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Research Note

Measuring the satisfaction and loyalty of Chinese smartphone users: A simple symbol-based decision-making method

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Abstract. User satisfaction and loyalty are very important in the mobile communications market, because mobile is frequently updated. Understanding and knowing well the level of customer satisfaction and loyalty needs a scientific assessment method. This paper intends to establish such a method for measuring the satisfaction and loyalty of Chinese smartphone users. First, combining the group decision-making and TOPSIS (technique for order preference by similarity to ideal solution) technique, a theoretical framework of evaluation was established. Second, the questionnaire survey was conducted by respondents, who were allowed to express their own opinions by using some simple symbols or leaving some measurement questions unanswered. Then, the information on the symbols and the nonresponses in questionnaires were fused into intuitionistic fuzzy information. Third, the preference orders of user satisfaction and loyalty were identified by using the TOPSIS technique and a projection measure. Finally, an experimental comparison was performed, the theoretical and practical implications of the current model were discussed, and the important limitations were recognized. In the end of this study, future research directions are suggested.

1. Introduction

Technology innovation is viewed as a continuously growing element in driving productivity and competitiveness of business units. It is reaching new heights day by day. The mobile market is an obvious example.

A smartphone is basically an ordinary looking mobile phone with the more common features of a handheld computer. It has a special intelligence that will allow the user to do things such as e-mailing, web browsing, audiovisual amusement, word processing, mobile videoing, and using the Global Positioning System (GPS).

China is the largest and fastest growing mobile market in the world. The cell phone can be perceived as a ubiquitous communication device. According to the statistical report released by China Internet Network Information Center (CNNIC) [1], China has had 724 million mobile Internet users by the end of June 2017. There are many best-selling brands in the mobile market in China with a fierce competition for gaining the dominance. Thus, retaining customers has become a great challenge for the companies. The ability to provide high degree of satisfaction is crucial for the brands to differentiate their position from their competitors, specifically in telecommunications

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market. Therefore, satisfying customer requirements and understanding the level of customer loyalty are two important concerns of a company [2], which need an in-depth study. Customer satisfaction is a key driver in the retail context [3], and it is considered as an antecedent of repurchase intention [4]. Understanding and knowing well the level of customer satisfaction and loyalty needs a scientific assessment method.

The existing studies have employed various approaches to evaluating the customer satisfaction and loyalty [5]. However, these studies have mostly assumed that there are linear relations between the key attributes and satisfaction as well as loyalty levels, and have mostly used regression-based models [6] and structural equation modeling methods [7] for estimation of these levels. There are several interesting exceptions in the literature. One is the study by Rahul and Majhi [2], who explored a nonlinear approach to estimation of consumer satisfaction and loyalty in mobile phone sector of India. Another interesting exception is the study by Li et al. [8], which used analytic hierarchy process to identify the most relevant services for consumers.

Statistical methods are very useful for examining the relationships between customer satisfaction and evaluation criteria, such as product quality, product service, on-time delivery, etc. However, there are some research gaps, which need in-depth investigation. We list these research gaps in the following section on problem description.

We consider that some evaluations might be very positive and some might be very negative in questionnaire survey. If the evaluations are aggregated into a collective decision of Decision Makers (DMs) or experts, the positive and negative information will be offset. To avoid this case, this paper attempts to propose a direct Group Decision-Making (GDM) method for measuring the satisfaction and loyalty. In addition, we note that the positive and negative evaluations are very useful information. To fully utilize them, this model intends to employ an extended TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [9] to compromise them. The Euclidean distance or the Hamming distance is commonly used in TOPSIS technique in order to obtain the relative closeness. In fact, the projection method [10,11] is a much better measure than the distance measures. It considers not only the distance but also the angle between the measured objects. To improve the TOPSIS technique, this paper will modify the TOPSIS model, in which the separation measure will be replaced by the projection measure.

The main contributions of this paper are as follows:

1. A new symbol-based approach contributes to the questionnaire survey;
2. A new information fusion method contributes to the evaluation methodology;
3. A decision-making technique contributes to the scientific assessment of user satisfaction and loyalty in the mobile communications market.

To this end, the rest of the paper is structured as follows. Section 2 reviews the related work. Section 3 presents descriptions for the problems. Section 4 briefly reviews the interval-valued intuitionistic fuzzy information and some relevant tools in GDM problems. Section 5 introduces the theoretical framework. Section 6 presents an evaluation methodology and algorithm based on the above idea. Section 7 presents an experimental comparison of projection measure with the Euclidean distance. Finally, Section 8 gives our discussion, conclusions, and future research.

2. Related work

Because of the tremendous potentiality of business in the mobile communications market, customer satisfaction and loyalty have received much academic interest. For example, Zhao et al. [12] explored the effects of service quality and justice on customer satisfaction. Bayraktar et al. [13] measured the efficiency of customer satisfaction and loyalty for mobile phone brands by using a data envelopment analysis method. Kim et al. [14] explored a mobile user engagement model to explain mobile user engagement intention through user's motivations, perceived value, and satisfaction. Haverila [15] investigated the mobile phone feature preferences among male respondents in Finland. Qi et al. [16] surveyed the effect of customer satisfaction and customer loyalty drivers of customer lifetime value on mobile data services. In addition, Bandarua et al. [17] explored a quantitative methodology for assessing customer satisfaction using evolutionary optimization. Kang and Park [18] introduced a review-based measurement of customer satisfaction in mobile service.

Due to the increasing complexity of the socio-economic environment, it may be impossible for a single DM to consider all relevant aspects of a problem. In this case, some decision-making problems require to be further extended to GDM [19-23]. Recently, the GDM has attracted great attention from researchers. For example, Pérez et al. [24] introduced a new consensus model for GDM problems with non-homogeneous experts. Hashemi et al. [25] explored a compromise ratio method with an application to water resources management in an intuitionistic fuzzy setting. Liu [26] proposed some Hamacher aggregation operators based on the Interval-Valued Intuitionistic Fuzzy Numbers (IVIFNs) and their application to GDM problems. Vahdani et al. [27] developed a GDM method based on a novel fuzzy modified TOPSIS technique. Morente-

Molinera et al. [28] introduced a systematic review and future trends of multi-granular fuzzy linguistic modeling in GDM problems. Mousavi et al. [29] modeled a fuzzy stochastic GDM approach to selection problems. Ebrahimnejad et al. [30] described a novel two-phase GDM approach to construction project selection in a fuzzy environment. Liao et al. [31] focused on multiplicative consistency of hesitant fuzzy preference relation and its application in GDM. Mousavi et al. [32] addressed a hierarchical GDM approach to new product selection in a fuzzy environment. Mousavi et al. [33] explored a fuzzy grey model based on the compromise ranking for GDM problems in manufacturing systems. Wan and Li [34] suggested an Atanassov's intuitionistic fuzzy programming method for heterogeneous GDM with Atanassov's intuitionistic fuzzy truth degrees. Mousavi et al. [35] observed a soft computing approach to the selection problem of material handling equipment based on a fuzzy grey group compromise solution. Mousavi et al. [36] presented an intuitionistic fuzzy grey model for selection problems with an application to the inspection planning in manufacturing firms. Meng and Chen [37] presented a new method for GDM with incomplete fuzzy preference relations. Gitinavard et al. [38] offered a new multi-criteria weighting and ranking model for GDM analysis based on interval-valued hesitant fuzzy sets in selection problems. Gitinavard et al. [39] introduced a distance-based decision model in interval-valued hesitant fuzzy setting for industrial selection problems. Zhang et al. [40] proposed a new method for ranking fuzzy numbers and its application to GDM. Xu and Shen [41] developed a new outranking choice method for GDM under Atanassov's interval-valued intuitionistic fuzzy environment. Yue [42] described a geometric approach to ranking interval-valued intuitionistic fuzzy numbers with an application to GDM problem.

At present, the intuitionistic fuzzy theory has extensively been applied to various areas [43]. Chen [44] developed an interval-valued intuitionistic fuzzy LINMAP method. Vahdani et al. [45] addressed a new design of the elimination and choice translating reality method for multi-criteria GDM in an intuitionistic fuzzy environment. Hashemi et al. [46] described an extended compromise ratio model under an interval-valued intuitionistic fuzzy environment. Dymova and Sevastjanov [47] introduced some operations on interval-valued intuitionistic fuzzy values in the framework of Dempster-Shafer theory. Verma et al. [48] proposed an improved intuitionistic fuzzy c-means clustering algorithm incorporating local information for brain image segmentation. Nguyen [49] developed a new interval-valued knowledge measure for interval-valued intuitionistic fuzzy sets with application to decision-making. Ouyang and Pedrycz [50] developed a new model for intuitionistic fuzzy Multi-Criteria Decision-Making (MCDM) problems. Mousavi et al. [51] designed a model of intuitionistic fuzzy VIKOR for GDM problems. Yue [52] developed a model for evaluating software quality based on symbol information and IVIFNs.

