A Holistic Day-ahead Distributed Energy Management Approach: Equilibrium Selection for Customers’ Game

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Abstract- In this paper, a new holistic distributed day-ahead energy management approach with desired equilibrium selection capability in a smart distribution grid is proposed. The interaction between customers and the distribution company is modeled as a single-leader multiple-follower Stackelberg game. The interaction among customers is modeled as a non-cooperative generalized Nash game because they meet a common constraint. Customers hold the average of the aggregate load in the appropriate domain to reshape it and improve the Load Factor. The strategy of the distribution company is day-ahead energy pricing obtained through maximizing its profit which is formulated as a stochastic conditional value at risk optimization to consider the uncertainty of the price of electricity in the wholesale market. Customers’ strategies are based on hourly consumption of deferrable loads and scheduled charge/discharge rates of energy storage devices in response to price. It is proved that the generalized Nash game has multiple equilibria; hence, the distributed proximal Tikhonov regularization algorithm is proposed here to achieve the desired equilibrium. The simulation results validate the performance of the proposed algorithm with 31.46% increase in the Load Factor besides 45.89 % and 14.23 % reduction in the maximum aggregate demand and aggregate billing cost, respectively.

Index terms- smart grid, energy management, generalized Nash game, load factor, proximal Tikhonov regularization algorithm.

1. Introduction

The significant growth in the electricity demand, limited fossil fuel resources and the increase in greenhouse gasses led the researchers to run studies on new demand-side management (DSM) programs. DSM improves the system’s energy consuming at the consumer’s side [1]. DSM includes enhancing energy efficiency [2, 3], smart energy pricing [4], demand response programs [5], optimal deployment of distributed energy resources [6] and energy storage devices at the consumer’s side [7].

Due to the smart control, smart measurement, and two-way communication capability of the smart grid, DSM can be employed effectively. These advantages allow an intelligent interaction between different entities of distribution grid by providing and exchanging necessary information. This network develops smart pricing schemes, particularly real-time pricing policy, in implementing DSM in an effective and efficient manner.

DSM can be employed in the centralized and decentralized approaches. In the centralized approach, a central controller,
basically at utility side, should collect all necessary information. This information includes many parameters and constraints from all agents of the smart grid to calculate optimal consumption schedule of customers. This approach is often impractical due to enormous computation and communication saturation in addition to privacy disclosure. To overcome these drawbacks, decentralized DSM methods are developed and adopted.

Many researchers have proposed distributed automated algorithmic DSM programs for implementing in the smart distribution grid. Most of them employed game theory as an analytical tool to model the optimization problem like the methods used in Atzeni et al. [6], Mohsenian-Rad et al. [8], Chen et al. [9], Deng et al. [10], Fadlullah et al. [11], Nguyen et al. [12] and Yaagoubi and Mouftah [13]. The major part of these studies is the customers who are price anticipating, locally run an iterative best-response algorithm to solve a distributed optimization problem. The result of these algorithms reveals the optimal energy consumption profile of every customer.

The implementation of efficient DSM programs depends on consumer and utility sides; thus, in the other group of papers, the customers and distribution company are decision makers in the DSM game. These researchers model the interaction of the customers and utility company as a Stackelberg game. Employing the Stackelberg game for modeling the interaction can be seen in Yang et al. [4], Chai et al. [5], Jia et al. [7], Soliman et al. [14] and Maharjan et al. [15]. In such approach, customers are price takers who accept the price as a fixed parameter.

In most studies, the energy management problem is modeled as a game with a unique equilibrium; but, depending on the structure of the optimization problem and the players’ interaction and behavior, the energy management game might have multiple equilibria. In these models, applying the distributed best-response algorithm is not responsive. However, recently, a promising advancement on solution computation of monotone games is developed by Scutari et al. [16, 17] that has motivated us to develop a monotone energy management approach in a smart distribution grid.

In this article, a new holistic distributed day-ahead energy scheduling for a smart distribution grid is proposed. Our proposed energy management mechanism addresses the interaction between utility grid and smart customers in a local distribution electricity market. Distribution company (DISCO) formulates and solves a conditional value at risk (CVaR) optimization-based problem to obtain the optimal day-ahead hourly price of energy in the presence of uncertainty of the electricity price in the wholesale market. This optimal price is announced to residential customers through a two-way communication facility.

The price taker customers are equipped with an energy management system (EMS) that receives the price and minimizes customer’s electricity bill while maximizing customers’ utility. Moreover, EMSs try to meet the shared constraint together which improves the appropriate shaping of the aggregate load, efficient utilization of the grid and decreasing the operation grid cost. Here, customers limit the average of the aggregate load (AAL) in the specified domain; therefore, the load factor (LF) can
be settled in an appropriate range. Under such circumstance, they satisfy a global constraint that couples the strategy space of customers to one another.

This holistic energy management is modeled as a single-leader multiple-follower Stackelberg game, where DISCO acts as the leader and customers are the followers. The interaction among customers can be modeled as a non-cooperative generalized Nash (GN) game [16] because of their self-interest behavior and shared constraint. The existence of a solution for the monotone game among customers, the GN equilibrium, is assessed here. A distributed proximal Tikhonov regularization algorithm (PTRA) is applied to reach the GN equilibrium, the essential section of this proposed algorithm for achieving the Stackelberg equilibrium (SE). The convergence conditions of this newly proposed energy management algorithm are subjected to the variational inequality (VI) theory. This algorithm selects the equilibrium which decreases the maximum aggregate demand and improves the LF at the global constraint among multiple equilibria of the GN game.

This paper is organized as follows. In Section 2, the model of the system is presented and the optimization problem is formulated. Then, the proper game framework which captures the energy management interaction, the game solution and its existence are introduced and discussed in Section 3. The distributed algorithm for achieving SE is proposed in Section 4. In Section 5, the implementation issues of the proposed energy management mechanism are discussed. Numerical results and performance evaluation of the proposed energy management algorithm are presented in Section 6. Finally, Section 7 concludes the paper.

