quad \forall\, j \in \{1, 2, \cdots, l\}, \qquad (7)$$

$$D_w(1) = \sum_{j=1}^{l} n^1_{j1}. \qquad (8)$$

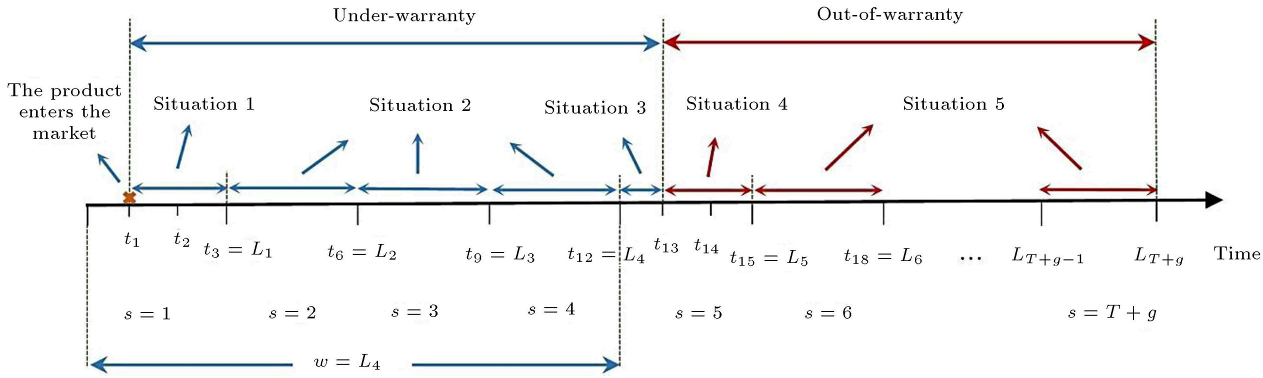


Figure 3. Situations of products.

With regard to the fact that products in Situation 2 are under warranty in the whole ICP, their failure probability is proportional to the length of ICP and calculated by $P_{js}^2 = P(f \leq L_s | f \geq L_{s-1}) = 1 - e^{-\lambda.(L_s - L_{s-1})}$. Additionally, the random variable denoting the number of failed products in Situation 2 follows the binomial distribution of $y_{js}^2 \sim b(S(j,p_j,w), P_{js}^2 = 1 - e^{-\lambda.(L_s - L_{s-1})})$. The characteristic of the 2nd to wth ICPs is that no product will get out of warranty, because it is assumed that warranty length is equal to w ICPs. Therefore, products in these ICPs are only in Situation 1 or 2, while all products produced before the sth ICP are in Situation 2 and those in the sth ICP are in Situation 1. Eqs. (9) and (10) calculate the maximum numbers of failed products in Situations 2 and 1, respectively, in the sth ICP. Also, Eq. (11) shows the total number of failed products in the sth ICP $(1 < s \leq w)$.

$$n_{js}^2 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^2.(1-P_{js}^2)} + S(j,P_j,w).P_{js}^2 - 0.5, \quad \forall\ j \in \{1,2,\cdots,(s-1).l\}, \quad s \in \{2,\cdots,w\}, \qquad (9)$$

$$n_{js}^1 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^1.(1-P_{js}^1)} + S(j,P_j,w).P_{js}^1 - 0.5, \quad \forall\ j \in \{(s-1).l,(s-1).l+1,\cdots,s.l\}, \quad s \in \{2,\cdots,w\}, \qquad (10)$$

$$D_w(s) = \sum_{j=1}^{(s-1).l} n_{js}^2 + \sum_{j=(s-1).l}^{s.l} n_{js}^1, \quad \forall\ s \in \{2,\cdots,w\}. \qquad (11)$$

For ICPs between w and T, the products can be in Situation 1, 2, or 3. Products produced in the sth ICP are in Situation 1, those produced in $s-w < j \leq s-1$ are in Situation 2, and those produced in $s-w-1 < j \leq s-w+1$ are in Situation 3.

For products in Situation 3, the probability of failure in the sth ICP is $P_{js}^3 = P(f \leq t_j + w | f \geq L_{s-1}) = 1 - e^{-\lambda(t_j + w - L_{s-1})}$ and the number of failures has the binomial distribution of $y_{js}^3 \sim b(S(j,p_j,w), P_{js}^3 = 1 - e^{-\lambda(t_j+w-L_{s-1})})$. Eqs. (12), (13), and (14) calculate the maximum numbers of failed products in Situations 3, 2, and 1, respectively, in the sth ICP when ICPs are between k and T. Eq. (15) calculates the total number of failed under-warranty products in the sth ICP.

$$n_{js}^3 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^3.(1-P_{js}^3)} + S(j,P_j,w).P_{js}^3 - 0.5, \quad \forall\ j \in \{(s-k-1).l,(s-k-1).l+1,\cdots,(s-k).l\}, \quad s \in \{w+1,\cdots,T\}, \qquad (12)$$

$$n_{js}^2 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^2.(1-P_{js}^2)} + S(j,P_j,w).P_{js}^2 - 0.5, \quad \forall\ j \in \{(s-k).l,(s-k).l+1,\cdots,(s-1).l\}, \quad s \in \{w+1,\cdots,T\}, \qquad (13)$$

$$n_{js}^1 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^1.(1-P_{js}^1)} + S(j,P_j,w).P_{js}^1 - 0.5, \quad \forall\ j \in \{(s-1).l,(s-1).l+1,\cdots,s.l\}, \quad s \in \{w+1,\cdots,T\}, \qquad (14)$$

$$D_w(s) = \sum_{j=(s-w-1).l}^{(s-w).l} n_{js}^3 + \sum_{j=(s-w).l}^{(s-1).l} n_{js}^2 + \sum_{t=(s-1).l}^{s.l} n_{js}^1, \quad \forall\ s \in \{w+1,\cdots,T\}. \qquad (15)$$

Because production is stopped at the end of T, all under-warranty products are in Situations 2 and 3 in $T < s \leq T+w$; products produced in $(s-w).l < j \leq T.l$ are in Situation 2 and those produced in $(s-w-1).l < j \leq (s-w).l$ are in Situation 3. Eqs. (16) and (17) calculate the maximum numbers of failed products in Situations 3 and 2, respectively, for $T < s \leq T+k$ and Eq. (18) calculates the total number of failed under-warranty products for the sth ICP.

$$n_{js}^3 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^3.(1-P_{js}^3)} + S(j,P_j,w).P_{js}^3 - 0.5, \quad \forall\ j \in \{(s-w-1).l,(s-w-1).l+1,\cdots,(s-w).l\}, \quad s \in \{T+1,\cdots,T+w\}, \qquad (16)$$

$$n_{js}^2 = \varphi^{-1}(\rho_w).\sqrt{S(j,P_j,w).P_{js}^2.(1-P_{js}^2)} + S(j,P_j,w).P_{js}^2 - 0.5, \quad \forall\ j \in \{(s-w).l,(s-w).l+1,\cdots,T.l\}, \quad s \in \{T+1,\cdots,T+w\}, \qquad (17)$$

$$D_w(s) = \sum_{t=(s-w-1).l}^{(s-w).l} n_{js}^3 + \sum_{j=(s-w).l}^{T.l} n_{js}^2,$$

$$\forall\ s \in \{T+1, \cdots, T+w\}. \qquad (18)$$

3.4. The number of failures for out-of-warranty products

$(w+1)$ is the first ICP in which the out-of-warranty products that have been produced in the 1st ICP are in Situation 4. The probability that products in Situation 4 fail in the sth ICP is equal to $P_{js}^4 = P(f \leq L_s | f \geq t_j + w) = 1 - e^{-\lambda.(L_{s.l} - t_{j+w.l})}$ and the number of failed products is binomially distributed as:

$$y_{js}^4 \sim b(S(j, p_j, w), \quad P_{js}^4 = 1 - e^{-\lambda.(L_{s.l} - t_{j+w.l})}).$$

