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Hartley-Ross type unbiased estimators of population mean using two auxiliary variables

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Abstract. In survey sampling, it is a well-established phenomenon that the efficiency of estimators increases with proper information on auxiliary variable(s). Keeping this fact in mind, the information on two auxiliary variables was utilized to propose a family of Hartley-Ross type unbiased estimators for estimating population mean under simple random sampling without replacement. Minimum variance of the new estimators was derived up to the first degree of approximation. Three real datasets were used to verify the efficient performance of the new family in comparison to the usual unbiased, Hartley and Ross, and other competing estimators.

1. Introduction

In the process of conducting sample surveys, researchers normally use ratio-type estimators when there is a need to estimate the unknown population parameters of the study variable by means of known population parameters of a correlated auxiliary variable(s). An eye view of literature on the estimation of population mean under simple random sampling without replacement (SRSWOR) with proper information on two auxiliary variables shows prominent studies of Abu-Dayyeh et al. [1], Kadilar and Cingi [2], Singh and Tailor [3], Lu and Yan [4], Lu et al. [5], Vishwakarma and Kumar [6], Sharma and Singh [7], Yasmeen et al. [8], Lu [9], Muneer et al. [10], and Shabbir and Gupta [11].

One eminent disadvantage of using ratio-type estimators is that they are typically biased. Hartely and Ross [12] initiated the concept of unbiased estimators for estimating population mean. Similar efforts for unbiased estimators were carried out by Robson [13], Murthy and Nanjamma [14], Biradar and Singh [15-17], Sahoo et al. [18], Singh et al. [19], Cekim and Kadilar [20], and Khan et al. [21]. Applying different sampling techniques, they used the information on a single auxiliary variable. Continuing these efforts, we proposed a new optimal family of Hartley-Ross type unbiased estimators with the novelty that the information on two auxiliary variables is used.

Consider $\Theta = \{\Theta_1, \Theta_2, \Theta_3, \cdots, \Theta_N\}$ as N units of a finite population. Let the ith unit of population, y_i, be the value of study variable y and x_i, z_i be the values of auxiliary variables x and z, respectively. n is the size of a sample selected from the population under SRSWOR scheme given that $n < N$. Some usual measures related to the study variable and auxiliary variables are presented in Table 1.

The following relative error terms are used to derive the expressions for the bias, variance, and minimum variance of the existing and suggested estimators:

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Table 1. Measures associated with study variable (y) and auxiliary variables (x and z).

Measure	Study variable y	Auxiliary variable x	Auxiliary variable z
Population mean	$\bar{Y}=N^{-1}\sum_{i=1}^{N} y_i$	$\bar{X}=N^{-1}\sum_{i=1}^{N} x_i$	$\bar{Z}=N^{-1}\sum_{i=1}^{N} z_i$
Sample mean	$\bar{y}=n^{-1}\sum_{i=1}^{n} y_i$	$\bar{x}=n^{-1}\sum_{i=1}^{n} x_i$	$\bar{z}=n^{-1}\sum_{i=1}^{n} z_i$
Population variance	$S_y^2=\frac{\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2}{(N-1)}$	$S_x^2=\frac{\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2}{(N-1)}$	$S_z^2=\frac{\sum_{i=1}^{N}\left(z_i-\bar{Z}\right)^2}{(N-1)}$
Coefficient of variation	$C_y=\bar{Y}^{-1}S_y$	$C_x=\bar{X}^{-1}S_x$	$C_z=\bar{Z}^{-1}S_z$

Table 2. Measures used in relative error terms.