2. System Modeling and Formulation

An automatic day-ahead energy management approach in a smart distribution system consisting of DISCO and its smart customers is studied. Every customer assumed to have renewable energy resources, elastic and inelastic loads and energy storage devices. The set of smart homes and the set of operation time slots are represented by \( \mathcal{N} \triangleq \{1,...,N\} \) and \( \mathcal{T} \triangleq \{1,...,T\} \), respectively, where it is assumed that \( T = 24 \).

Every customer is equipped with an EMS connected to both the electrical feeder and communication link as shown in Fig. 1. This EMS could share real-time information through bidirectional communications with one another and DISCO. EMS schedules energy consumption of a smart home in response to the price information.

DISCO sends the hourly price to its customers. Based on this price vector, every EMS decides on its daily load profile, \( I_n = [I^1_n,...,I^T_n] \), through storage devices and flexible appliances scheduling. Next, every EMS sends the load profile back to DISCO; consequently, DISCO will compute the new optimal price again based on the current best-response strategy of customers. This procedure is repeated until an equilibrium is achieved.
In this smart distribution grid, it is assumed that the customers agree with DISCO on meeting the global constraint, AAL, voluntarily or through incentive contract. By adjusting the AAL parameter, LF increases and the proper aggregate load shaping could be achieved. LF is calculated as follows

\[
LF = \frac{L_{ave}}{L_{peak}} = \frac{\sum_{t \in T} l'}{\max_{t \in T} l'},
\]

where \( l' = \sum_{n \in N} l'_n \) is the aggregate load of the customers at the time slot \( t \). It is also assumed that every customer follows its committed day-ahead consumption profile strictly. Following, we address the objective models for DISCO and smart homes.

### 2.1. DISCO’s Objective Model

DISCO must provide electricity demand of its clients from the wholesale market in all operations time horizons. DISCO could learn the demand behavior of its customers through different demand response programs with the help of amenities and facilities along with the smart grid.

DISCO predicts day-ahead energy prices \( \pi \) in the wholesale day-ahead market based on the historical data. Next, by maximizing its profit, DISCO derives the optimal day-ahead sale price \( \rho \) and sends it to its customers; accordingly, DISCO is exposed to the following optimization problem

\[
\begin{align*}
\max_{\rho \in \mathcal{P}} & \quad f_D(\rho, l) = (1-\mu) \sum_{t \in T} \mathbb{E}\{(\rho' - \pi')l'\} + \mu \text{CVaR}_\alpha \left\{ \sum_{t \in T} (\rho' - \pi')l' \right\}, \\
\mathcal{P} & \triangleq \left\{ (\rho')_{t \in T} : \rho'_{\min} \leq \rho' \leq \rho'_{\max}, \frac{1}{T} \sum_{t \in T} \rho' \leq \rho_{ave} \right\},
\end{align*}
\]

where \( \text{CVaR}_\alpha \) is the expected profit in the \((1-\alpha)\times100\%\) worst scenarios and \( \alpha \in [0,1] \) is a confidence level. \( \mu \) is a weighting factor that determines how much DISCO is a risk-neutral or risk-averse agent. In fact, DISCO uses CVaR optimization framework to handle the uncertainty in the prediction of the day-ahead wholesale market electricity prices and guarantees a certain amount of profit for itself.

DISCO chooses an optimal price from the strategy set \( \mathcal{P} \) based on the customers’ best-response. Due to the grid configuration limitations, the load should not exceed a certain amount and the minimum day-ahead price \( \rho'_{\min} \) should be specified. Furthermore, the price should not be set too high until the minimum load \( l'_{\min} \) is met. This fact indicates that the price is restricted by an upper bound which is \( \rho'_{\max} \). To alleviate the market power of DISCO, an upper limit \( \rho_{ave} \) is forced on the average price by the system regulator. With such a scheme, the customers’ consumption behavior and DISCO activity could be
Eq. (2) is difficult to evaluate in the continuous profit distribution. To deal with this problem, Monte Carlo Sampling is used to draw K i.i.d. samples from the profit distribution so that Eq. (2) can be rewritten in a tractable form as

\[
\begin{align*}
\text{Max} \quad f_{\rho}(\rho, I) &= \sum_{t \in T} \left[ (1 - \mu) \left\{ \left( \rho' - \frac{1}{K} \sum_{t' = 1}^{K} (\pi_{t'}') \right) I' \right\} + \mu \left\{ c' - \frac{1}{1 - \alpha} \sum_{t' = 1}^{K} z_{t'}' \right\} \right], \\
\text{S.t.:} \quad &\quad \rho \in \mathcal{P}, \quad \mathcal{P} \triangleq \left\{ (\rho)^{\top}_{t} : \rho_{\text{min}}' \leq \rho' \leq \rho_{\text{max}}', \frac{1}{T} \sum_{t \in T} \rho' \leq \rho_{\text{ave}} \right\}, \\
&\quad z_{t}'' \geq \sum_{t \in T} c' - (\rho' - \pi_{t}'), \quad z_{t}'' \geq 0, \quad k = 1, ..., K,
\end{align*}
\]

where \( K \) denotes the number of random day-ahead price in the wholesale market. \( \pi_{t}' \) is the realization of the day-ahead wholesale price under scenario \( k \) at the time slot \( t \). Additionally, \( z_{t}'' \) and \( c' \) are auxiliary variables.

2.2. Smart Home Model

A smart residential home has renewable energy resources, responsive loads, and energy storage devices which are described individually following.

2.2.1 Renewable Energy Resource

Every customer utilizes a renewable energy resource such as the wind or solar energy that the generation of which follows the stochastic process. Here, it is assumed that every customer can predict its renewable energy production vector \( P_n \) on the next operation day. For each renewable resource, \( 0 \leq P_n' \leq P_{n,\text{max}} \), where \( P_n' \) is the energy production from the renewable energy resource of the customer \( n \) at time slot \( t \) which always is smaller than nominal capacity \( P_{n,\text{max}} \).