Eq. (19) calculates the maximum number of failed out-of-warranty products in Situation 4 produced in $j \in \{1, 2, \cdots, l\}$ and Eq. (20) calculates the total number of failed out-of-warranty products in $s = w + 1$.

$$n_{js}^4 = \varphi^{-1}(\rho_{pw}).\sqrt{S(j, P_j, w).P_{js}^4.(1 - P_{js}^4)} + S(j, P_j, w).P_{js}^4 - 0.5,$$

$$\forall\ j \in \{1, 2, \cdots, l\}, \qquad s = w + 1, \qquad (19)$$

$$D_{pw}(w+1) = \sum_{j=1}^{l} n_{js}^4, \qquad s = w + 1. \qquad (20)$$

In $w + 1 < s \leq T + w$, out-of-warranty products are in Situations 4 and 5, so the products produced in $(s - w - 1).l < j \leq (s - w).l$ are in Situation 4 and those produced in $1 < j \leq (s - w - 1).l$ are in Situation 5.

The probability of failure of products in Situation 5 is equal to $P_{js}^5 = P(f \leq L_s | f \geq L_{s-1}) = 1 - e^{-\lambda.(L_s - L_{s-1})}$ and the random variable of the number of failed products in Situation 5 is $y_{js}^5 \sim b(S(j, p_j, w), P_{js}^5 = 1 - e^{-\lambda.(L_s - L_{s-1})})$. Using Eq. (21), we can compute the maximum number of failures for products in Situation 5 produced in $j \in \{1, 2, \cdots, (s - (w + 1)).l\}$, while Eq. (22) calculates the maximum number of failed out-of-warranty products in Situation 4 produced in $\{(s - (w + 1)).l, (s - (w + 1)).l + 1, \cdots, (s - w).l\}$. Also, Eq. (23) calculates the total number of failed out-of-warranty products in $s \in \{w + 2, \cdots, T + w\}$.

$$n_{js}^5 = \varphi^{-1}(\rho_{pw}).\sqrt{S(j, P_j, w).P_{js}^5.(1 - P_{js}^5)} + S(j, P_j, w).P_{js}^5 - 0.5,$$

$$\forall\ j \in \{1, 2, \cdots, (s - (w + 1)).l\},$$

$$s \in \{w + 2, \cdots, T + w\}, \qquad (21)$$

$$n_{js}^4 = \varphi^{-1}(\rho_{pw}).\sqrt{S(j, P_j, w).P_{js}^4.(1 - P_{js}^4)} + S(j, P_j, w).P_{js}^4 - 0.5,$$

$$\forall\ j \in \{(s-(w+1)).l, (s-(w+1)).l+1, \cdots, (s-k).l\},$$

$$s \in \{w + 2, \cdots, T + w\}, \qquad (22)$$

$$D_{pw}(s) = \sum_{j=1}^{[s-(w+1)].l} n_{js}^5 + \sum_{j=[s-(w+1)].l}^{(s-w).l} n_{js}^4,$$

$$\forall\ s \in \{w + 2, \cdots, T + w\}. \qquad (23)$$

Since at the end of $T + w$, they have become out of warranty, all products are in Situation 5. For these products, the maximum number of failures can be calculated by Eq. (24) and the total number of failures in $s \in \{T + w + 1, \cdots, T + g\}$ is calculated by Eq. (25).

$$n_{js}^5 = \varphi^{-1}(\rho_{pw}).\sqrt{S(j, P_j, w).P_{js}^5.(1 - P_{js}^5)} + S(j, P_j, w).P_{js}^5 - 0.5,$$

$$\forall\, j \in \{1, 2, \cdots, T.l\}, \quad s \in \{T+w+1, \cdots, T+g\}, \ (24)$$

$$D_{pw}(s) = \sum_{j=1}^{T.l} n_{js}^5, \quad \forall\ s \in \{T+w+1, \cdots, T+g\}. \ (25)$$

3.5. Spare parts inventory control

In this section, we present the framework for spare parts inventory control based on the number of failures in each ICP as calculated in the previous section. The number of components to be sent to the refurbishing center is a proportion of the number of failed products.

As mentioned previously, failures are associated with under-warranty products until the wth ICP. From the $(w + 1)$th ICP to the $(T + w)$th ICP, failures are for both under- and out-of-warranty products. Finally, after the $(T + w + 1)$th ICP, failures are only associated with out-of-warranty products. Eqs. (26)-(28) calculate the number of component i to be sent to the refurbishing center in each ICP.

$$F_i(s) = \delta_i \times (\alpha_i.D_w(s)) \qquad \forall\ s \in \{1, \cdots, w\}, \quad (26)$$

$$F_i(s) = \delta_i \times (\alpha_i.(D_w(s) + D_{pw}(s)))$$

$$\forall\ s \in \{w + 1, \cdots, T + w\}, \qquad (27)$$

$$F_i(s) = \delta_i \times (\alpha_i.D_{pw}(s))$$

$$\forall\ s \in \{T + w + 1, \cdots, T + g - 1\}. \qquad (28)$$

In the refurbishing center, some percentage of the components are successfully refurbished and taken into account as as-good-as-new component inventory, while others are disposed. Eqs. (29) and (30) calculate the

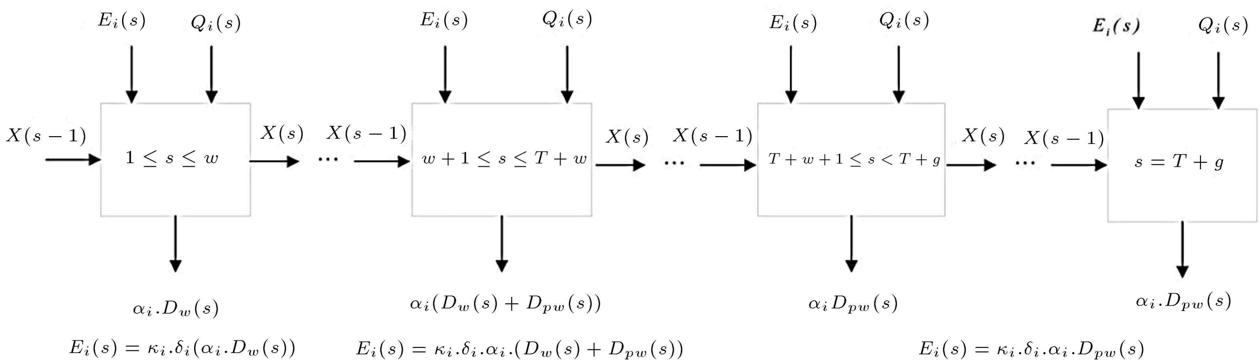


Figure 4. Spare part inventory system for under-warranty and out-of-warranty products.

numbers of component i sent for refurbishment and disposal, respectively.

$$E_i(s) = \kappa_i \times F_i(s) \qquad \forall\ s \in \{1, 2, \cdots, T+g\}, \quad (29)$$

$$V_i(s) = \left[\frac{(1-\delta_i)}{\delta_i} + (1-\kappa_i)\right] \times F_i(s)$$

$$\forall\ s \in \{1, 2, \cdots, T+g\}. \quad (30)$$

Figure 4 demonstrates the flow of spare parts inventory for component i in each ICP of the planning horizon. In each ICP, the on-hand inventory consists of the remaining inventory of the previous ICP and the amount of products manufactured and remanufactured minus the demand for the component in the current ICP.