Description	Sample		Population	
Covariance between y and x	$s_{yx}=(n-1)^{-1}\sum\limits_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})$		$S_{yx}=(N-1)^{-1}\sum\limits_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right)$	
Mean of transformed x	$\bar{x}^{(1)}=C_x\bar{x}+\rho_{yx}$ $\bar{x}^{(2)}=C_x\bar{x}+\beta_{2(x)}$ $\bar{x}^{(3)}=\alpha\bar{x}+\beta$		$\bar{X}^{(1)}=C_x\bar{X}+\rho_{yx}$ $\bar{X}^{(2)}=C_x\bar{X}+\beta_{2(x)}$ $\bar{X}^{(3)}=\alpha\bar{X}+\beta$	
Ratio of y to the transformed x	$r_i^{(0)}=\frac{y_i}{x_i}$, $r_i^{(1)}=\frac{y_i}{C_x x_i+\rho_{yx}}$, $r_i^{(2)}=\frac{y_i}{C_x x_i+\beta_{2(x)}}$, $r_i^{(3)}=\frac{y_i}{\alpha x_i+\beta}$,	$i=1,2,\cdots,n$ $i=1,2,\cdots,n$ $i=1,2,\cdots,n$ $i=1,2,\cdots,n$	$R_i^{(0)}=\frac{y_i}{x_i}$, $R_i^{(1)}=\frac{y_i}{C_x x_i+\rho_{yx}}$, $R_i^{(2)}=\frac{y_i}{C_x x_i+\beta_{2(x)}}$, $R_i^{(3)}=\frac{y_i}{\alpha x_i+\beta}$,	$i=1,2,\cdots,N$ $i=1,2,\cdots,N$ $i=1,2,\cdots,N$ $i=1,2,\cdots,N$
Mean of the ratio of y to transformed x	$\bar{r}^{(k)}=n^{-1}\sum\limits_{i=1}^{n} r_i^{(k)}$	for $k=0,1,2,3$	$\bar{R}^{(k)}=N^{-1}\sum\limits_{i=1}^{N} r_i^{(k)}$	for $k=0,1,2,3$

$$\varepsilon_0=\frac{\bar{y}}{\bar{Y}}-1,\qquad \varepsilon_1=\frac{\bar{x}}{\bar{X}}-1,$$

$$\varepsilon_2=\frac{\bar{z}}{\bar{Z}}-1,\qquad \varepsilon_3=\frac{s_{yx}}{S_{yx}}-1,$$

$$\varepsilon_4=\frac{\bar{x}^{(j)}}{\bar{X}^{(j)}}-1,\qquad \text{for } j=1,2,3,$$

$$\varepsilon_5=\frac{\bar{r}^{(k)}}{\bar{R}^{(k)}}-1,\qquad \text{for } k=0,1,2,3. \tag{1}$$

Table 2 presents a detailed description of some terms used in Eq. (1).

Remark 1.1. s_{yx}, $\bar{x}^{(j)}$, and $\bar{r}^{(k)}$ are the unbiased estimators of their population parameters S_{yx}, $\bar{X}^{(j)}$, and $\bar{R}^{(k)}$, respectively.

Remark 1.2. The expectations of the relative errors given in Eq. (1) are as follows [19,20]:

$$E(\varepsilon_i)=0,\qquad \text{for } i=0,1,2,3,4,5,$$

$$E(\varepsilon_0^2)=\psi C_y^2,\qquad E(\varepsilon_1^2)=\psi C_x^2,$$

$$E(\varepsilon_2^2)=\psi C_z^2,\qquad E(\varepsilon_3^2)=\psi\left(\frac{\theta_{22x}}{\rho_{yx}}-1\right),$$

$$E(\varepsilon_4^2)=\psi C_{x^{(j)}}^2,\qquad E(\varepsilon_5^2)=\psi C_{r^{(k)}}^2,$$

$$E(\varepsilon_0\varepsilon_1)=\psi\rho_{yx}C_yC_x=\psi C_{yx},$$

$$E(\varepsilon_0\varepsilon_2)=\psi\rho_{yz}C_yC_z=\psi C_{yz},$$

$$E(\varepsilon_0\varepsilon_3)=\psi\left(\frac{C_y\theta_{21x}}{\rho_{yx}}\right),$$

$$E(\varepsilon_1\varepsilon_2)=\psi\rho_{xz}C_xC_z=\psi C_{xz},$$

$$E(\varepsilon_1\varepsilon_3)=\psi\left(\frac{C_x\theta_{12x}}{\rho_{yx}}\right),$$