2.2.2 Responsive Loads

The consumption of flexible loads could be shifted to off-peak hours, where the energy price is low. These loads could be supplied either by distribution grid or by customer’s own storage devices and their renewable energy resources. For every customer, \( A_n \) is the set of elastic home appliances. The consumption profile of each \( a_n \in A_n \) is \( D_{n,a_n} = [d_{n,a_n}^{'1}, ..., d_{n,a_n}^{'I}] \), where \( d_{n,a_n}^{i} \) is the hourly scheduled energy consumption of appliance \( a_n \) at day slot \( t \) by EMS at time slot \( t \). Here, it is assumed that every customer knows his/her total daily energy consumption, \( E_{n,a_n} \), of its appliance in advance as considered in [8-12]. The \( E_{n,a_n} \) must be available during specified time interval, \( T_{n,a_n} \triangleq \{ \theta_{n,a_n}, ..., \varphi_{n,a_n} \} \), indicating that the appliance can perform its task on
time. The value of $d_{n,a_t}^t$ is limited to the minimum and maximum levels in $t \in T_{n,a_t}$, i.e., $d_{n,a_t,min}^t$ and $d_{n,a_t,max}^t$, thus, the following constraints should be considered for each appliance

$$\sum_{t \in T_{n,a_t}} d_{n,a_t}^t = E_{n,a_t}, \quad d_{n,a_t,min}^t \leq d_{n,a_t}^t \leq d_{n,a_t,max}^t, \quad \forall t \in T_{n,a_t},$$

and

$$d_{n,a_t}^t = 0, \quad \forall t \notin T_{n,a_t}.$$  \hspace{1cm} (4)

At this stage, by introducing $D_n = (d_n^t)^T_{t=1}$ as the energy consumption vector of elastic loads, the feasible strategy set $\mathcal{D}_n$ could be defined for responsive loads as

$$\mathcal{D}_n \triangleq \{(d_n^t)^T_{t=1} \mid \text{Constraints (4), (5)}\},$$

$\mathcal{D}_n \triangleq \{(d_n^t)^T_{t=1} \mid d_n^t = (d_{n,1}^t, \ldots, d_{n,K}^t); d_{n,a_t}^t \in \mathcal{D}_{n,a_t}\}$.  \hspace{1cm} (6)

2.2.3 Energy Storage Model

In this system, storage devices of the customer $n$ are charged through distribution grid at low price period and are discharged for supplying customer’s appliances when the energy price is high. Let $b_n^t$ is the charging/discharging strategy of the customer $n$ for its energy storages at time slot $t$. When $b_n^t < 0$, the storage is discharged for the internal appliances, otherwise, it is charged from the grid. In such a case, the daily scheduled vector of storage devices can be written as $B_n = [b_n^1, \ldots, b_n^T]$. For reliable and efficient performance of the storage devices; the $b_n^t$ should lie in a specified interval

$$-b_{n,max} \leq b_n^t \leq b_{n,max},$$

where $b_{n,max}$ is the maximum charging/discharging rate of storage devices. The $\text{Soc}_n^t$ parameter denotes the state of charge of storage devices at the beginning of time slot $t$ that depends not only on $b_n^t$ but also on scheduling strategy on previous time slots. The $\text{Soc}_n^t$ should be smaller than energy storage devices’ maximum capacity ($E_{n,h_{max}}$) of the customer $n$ for all the time slots that expressed as

$$0 \leq \text{Soc}_n^t + \sum_{j=1}^{t} b_n^j \leq E_{n,h_{max}}, \quad \forall t \in T.$$  \hspace{1cm} (7)

The assumption is that every customer knows its desired charge level at the end of scheduling time horizon. $\text{Soc}_n^T$; therefore,
the total daily energy requirement, $E^b_n$, should be provided for charging the storage devices according to Eq.(9)

$$E^b_n = \text{Soc}^T_n - \text{Soc}^0_n, \quad \sum_{i=1}^T b'_n = E^b_n. \tag{9}$$

Consequently, the feasible strategy set of the storage devices is

$$\mathcal{B}_n = \{(b'_n)^T \mid \text{Constraints(7),(8),(9)}\}. \tag{10}$$

2.2.4 Customer’s Objective Model

The customer $n$ should purchase load demand, $l'_n = D_{n,\text{inl}} + b'_n + \sum_{a \in A_n} d'_{n,a} - P_{n}'$, from the grid on each time slot, where $D_{n,\text{inl}}$ indicates the consumption of customer’s inelastic loads at time slot $t$. It is assumed that customers cannot inject energy into the grid and the load demand cannot exceed a certain value, $l_{n,\text{max}}$. At this phase, the feasible strategy set of every customer is presented as

$$\mathcal{L}_n = \{(d'_n, b'_n)^T \mid d'_n \in \mathcal{D}_n, b'_n \in \mathcal{B}_n, 0 \leq l'_n \leq l_{n,\text{max}}\}. \tag{11}$$

In this model, customers want to maximize their utility while minimizing their electrical energy bill. As previously noted, customers may collaborate in AAL adjusting program that is beneficial for both DISCO and customers; thus, the customers in addition to holding their individual constraints, together adjust the AAL global constraint. Hence, the objective function of each customer is formulated as

$$\text{Min}_{l_n,\rho_n} f_n(l_n, \rho) = \sum_{i=1}^T \mathbb{E}\{\rho l'_n - U'_n\},$$

$$\mathcal{L}_n = \{(l'_n)^T \mid l'_n \in \mathcal{L}, l_{\text{ave}} \leq L_{\text{ave}}(l) \leq L_{\text{ave}}^\text{max}\}, \tag{12}$$

where $L_{\text{ave}}^\text{min}$ and $L_{\text{ave}}^\text{max}$ are the lower and upper thresholds of AAL, respectively. These values should be set in a manner that the aggregate load profile becomes smoother. The AAL constraint can be broken into two constraints of: $L_{\text{ave}}^\text{min} - L_{\text{ave}} \leq 0$ and $L_{\text{ave}}^\text{max} - L_{\text{ave}} \leq 0$, which are reduced in $L_{\text{ave}}(l) \leq 0$ form.