Now, we have completed the task of calculating different components of the profit function besides the required inventory balance equations for different ICPs. The optimization problem is summarized as follows:

$$\max\ z = \sum_{j=1}^{l.T} (P_j - c).S(j, P_j, w)$$

$$+ \sum_{i=1}^{I} pc_i. \sum_{s=w+1}^{T+g} (\alpha_i D_{pw}(s))$$

$$+ \sum_{i=1}^{I} po_i.X_i(T+g) - \sum_{i=1}^{I} o_i. \sum_{s=1}^{T+g} E_i(s)$$

$$- \sum_{i=1}^{I} v_i. \sum_{s=1}^{T+g} V_i(s) - \sum_{i=1}^{I} h_i. \sum_{s=1}^{T+g} X_i(s)$$

$$- \sum_{i=1}^{I} c'_i. \sum_{s=1}^{T+g} Q_i(s), \quad (31)$$

$$P_{j-1} \ge P_j \qquad \forall\ j \in \{1, \cdots, (T+g) \times l\}, \quad (32)$$

$$P_{\min} \le p_j \le P_{\max} \qquad \forall\ j \in \{1, \cdots, (T+g) \times l\}, \quad (33)$$

$$W_{\min} \le w \le W_{\max}, \quad (34)$$

$$X_i(0) = 0 \qquad \forall\ i \in \{1, \cdots, I\}, \quad (35)$$

$$X_i(s) = X_i(s-1) - \alpha_i D_w(s) + E_i(s) + Q_i(s)$$

$$\forall\ i \in \{1, \cdots, I\}, \quad s \in \{1, \cdots, w\}, \quad (36)$$

$$X_i(s) = X_i(s-1) - \alpha_i(D_w(s) + D_{pw}(s)) + E_i(s) + Q_i(s)$$

$$\forall\ i \in \{1, \cdots, I\}, \quad s \in \{w+1, \cdots, T+w\}, \quad (37)$$

$$X_i(s) = X_i(s-1) - \alpha_i D_{pw}(s) + E_i(s) + Q_i(s)$$

$$\forall\ i \in \{1, \cdots, I\}, \quad s \in \{T+w+1, \cdots, T+w\}, \quad (38)$$

$$X_i(s), Q_i(s) \ge 0$$

$$\forall\ i \in \{1, \cdots, I\}, \quad s \in \{1, \cdots, T+w\}. \quad (39)$$

Eq. (35) states that the inventory of all components at the beginning of the planning horizon is zero. Eqs. (36)-(38) are inventory balance equations for different ICPs (see Figure 4). Finally, Eq. (39) ensures that no shortage is encountered in all ICPs.

4. Solution method

In this section, we propose an effective algorithm to solve the problem already described. The objective function to be minimized and the constraints are all nonlinear with respect to prices (P_j) and warranty length (w) variables. Once we fix the values of these variables, the remaining variables, which are ($Q_i(s)$), can be found via translating the reduced problem to a minimum cost network flow problem. Thus, we can effectively reduce the search to the price-warranty length space for finding good quality solutions to the problem.

With the above considerations and given the prices (in each PP) and warranty length, the amount of products sold in each PP can be calculated. Then,

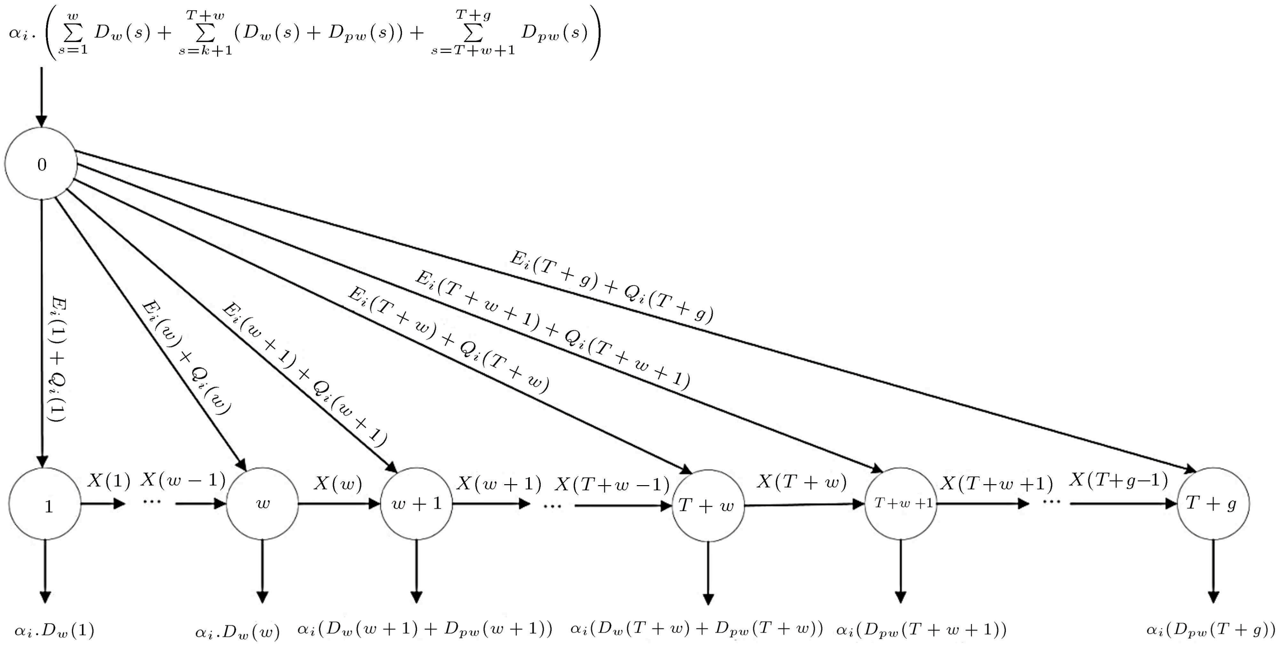


Figure 5. The min cost network flow problem for spare parts inventory control.

```
Begin
π := 0;
Establish a feasible flow x in the network;
Define the residual network G(x) and compute the kilter number of arcs;
While the network contains an out-of-kilter arc do
        Begin
        Select an out-of-kilter arc (p, q) in G(x);
        Define the length of each arc (i, j) in G(x) as max{0, c^π_ij};
        Let d(.) denotes the shortest path distances from nodes q to all other nodes in
        G(x) − {(p, q)} and let P denote a shortest path from node q to node p;
        Update π' = π(i) − d(i) for all i ∈ N;
        If c^π'_ij < 0 then
             Begin
             W = P ∪ {(p, q)};
             δ = min{r_ij : (I, j) ∈ W};
             Augment δ units of flow along W;
             Update x, G(x), and the reduced costs;
        End;
End.
```