$$E(\varepsilon_2\varepsilon_3)=\psi\left(\frac{C_z\theta_{12z}}{\rho_{yz}}\right),$$

where:

$$\psi=\left(\frac{1}{n}-\frac{1}{N}\right)$$

is finite population correction factor;

$$C_x^{(j)}=\left(\bar{X}^{(j)}\right)^{-1}S_{x^{(j)}}$$

is coefficient of variation for transformed x;

$$C_r^{(k)}=\left(\bar{R}^{(k)}\right)^{-1}S_{r^{(k)}}$$

is coefficient of variation for ratio of y to transformed x;

$$S_{x^{(j)}} = \sqrt{(N-1)^{-1} \sum_{i=1}^{N} \left(x_i^{(j)} - \bar{X}^{(j)}\right)^2}$$

is standard deviation of transformed x

$$S_{r^{(k)}} = \sqrt{(N-1)^{-1} \sum_{i=1}^{N} \left(r_i^{(k)} - \bar{R}^{(k)}\right)^2}$$

is standard deviation of ratio of y to transformed x;

$$S_{yx^{(j)}} = \sqrt{(N-1)^{-1} \sum_{i=1}^{N} \left(y_i - \bar{Y}\right) \left(x_i^{(j)} - \bar{X}^{(j)}\right)}$$

is covariance between y and transformed x (for all above expressions $j = 1, 2, 3$ and $k = 0, 1, 2, 3$);

$$\rho_{yx} = \left(S_y S_x\right)^{-1} S_{yx}$$

$$\rho_{yz} = \left(S_y S_z\right)^{-1} S_{yz}$$

$$\rho_{xz} = \left(S_x S_z\right)^{-1} S_{xz}$$

are correlation coefficients between y & x, y & z, and x & z, respectively;

$$S_{yx} = (N-1)^{-1} \sum_{i=1}^{N} \left(y_i - \bar{Y}\right) \left(x_i - \bar{X}\right)$$

$$S_{yz} = (N-1)^{-1} \sum_{i=1}^{N} \left(y_i - \bar{Y}\right) \left(z_i - \bar{Z}\right)$$

$$S_{xz} = (N-1)^{-1} \sum_{i=1}^{N} \left(x_i - \bar{X}\right) \left(z_i - \bar{Z}\right)$$

are covariances between y & x, y & z, and x & z, respectively.

Values of θ_{21x}, θ_{12x}, θ_{22x}, and θ_{12z} can be obtained through the following expressions:

$$\theta_{pqx} = \frac{\mu_{pqx}}{\left(\mu_{20x}^{\frac{p}{2}}\right)\left(\mu_{02x}^{\frac{q}{2}}\right)}, \quad \mu_{pqx} = \frac{\sum_{i=1}^{N} \left(y_i - \bar{Y}\right)^p \left(x_i - \bar{X}\right)^q}{N},$$

$$\theta_{pqz} = \frac{\mu_{pqz}}{\left(\mu_{20z}^{\frac{p}{2}}\right)\left(\mu_{02z}^{\frac{q}{2}}\right)}, \quad \mu_{pqz} = \frac{\sum_{i=1}^{N} \left(y_i - \bar{Y}\right)^p \left(z_i - \bar{Z}\right)^q}{N},$$

for $p, q = 0, 1, 2$.

2. Review of literature

This section presents some estimators from the literature when estimating population mean under simple random sampling.

- The traditional unbiased estimator for unknown population mean along with its variance is:

$$\bar{y}_0^{(u)} = \bar{y}, \tag{2}$$

$$\mathrm{Var}\left(\bar{y}_0^{(u)}\right) = \psi \bar{Y}^2 C_y^2. \tag{3}$$

- Hartley and Ross [12] initiated an unbiased ratio-type estimator as given below:

$$\bar{y}_{HR}^{(u)} = \bar{r}^{(0)} \bar{X} + \frac{n(N-1)}{N(n-1)} \left(\bar{y} - \bar{r}^{(0)} \bar{x}\right), \tag{4}$$

where:

$$\bar{r}^{(0)} = n^{-1} \sum_{i=1}^{n} r_i^{(0)}, \qquad r_i^{(0)} = \frac{y_i}{x_i}.$$