The $U'_n$ is customer $n$’s utility function at time slot $t$. The customer satisfaction level on electricity consumption could be quantified as a utility function. In this article, the quadratic utility function is applied in modeling the customers’ preference that is used in [5, 10]. This function captures the required characteristics for a utility function like non-decreasing property and non-increasing marginal benefit. This function for the customer $n$ is
where $\omega_n^l$ and $\kappa_n^l$ are the predetermined variables and vary among customers. By the proper setting of these parameters, it is possible to model the value of electrical energy for each customer.

3. Energy Management Game

In this holistic energy management approach, the interactions of different agents are divided into two parts: a) the interaction between DISCO and customers b) customers’ interaction due to the common constraint. In the following, the proper games that capture these interactions are introduced; besides, the games’ solutions and their properties are discussed.

3.1. Stackelberg Game: DISCO and Customers Interaction

In the proposed energy management approach: first, DISCO decides on optimal day-ahead energy price by considering the customers’ reaction (i.e., their consumption), next, EMS units provide scheduling for customers’ consumption based on the received optimal price. Thus, the interaction between DISCO and customers is modeled as a Stackelberg game.

**Definition 1:** The Stackelberg, day-ahead energy management game, $\mathcal{G}$, can be described through the tuple $\mathcal{G} = \{\mathcal{N} \cup \{\text{DISCO}\}, \mathcal{P}, \{\mathcal{L}_n\}_{n \in \mathcal{N}}, f_D, \{f_n\}_{n \in \mathcal{N}}\}$, where

- $\mathcal{N} \cup \{\text{DISCO}\}$ is the set of players in the Stackelberg game, where DISCO is the leader and $\mathcal{N}$ is the set of the followers;
- $\mathcal{P}$ is the feasible strategy set of DISCO as defined through Eq. (3);
- $\mathcal{L}_n$ is the feasible strategy set of each customer as defined through Eq. (12) and
- $f_D$ and $f_n$ are the objective functions of DISCO and every customer as defined through Eq. (3) and Eq. (12), respectively.

3.2. Non-cooperative GN Game: Customers Interaction

Every customer plays a non-cooperative GN game with other customers in response to the optimal price because the strategy space of customers is coupled within common constraint.

**Definition 2:** The non-cooperative GN game, $\mathcal{G}$, is defined by the tuple $\mathcal{G} = \{\mathcal{N}, \{\mathcal{L}_n\}_{n \in \mathcal{N}}; \{f_n\}_{n \in \mathcal{N}}\}$.
3.3. GN and Stackelberg Equilibrium

In order to compute the equilibrium of $\mathcal{G}$ game, the scheme is proposed by Scutari et al. [16] is applied here. In this scheme, the $\mathcal{G}$ game is substituted with its equivalent KKT system and turned into an augmented Nash problem with $N+1$ players. DISCO acts as $(N+1)^{th}$ player and controls the overprice variable $\lambda$ which is the corresponding multiplier of the common constraint $L_{\text{ave}}(l) \leq 0$. DISCO treats the price complementary condition as an optimizing problem, $\min_{k \in \Phi} \lambda^T L_{\text{ave}}(l)$, but at this point, it acts at the same level with customers. DISCO forces the customers to meet shared constraint by assigning $\lambda$. In this new extended Nash game, the objective functions of $N+1$ players is expressed as

$$\min_{l \in \mathbb{L}} f_{n,\text{aug}} = f_n(l_n, l_n) + \lambda^T L_{\text{ave}}(l), \quad \forall n = 1, \ldots, N,$$

$$\min_{\lambda \in \mathbb{L}} f_{N+1} = -\lambda^T L_{\text{ave}}(l).$$

In response to optimal price vector $\rho^*$ which is derived from Eq. (3), every customer plays its best strategy $l^*_n$ by considering the overprice $\lambda^*$; thus, the strategy set $(l^*, \lambda^*)$ is the GN equilibrium of $\mathcal{G}$. At this point, the SE $(\rho^*, l^*, \lambda^*)$ occurs which is referred to as an equilibrium solution for this proposed day-ahead energy management game.

**Definition 3:** In $\mathcal{G}$, the strategy set $(\rho^*, l^*, \lambda^*)$ generates the SE if and only if it meets the following set of inequalities

$$f_{n,\text{aug}}(l^*_n, l^*_n, \rho^*, \lambda^*) \leq f_{n,\text{aug}}(l_n, l_n, \rho^*, \lambda^*), \quad \forall n = 1, \ldots, N,$$

$$f_{N+1}(\lambda^*) \leq f_{N+1}(\lambda),$$

$$f_{D}(\rho^*, l^*) \leq f_{D}(\rho, l^*),$$

where $l_n$ is the strategy set of all customers without the customer $n$.

3.4. Equilibrium Existence and Properties

Because the optimization problem of DISCO is convex, the optimal day-ahead price can be calculated by any of the convex optimization methods [18]. A capable and proper tool, VI technique, is applied for studying the solution and properties of the non-cooperative games [17]; however, GN game is difficult to solve and VI method is not applicable to all cases. In some situations, GN games can be reformulated as a VI($\mathcal{L}, F$) problem [16] which finds the point $l^* \in \mathcal{L}$ by meeting

$$(l - l^*)^T F(l^*) \geq 0, \quad \forall l \in \mathcal{L}.$$
The solution of $\text{VI}(\mathcal{L}, F)$ problem is named variational solution and is computed by specifying the monotonicity property of the vector-valued function $F$. If the next lemma is satisfied, a solution of $\tilde{G}$ game is yielded by solving its equivalent VI problem [16].