TOPSIS [53] is a very useful technique for decision-making problems. For example, Mokhtarian et al. [54] described a new flexible and reliable IVF-TOPSIS method based on uncertainty risk reduction in decision-making process. Beikkhakhian et al. [55] focused an ISM model in evaluating agile suppliers selection criteria and ranking suppliers using fuzzy TOPSIS-AHP methods. Roszkowska and Wachowicz [56] explored an application of fuzzy TOPSIS to scoring the negotiation offers in ill-structured negotiation problems. TOPSIS technique is a compromise method [57], which highlights the positive and the negative information, and compromises them by a relative closeness. The relative closeness is composed of some separation measures between alternative decisions and the ideal decisions. As mentioned in the Introduction, the projection [58] is a more comprehensive measure than the distance measures. In this model, the separation measure will be replaced by the projection measure.

3. Problems description

The research questions established in this study include the following three aspects:

1. The existing literature lacks the rating methods of customer satisfaction for different mobile brands with conflicting and incommensurable criteria;
2. One of the major sources of uncertainty is nonresponse in questionnaire survey, which occurs when a portion of the individuals sampled cannot or will not participate in the survey. Nonresponse to the survey or a part of questions in the survey may occur by hesitation, uncertainty, or negligence; identifying this cause also provides us with useful information. The current methods, to the best of our knowledge, fail to handle it;
3. Many questionnaires require the participants to respond to too many questions, including some items and options. Some participants often complain that they are pressed for time. They prefer the questionnaires with fewer options, which are answered by using simple symbols. For example, the questionnaire is answered by using symbols $\surd$, $\times$, and $\bigcirc$, which denote satisfaction, dissatisfaction, and hesitation or abstention, respectively.

To fill these research gaps, this paper intends to employ a decision-making method to evaluate the customer satisfaction and loyalty in mobile sector.

MCDM is one of the most complicated administrative processes in management, which is the procedure for ranking the feasible alternatives with conflicting and incommensurable criteria. GDM is an extension of MCDM, which can consider various views of DMs. Thus, this paper employs the GDM method to solve the problem of ranking customer satisfaction for different mobile brands.

To collect and aggregate the nonresponse items in tested questionnaires, the fuzzy logic is more suitable. Fuzzy logic originates in the theory of fuzzy set [59], which is one of the techniques of soft computing and can deal with the inherent subjectivity, imprecision, and vagueness in the articulation of opinions. Intuitionistic fuzzy theory [60] is a generalization of fuzzy set. The intuitionistic fuzzy number [61] is a special case of intuitionistic fuzzy set. An intuitionistic fuzzy number can comprehensively measure an evaluation object with three parameters (μ, ν, π): The first one is the proportion of DMs who are supporting ($\surd$) the evaluation object; the second one is the proportion of DMs who are against ($\times$) the evaluation object; and the third one is the proportion of DMs who are uncertain ($\bigcirc$) about the evaluation object. The third parameter, π, includes the nonresponse information. For measuring the nonresponse information and symbol-based evaluations, our model attempts to employ the intuitionistic fuzzy theory to develop the theoretical framework.

4. Preliminaries

As a generalization of Zadeh's fuzzy set [59], Atanassov [60] introduced the intuitionistic fuzzy set. Later, Atanassov et al. [62] extended the notion of intuitionistic fuzzy set to interval-valued intuitionistic fuzzy set.

Xu and Chen [63] called the pair $\tilde{\alpha} = (\mu_{\tilde{a}}, \nu_{\tilde{\alpha}})$ an IVIFN, which is written as:

$$\tilde{\alpha} = ([\mu^l_{\tilde{\alpha}}, \mu^u_{\tilde{\alpha}}], [\nu^l_{\tilde{\alpha}}, \nu^u_{\tilde{\alpha}}]), \tag{1}$$

where $[\mu^l_{\tilde{\alpha}}, \mu^u_{\tilde{\alpha}}], [\nu^l_{\tilde{\alpha}}, \nu^u_{\tilde{\alpha}}], [\pi^l_{\tilde{\alpha}}, \pi^u_{\tilde{\alpha}}] \subseteq [0,1], \mu^u_{\tilde{\alpha}} + \nu^u_{\tilde{\alpha}} \leq 1, \pi^l_{\tilde{\alpha}} = 1 - \mu^u_{\tilde{\alpha}} - \nu^u_{\tilde{\alpha}}$, and $\pi^u_{\tilde{\alpha}} = 1 - \mu^l_{\tilde{\alpha}} - \nu^l_{\tilde{\alpha}}$.

Example 1. If an alternative, A_i, with respect to attribute (u_j) is measured by an IVIFN $([\mu^l_{ij}, \mu^u_{ij}], [\nu^l_{ij}, \nu^u_{ij}])$,then $[\mu^l_{ij}, \mu^u_{ij}]$ represents the satisfactory interval, $[\nu^l_{ij}, \nu^u_{ij}]$ represents the dissatisfactory interval, and $[\pi^l_{ij}, \pi^u_{ij}]$ represents the indeterminacy or hesitation interval.

Xu and Chen [63] introduced the following operations.

Definition 1. Let $\tilde{\alpha} = ([\mu^l_{\tilde{\alpha}}, \mu^u_{\tilde{\alpha}}], [\nu^l_{\tilde{\alpha}}, \nu^u_{\tilde{\alpha}}])$, and $\tilde{\beta} = ([\mu^l_{\tilde{\beta}}, \mu^u_{\tilde{\beta}}], [\nu^l_{\tilde{\beta}}, \nu^u_{\tilde{\beta}}])$ be two IVIFNs, and λ be a real number; then,

$$\tilde{\alpha} + \tilde{\beta} =([\mu^l_{\tilde{\alpha}} + \mu^l_{\tilde{\beta}} - \mu^l_{\tilde{\alpha}}\mu^l_{\tilde{\beta}}, \mu^u_{\tilde{\alpha}} + \mu^u_{\tilde{\beta}} - \mu^u_{\tilde{\alpha}}\mu^u_{\tilde{\beta}}], [\nu^l_{\tilde{\alpha}}\nu^l_{\tilde{\beta}}, \nu^u_{\tilde{\alpha}}\nu^u_{\tilde{\beta}}]);$$

$$\lambda\tilde{\alpha} = ([1 - (1 - \mu^l_{\tilde{\alpha}})^\lambda, 1 - (1 - \mu^u_{\tilde{\alpha}})^\lambda], [(\nu^l_{\tilde{\alpha}})^\lambda, (\nu^u_{\tilde{\alpha}})^\lambda]), \quad \lambda > 0;$$

$$\tilde{\alpha}^c = ([\nu^l_{\tilde{\alpha}}, \nu^u_{\tilde{\alpha}}], [\mu^l_{\tilde{\alpha}}, \mu^u_{\tilde{\alpha}}]),$$

where $\tilde{\alpha}^c$ is the complement of $\tilde{\alpha}$.

Definition 2. Let $\tilde{\alpha} = ([\mu^l_{\tilde{\alpha}}, \mu^u_{\tilde{\alpha}}], [\nu^l_{\tilde{\alpha}}, \nu^u_{\tilde{\alpha}}])$ and $\tilde{\beta} = ([\mu^l_{\tilde{\beta}}, \mu^u_{\tilde{\beta}}], [\nu^l_{\tilde{\beta}}, \nu^u_{\tilde{\beta}}])$ be two IVIFNs, then:

$$Proj_{\tilde{\beta}}(\tilde{\alpha}) = \frac{\tilde{\alpha} \cdot \tilde{\beta}}{|\tilde{\beta}|} \tag{2}$$

is called the projection of $\tilde{\alpha}$ on $\tilde{\beta}$, where $\tilde{\alpha} \cdot \tilde{\beta} = \mu^l_{\tilde{\alpha}}\mu^l_{\tilde{\beta}} + \mu^u_{\tilde{\alpha}}\mu^u_{\tilde{\beta}} + \nu^l_{\tilde{\alpha}}\nu^l_{\tilde{\beta}} + \nu^u_{\tilde{\alpha}}\nu^u_{\tilde{\beta}} + \pi^l_{\tilde{\alpha}}\pi^l_{\tilde{\beta}} + \pi^u_{\tilde{\alpha}}\pi^u_{\tilde{\beta}}$ is the inner/scalar product between α and β; $|\tilde{\beta}| = ((\mu^l_{\tilde{\beta}})^2 + (\mu^u_{\tilde{\beta}})^2 + (\nu^l_{\tilde{\beta}})^2 + (\nu^u_{\tilde{\beta}})^2 + (\pi^l_{\tilde{\beta}})^2 + (\pi^u_{\tilde{\beta}})^2)^{1/2}$ is the module of $\tilde{\beta}$; and by Eq. (1), $\pi^l = 1 - \mu^u - \nu^u, \pi^u = 1 - \mu^l - \nu^l, \pi^u_{\tilde{\alpha}} = 1 - \mu^l_{\tilde{\alpha}} - \nu^l_{\tilde{\alpha}}, \pi^l_{\tilde{\beta}} = 1 - \mu^u_{\tilde{\beta}} - \nu^u_{\tilde{\beta}}$, and $\pi^u_{\tilde{\beta}} = 1 - \mu^l_{\tilde{\beta}} - \nu^l_{\tilde{\beta}}$.