Algorithm 1. The out-of-kilter algorithm.

the numbers of failed under- and out-of-warranty products in each ICP are calculated. In this way, the demand for spare parts in each ICP is determined. The remaining is a spare parts inventory control subproblem, which, as will be shown, can be modeled as a network problem. Figure 5 demonstrates the network counterpart, which can be solved by a minimum-cost network flow algorithm such as out-of-kilter. This method was first introduced by Fulkerson [28]. It works on both the primal problem (edges of the network) and the dual problem (nodes) in successive phases first to find a feasible solution and then, to optimize the problem. The pseudo code of the out-of-kilter is shown in Algorithm 1.

Now, we can develop an algorithm for the problem to scale first, the prices and warranty length with the aid of a search based algorithm, e.g., the recently proposed Optics Inspired Optimization (OIO), and then, the production amount for spare parts of each component, relevant to the given prices and warranty length, by optimally solving a minimum cost network flow problem. The latter step is taken to compute the fitness relevant to the individuals of OIO. For comparison, we also use the Improved Particle Swarm Optimization (IPSO). Our reason for using these algorithms comes from the point that Particle Swarm Optimization (PSO) is a classic and popular algorithm for solving optimization problems. On the other hand, to include a relatively newer and modern algorithm, we use the recently proposed OIO algorithm, which has proven effective and needs few parameters. Accordingly, we think that it may be useful to compare the results of an older algorithm like PSO with a newer one.

4.1. Solution representation and fitness function

To solve the problem with the aid of OIO, an individual is a vector of length $l.T + 1$, in which the first $l.T$ elements are prices sorted in descending order and the last element is the warranty length. All prices and warranty length should be generated between their upper and lower bounds. When computing the fitness function relevant to a given individual, i.e., the objective function value, first, decision variables relevant to the production amount for spare parts of each component in the sth ICP are set optimally via solving the relevant minimum-cost network flow problem for spare parts inventory control and then, the fitness value is calculated (see Figure 6).

4.2. The Optics Inspired Optimization (OIO) algorithm

OIO is an optics inspired population-based evolutionary algorithm that was first proposed by Husseinzadeh Kashan [29]. The algorithm assumes that a number of artificial light points (points in R^{n+1} whose mapping on R^n represents potential solutions to the problem) are sitting in front of an artificial wavy mirror reflecting their images. OIO treats the surface of the function to be optimized as the reflecting mirror composed of peaks and valleys. Each peak is treated as a convex reflective surface and each valley is treated as a concave reflective surface. In this way, the artificial ray glittering from the artificial light point is reflected back artificially by the function surface, given that the reflecting surface is a part of a peak or a part of a valley, and the artificial image point (a new point in R^{n+1}, which is mapped on R^n as a new solution in the search domain) is formed upright (toward the light point position in the search space) or inverted (outward the light point position in the search space). Recently, several studies have used this algorithm in problem optimization [30–33].

Figure 7 illustrates how the new solution is generated in OIO in the one-dimensional search space. In this figure, it is assumed that an artificial light point in the joint search and objective space (i.e., R^{n+1}) is in front of the function surface (mirror) in a particular distance from the vertex (values on the X-axis form the search/solution space and values on the $f(X)$-axis form the objective space; the set of all points in the $X - f(X)$-coordinate system forms the joint search and objective space). Using the mirror equations of physics, the artificial image is formed in the joint search and objective space. Then, the new solution is generated in the search space through mapping the artificial image position onto the search space. The procedure of generating new solutions is directly dependent on the reflecting part of the function surface (convex or concave) and the position of the artificial light point in the joint search as well as objective space. Figure 7 shows four different situations which may occur in generating new solutions.

The above process for generation of a new solution can be translated in an algorithmic manner as follows. For a given individual solution O in the population, a different solution F (vertex point) is selected randomly from the population. If F has a worse fitness value than O, it is assumed that the surface is convex and a new solution is generated upright somewhere toward O on the line connecting O and F (see Figure 7(a)). If F has a better fitness value than O, then it is assumed that the surface is concave and the new solution is generated upright toward (see Figure 7(b)) or inverted outward (see Figure 7(c) and (d)) O on the line connecting O and F in the search space.

Through the procedure of generating new solutions described conceptually in Figure 8, OIO is able to perform both exploration and exploitation during the search process. The exploration ability is achieved by adopting a larger jump in the solution space (see Figure 7(b) and (c)) while exploitation is performed by adopting a smaller jump over the base solutions (see Figure 7(a) and (d)). The detailed and ready-to-implement flowchart of OIO is shown in Figure 8. The notation adopted in Figure 8 is described as follows:

- $\vec{O}_j^t = [o_{j1}^t o_{j2}^t \cdots o_{jn}^t]_{1\times n}$ is the position of artificial light point j in the n-dimensional search space in iteration t (i.e., the jth solution in the population);
- $\vec{F}_j^t = [f_{j1}^t f_{j2}^t \cdots f_{jn}^t]_{1\times n}$ is a different point in the search space (i.e., an individual in the population) through which the artificial principal axis passes;