**Lemma 1:** In $\tilde{G}$, if the following conditions are met: 1) The individual strategy set $\mathcal{L}_n$ is closed and convex; 2) The customers’ objective $f_n$ is convex and twice continuously differentiable in $l \in \mathcal{L}$ for every fixed $l_n \in \mathcal{L}_n$; 3) The common constraint $AAL$ is continuously differentiable and jointly convex in $l \in \mathcal{L}$; then every solution of the $\text{VI}(\mathcal{L}, F)$ problem is a solution of GN game (not vice versa), where $\mathcal{L} \triangleq \prod_{n=1}^{N} \mathcal{L}_n$ and $F(l) \triangleq (\nabla_{l} f_n(l))_{n=1}^{N}$. For the Proof, see Appendix A.

Now that, all conditions of Lemma 1 are met in the $\tilde{G}$ game, the associated $\text{VI}(\mathcal{L}, F)$ problem is assessed for the variational solution of $\tilde{G}$ game as presented in Theorem 1.

**Theorem 1:** The customer’s game among EMS units, $\tilde{G}$, in response to optimal price setting by DISCO has multiple Nash equilibria. For the Proof, see Appendix B.

As stated before, equilibriums of $\tilde{G}$ is computed through solving the equivalent augmented Nash problem (Eq. (14), (15)). With this approach, in the next section, we can design a distributed algorithm for converging to SE.

4. Distributed Algorithms

In this section, we devise an algorithm for reaching the SE where should be executed by both DISCO and customers. As proved in Theorem 1, the $\tilde{G}$ game among EMS units has multiple equilibria; thus, the best-response algorithm may fail to converge here.

To address this issue, the DISCO and customers, in augmented Nash game should optimize a regularized problem at every algorithm’s iteration. By regularizing the players’ objective, the monotone $\text{VI}(\mathcal{L}, F)$ problem becomes strongly monotone; as a result, the game converges to one of the primal game’s equilibria. The regularized $\tilde{G}$ game is

$$\min_{l_n \in \mathcal{L}_n} f_n + (\lambda^{(i)})^{T} L_{\text{ave}} (l_n, l_n^{(i)}) + \frac{\tau}{2} \|l_n - l_n^{(i)}\|^2, \quad \forall n = 1, ..., N, \tag{19}$$

$$\min_{\lambda^{(i)}} L_{\text{ave}} (l)^{T} \lambda^{(i)} + \frac{\tau}{2} \|\lambda^{(i)}\|^2, \tag{20}$$

where $\tau$ is a regularization parameter and the superscript $i$ represents the $i$th iteration.

The parameter $\tau$ must be chosen large enough for the convergence of the developed best-response based algorithm named as
proximal decomposition algorithm (PDA). By applying this algorithm, the customers reach one of the $\bar{G}$ equilibria without the possibility of selecting among equilibria which leads to uncertainty in the resultant system performance. Here, a PTRA algorithm is developed with the desired equilibrium selection ability. The prominence of this PTRA is revealed through computation and signaling increment cost among customers.

To drive the system to the desired equilibrium, a criterion function $\phi(l_n, l_w)$ should be defined aligned with the equilibrium of the interest herein. The merit function $\phi$ represents the degree of signaling and performance, without which, customers do not exchange any information; otherwise, they should cooperate. According to [17-Theorem 21], $\phi$ must have the following properties: 1) Continuously differentiable and convex on $L$; 2) Having bounded level sets on the solutions of GN game; 3) $\nabla \phi$ is a Lipschitz continuous on $L$. Accordingly, the criterion function in this algorithm is proposed as $\phi(l_n, l_w) = \|w\|_{\infty} = L_{\text{peak}}$ because it embodies all necessary conditions. In fact, customers should only agree to consider $\min\|p\|_{\infty}$ in their optimization problem at the beginning of $\bar{G}$ game. With such a merit function, the $\bar{G}$ game, among the equilibriums, converges to the one which reduces the maximum aggregate load; subsequently, improves LF. With these scheming, the $\bar{G}$ game converges to the considered desired equilibrium; consequently, every customer should solve a sequence of standard regularized problem as

$$\min_{l_n \in L} f_n + (\lambda^{(i)})^2 L_{\text{ave}}(l_n, l_w^{(i)}) + \|g^{(i)}\|_{\infty} + \frac{\varepsilon}{2}\|l_n - l_w^{(i)}\|^2,$$  \hspace{1cm} \text{(21)}

where $\{g^{(i)}\}$ is a sequence with $g^{(i)} > 0$, $\{g^{(i)}\} \rightarrow 0$, and $\sum_{i=0}^{\infty} g^{(i)} = \infty$ [17].

At this stage, the holistic energy management algorithm is ready to be exhibited. Initially, DISCO computes $\tau$ as given in Theorem 2 and broadcasts to customers. Before every playing sequence of regularized augmented Nash game, DISCO solves Eq. (3) by any convex optimization method [18] and forwards optimal day-ahead price and daily aggregate load to the customers. Then, DISCO and customers run an iterative PTRA for achieving the desired GN equilibrium. At each iteration $i$, DISCO and every customer optimize problems (20), (21), respectively. The $\bar{G}$ game ends when a termination criterion (e.g. $\|p \tau - T\|_{\text{p}} \leq \varepsilon$) is met. At this point, every customer sends its consumption profile to DISCO for the new optimal price calculating. This energy management algorithm is ended and reaches SE when the calculated price does not significantly change between two consecutive iterations. The algorithm procedure and its convergence conditions are given in Theorem 2 and Algorithm 1, respectively.

\textit{Theorem 2:} If in the mentioned monotone $\bar{G}$ game, the regularization parameter $\tau$ meets
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\[ \tau > \sqrt{N + A_1 + \ldots + A_N}, \]

(22)

and \( \{\mathcal{G}^{(i)}\} \) is in \( \mathcal{G}^{(i)} > 0 \), \( \{\mathcal{G}^{(i)}\} \to 0 \) and \( \sum_{i=0}^{\infty} \mathcal{G}^{(i)} = \infty \) form, then, any sequence \( \{a_n^{(i)}, b_n^{(i)}, \mathcal{A}^{(i)}\}_n \) generated by PTRA converges to a variational solution of the \( \mathcal{G} \) game and finally, the SE is yielded through Algorithm 1. For the Proof, see Appendix C.