In general, the larger the value of $Proj_{\tilde{\beta}}(\tilde{\alpha})$, the closer $\tilde{\alpha}$ is to $\tilde{\beta}$.

Definition 3. Let $X = (\tilde{x}_{ij})_{m\times n}$ be a matrix. If all $\tilde{x}_{ij}$ elements are IVIFNs, then X is called an interval-valued intuitionistic fuzzy matrix.

Definition 4. Let $X = (([\mu^l_{ij}, \mu^u_{ij}], [\nu^l_{ij}, \nu^u_{ij}]))_{m\times n}$ be an interval-valued intuitionistic fuzzy matrix, then:

$$X^c = (([\nu^l_{ij}, \nu^u_{ij}], [\mu^l_{ij}, \mu^u_{ij}]))_{m\times n} \tag{3}$$

is called the complement of X.

Similar to Eq. (2), we have the projection of an interval-valued intuitionistic fuzzy matrix on another one, as the following definition shows.

Definition 5. Let $X = (([\mu^l_{ij}, \mu^u_{ij}], [\nu^l_{ij}, \nu^u_{ij}]))_{m\times n}$ and $Y = (([\xi^l_{ij}, \xi^u_{ij}], [\eta^l_{ij}, \eta^u_{ij}]))_{m\times n}$ be two interval-valued intuitionistic fuzzy matrices; then:

$$Proj_Y(X) = \frac{XY}{|Y|} \tag{4}$$

is called the projection of X on Y, where:

$$XY = \sum_{i=1}^{m}\sum_{j=1}^{n}(\mu^l_{ij}\xi^l_{ij} + \mu^u_{ij}\xi^u_{ij} + \nu^l_{ij}\eta^l_{ij} + \nu^u_{ij}\eta^u_{ij} + \pi^l_{ij}\rho^l_{ij} + \pi^u_{ij}\rho^u_{ij}),$$

and:

$$|Y| =(\sum_{i=1}^{m}\sum_{j=1}^{n}(\xi_{ij}^l)^2 + (\xi_{ij}^u)^2 + (\eta_{ij}^l)^2 + (\eta_{ij}^u)^2 + (\rho_{ij}^l)^2 + (\rho_{ij}^u)^2)^{1/2}.$$

By Eq. (1):

$$\pi_{ij}^l = 1 - \mu_{ij}^u - \nu_{ij}^u,$$

$$\pi_{ij}^u = 1 - \mu_{ij}^l - \nu_{ij}^l,$$

$$\rho_{ij}^l = 1 - \xi_{ij}^u - \eta_{ij}^u,$$

and:

$$\rho_{ij}^u = 1 - \xi_{ij}^l - \eta_{ij}^l (i \in M, j \in N).$$

5. Methodology

For convenience, the terminologies and notations are explained as follows:

1. *Alternative, i.e., evaluation object.* A set of m feasible alternatives is written as $A = \{A_1, A_2, \cdots, A_i, \cdots, A_m\}$, and $i \in M = \{1, 2, \ldots, m\}$;
2. *Criterion, i.e., evaluation attribute.* A set of criteria is written as $U = \{u_1, u_2, \cdots, u_j, \cdots, u_n\}$, and $j \in N = \{1, 2, \ldots, n\}$;
3. *Weight of criterion, i.e., importance of criterion.* A weight vector of criteria is written as $w = (w_1, w_2, \cdots, w_j, \cdots, w_n)$, with $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$;
4. *DM, i.e., expert, who takes part in decision process.* A set of DMs is written as $D = \{d_1, d_2, \cdots, d_k, \cdots, d_t\}$, and $k \in T = \{1, 2, \ldots, t\}$.

Suppose that the tested mobile brands are regarded as alternatives $A_i (i \in M)$, and the respondents or participants are regarded as DMs $d_k (k \in T)$. Each d_k represents a class (group) of participants and s_k is the total number of DMs d_k.

For convenience, we divide each class d_k into l subclasses $H = \{1, 2, \ldots, l\}$ according to the educational background of respondents. Moreover let s_k^{ih} be the total number of d_k with hth $(h \in H)$ educational background; they evaluate the ith alternative. The symbols $\surd$ and $\times$ are respectively collected from criteria $u_j (j \in N)$ in the questionnaires. The number of symbols $\surd$ is written as m_{kj}^{ih}, and the number of symbols $\times$ is written as n_{kj}^{ih}. To obtain an IVIFN from the number of symbols $\surd$ and $\times$, we first model them by:

$$\xi_{kj}^{ih} = \frac{m_{kj}^{ih}}{s_k^{ih}}, \qquad \eta_{kj}^{ih} = \frac{n_{kj}^{ih}}{s_k^{ih}},$$

$$i \in M, \quad j \in N, \quad k \in T, \quad h \in H. \tag{5}$$

Then, an interval-valued intuitionistic fuzzy score is obtained as an IVIFN as follows:

$$x_{kj}^i = ([\mu_{kj}^{il}, \mu_{kj}^{iu}], [\nu_{kj}^{il}, \nu_{kj}^{iu}]),$$

$$i \in M, \quad j \in N, \quad k \in T, \tag{6}$$

where:

$$\mu_{kj}^{il} = \xi_{kj}^{il}/\sigma_{kj}^{iu}, \qquad \mu_{kj}^{iu} = \xi_{kj}^{iu}/\sigma_{kj}^{iu},$$

$$\nu_{kj}^{il} = \eta_{kj}^{il}/\sigma_{kj}^{iu}, \qquad \nu_{kj}^{iu} = \eta_{kj}^{iu}/\sigma_{kj}^{iu},$$

and:

$$\sigma_{kj}^{iu} = \xi_{kj}^{iu} + \eta_{kj}^{iu}, \qquad \xi_{kj}^{il} = \min_{h \in H}\{\xi_{kj}^{ih}\},$$

$$\xi_{kj}^{iu} = \max_{h \in H}\{\xi_{kj}^{ih}\}, \qquad \eta_{kj}^{il} = \min_{h \in H}\{\eta_{kj}^{ih}\},$$

$$\eta_{kj}^{iu} = \max_{h \in H}\{\eta_{kj}^{ih}\}.$$

It is clear that μ_{kj}^{iu} and ν_{kj}^{iu} in Eq. (6) satisfy the condition $\mu_{kj}^{iu} + \nu_{kj}^{iu} \leq 1$ $(i \in M, j \in N, k \in T)$ shown in Eq. (1).

A GDM problem with t DMs, m alternatives, and n criteria can be characterized by the following interval-valued intuitionistic fuzzy matrix:

$$X_i = (x_{kj}^i)_{t \times n}, i \in M, \tag{7}$$

where x_{kj}^i are shown in Eq. (6)

The aim of GDM is to rank the alternatives A_i $(i \in M)$ according to X_i $(i \in M)$.

Suppose that $w = (w_1, w_2, \ldots, w_n)$ is the weight vector of criteria; then:

$$Y_i = (y_{kj}^i)_{t \times n}, \quad i \in M, \quad k \in T, \quad j \in N, \tag{8}$$

is the weighted alternative decision of X_i, where $y_{kj}^i = w_j x_{kj}^i = ([\tau_{kj}^{il}, \tau_{kj}^{iu}], [\upsilon_{kj}^{il}, \upsilon_{kj}^{iu}])$, and $\tau_{kj}^{il} = 1 - (1 - \mu_{kj}^{il})^{w_j}, \tau_{kj}^{iu} = 1 - (1 - \mu_{kj}^{iu})^{w_j}, \upsilon_{kj}^{il} = (\nu_{kj}^{il})^{w_j}, \upsilon_{kj}^{iu} = (\nu_{kj}^{iu})^{w_j} (i \in M, k \in T, j \in N)$ by Definition 1.

According to the TOPSIS technique, we let:

$$Y_+ = (y_{kj}^+)_{t \times n}, \tag{9}$$

be the Positive Ideal Decision (PID) among all Y_i $(i \in M)$, where, the $y_{kj}^+ = ([\tau_{kj}^{+l}, \tau_{kj}^{+u}], [\upsilon_{kj}^{+l}, \upsilon_{kj}^{+u}])$ and $\tau_{kj}^{+l} = \max_{i \in M}\{\tau_{kj}^{il}\}, \tau_{kj}^{+u} = \max_{i \in M}\{\tau_{kj}^{iu}\}, \upsilon_{kj}^{+l} = \min_{i \in M}\{\upsilon_{kj}^{il}\}$ and $\upsilon_{kj}^{+u} = \min_{i \in M}\{\upsilon_{kj}^{iu}\}$ $(k \in T, j \in N)$.