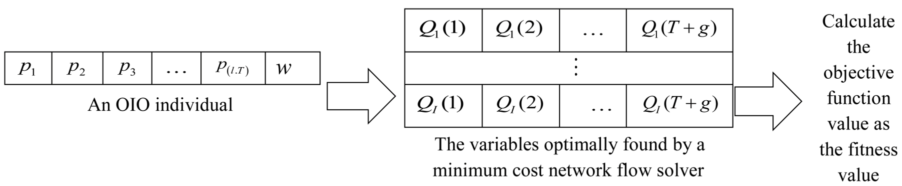
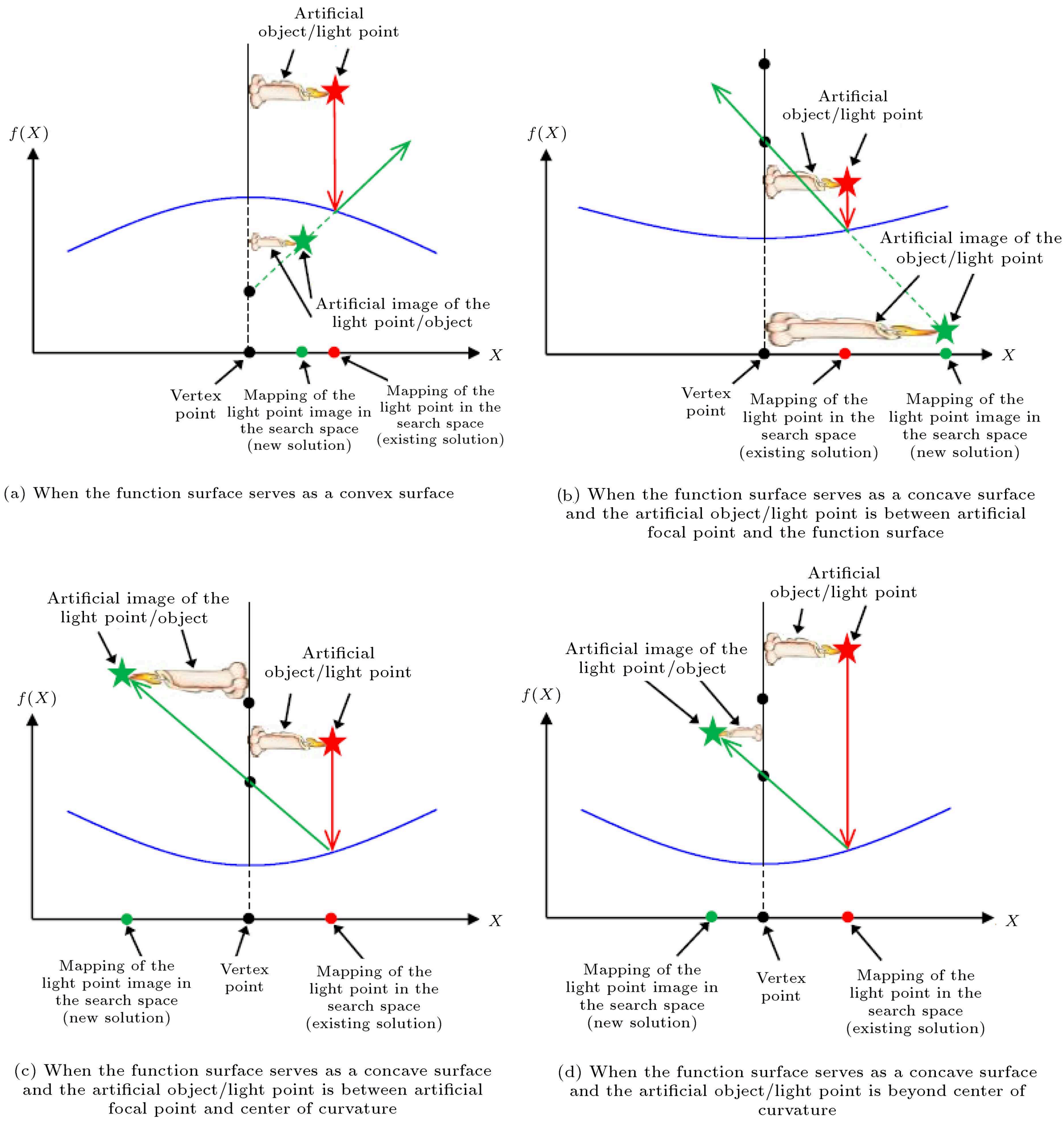


Figure 6. Solution representation.



(a) When the function surface serves as a convex surface

(b) When the function surface serves as a concave surface and the artificial object/light point is between artificial focal point and the function surface

(c) When the function surface serves as a concave surface and the artificial object/light point is between artificial focal point and center of curvature

(d) When the function surface serves as a concave surface and the artificial object/light point is beyond center of curvature

Figure 7. The idea behind generation of the new solutions in Optic Inspired Optimization (OIO).

- $\vec{I}_j^t = [i_{j1}^t i_{j2}^t \cdots i_{jn}^t]_{1\times n}$ is an image position of the artificial light point j in the search space in iteration t. The artificial image is formed by the artificial mirror whose principal axis passes through $\vec{F}_{i_k}^t$;

- s_{j,i_k}^t is the position of the artificial light point j (whose image is formed by the artificial mirror) on the function/objective axis (objective space) in iteration t. The position of artificial light point j in the joint search and objective space is thus given by the vector $[o_{j1}^t o_{j2}^t \cdots o_{jn}^t]$;

- p_{j,i_k}^t is the distance between the position of artificial light point j on the function/objective axis and the position of artificial mirror vertex on the function/objective axis in iteration t;

- q_{j,i_k}^t is the distance between the image position of the artificial light point j on the function/objective axis and the position of artificial mirror vertex on the function/objective axis in iteration t;

- $r_{i_k}^t$ is the radius of curvature of the artificial mirror whose center of curvature is on the principal axis, which passes through $\vec{F}_{i_k}^t$;

- $m_{i_k}^t$ is the position of the center of curvature on the function/objective axis (objective space);

- HO_{j,i_k}^t is the height of the artificial light point j from artificial principal axis in iteration t;