**Algorithm 1:** Algorithm to reach SE

**Data:** Set \( i = 0 \) and the initial centroids \( (\mathcal{T}_n = 0)_{n=1}^N \) and \( \mathcal{K} = 0 \). Given \( \tau, \{\mathcal{G}^{(i)}\} \to 0, L_{\text{min}}, L_{\text{max}}, \) any feasible starting point \( (L_n^{(0)})_{n=1}^N, \mathcal{A}^{(0)} \geq 0, l^{(0)}, \overline{J} = \mathcal{G}^{(0)} \).

**Step 1:** DISCO computes optimal day-head price \( \rho^* \), as \( \rho^* = \arg \min_{\rho \in D} \{ f_D \} \).

**Step 2:** If \( \|\rho^{(h)} - \rho^{(h-1)}\|_2 / \|\rho^{(h)}\|_2 \leq \varepsilon, STOP. \) The game reaches to SE.

\( h \leftarrow h + 1. \) DISCO sends optimal day-head price \( \rho^* \) and aggregate load vector \( l \) to customers.

**Step 3:** If \( \|l^{(i)} - l^{(i-1)}\|_2 / \|l^{(i)}\|_2 \leq \varepsilon, \) The game reaches to GN equilibrium. Go to **Step 1**.

**Step 4:** Customers play a GN game in response to \( \rho \) using PTRA.

**PTRA:** For \( n \in \mathcal{N} \), each customer calculates \( l_n^{(i+1)} \triangleq \{D_n^{(i+1)}, B_n^{(i+1)}\} \) as

\[
l_n^{(i+1)} = \arg \min_{l_n \in \mathbb{Z}^+} \left\{ f_n + (\mathcal{A}^{(i)})^T L_{\text{min}} (l_n, l_{\text{max}}) + \overline{J} \|l_n\|_\varepsilon + \frac{\tau}{2} \|l_n - l_n\|_2^2 \right\}, \]

DISCO computes \( \mathcal{A}^{(i+1)} \) as

\[
\mathcal{A}^{(i+1)} = \arg \min_{\mathcal{A}^{(i)}} \left\{-\mathcal{A}^T L_{\text{min}} (l) + \frac{\tau}{2} \|\mathcal{A} - \mathcal{A}\|_2^2 \right\}
\]

If equilibrium is reached, then customers and DISCO update their centroids \( \mathcal{T}_n = l_n^{(i+1)}, \overline{J} = \mathcal{G}^{(i+1)} \) and \( \mathcal{K} = \mathcal{A}^{(i+1)} \), respectively.

**Step 5:** \( i \leftarrow i + 1; \) Go to **Step 3**.

5. Implementation of Proposed Energy Management Mechanism

The proposed energy management mechanism is only applicable in the smart grid platform. All of the smart agents of this grid must be equipped with AMI, sensors, controllers and bi-directional communication system to achieve optimality in grid goals. In such context, some new technical and socio-economic challenges which include data management, communication issue, cybersecurity, customer privacy and active participation of customers in energy management program arise.

Existing the large number of meters and computational units in the smart grid leads to a vast amount of data and information which should be collected, stored, retrieved and analyzed. Management of these big data in the smart grid is a vital issue where may decrease the processing speed of data aggregation and analysis. Cloud-based big data analytics technology seems to be a
solution to this problem. Furthermore, a proper communication protocol and media should be defined and applied to transfer data among different parties. Although wide ranges of communication technology for deployment in the smart grid exist, they have their limitations. The optical fiber has high bandwidth and low attenuation and interface but is very expensive and complicated.

On the other hand, cybersecurity should address information availability, integrity, and confidentiality to avoid any potential threat to system operation and customer's privacy. In fact, cybersecurity is the other essential feature of the reliable smart grid operation. Every layer of smart grid should demand specific security which alone is not sufficient. The cybersecurity besides operation policy of operator should be in such a way that customer's privacy is kept intact and consequently customer accepts the smart grid technology. In our proposed energy management mechanism, customer’s consumption behavior and his preferences are the customer's private information, and DISCO only has access to them. In other words, every customer only receives the aggregated load of the grid, and the personal habits and behavior of customers don't reveal to other customers, and consequently, their privacy is kept.

To successfully implement the different operation programs in the smart grid, the active participation of customers in these programs is needed. Utilities with the help of amenities and facilities along with smart grid can design different operational aspects such as tariff schemes and operational policies which may not be accepted by customers. In this situation, the grid operator with precise regulations and incentive policies can motivate the customers to take part actively in different operation programs or accepting the new tariff scheme willingly.

6. Simulation Results

In this section, the performance of the proposed day-ahead energy management approach by numerical simulation is evaluated. A smart distribution grid with $N = 20$ customers is simulated in MATLAB software. To solve optimization problems, CVX, a package for specifying and solving convex programs is applied [19].

Every customer has some shiftable and non-shiftable loads. PHEV, washer, dryer and dishwasher are examples of shiftable appliances that are flexible in their scheduling consumption. The second group is named non-shiftable appliances for example TV, refrigerator, lighting, and heating has a strict scheduling consumption constraint. The daily consumption of customers before management and daily inelastic demand are generated based on the model introduced in [20]. It is assumed that every customer has four shiftable appliances as mentioned before, and their feasible operation is set based on the real consumption pattern in residential customers. For instance, PHEV owners practically charge their cars when they are at home at late night to early morning, or washing machine usually works during the day before dryer start to work. As stated, every customer is
equipped with energy storage devices the capacity of which is generated in a random manner within [2 kWh 3 kWh] range. The initial and final charge levels of customers’ storage devices are among [0.2 0.3] and [0.5 0.6] of the devices’ storage capacity, respectively. In addition to this, every customer utilizes a renewable energy resource the maximum energy production of which is within [1 kWh 2 kWh] range. The estimated expected wholesale price is bar charted in Fig. 2.