A Negative Ideal Decision (NID) should have the maximum separation from the PID, so we let:

$$Y_- = (y_{kj}^-)_{t \times n}, \tag{10}$$

be an NID of all Y_i ($i \in M$), where $y^-_{kj} = ([\tau^{-l}_{kj}, \tau^{-u}_{kj}], [\upsilon^{-l}_{kj}, \upsilon^{-u}_{kj}])$, and $\tau^{-l}_{kj} = \min_{i\in M}\{\tau^{il}_{kj}\}, \tau^{-u}_{kj} = \min_{i\in M}\{\tau^{iu}_{kj}\}, \upsilon^{-l}_{kj} = \max_{i\in M}\{\upsilon^{il}_{kj}\}$ and $\upsilon^{-u}_{kj} = \max_{i\in M}\{\upsilon^{iu}_{kj}\}$ ($k \in T, j \in N$).

In addition, the complement $(Y_+)^c$ of Y_+ should have the maximum separation from Y^+, so we let:

$$Y_c = (Y_+)^c \tag{11}$$

be another NID of all Y_i ($i \in M$), where $(Y_+)^c = ((y^+_{kj})^c)_{t\times n}$ and $(y^+_{kj})^c = ([\tau^{+l}_{kj}, \tau^{+u}_{kj}], [\upsilon^{+l}_{kj}, \upsilon^{+u}_{kj}])^c = ([\upsilon^{+l}_{kj}, \upsilon^{+u}_{kj}], [\tau^{+l}_{kj}, \tau^{+u}_{kj}])$ ($k \in T, j \in N$) by Definition 1.

By Eq. (4), the separation of each decision, Y_i, from its PID Y_+, S^+_i, is given by the projection between Y_i and Y_+ as follows:

$$S^+_i = Proj_{Y_+}(Y_i), \quad i \in M, \tag{12}$$

where:

$$Proj_{Y_+}(Y_i) = Y_iY_+/|Y_+| = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tau^{il}_{kj}\tau^{+l}_{kj} + \tau^{iu}_{kj}\tau^{+u}_{kj} + \upsilon^{il}_{kj}\upsilon^{+l}_{kj} + \upsilon^{iu}_{kj}\upsilon^{+u}_{kj} + \pi^{il}_{kj}\pi^{+l}_{kj} + \pi^{iu}_{kj}\pi^{+u}_{kj})/(\sum_{i=1}^{m}\sum_{j=1}^{n}((\tau^{+l}_{kj})^2 + (\tau^{+u}_{kj})^2 + (\upsilon^{+l}_{kj})^2 + (\upsilon^{+u}_{kj})^2 + (\pi^{+l}_{kj})^2 + (\pi^{+u}_{kj})^2))^{1/2},$$

and: $\tau^{+l}_{kj}, \tau^{+u}_{kj}, \upsilon^{+l}_{kj}$, and υ^{+u}_{kj} are same as in Eq. (9), $\pi^{il}_{kj} = 1-\tau^{iu}_{kj}-\upsilon^{iu}_{kj}, \pi^{iu}_{kj} = 1-\tau^{il}_{kj}-\upsilon^{il}_{kj}, \pi^{+l}_{kj} = 1-\tau^{+u}_{kj}-\upsilon^{+u}_{kj}$ and $\pi^{+u}_{kj} = 1-\tau^{+l}_{kj}-\upsilon^{+l}_{kj} (i \in M, k \in T, j \in N)$ by Eq. (1).

Similarly, the separations of each Y_i from its NIDs Y_- and Y_c, S^-_i and S^c_i are given by:

$$S^-_i = Proj_{Y_-}(Y_i), S^c_i = Proj_{Y_c}(Y_i), i \in M, \tag{13}$$

where:

$$Proj_{Y_-}(Y_i) = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tau^{il}_{kj}\tau^{-l}_{kj} + \tau^{iu}_{kj}\tau^{-u}_{kj} + \upsilon^{il}_{kj}\upsilon^{-l}_{kj} + \upsilon^{iu}_{kj}\upsilon^{-u}_{kj} + \pi^{il}_{kj}\pi^{-l}_{kj} + \pi^{iu}_{kj}\pi^{-u}_{kj})/(\sum_{i=1}^{m}\sum_{j=1}^{n}((\tau^{-l}_{kj})^2 + (\tau^{-u}_{kj})^2 + (\upsilon^{-l}_{kj})^2 + (\upsilon^{-u}_{kj})^2 + (\pi^{-l}_{kj})^2 + (\pi^{-u}_{kj})^2))^{1/2},$$

$$Proj_{Y_c}(Y_i) = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tau^{il}_{kj}\upsilon^{+l}_{kj} + \tau^{iu}_{kj}\upsilon^{+u}_{kj} + \upsilon^{il}_{kj}\tau^{+l}_{kj} + \upsilon^{iu}_{kj}\tau^{+u}_{kj} + \pi^{il}_{kj}\pi^{+l}_{kj} + \pi^{iu}_{kj}\pi^{+u}_{kj})/(\sum_{i=1}^{m}\sum_{j=1}^{n}((\tau^{+l}_{kj})^2 + (\tau^{+u}_{kj})^2 + (\upsilon^{+l}_{kj})^2 + (\upsilon^{+u}_{kj})^2 + (\pi^{+l}_{kj})^2 + (\pi^{+u}_{kj})^2))^{1/2},$$

$\tau^{-l}_{kj}, \tau^{-u}_{kj}, \upsilon^{-l}_{kj}$ and υ^{-u}_{kj} are same as in Eq. (10); $\pi^{il}_{kj}, \pi^{iu}_{kj}, \pi^{+l}_{kj}$ and π^{+u}_{kj} are same as in Eq. (12); and $\pi^{-l}_{kj} = 1-\tau^{-u}_{kj}-\upsilon^{-u}_{kj}$ and $\pi^{-u}_{kj} = 1-\tau^{-l}_{kj}-\upsilon^{-l}_{kj} (i \in M, k \in T, j \in N)$ by Eq. (1).

For each decision Y_i, an extended relative closeness in TOPSIS technique is calculated by Yue [65]:

$$RC_i = \frac{S^+_i}{S^+_i + S^c_i + S^-_i}, \quad i \in M. \tag{14}$$

6. Evaluation procedure

6.1. Identification of alternatives, criteria and decision makers

In this subsection, we first determine the alternatives, criteria, DMs, and questionnaire. As aforementioned, here, three brands of top-selling smartphones are considered as alternatives, which are available to Chinese consumers. That is to say, the set of evaluation objects is $A = \{A_1, A_2, A_3\}$={brand 1, brand 2, brand 3}. The survey of customer satisfaction is randomly sampled from the current smartphone users. Evaluation criteria in this paper are based on the literature [66], which are $U = \{u_1, u_2, u_3\}$={customer equity, product function, mobile convenience}. The DMs are respondents (evaluators) in this model. A DM is a group of respondents classified based on their age grades. More specifically, the set of DMs is $D = \{d_1, d_2, d_3, d_4\}$={the respondents younger than 20, the respondents between the ages of 21 and 35, the respondents between the ages of 36 and 50, the respondents older than 50}. The survey questionnaire is applied especially to consumers who are users in Guangdong, China.

Application data are gathered from 8705 customers who are using at least one of three mobile brands; here, they are the DMs.

The questionnaire is shown in Table 1, in which each option □ is marked by the respondent with only one of the three $\surd, \times, \bigcirc$ symbols. The questionnaires are collected through the Internet, e-mail, and phone. The data collection, aggregation, and measurement are elaborated on in the following section.

6.2. Data collection, aggregation and measurement

The detailed descriptions are shown in the following steps:

Step 1. Collection data and statistics.

Table 1. Questionnaire for three smartphone brands.

Brand	Age	Option	Education	Option	Criterion	Option
Brand 1	<25	□	A	□	Customer equity	□
	26-40	□	B	□	Product function	□
	41-55	□	C	□	Mobile convenience	□
	>56	□	D	□		
Brand 2	<25	□	A	□	Customer equity	□
	26-40	□	B	□	Product function	□
	41-55	□	C	□	Mobile convenience	□
	>56	□	D	□		
Brand 3	<25	□	A	□	Customer equity	□
	26-40	□	B	□	Product function	□
	41-55	□	C	□	Mobile convenience	□
	>56	□	D	□		

Notes: A: Junior high school; B: High school or less; C: Some college; D: College degree or more.