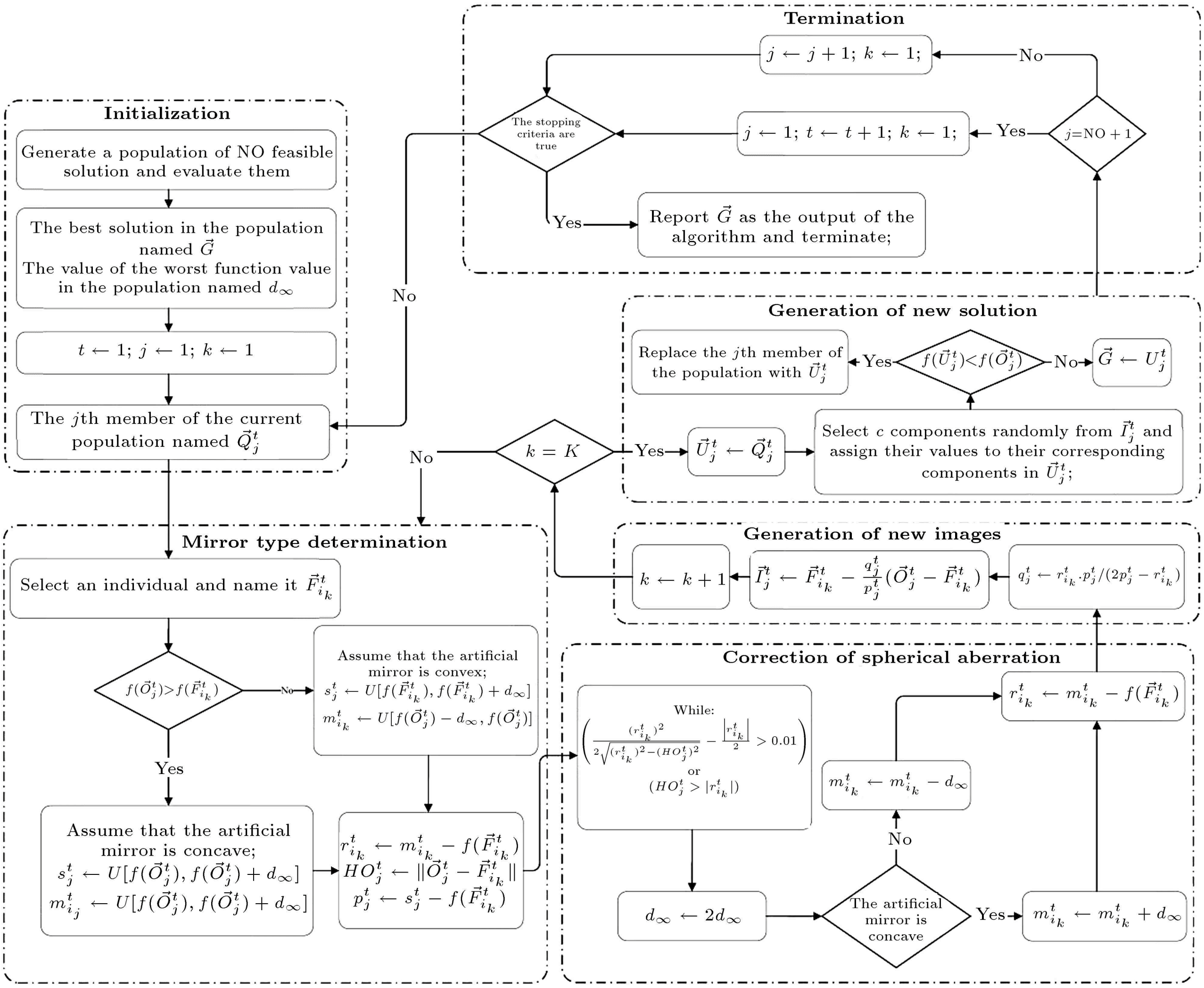


Figure 8. Flowchart of the Optic Inspired Optimization (OIO) algorithm.

- HI_{j,i_k}^t is the image height of the artificial light point j from artificial principal axis in iteration t;
- κ_{j,i_k}^t is the value of lateral aberration relevant to the artificial mirror which is reflecting the image of the artificial light point j in iteration t.

4.3. The Improved Particle Swarm Optimization (IPSO) algorithm

In the literature, many studies can be found that use algorithms based on PSO to solve similar problems [34–39]. Accordingly, we use an IPSO, first proposed by Jiang et al. [40], for comparison with the results achieved by the OIO algorithm.

PSO is a population-based metaheuristic algorithm proposed by Kennedy and Eberhart [41]. Its concept originates in the social behavior of swarms. A particle in the swarm starts from an initial position and moves in the search space according to the effects of two sources, namely the personal best (P_{best}) and global best (g_{best}). Specifically, velocity of each particle changes based on its distance from the best position (solution) it has achieved so far (i.e., p_{best}) as well as its distance from the best position obtained by the swarm (g_{best}). The equations for updating velocity and position of each particle are:

$$v_{i+1}=\omega\times v_i+C_1\times(P_{\text{best}}-x_i)+C_2\times(g_{\text{best}}-x_i), \quad (40)$$

$$x_{i+1} = x_i + v_{i+1}. \quad (41)$$

The acceleration constants C_1 and C_2 in Eq. (40) are those that control the effect of P_{best} and g_{best} positions on the velocity. On the other hand, ω is the inertia factor, which is reduced throughout the search. The calculated velocities can be at most $v_{\max}$.

Before utilizing the IPSO algorithm, a random population is selected, which is clustered into some sub-populations. Then, the algorithm is applied to these sub-populations. At certain points in time, the sub-populations are merged for sharing information and reclustered.

Steps of the IPSO algorithm are as follows:

Step 1: Choose $p \geq 1$, $m \geq 1$, where p is the number of sub-swarms and m is the number of particles in each sub-swarm. Set the sample $s = pm$ and then, calculate the objective function for each particle X_i.

Step 2: Sort the function values of particles in ascending order and put them in an array $E = \{X_i, f_i | i = 1, \cdots, s\}$.

Step 3: Partition E into p sub-swarms $A^1, A^2, \cdots, A^p$ such that:

$$A^k = \{X_j^k, f_j^k | X_j^k = X_{k+p(j-1)}, f_j^k = f_{k+p(j-1)},$$

$$j = 1, \cdots, m\}, \qquad k = 1, \cdots, p.$$

Step 4: Evolve each A^k by PSO.

Step 4.1: Determine the population size (q) and the maximum iteration (T).

Step 4.2: Select q particles $Y_1^k, \cdots, Y_q^k$ from A^k by this strategy in a way that the particles with better objective functions have more probability to be selected. Store them in $F^k = \{Y_i^k, V_i^k, u_i^k | i = 1, \cdots, q\}$, where V_i^k is the velocity for particle Y_1^k and u_i^k is the corresponding function value. Set G^k as the best individual of the whole swarm.

Step 4.3: Evaluate the function values of Y_i^k and P_i^k. If Y_i^k is better, then set $P_i^k = Y_i^k$. Evaluate the function values of Y_i^k and G^k; if Y_i^k is better, then set $G^k = Y_i^k$.

Step 4.4: Update the position and velocity of each particle according to Eqs. (40) and (41).

Step 5: Substitute $A^1, A^2, \cdots, A^p$ into E.

Step 6: If convergence criteria are satisfied, stop. Otherwise, go to Step 4.

5. Numerical example and sensitivity analysis

In order to observe and understand the patterns of optimal dynamics of key variables, we conduct numerical analysis for the LED 32″ that is produced by SANAM electronic company. SANAM electronic is one of the leader companies in the electronic industry of Iran that began producing color-television sets name-branded SANAM in 1993. The company offered the following information:

1. Based on the price skimming strategy, the price starts from \$280 (it is the maximum price) and decreases to \$200;
2. The unit production cost for LED 32″ is about \$150;
3. According to historical data and opinions of the experts, mainboard and panel are two key components. Production cost, refurbishing cost, disposal cost, and holding cost for mainboard are about \$28 (per unit), \$17 (per unit), \$2 (per unit), and \$2 (per unit/month), respectively, and these costs for panel are \$18 (per unit), \$8 (per unit), \$1 (per unit), and \$1 (per unit/month), respectively;
4. Selling prices for mainboard and panel for out-of-warranty products are \$41 and \$26, respectively. Other parameters are shown in Table 3.

Table 3. Parameter values for LED 32″.

I	l	$W_{\min}$	$W_{\max}$	po_1	po_2	D_0	U
2	1	12	27	\$11	\$5	500	2000
ρ_w	ρ_{pw}	α_1	α_2	δ_1	δ_2	κ_1	κ_2
0.9	0.8	0.40	0.25	0.4	0.5	6	10

In order to solve and analyze the case study problem, parameters of the OIO algorithm and IPSO algorithm are shown in Tables 4 and 5, respectively.

After solving the problem, there are important dynamic relationships that can be derived from the numerical analysis. We first look into the relationship of T and g with optimal warranty length and company profit. We also present the price trends, the production amount for the LED 32″ products, and the production amount for spare parts. Finally, the impact of the failure rate of products on total profit and warranty length is shown.

5.1. *The optimal profit and warranty length for given T and g*

Table 6 shows optimal profits for various Ts and gs obtained by two algorithms, namely OIO and PSO.

According to historical data, the failure rate of products is 0.07 and the price and warranty coefficients (k_1 and k_2) are estimated by the company at 6 and 10. Currently, the company offers two-year warranty for all products. We show that for each combination of T and g, the company must choose different warranty periods. We ran the proposed algorithm 10 times for each combination of T and g (720 problems in total) using MATLAB on a Pentium 4 computer with 8 GB RAM and Corei7 3.61 GHz CPU. The results have been reported in Table 3. The best, the worst, and the mean objective function obtained as well as standard deviation and average time are reported in this table.