Seven scenarios are assessed that in all of them, the customers are equipped with the flexible loads and storage devices, but only in the last four scenarios, the customers utilize the renewable energy resources. In the first and fourth scenarios, no energy management is considered. In the second and fifth, every smart home benefits from PTRA energy management approach in which storage devices with only charge capability is of concern. In the third and sixth scenarios, every customer employs PTRA, but the storage devices can be charged and discharged. The condition in the last is similar to that of the sixth with applying a different management algorithm which is PDA management approach with no predictable performance.

The aggregate load during a day through the first three scenarios is illustrated in Fig. 3, where the smoother load profile and lower maximum demand are obtained through this PTRA proposed energy management approach, indicating a greater LF and a reduction in customers’ bill. Storage devices charging in off-peak hours and discharging in peak hours in scenario 3 further decrease the maximum aggregate load in comparison with scenario 2.

The renewable resources, due to their stochastic nature, have variable and uncontrollable power generation. These characteristics, when applied in distribution grid, disturb the load profile shaping and reduce LF. The aggregate demand in scenarios 4-6 is shown in Fig. 4. As observed, the PTRA approach improves LF by proper scheduling of storage devices and flexible loads. In scenario 5, since the discharging ability of the storage devices is canceled, the peak aggregate demand is more than scenario 6.

This proposed PTRA energy management framework converges to equilibrium which would improve the LF with a reduction in aggregate peak demand. The performance of PTRA and PDA approaches are compared in Fig. 5. It is obvious that the maximum aggregate load reduces in PTRA approach and LF increases in comparison with PDA.

The LF, the maximum aggregate demand values, and the aggregate billing cost through different scenarios are tabulated in Table 1, which for smart homes that contain flexible loads and storage devices, the amount of LF becomes significantly high by utilizing energy management program. LF increases by 24.72% and 27.35% in the second and third scenarios, respectively, in comparison with the first scenario. In a similar manner, LF increases in the presence of fluctuating renewable resources by implementing PTRA approach, as can be seen in Table 1. (i.e., 28.74% and 31.46% greater in the fifth and sixth scenarios compared to the fourth scenario). The storage devices with their discharging ability contribute to an increase in LF. As expected, the PTRA has a higher capability in the improvement of the aggregate load shaping in comparison with PDA. This is due to
equilibrium selection capability of PTRA which is aligned with LF improvement.

As observed in Table 1, PTRA approach reduces the aggregate billing cost of customers by redistributing the elastic loads and storage scheduling in the low price hours. The total billing cost of customers is reduced by 13% in the third scenario compared to that of the first scenario and by 14.23% in the sixth scenario compared to that of the fourth scenario. It is obvious from Table I that PDA reduces the total billing cost of the system by 11.85% which is approximately 2.5 percent less than that of the PTRA.

Although PTRA decreases the aggregate billing cost of customers, an individual customer should be motivated enough to participate in PTRA management program. For this reason, the daily individual customers’ bills in the fourth and fifth scenarios are compared in Fig. 6, where it is clear, all customers pay less to DISCO and they benefit from participating in PTRA approach. From the obtained results, it can be deduced that by a proper scheduling of responsive loads and storage devices, the aggregate load shaping is improved especially in the presence of renewable energy resources. The effect of the flexible appliances’ and storage devices’ capacity on the aggregate billing cost and LF is shown in Fig. 7. As expected, an increase in their capacity enhances the aggregate load profile shape and reduces the aggregate billing cost.

7. Conclusion

In this paper, a holistic day-ahead distributed energy management approach in a smart distribution grid is proposed, where both DISCO and its clients are involved. Energy management game and the interactions between DISCO and customers are modeled as a Stackelberg game. DISCO derives day-ahead optimal price by maximizing its profit and sends it to its customers. Customers receive the price and schedule their flexible loads usage and charge/discharge storage devices profiles while meeting the common constraint AAL; consequently, the interaction among customers is modeled as a non-cooperative GN game.

In addition, customers agree with each other to reduce the maximum aggregate load by adding a common extra term $\min \{ \phi(L_n, L_{-n}) = \|L_{-n}\|_c = L_{\text{peak}} \}$ to their objectives. By adopting this approach, it is proved that the GN game converges to an equilibrium which is in the alignment with the desired system performance (i.e., increasing LF).

Considering both common constraint AAL and common function $\|L\|_c$ in the customer optimization problem, the appropriate shape of the aggregate load in and without the presence of fluctuating production of renewable resources is achieved. Consequently, an increase in the LF leads to a decrease in the customers’ billing cost and efficient DISCO’s facility utilization. Numerical results verify the theoretical outcomes and illustrate that this proposed energy management approach is applicable in practical smart grid situation.
APPENDIX A

(1) Proof of Lemma 1

This lemma is based on the results obtained in [16], [17]. The set $\mathcal{C}_n$ is closed and convex because it is polyhedra [18]. The function $f_n$ is convex if the Hessian matrix $H_n' \triangleq \nabla^2 f'_n$ is positive semi-definite [18]. The $H'_n$ is obtained as follows

$$H'_n(I') = \begin{bmatrix} \frac{\partial^2 f'_n}{\partial d_n^2} & \frac{\partial^2 f'_n}{\partial d_n^2} & \cdots & \frac{\partial^2 f'_n}{\partial b_m^2} \\ \frac{\partial^2 f'_n}{\partial d_n^2} & \frac{\partial^2 f'_n}{\partial d_n^2} & \cdots & \frac{\partial^2 f'_n}{\partial b_m^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f'_n}{\partial d_n^2} & \frac{\partial^2 f'_n}{\partial d_n^2} & \cdots & \frac{\partial^2 f'_n}{\partial b_m^2} \end{bmatrix}$$

(A.1)

It is clear that the eigenvalues of $H'_n$ are non-negative, indicating that $H'_n$ is positive semi-definite. The common constraint AAL is expressed as intersection of the two half-spaces, hence, a convex set [18].