For the tested mobile brand, A_i $(i = 1, 2, 3)$, data collection is conducted by 50 professionals. The symbols $\surd$ and $\times$ are respectively collected from three criteria $\{u_j | j = 1, 2, 3\}$ of questionnaires and the other forms. The number of symbols $\surd$ is written as m_{kj}^{ih}, and the number of symbols $\times$ is written as n_{kj}^{ih}. The statistics are shown in Table 2.

The information $\bigcirc$ is considered same as the nonresponse in statistics. Table 2 shows that the symbol $\bigcirc$ has not been included in the statistical information. However, it is not neglected. In fact, it may show that the respondent does not neglect this evaluation criterion; however, they consider it hesitantly or with abstention. For convenience, the information $\bigcirc$ and the nonresponse will be quantified and aggregated into hesitancy degrees π_{kj}^{il} and π_{kj}^{iu} in Eqs. (12) and (13), although they are not completely equivalent to hesitancy.

Example 2. Table 2 shows that the set related to symbol "$\surd$" is $\{m_{11}^{11}, m_{11}^{12}, m_{11}^{13}, m_{11}^{14}\} = \{85, 78, 40, 41\}$; and the set related to symbol "$\times$" is $\{n_{11}^{11}, n_{11}^{12}, n_{11}^{13}, n_{11}^{14}\} = \{66, 176, 115, 76\}$. If $s_1^{11} = s_1^{12} = s_1^{13} = s_1^{14}$, we have the satisfactory interval $[40, 85]$ and dissatisfactory interval $[66, 176]$. A comprehensive value $([40, 85], [66, 176])$ is obtained, which is given by the 1st DM d_1 for the 1st alternative, A_1, with respect to the 1st criterion u_1.

Step 2. Normalize the data.

All data in Table 2 are normalized by Eq. (5), in which s_{kj}^{ih}, m_{kj}^{ih} and n_{kj}^{ih} are shown in Table 2; and $i, j = 1, 2, 3, k, h = 1, 2, 3, 4$. The normalized data are shown in Table 3.

To understand Eq. (5), the following example gives a specific calculation.

Example 3. For the comprehensive value $([40, 85], [66, 176])$ in Example 2, if the numbers $s_1^{1h} (h = 1, 2, 3, 4)$ are unequal, then the set related to symbol "$\surd$" can be transformed by Eq. (5) to:

$$\{m_{11}^{11}/s_1^{11}, m_{11}^{12}/s_1^{12}, m_{11}^{13}/s_1^{13}, m_{11}^{14}/s_1^{14}\}$$

$$= \{85/250, 78/280, 40/241, 41/165\}$$

$$= \{0.3400, 0.2786, 0.1660, 0.2485\}.$$

The set related to symbol "$\times$" can be transformed by Eq. (5) to:

$$\{n_{11}^{11}/s_1^{11}, n_{11}^{12}/s_1^{12}, n_{11}^{13}/s_1^{13}, n_{11}^{14}/s_1^{14}\}$$

$$= \{66/250, 176/280, 115/241, 76/165\}$$

$$= \{0.2640, 0.6286, 0.4772, 0.4606\}.$$

We have the satisfactory interval:

$$[\xi_{11}^{1l}, \xi_{11}^{1u}] = [\min\{0.3400, 0.2786, 0.1660, 0.2485\},$$

$$\max\{0.3400, 0.2786, 0.1660, 0.2485\}]$$

$$= [0.1660, 0.3400],$$

and dissatisfactory interval:

$$[\eta_{11}^{1l}, \eta_{11}^{1u}] = [\min\{0.2640, 0.6286, 0.4772, 0.4606\},$$

$$\max\{0.2640, 0.6286, 0.4772, 0.4606\}] =$$

$$[0.2640, 0.6286].$$

Table 2. Statistics of assessment information

Brand	DM	Education	s_k^{ih}	u_1		u_2		u_3	
				$m_{k1}^{ih}(\surd)$	$n_{k1}^{ih}(\times)$	$m_{k2}^{ih}(\surd)$	$n_{k2}^{ih}(\times)$	$m_{k3}^{ih}(\surd)$	$n_{k3}^{ih}(\times)$
A_1	d_1	A	250	85	66	80	122	104	129
		B	280	78	176	92	82	120	145
		C	241	40	115	132	107	30	110
		D	165	41	76	61	98	42	78
	d_2	A	190	81	80	84	74	88	85
		B	210	78	75	94	77	132	69
		C	258	125	124	132	88	84	130
		D	262	54	88	65	72	66	89
	d_3	A	190	58	87	88	78	99	83
		B	216	124	80	87	77	95	84
		C	250	89	105	127	99	120	120
		D	162	68	68	70	73	75	77
	d_4	A	240	89	84	126	108	78	85
		B	238	83	93	87	131	98	126
		C	216	70	112	137	79	67	128
		D	192	61	89	78	58	92	86
A_2	d_1	A	250	89	152	67	93	30	55
		B	230	83	76	86	132	66	79
		C	270	132	130	80	135	102	76
		D	170	76	84	68	78	43	94
	d_2	A	241	85	62	80	122	104	127
		B	268	78	180	92	72	120	135
		C	239	40	125	132	97	30	110
		D	176	41	61	61	95	42	74
	d_3	A	245	90	60	86	120	110	125
		B	268	81	177	93	69	125	133
		C	246	36	120	137	99	25	108
		D	168	41	59	65	93	37	72
	d_4	A	190	59	57	89	88	92	85
		B	190	81	74	83	93	88	91
		C	188	52	126	70	104	73	102
		D	183	83	90	61	85	61	82
A_3	d_1	A	190	81	79	84	70	88	84
		B	213	78	73	94	76	132	65
		C	260	125	119	132	85	84	131
		D	158	54	87	65	72	66	83
	d_2	A	180	56	85	86	76	97	80
		B	213	122	79	89	75	93	81
		C	260	87	102	129	97	122	118
		D	158	66	67	68	70	73	76
	d_3	A	250	86	18	66	96	27	58
		B	223	80	79	83	135	63	82
		C	270	129	132	77	138	99	79
		D	178	73	87	65	80	38	91
	d_4	A	240	126	107	76	83	57	63
		B	233	87	131	96	124	78	77
		C	216	137	76	65	126	48	129
		D	188	78	58	90	81	80	93

Notes: A, B, C and D are same as in Table 1.

Table 3. Normalization of assessment information.