Table 6 shows that as life cycle of the products and guarantee period for spare parts availability increase, the profit of the manufacturer increases. However, this does not necessarily mean that the manufacturer can always select a longer life cycle or guarantee period, because the competition situation is very complex and change in factors such as technology leads to changes in customer interest, forcing the company to shift to new products.

Table 4. Optic Inspired Optimization (OIO) parameters.

OIO algorithm	**Maximum number of function evaluations**	**Number of light points (population size)**
	2000	30

Table 5. Improved Particle Swarm Optimization (IPSO) parameters.

IPSO algorithm	**Maximum number of function evaluations**	**Population size**	**Acceleration constants (c_1 and c_2)**	**Inertia factor (ω^*)**	**Number of sub-swarms (p)**
	2000	20	0.2	Linearly decreasing from 0.9 to 0.4	2

*: The parameter ω is suggested by Shi and Eberhart [46].

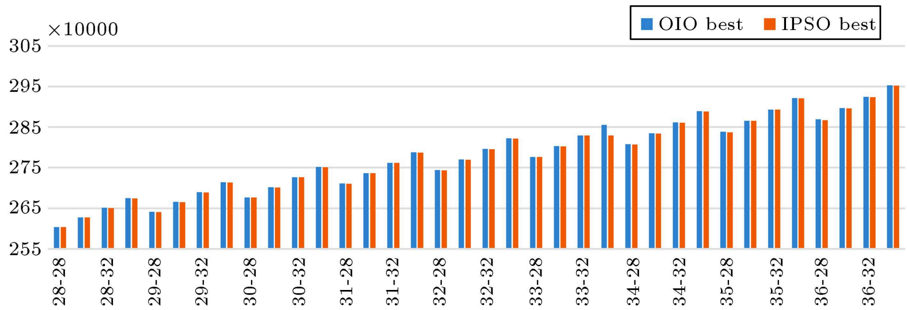


Figure 9. Comparison of Optic Inspired Optimization (OIO) and Particle Swarm Optimization (PSO) best solutions.

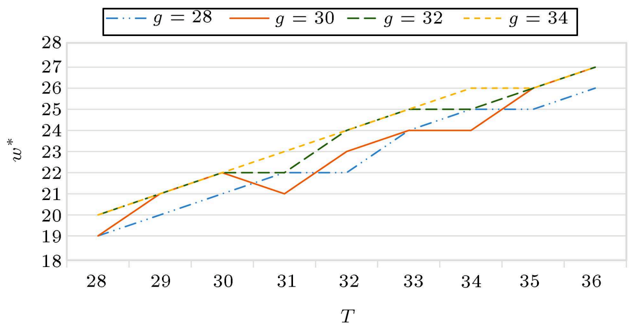


Figure 10. Warranty length comparison for different Ts and gs.

As Figure 9 demonstrates, the best solutions with the OIO algorithm are greater than or equal to the best solutions obtained by the IPSO algorithm, which shows the better performance of OIO in terms of the objective function (profit). Furthermore, it shows the impact of g and T changes on the profit of the manufacturer.

Figure 10 compares the optimal warranty periods for different life cycles and guarantee periods. It illustrates that the optimal warranty period for $g = 34$ is greater than or equal to other cases. On the other hand, longer life cycles have greater warranty periods. Thus, it can be an important achievement for SANAM company, because they often considered a fixed warranty period for products even if life cycles of products differed.

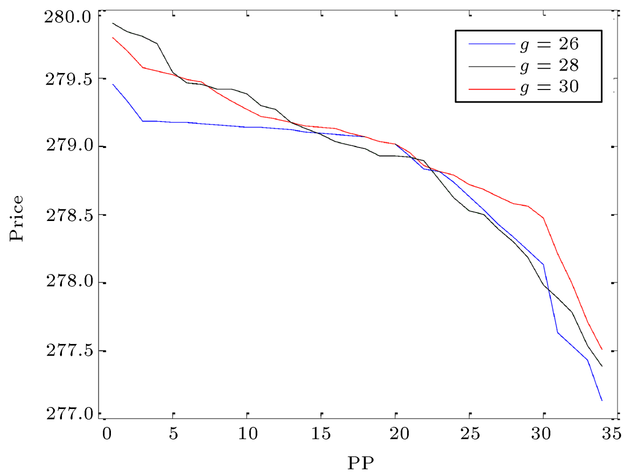


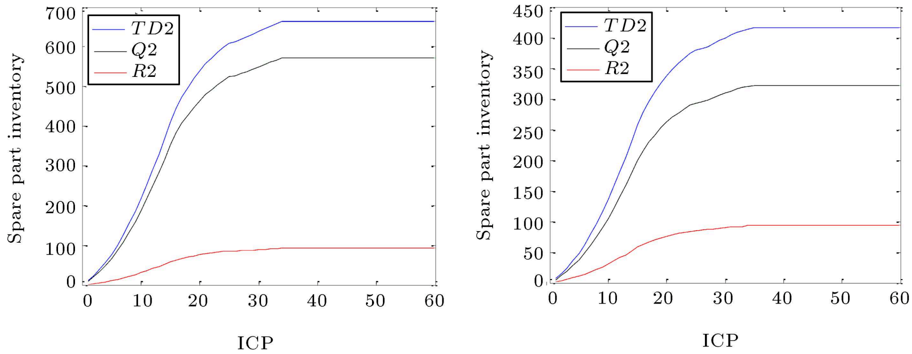
Figure 11. Price trend for $T = 32$.

According to the current policy of the company, life cycle for LED 32″ is 32 months ($T = 32$). Accordingly, the optimal prices for this value of T and different gs are depicted in Figure 11.

It is worth noting that in the final PPs, the slopes sharply increase. This is due to the fact that products produced in these periods have lower chance of failing as out-of-warranty products; Therefore, it would be

Table 6. Optimal profits with Optic Inspired Optimization (OIO) and Particle Swarm Optimization (PSO) algorithm.