The variance of this estimator in the first order of approximation is equal to the mean square error of the usual ratio estimator (see Singh and Mangat [22]).

$$\mathrm{Var}\left(\bar{y}_{HR}^{(u)}\right) \cong \psi \bar{Y}^2 \left[C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x\right]. \tag{5}$$

- Grover and Kaur [23] followed the lines of Gupta and Shabbir [24] and Shabbir and Gupta [25] and suggested a generalized class based on ratio-type exponential estimators as follows:

$$\bar{y}_{GK} = \left[q_1 \bar{y} + q_2 \left(\bar{X} - \bar{x}\right)\right] \exp\left[\frac{\alpha\left(\bar{X} - \bar{x}\right)}{\alpha\left(\bar{X} + \bar{x}\right) + 2\beta}\right], \tag{6}$$

where q_1 and q_2 are the suitable weights to be chosen. $\alpha(\neq 0)$ and β are either known quantities or functions of any known population parameters, including coefficient of variation, coefficient of skewness, coefficient of Kurtosis, coefficient of correlation, etc.

The optimal values of q_1 and q_2 are shown in Box I.

We get the minimum MSE by utilizing the optimal values shown in Box I. The expression for minimum MSE up to the first degree of approximation is shown in Box II.

- Singh et al. [19] suggested two Hartley-Ross type estimators by considering the estimators of Kadilar and Cingi [26] and Upadhyaya and Singh [27]:

$$\bar{y}_{S1}^{(u)} = \bar{r}^{(1)} \bar{X}^{(1)} + \frac{n(N-1)}{N(n-1)} \left(\bar{y} - \bar{r}^{(1)} \bar{x}^{(1)}\right), \tag{8}$$

$$\bar{y}_{S2}^{(u)} = \bar{r}^{(2)} \bar{X}^{(2)} + \frac{n(N-1)}{N(n-1)} \left(\bar{y} - \bar{r}^{(2)} \bar{x}^{(2)}\right), \tag{9}$$

where:

$$\bar{r}^{(1)} = n^{-1} \sum_{i=1}^{n} r_i^{(1)}, \qquad r_i^{(1)} = \frac{y_i}{C_x x_i + \rho_{yx}} = \frac{y_i}{x_i^{(1)}},$$

$$q_{1(opt)}=\frac{8-\psi\lambda^2C_x^2}{8\left[1+\psi C_y^2\left(1-\rho_{yx}^2\right)\right]},$$

$$q_{2(opt)}=\frac{\bar{Y}\left[\psi\lambda^3C_x^3+8C_y\rho_{yx}-\psi\lambda^2C_x^2C_y\rho_{yx}-4\lambda C_x\left\{1-\psi C_y^2(1-\rho_{yx}^2)\right\}\right]}{8\bar{X}C_x\left[1+\psi C_y^2\left(1-\rho_{yx}^2\right)\right]},$$

where:

$$\lambda=\frac{\alpha\bar{X}}{\alpha\bar{X}+\beta}.$$

Box I

$$MSE_{\min}\left(\bar{y}_{GK}\right)\cong\frac{\psi\bar{Y}^2\left[64C_y^2\left(1-\rho_{yx}^2\right)-\psi\lambda^4C_x^4-16\psi\lambda^2C_x^2C_y^2\left(1-\rho_{yx}^2\right)\right]}{64\left[1+\psi C_y^2\left(1-\rho_{yx}^2\right)\right]}. \tag{7}$$

Box II

$$\bar{X}^{(1)}=C_x\bar{X}+\rho_{yx},$$

$$\bar{r}^{(2)}=n^{-1}\sum_{i=1}^{n}r_i^{(2)},\qquad r_i^{(2)}=\frac{y_i}{C_xx_i+\beta_{2(x)}}=\frac{y_i}{x_i^{(2)}},$$

$$\bar{X}^{(2)}=C_x\bar{X}+\beta_{2(x)},$$

where ρ_{yx} is the coefficient of correlation between study variable y and auxiliary variable x, and $\beta_{2(x)}$ is the coefficient of kurtosis of auxiliary variable x.