Brand	DM	Education and [min, max]	u_1 $\xi_{k1}^{ih}(\checkmark)$	u_1 $\eta_{k1}^{ih}(\times)$	u_2 $\xi_{k2}^{ih}(\checkmark)$	u_2 $\eta_{k2}^{ih}(\times)$	u_3 $\xi_{k3}^{ih}(\checkmark)$	u_3 $\eta_{k3}^{ih}(\times)$
A_1	d_1	A	0.3400	0.2640	0.3200	0.4880	0.4160	0.5160
		B	0.2786	0.6286	0.3286	0.2929	0.4286	0.5179
		C	0.1660	0.4772	0.5477	0.4440	0.1245	0.4564
		D	0.2485	0.4606	0.3697	0.5939	0.2545	0.4727
		[min, max]	[0.1660, 0.3400]	[0.2640, 0.6286]	[0.3200, 0.5477]	[0.2929, 0.5939]	[0.1245, 0.4286]	[0.4564, 0.5179]
	d_2	A	0.4263	0.4211	0.4421	0.3895	0.4632	0.4474
		B	0.3714	0.3571	0.4476	0.3667	0.6286	0.3286
		C	0.4845	0.4806	0.5116	0.3411	0.3256	0.5039
		D	0.2061	0.5333	0.2481	0.4364	0.2519	0.5394
		[min, max]	[0.2061, 0.4845]	[0.3571, 0.5333]	[0.2481, 0.5116]	[0.3411, 0.4364]	[0.2519, 0.6286]	[0.3286, 0.5394]
	d_3	A	0.3053	0.4579	0.4632	0.4105	0.5211	0.4368
		B	0.5741	0.3704	0.4028	0.3565	0.4398	0.3889
		C	0.3560	0.4200	0.5080	0.3960	0.4800	0.4800
		D	0.4198	0.4198	0.4321	0.4506	0.4630	0.4753
		[min, max]	[0.3053, 0.5741]	[0.3704, 0.4579]	[0.4028, 0.5080]	[0.3565, 0.4506]	[0.4398, 0.5211]	[0.3889, 0.4800]
	d_4	A	0.3708	0.3500	0.5250	0.4500	0.3250	0.3542
		B	0.3487	0.3908	0.3655	0.5504	0.4118	0.5294
		C	0.3241	0.5185	0.6343	0.3657	0.3102	0.5926
		D	0.3177	0.4635	0.4063	0.3021	0.4792	0.4479
		[min, max]	[0.3177, 0.3708]	[0.3500, 0.5185]	[0.3655, 0.6343]	[0.3021, 0.5504]	[0.3102, 0.4792]	[0.3542, 0.5926]
A_2	d_1	A	0.3560	0.6080	0.2680	0.3720	0.1200	0.2200
		B	0.3609	0.3304	0.3739	0.5739	0.2870	0.3435
		C	0.4889	0.4815	0.2963	0.5000	0.3778	0.2815
		D	0.4471	0.4941	0.4000	0.4588	0.2529	0.5529
		[min, max]	[0.3560, 0.4889]	[0.3304, 0.6080]	[0.2680, 0.4000]	[0.3720, 0.5739]	[0.1200, 0.3778]	[0.2200, 0.5529]
	d_2	A	0.3527	0.2573	0.3320	0.5062	0.4315	0.5270
		B	0.2910	0.6716	0.3433	0.2687	0.4478	0.5037
		C	0.1674	0.5230	0.5523	0.4059	0.1255	0.4603
		D	0.2330	0.3466	0.3466	0.5398	0.2386	0.4205
		[min, max]	[0.1674, 0.3527]	[0.2573, 0.6716]	[0.3320, 0.5523]	[0.2687, 0.5398]	[0.1255, 0.4478]	[0.4205, 0.5270]
	d_3	A	0.3673	0.2449	0.3510	0.4898	0.4490	0.5102
		B	0.3022	0.6604	0.3470	0.2575	0.4664	0.4963
		C	0.1463	0.4878	0.5569	0.4024	0.1016	0.4390
		D	0.2440	0.3512	0.3869	0.5536	0.2202	0.4286
		[min, max]	[0.1463, 0.3673]	[0.2449, 0.6604]	[0.3470, 0.5569]	[0.2575, 0.5536]	[0.1016, 0.4664]	[0.4286, 0.5102]
	d_4	A	0.3105	0.3000	0.4684	0.4632	0.4842	0.4474
		B	0.4263	0.3895	0.4368	0.4895	0.4632	0.4789
		C	0.2766	0.6702	0.3723	0.5532	0.3883	0.5426
		D	0.4536	0.4918	0.3333	0.4645	0.3333	0.4481
		[min, max]	[0.2766, 0.4536]	[0.3000, 0.6702]	[0.3333, 0.4684]	[0.4632, 0.5532]	[0.3333, 0.4842]	[0.4474, 0.5426]
A_3	d_1	A	0.4263	0.4158	0.4421	0.3684	0.4632	0.4421
		B	0.3662	0.3427	0.4413	0.3568	0.6197	0.3052
		C	0.4808	0.4577	0.5077	0.3269	0.3231	0.5038
		D	0.3418	0.5506	0.4114	0.4557	0.4177	0.5253
		[min, max]	[0.3418, 0.4808]	[0.3427, 0.5506]	[0.4114, 0.5077]	[0.3269, 0.4557]	[0.3231, 0.6197]	[0.3052, 0.5253]
	d_2	A	0.3111	0.4722	0.4778	0.4222	0.5389	0.4444
		B	0.5728	0.3709	0.4178	0.3521	0.4366	0.3803
		C	0.3346	0.3923	0.4962	0.3731	0.4692	0.4538
		D	0.4177	0.4241	0.4304	0.4430	0.4620	0.4810
		[min, max]	[0.3111, 0.5728]	[0.3709, 0.4722]	[0.4178, 0.4962]	[0.3521, 0.4430]	[0.4366, 0.5389]	[0.3803, 0.4810]
	d_3	A	0.3440	0.0720	0.2640	0.3840	0.1080	0.2320
		B	0.3587	0.3543	0.3722	0.6054	0.2825	0.3677
		C	0.4778	0.4889	0.2852	0.5111	0.3667	0.2926
		D	0.4101	0.4888	0.3652	0.4494	0.2135	0.5112
		[min, max]	[0.3440, 0.4778]	[0.0720, 0.4889]	[0.2640, 0.3722]	[0.3840, 0.6054]	[0.1080, 0.3667]	[0.2320, 0.5112]
	d_4	A	0.5250	0.4458	0.3167	0.3458	0.2375	0.2625
		B	0.3734	0.5622	0.4120	0.5322	0.3348	0.3305
		C	0.6343	0.3519	0.3009	0.5833	0.2222	0.5972
		D	0.4149	0.3085	0.4787	0.4309	0.4255	0.4947
		[min, max]	[0.3734, 0.6343]	[0.3085, 0.5622]	[0.3009, 0.4787]	[0.3458, 0.5833]	[0.2222, 0.4255]	[0.2625, 0.5972]

Notes: A, B, C and D are same as in Table 1.

A comprehensive value:

$$([\xi_{11}^{1l},\xi_{11}^{1u}],[\eta_{11}^{1l},\eta_{11}^{1u}]) = ([0.1660, 0.3400], [0.2640, 0.6286])$$

is obtained, which is given by d_1 for A_1 with respect to u_1.

Step 3. Determine the interval-valued intuitionistic fuzzy information of assessment criteria.

For each assessment criterion, u_j, of alternative A_i with respect to d_k, the evaluation value x_{kj}^i, calculated by Eq. (6), is shown in Table 4, where $M = N = \{1,2,3\}$, and $T = \{1,2,3,4\}$.

To understand Eq. (6), the following example gives a specific calculation.

Example 4. For the mentioned comprehensive value:

$$([\xi_{11}^{1l},\xi_{11}^{1u}],[\eta_{11}^{1l},\eta_{11}^{1u}]) =([0.1660, 0.3400], [0.2640, 0.6286]).$$

in Example 3, the condition $\xi_{11}^{1u} + \eta_{11}^{1u} \leq 1$ is not always satisfied. Therefore, we must transform it to satisfy the condition of an IVIFN in Eq. (1). By Eq. (6), we have:

$$\sigma_{11}^{1u} = \xi_{11}^{1u} + \eta_{11}^{1u} = 0.3400 + 0.6286 = 0.9686.$$

So:

$$\mu_{11}^{1l} = \xi_{11}^{1l}/\sigma_{11}^{1u} = 0.1660/0.9686 = 0.1714,$$

$$\mu_{11}^{1u} = \xi_{11}^{1u}/\sigma_{11}^{1u} = 0.3400/0.9686 = 0.3510,$$

$$\nu_{11}^{1l} = \eta_{11}^{1l}/\sigma_{11}^{1u} = 0.2640/0.9686 = 0.2726,$$

$$\nu_{11}^{1u} = \eta_{11}^{1u}/\sigma_{11}^{1u} = 0.6286/0.9686 = 0.6490.$$

Therefore:

$$x_{11}^1 =([\mu_{11}^{1l},\mu_{11}^{1u}],[\nu_{11}^{1l},\nu_{11}^{1u}]) = ([0.1714, 0.3510], [0.2726, 0.6490]),$$

in X_1, which is shown in Table 4.

Step 4. Construct the weighted decision matrices.

The weight vector of the criteria given here is based on the literature [66], which is:

$$w = (w_1, w_2, w_3) = (0.3490, 0.3570, 0.2940).$$

The weighted decision matrices Y_i are constructed by Eq. (8) and shown in Table 5, in which $M = N = \{1,2,3\}$ and $T = \{1,2,3,4\}$.

Table 4. Assessment matrices with interval-valued intuitionistic fuzzy information

Matrix	DM	u_1	u_2	u_3
X_1	d_1	([0.1714, 0.3510], [0.2726, 0.6490])	([0.2803, 0.4798], [0.2565, 0.5202])	([0.1315, 0.4528], [0.4823, 0.5472])
	d_2	([0.2025, 0.4760], [0.3509, 0.5240])	([0.2617, 0.5397], [0.3598, 0.4603])	([0.2157, 0.5382], [0.2813, 0.4618])
	d_3	([0.2958, 0.5563], [0.3589, 0.4437])	([0.4202, 0.5299], [0.3719, 0.4701])	([0.4394, 0.5205], [0.3885, 0.4795])
	d_4	([0.3572, 0.4170], [0.3935, 0.5830])	([0.3086, 0.5354], [0.2550, 0.4646])	([0.2894, 0.4471], [0.3305, 0.5529])
X_2	d_1	([0.3246, 0.4457], [0.3012, 0.5543])	([0.2752, 0.4107], [0.3820, 0.5893])	([0.1289, 0.4059], [0.2364, 0.5941])
	d_2	([0.1634, 0.3443], [0.2511, 0.6557])	([0.3040, 0.5057], [0.2460, 0.4943])	([0.1288, 0.4594], [0.4314, 0.5406])
	d_3	([0.1424, 0.3574], [0.2383, 0.6426])	([0.3125, 0.5015], [0.2318, 0.4985])	([0.1041, 0.4776], [0.4388, 0.5224])
	d_4	([0.2461, 0.4036], [0.2670, 0.5964])	([0.3263, 0.4585], [0.4534, 0.5415])	([0.3246, 0.4716], [0.4357, 0.5284])
X_3	d_1	([0.3314, 0.4661], [0.3323, 0.5339])	([0.4270, 0.5270], [0.3393, 0.4730])	([0.2822, 0.5412], [0.2665, 0.4588])
	d_2	([0.2977, 0.5481], [0.3549, 0.4519])	([0.4449, 0.5283], [0.3749, 0.4717])	([0.4281, 0.5284], [0.3729, 0.4716])
	d_3	([0.3559, 0.4943], [0.0745, 0.5057])	([0.2701, 0.3807], [0.3928, 0.6193])	([0.1230, 0.4177], [0.2643, 0.5823])
	d_4	([0.3121, 0.5301], [0.2578, 0.4699])	([0.2833 ,0.4508], [0.3256, 0.5492])	([0.2173, 0.4161], [0.2567, 0.5839])