APPENDIX B

(1) Proof of Theorem 1

Based on [16-Theorem 1], if $F$ is monotone on $\mathcal{L}$, then $VI(\mathcal{L},F)$ (the $\mathcal{G}$ game) has a convex solution set. The monotonicity property of $F$ is realized by determining the definiteness property of Jacobian matrix $JF(I) \triangleq (J_{F_n}F_n(I))_{n=1}^N$ [16], which is calculated as

$$J_{F_n}F_n'(I') = H_n'(I')$$

$$J_{F_n}F_n'(I') = \begin{bmatrix} \frac{\partial^2 f'_n}{\partial d_n^2} & \frac{\partial^2 f'_n}{\partial d_n^2} & \cdots & \frac{\partial^2 f'_n}{\partial b_m^2} \\ \frac{\partial^2 f'_n}{\partial d_n^2} & \frac{\partial^2 f'_n}{\partial d_n^2} & \cdots & \frac{\partial^2 f'_n}{\partial b_m^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f'_n}{\partial d_n^2} & \frac{\partial^2 f'_n}{\partial d_n^2} & \cdots & \frac{\partial^2 f'_n}{\partial b_m^2} \end{bmatrix}$$

(B.1)

Here, all eigenvalues of the Jacobian matrix $JF$ are non-negative, thus, the matrix $JF$ is positive semi-definite; consequently, the $F$ is monotone on $\mathcal{L}$. This result indicates that the $\mathcal{G}$ game has multiple equilibria and the Theorem 1 is proved.

APPENDIX C

(1) Proof of Theorem 2

According to [16], [17], the PTRA converges if the $(N+1) \times (N+1)$ matrix $\tilde{Y}_{F,\mathcal{G},\mathcal{R}}$, with
\[
\tilde{Y}_{F,\tau} \triangleq \begin{bmatrix}
Y_{F,\tau} + \tau I_N & -\gamma \\
-\gamma & \tau
\end{bmatrix},
\quad (C.1)
\]

is a positive definite, where

\[
\begin{bmatrix}
Y_{F,\tau}
\end{bmatrix}_{nm} \triangleq \begin{cases}
\alpha_{nm}^{\min}, & \text{if } n = m \\
-\beta_{nm}^{\max}, & \text{if } n \neq m
\end{cases}, \quad \gamma \triangleq (\gamma_s)_{s=1}^N, \quad \gamma_n \triangleq \sup_{l \in L} \| \nabla_l L_{m_{av}} \|, \\
\alpha_n^{\min} \triangleq \inf_{l \in L} \lambda_{\min}(J_{n,\tau}(F_n + \overline{\mathbf{F}}_{n,\tau})), \quad \beta_{nm}^{\max} \triangleq \sup_{l \in L} \| J_{n,\tau}(F_n + \overline{\mathbf{F}}_{n,\tau}) \|.
\]
\quad (C.2)

\lambda_{\min} \{ \cdot \} \text{ and } \| \cdot \| \text{ indicate the smallest eigenvalue of the matrix argument and the spectral norm of the matrix argument, respectively. The } \tilde{Y}_{F,\tau} \text{ is calculated as follows}

\[
\nabla_l \phi(I) = \begin{bmatrix}
[I]_{1:A_l} & I = i_{\max} \geq 0 \\
0, & \text{otherwise}
\end{bmatrix} \quad \Rightarrow \alpha_n^{\min} = 0, \beta_{nm}^{\max} = 0,
\quad (C.3)
\]

\[
Y_{F,\tau} = \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0
\end{bmatrix}_{n \times N}, \quad \nabla_l L_{m_{av}}(I) = \begin{bmatrix}
0 & L_{m_{av}}^{\min} - L_{m_{av}} \leq 0 \\
\frac{1}{T}[I]_{1:A_l} & L_{m_{av}}^{\min} - L_{m_{av}} \leq 0 \\
0 & L_{m_{av}}^{\max} - L_{m_{av}} \leq 0
\end{bmatrix},
\quad (C.4)
\]

\[
\gamma_n \triangleq \sup_{l \in L} \| \nabla_l L_{m_{av}}(I) \| \quad \Rightarrow \gamma_n \leq \sqrt{(1+A_n)}.
\]

Here, it can be claimed that \( \tilde{Y}_{F,\tau} \preceq \tilde{Y}_{F,\tau} \), where

\[
\tilde{Y}_{F,\tau} \triangleq \begin{bmatrix}
\tau & \text{if } n = m \\
0 & \text{if } n \neq m \text{ and } n,m \neq N+1 \\
-\sqrt{(1+A_n)} & \text{otherwise}
\end{bmatrix},
\quad (C.5)
\]

It is easily obtained that \( \tilde{Y}_{F,\tau} \) is positive definite if following condition satisfies: \( \tau > \sqrt{N + A_1 + \ldots + A_N} \). ■

8. References


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Figure captions

Figure 1. Smart distribution grid.

Figure 2. Expected day-ahead wholesale electricity prices.

Figure 3. Daily aggregate load in the first three scenarios.

Figure 4. Daily aggregate load in the second three scenarios.

Figure 5. Daily aggregate load in PDA and PTRA.

Figure 6. Daily individual customers’ bill.

Figure 7. Aggregate billing cost and LF. a) The effect of aggregate storage devices’ capacity. b) The effect of aggregate flexible loads’ capacity.

Table caption

Table 1. LF, maximum aggregate demand and aggregate billing cost in studied scenarios

Figures
Figure 1.

Figure 2.
Figure 3.

Figure 4.

Figure 5.
Table

Table 1.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>LF</th>
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<th>Aggregate Billing cost ($)</th>
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