		OIO & out-of-kilter					IPSO & out-of-kilter				
T	***g***	**Best sol**	**Worst sol**	**Mean**	**std**	**Time**	**Best sol**	**Worst sol**	**Mean**	**std**	**Time**
28	28	2603760	2603029	2603480	287	1623	2603682	2580047	2598700	10429	1753
	30	2627564	2626634	2627318	386	1618	2627179	2571533	2605916	22678	1805
	32	2651245	2650189	2650949	442	1719	2650675	2650159	2650453	193	1853
	34	2675181	2674469	2674902	275	1634	2674613	2654458	2669932	8680	1928
29	28	2641285	2640862	2640997	172	1647	2640814	2608935	2633207	13733	1762
	30	2665486	2664603	2665198	356	1763	2665099	2646785	2660982	7950	1846
	32	2689975	2689284	2689706	256	1770	2689225	2632229	2669506	26224	1906
	34	2714284	2713948	2714091	135	1781	2713822	2688432	2708214	11072	1962
30	28	2676853	2676370	2676623	216	1786	2676244	2636554	2667915	17537	1786
	30	2701659	2700194	2701061	565	1697	2701026	2629962	2669787	26633	1830
	32	2726729	2724816	2726130	777	1601	2726155	2689984	2712019	1494	1921
	34	2751747	2750602	2751467	486	1431	2751107	2734382	2747066	7165	2001
31	28	2711353	2710218	2711045	468	1539	2710749	2659509	2697099	2217	1829
	30	2736510	2734241	2735751	874	1755	2736189	2735491	2735957	285	1880
	32	2762190	2760639	2761532	699	1669	2761697	2716043	2734046	1884	1945
	34	2787796	2786258	2787153	788	1672	2787204	2717752	2763231	3181	2004
32	28	2744349	2742953	2743928	596	1607	2743561	2680216	2722400	2582	1848
	30	2770476	2769037	2769690	561	1721	2769300	2733869	2755452	1694	1932
	32	2796483	2795518	2796102	398	1644	2795773	2708293	2748868	3675	1976
	34	2822631	2821717	2822331	354	1760	2821783	2726735	2789165	3969	2032
33	28	2776137	2775296	2775878	342	1637	2776298	2718769	2753725	2994	1862
	30	2802966	2800939	2802436	845	1600	2802147	2768597	2786827	1659	1905
	32	2829426	2828731	2829023	295	1619	2829125	2781050	2814746	2122	1977
	34	2855426	2828731	2829023	294	1795	2835500	2799585	2838047	2290	1976
34	28	2808021	2807000	2807586	454	1656	2807499	2775615	2796023	1508	1949
	30	2834658	2833066	2834083	599	1676	2834238	2801037	2818037	1190	2015
	32	2861885	2860321	2861026	590	1695	2860977	2786200	2837268	3004	2080
	34	2888951	2887356	2888315	622	1714	2888359	2850460	2874800	1862	2146
35	28	2838657	2837403	2838101	457	1640	2837426	2687972	2791282	6120	1977
	30	2865837	2864979	2865373	335	1752	2865189	2832093	2856753	1420	2067
	32	2893384	2892833	2893193	212	1771	2893180	2843405	2882782	2201	2101
	34	2921338	2919845	2920777	634	1690	2920911	2878023	2905222	2141	2179
36	28	2868936	2867877	2868299	431	1808	2867355	2827115	2848968	1671	1962
	30	2896872	2895765	2896364	406	1827	2895943	2821879	2874628	3027	2011
	32	2924787	2923771	2924311	396	1750	2924194	2890643	2917349	1493	2135
	34	2952704	2951627	2952265	404	1820	2952171	2908783	2922819	1691	1861



(a) Spare part inventory for component 1 (b) Spare part inventory for component 2

Figure 12. Spare part inventory.

Table 7. Relationship between failure rate, warranty length, and total profit.

Rate of failure of products	w^*	Total profit $\times 10^3$
0.04	27	2985.041
0.045	27	2958.401
0.05	26	2933.135
0.055	26	2908.565
0.06	25	2883.826
0.065	24	2857.136
0.07	22	2834.658
0.075	20	2820.776
0.08	16	2830.339
0.085	13	2842.540
0.09	12	2855.491
0.095	12	2867.065
0.1	12	2878.431
0.12	12	2925.103
0.13	12	2943.647

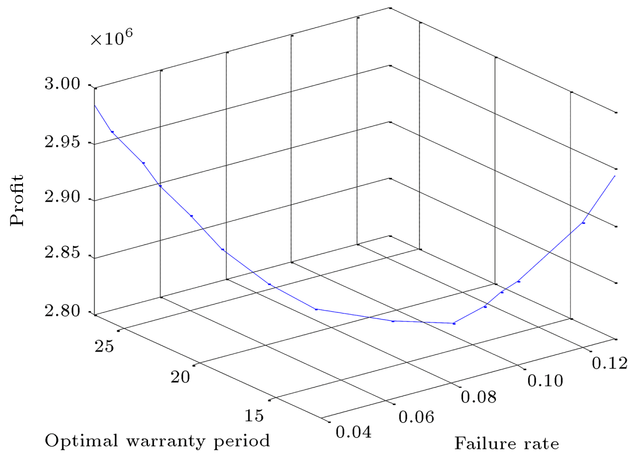


Figure 13. Failure rate analysis.

better for the manufacturer to sell more products by decreasing prices.

Total demands for spare parts are depicted in Figure 12, where $TD1$ is for component 1 (Figure 11(a)) and $TD2$ is for component 2 (Figure 11(b)). Demand is met from two sources, namely refurbishing ($R1$ and $R2$) and manufacturing. $Q1$ and $Q2$ represent the amounts of product that should be produced. The increasing trend in these values is due to the cumulative demand for all products until a given time.

5.2. Sensitivity analysis and changes in failure rate of products

In this section, we focus on how changes in failure rate (λ) affect optimal warranty period and total profit. In order to observe the dynamics more clearly, we need to use fixed product life cycle (T) and spare parts availability period (g). For the ensuing analysis, we consider $T = 32$ and $g = 30$. Table 7 shows the optimal values of warranty length and profit for various failure rates.

As it can be observed in Figure 13, profits of the manufacturer increase when failure rate increases or decreases. When failure rate decreases, warranty costs decrease. Thus, the manufacturer can propose higher warranty lengths and sell more products. On the other hand, with larger values of failure rate, the manufacturer proposes smaller warranty lengths, because shorter warranty length leads to lower warranty cost and more profit from selling spare parts for out-of-warranty products. It should be noted that shorter warranty length and higher failure rate will cause customer dissatisfaction, which weaken competitiveness of the company and lessen its market share.

6. Conclusion

The main purpose of the current study was to develop a new nonlinear model to integrate and optimize price of products, warranty length, and spare part inventory control decisions. Since the sale of spare parts for out-of-warranty products has sensible effects on the profit of a company, a unique capability has been proposed in the presented model to calculate the demand for out-of-warranty spare parts and optimize spare part inventory decisions, which were not considered in the previous studies. In order to solve the model, a new optimization approach was proposed that hybridized the metaheuristic algorithm with a minimum cost network flow optimizer.

We solved the model with real data for LED 32″ by two types of algorithm. The first one was a combination of Optic Inspired Optimization (OIO) and MCNFP and the second one was a combination of Improved Particle Swarm Optimization (IPSO) and MCNFP. Experimental analyses showed that if the company decided to set a longer life cycle for the products, it was more profitable to propose longer warranty lengths. Additionally, it was recommended that the company decrease the prices more sharply in the final periods of the life cycle than in the initial periods in order to benefit from increased sale in the final periods. Finally, we found that failure rate of products was inversely proportional to warranty length, since increase in failure rate would lead to an increase in warranty costs, making it reasonable to decrease the warranty length.

As a future direction for research, it is interesting to conduct pricing for selling spare parts for out-of-warranty products, since this will lead to modeling the real-world conditions more accurately. The model can also be extended for considering two-dimensional warranty, which will make it applicable to other fields such as automobile industry. Other considerations such as shortage and lost sales can also be incorporated in the inventory control problem, while minimizing shortage can be taken into account as another important objective along with maximizing profit.

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