Table 5. Weighted decision matrices with interval-valued intuitionistic fuzzy information

Matrix	DM	u_1	u_2	u_3
Y_1	d_1	([0.0635, 0.1401], [0.6353, 0.8599])	([0.1108, 0.2081], [0.6153, 0.7919])	([0.0406, 0.1625], [0.8070, 0.8375])
	d_2	([0.0759, 0.2019], [0.6938, 0.7981])	([0.1027, 0.2419], [0.6942, 0.7581])	([0.0689, 0.2032], [0.6888, 0.7968])
	d_3	([0.1152, 0.2469], [0.6993, 0.7531])	([0.1768, 0.2362], [0.7025, 0.7638])	([0.1564, 0.1943], [0.7573, 0.8057])
	d_4	([0.1429, 0.1716], [0.7222, 0.8284])	([0.1234, 0.2394], [0.6139, 0.7606])	([0.0956, 0.1599], [0.7221, 0.8401])
Y_2	d_1	([0.1280, 0.1861], [0.6579, 0.8139])	([0.1085, 0.1720], [0.7092, 0.8280])	([0.0398, 0.1419], [0.6544, 0.8581])
	d_2	([0.0604, 0.1370], [0.6174, 0.8630])	([0.1213, 0.2224], [0.6061, 0.7776])	([0.0397, 0.1654], [0.7810, 0.8346])
	d_3	([0.0522, 0.1430], [0.6062, 0.8570])	([0.1252, 0.2201], [0.5934, 0.7799])	([0.0318, 0.1738], [0.7849, 0.8262])
	d_4	([0.0939, 0.1650], [0.6307, 0.8350])	([0.1315, 0.1967], [0.7540, 0.8033])	([0.1090, 0.1710], [0.7833, 0.8290])
Y_3	d_1	([0.1311, 0.1967], [0.6808, 0.8033])	([0.1803, 0.2345], [0.6799, 0.7655])	([0.0929, 0.2047], [0.6779, 0.7953])
	d_2	([0.1160, 0.2421], [0.6966, 0.7579])	([0.1895, 0.2353], [0.7045, 0.7647])	([0.1515, 0.1983], [0.7482, 0.8017])
	d_3	([0.1423, 0.2117], [0.4040, 0.7883])	([0.1063, 0.1572], [0.7163, 0.8428])	([0.0379, 0.1470], [0.6762, 0.8530])
	d_4	([0.1224, 0.2317], [0.6231, 0.7683])	([0.1121, 0.1926], [0.6699, 0.8074])	([0.0695, 0.1463], [0.6704, 0.8537])

Table 6. Ideal decision among all weighted decisions.

Decision	DM	u_1	u_2	u_3
Y_+	d_1	([0.1311, 0.1967], [0.6353, 0.8033])	([0.1803, 0.2345], [0.6153, 0.7655])	([0.0929, 0.2047], [0.6544, 0.7953])
	d_2	([0.1160, 0.2421], [0.6174, 0.7579])	([0.1895, 0.2419], [0.6061, 0.7581])	([0.1515, 0.2032], [0.6888, 0.7968])
	d_3	([0.1423, 0.2469], [0.4040, 0.7531])	([0.1768, 0.2362], [0.5934, 0.7638])	([0.1564, 0.1943], [0.6762, 0.8057])
	d_4	([0.1429, 0.2317], [0.6231, 0.7683])	([0.1315, 0.2394], [0.6139, 0.7606])	([0.1090, 0.1710], [0.6704, 0.8290])
Y_-	d_1	([0.0635, 0.1401], [0.6808, 0.8599])	([0.1085, 0.1720], [0.7092, 0.8280])	([0.0398, 0.1419], [0.8070, 0.8581])
	d_2	([0.0604, 0.1370], [0.6966, 0.8630])	([0.1027, 0.2224], [0.7045, 0.7776])	([0.0397, 0.1654], [0.7810, 0.8346])
	d_3	([0.0522, 0.1430], [0.6993, 0.8570])	([0.1063, 0.1572], [0.7163, 0.8428])	([0.0318, 0.1470], [0.7849, 0.8530])
	d_4	([0.0939, 0.1650], [0.7222, 0.8350])	([0.1121, 0.1926], [0.7540, 0.8074])	([0.0695, 0.1463], [0.7833, 0.8537])
Y_c	d_1	([0.6353, 0.8033], [0.1311, 0.1967])	([0.6153, 0.7655], [0.1803, 0.2345])	([0.6544, 0.7953], [0.0929, 0.2047])
	d_2	([0.6174, 0.7579], [0.1160, 0.2421])	([0.6061, 0.7581], [0.1895, 0.2419])	([0.6888, 0.7968], [0.1515, 0.2032])
	d_3	([0.4040, 0.7531], [0.1423, 0.2469])	([0.5934, 0.7638], [0.1768, 0.2362])	([0.6762, 0.8057], [0.1564, 0.1943])
	d_4	([0.6231, 0.7683], [0.1429, 0.2317])	([0.6139, 0.7606], [0.1315, 0.2394])	([0.6704, 0.8290], [0.1090, 0.1710])

Table 7. Separation, relative closeness and ranking of smartphones.

Brand	S_i^+	Ranking	S_i^-	Ranking	S_i^c	Ranking	RC_i	Ranking
A_1	3.7918	2	3.4043	2	3.2352	2	0.3635	2
A_2	3.8400	1	3.4947	1	3.2278	3	0.3636	1
A_3	3.7771	3	3.3910	3	3.2819	1	0.3614	3

Step 5. Determine the ideal decisions.

For $Y_i (i \in M)$ in Eq. (8), the PID and NIDs are determined by Eqs. (9)-(11). The ideal decisions are shown in Table 6.

Step 6. Calculate the separations of each decision from its ideal decisions.

The separations of each decision Y_i from its ideal decisions are calculated by Eqs. (12) and (13), where $M = N = \{1, 2, 3\}$ and $T = \{1, 2, 3, 4\}$.

Step 7. Calculate the relative closeness.

For each decision Y_i, the relative closeness is calculated by Eq. (14).

Step 8. Rank the preference order of alternatives.

All alternatives are ranked in descending order in accordance with their relative closeness.

The separations, relative closeness, and ranking of alternatives are summarized in Table 7.

Table 7 shows that the order of customer satisfaction and loyalty for the three tested smartphone brands is as follows:

$A_2 \succ A_1 \succ A_3$.

As shown, A_2 is the best, followed by A_1, and the worst smartphone is A_3.

7. Experimental comparison with another measure

In this section, we show an experimental comparison of the projection measure with the Euclidean distance to illustrate the advantages of the introduced method in this paper.

Corresponding to Eq. (12), the separation of each decision Y_i from its PID Y_+, S_{Ei}^+, is replaced by the Euclidean distance as follows:

$$S_{Ei}^+ = d_E(Y_i, Y_+), i \in M, \tag{15}$$

where $d_E(Y_i, Y_+) = (\sum_{k=1}^t \sum_{j=1}^n ((\tau_{kj}^{il} - \tau_{kj}^{+l})^2 + (\tau_{kj}^{iu} - \tau_{kj}^{+u})^2 + (\upsilon_{kj}^{il} - \upsilon_{kj}^{+l})^2 + (\upsilon_{kj}^{iu} - \upsilon_{kj}^{+u})^2 + (\pi_{kj}^{il} - \pi_{kj}^{+l})^2 + (\pi_{kj}^{iu} - \pi_{kj}^{+u})^2))^{1/2}$.

Corresponding to Eq. (13), the separations of each decision Y_i from its NIDs Y_- and Y_c, S_{Ei}^- and S_{Ei}^c, are replaced by the Euclidean distance as follows:

$$S_{Ei}^- = d_E(Y_i, Y_-), S_{Ei}^c = d_E(Y_i, Y_c), i \in M, \tag{16}$$

where:

$$\begin{aligned} d_E(Y_i, Y_-) =& (\sum_{k=1}^t \sum_{j=1}^n ((\tau_{kj}^{il} - \tau_{kj}^{-l})^2 + (\tau_{kj}^{iu} - \tau_{kj}^{-u})^2 \\ & + (\upsilon_{kj}^{il} - \upsilon_{kj}^{-l})^2 + (\upsilon_{kj}^{iu} - \upsilon_{kj}^{-u})^2 \\ & + (\pi_{kj}^{il} - \pi_{kj}^{-l})^2 + (\pi_{kj}^{iu} - \pi_{kj}^{-u})^2))^{1/2} \end{aligned}$$

and:

$$\begin{aligned} d_E(Y_i, Y_c) =& (\sum_{k=1}^t \sum_{j=1}^n ((\tau_{kj}^{il} - \upsilon_{kj}^{+l})^2 + (\tau_{kj}^{iu} - \upsilon_{kj}^{+u})^2 \\ & + (\upsilon_{kj}^{il} - \tau_{kj}^{+l})^2 + (\upsilon_{kj}^{iu} - \tau_{kj}^{+u})^2 \\ & + (\pi_{kj}^{il} - \pi_{kj}^{+l})^2 + (\pi_{kj}^{iu} - \pi_{kj}^{+u})^2))^{1/2}. \end{aligned}$$

Eq. (14) is modified as follows [67]:

$$RC_{Ei} = \frac{S_{Ei}^- + S_{Ei}^c}{S_{Ei}^+ + S_{Ei}^- + S_{Ei}^c}, \ i \in M. \tag{17}$$

Table 8. Separation, relative closeness and ranking of smartphones based on the Euclidean distance.

Brand	S^{+}_{Ei}	Ranking	S^{-}_{Ei}	Ranking	S^{c}_{Ei}	Ranking	RC_{Ei}	Ranking
A_1	0.5794	3	0.4278	2	3.8980	2	0.8819	3
A_2	0.5463	2	0.4012	3	3.9862	1	0.8893	2
A_3	0.3708	1	0.6029	1	3.8143	3	0.9225	1

Next, Eqs. (15) and (16) are used to calculate the separations, which are shown in Table 8.

Table 8 displays that the ranking of three bands of smartphones is $A_3 \succ A_2 \succ A_1$, which is different from the ranking based on the projection measure in Table 7. To show which ranking is ideal, we now further investigate all the rankings based on the related measures. According to the literature [68], generally the greater the number of items in a ranking, the more its robustness, and the higher its credibility will be.

We first review the measures related to projection. It is noteworthy that $S^{+}_{(\cdot)}$ in Eq. (12) is also a measure, which can measure the alternatives Y_i. From Eqs. (2) and (3), we know that the larger the value of S^{+}_{i}, the better the alternative A_i is. Similarly, $S^{-}_{(\cdot)}$ and $S^{c}_{(\cdot)}$ in Eq. (13) are the measures to calculate Y_i. The smaller the values of S^{-}_{i} and/or S^{c}_{i}, the better the alternative A_i is. The rankings based on $S^{+}_{(\cdot)}$, $S^{-}_{(\cdot)}$ and $S^{c}_{(\cdot)}$ are also shown in Table 7.

Also, the $S^{+}_{E(\cdot)}$ in Eq. (15) is a measure. The smaller the value of S^{+}_{Ei}, the better alternative A_i is. Moreover, $S^{-}_{E(\cdot)}$ and $S^{c}_{E(\cdot)}$ in Eq. (16) are measures. The larger the values of $S^{-}_{E(\cdot)}$ and/or $S^{c}_{E(\cdot)}$, the better the alternative A_i is. The rankings based on Eq. (15) are also shown in Table 8.

To display the most preferred ranking, we list these rankings in Table 9.

Table 9 displays that the ranking $A_2 \succ A_1 \succ A_3$ is the most preferred ranking, which appears four times, and other rankings appear two times. Thus, $A_2 \succ A_1 \succ A_3$ is an ideal ranking, which is consistent with the projection measure. This result again illustrates the superiority of the projection measure provided in this paper.

8. Discussion and conclusions

8.1. Implications

The most significant contribution of this study can be explained primarily in both theoretical and practical perspectives namely managerial, marketing, and educational implications.

Studies on customer satisfaction and loyalty of smartphone users have advanced in both number and variety. However, there are some research gaps in theory and practice. In particular, simple and straightforward methods with well-founded theory have not been developed. Our research has filled a part of the research gaps.

In theoretical perspective, this study has investigated, proposed, and tested a scientific assessment method for the satisfaction and loyalty of Chinese smartphone users. It employs a decision-making method to evaluate the levels of customer satisfaction and loyalty in mobile sector. With the aid of intuitionistic fuzzy theory and GDM methods, this model solves the information fusion problem for symbols and nonresponse options in questionnaires. This finding has universal significance in assessment methods and related areas, including engineering and management applications. Therefore, this method is a well-founded and conclusive one.

This study has important implications for practitioners. Without doubt, the competition for greater market share is being intensified within the mobile phone industry in China. A more specific approach to building up a novel competitive edge is vital for smartphone companies. One of the best and obvious ways of achieving this is through a scientifically sound marketing and customer retention strategy. It is

Table 9. Statistics of rankings based on eight measures.

Measure	$A_2 \succ A_1 \succ A_3$	$A_3 \succ A_2 \succ A_1$	$A_2 \succ A_3 \succ A_1$
$S^{+}_{(\cdot)}$	$\surd$		
$S^{-}_{(\cdot)}$	$\surd$		
$S^{c}_{(\cdot)}$			$\surd$
$RC(\cdot)$	$\surd$		
$S^{+}_{E(\cdot)}$		$\surd$	
$S^{-}_{E(\cdot)}$			$\surd$
$S^{c}_{E(\cdot)}$	$\surd$		
$RC_{E(\cdot)}$		$\surd$	

costlier to attract new customers than to retain the existing ones. Therefore, the key focus in managing customer satisfaction and loyalty is on identifying the main satisfaction determinants from the user's perspective and then, on assessing the company's performance in addressing each of these determinants. Also, it is vital to build up a scientific assessment method for assisting the company in understanding and knowing well the trend of customers.

This study has provided a useful methodology built on GDM and fuzzy information. It can assist managers in making accurate and timely decisions by measuring user satisfaction and loyalty. This method is simple in both questionnaire survey and information aggregation. The questionnaire is implemented easily by marking simple symbols. The statistics and aggregation are implemented easily on a computer. GDM is a comprehensive decision-making method. Intuitionistic fuzzy information is a quantification through human beings' thinking. Thus, this method is comprehensive and practical. For long relationship building, we hope that our research can persistently improve the quality of infrastructure and mobile applications, and reduce the costs of accessing and using these applications. Such efforts would be helpful to cultivate cumulative satisfaction and customer loyalty over time.

8.2. Limitations and future research directions

To accomplish our development objective, we should recognize some important limitations. First, we should consider whether the information is from representative samples of the users. The sample in this paper does not represent a variety of users. In future research, we hope to extend the sample size. Second, we should consider the issue of criteria. The current model refers to the literature [66] and employs three dimensions as evaluation criteria. The evaluation criteria in this paper can be extended to all the criteria in the literature [66] as sub-criteria, such as value equity, memory, processor, remote control services, and location based services. Third, we should consider the weight of DM. The weights of DMs are the same for all DMs in our model. In some cases, they are different, which can be considered in the future research.

In addition to the above-mentioned limitations, the future research should investigate the wider applicability of our method.

9. Conclusions

We studied the assessment issues related to consumer satisfaction and loyalty in the mobile telecom sector in China. The present study confirms that an intuitionistic fuzzy GDM approach is an applicable method to measure the user satisfaction and loyalty in mobile telecommunication domain.

This study makes five specific contributions to the existing evaluation methods, which are listed as follows:

1. Easy-to-operate survey. The questionnaire was answered by using some simple symbols, which made it easy for the respondents to complete the survey.
2. Comprehensive information carrier. The symbolic information in the survey was fused into intuitionistic fuzzy information in a GDM setting. The intuitionistic fuzzy information is a comprehensive information carrier.
3. Comprehensive utilization of information resources. This paper solved an information fusion problem of nonresponse options in questionnaires. It identified the nonresponse options as information related to the customer satisfaction, and the nonresponse options were fused into intuitionistic fuzzy information.
4. Comprehensive measure. The separation measure in TOPSIS technique was replaced by the projection measure, which was a more comprehensive measure.
5. Extended evaluation field. This methodology extended the evaluation field to a new context.

   Specifically, it developed the previous research on the relationship between factors and customer satisfaction into a new research on the ranking of evaluation objects.

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