Given below are the variances of $\bar{y}_{S1}^{(u)}$ and $\bar{y}_{S2}^{(u)}$:

$$\mathrm{Var}\left(\bar{y}_{S1}^{(u)}\right)\cong\psi\left[S_y^2+\left(\bar{R}^{(1)}S_{x^{(1)}}\right)^2-2\bar{R}^{(1)}S_{yx^{(1)}}\right], \tag{10}$$

$$\mathrm{Var}\left(\bar{y}_{S2}^{(u)}\right)\cong\psi\left[S_y^2+\left(\bar{R}^{(2)}S_{x^{(2)}}\right)^2-2\bar{R}^{(2)}S_{yx^{(2)}}\right], \tag{11}$$

where $\bar{R}^{(1)}$ and $\bar{R}^{(2)}$ are defined in Table 2, and $S_{x^{(1)}}$, $S_{x^{(2)}}$, $S_{yx^{(1)}}$, and $S_{yx^{(2)}}$ are defined in Remark 1.2.

- A special version of estimators of Khoshnevisan et al. [28] was utilized by Cekim and Kadilar [20] to form a general family of Hartley-Ross type unbiased estimators as follows:

$$\bar{y}_{CK1}^{(u)}=\bar{r}^{(3)}\bar{X}^{(3)}+\frac{n(N-1)}{N(n-1)}\left(\bar{y}-\bar{r}^{(3)}\bar{x}^{(3)}\right), \tag{12}$$

where:

$$\bar{r}^{(3)}=n^{-1}\sum_{i=1}^{n}r_i^{(3)},\qquad r_i^{(3)}=\frac{y_i}{\alpha x_i+\beta}=\frac{y_i}{x_i^{(3)}},$$

$$\bar{X}^{(3)}=\alpha\bar{X}+\beta.$$

α and β are explained earlier.

The variance of $\bar{y}_{CK1}^{(u)}$ is given by:

$$\mathrm{Var}\left(\bar{y}_{CK1}^{(u)}\right)\cong\psi\left[S_y^2+\left(\bar{R}^{(3)}S_{x^{(3)}}\right)^2-2\bar{R}^{(3)}S_{yx^{(3)}}\right], \tag{13}$$

where $\bar{R}^{(3)}$ is defined in Table 2, and $S_{x^{(3)}}$ and $S_{yx^{(3)}}$ are defined in Remark 1.2.

Remark 2.1. Some important considerations can be noticed here. If we have:

i) $\alpha=C_x$ and $\beta=\rho_{yx}$ in $r_i^{(3)}$, then

$r_i^{(3)}=r_i^{(1)}$ and $\bar{y}_{CK1}^{(u)}=\bar{y}_{S1}^{(u)}$;

ii) $\alpha=C_x$ and $\beta=\beta_{2(x)}$ in $r_i^{(3)}$, then

$r_i^{(3)}=r_i^{(2)}$ and $\bar{y}_{CK1}^{(u)}=\bar{y}_{S2}^{(u)}$.

- Another family of Hartley-Ross type unbiased estimators from a special version of Koyuncu and Kadilar [29] was proposed by Cekim and Kadilar [20] and is defined as follows:

$$\begin{aligned}\bar{y}_{CK2}^{(u)}=&q_3\bar{y}\left[\frac{\alpha\bar{X}+\beta}{\gamma\left(\alpha\bar{x}+\beta\right)+(1-\gamma)\left(\alpha\bar{X}+\beta\right)}\right]^t\\&-q_3\bar{y}\psi\left[\frac{t(t+1)}{2}\gamma^2\lambda^2C_x^2-t\gamma\lambda\frac{s_{yx}}{\bar{y}\bar{X}}\right]\\&-(q_3-1)\bar{y},\end{aligned} \tag{14}$$

where q_3 is the suitable weight that minimizes the variance, $t = 1$, $\gamma = 1$, $\lambda = \frac{\alpha\bar{X}}{\alpha\bar{X}+\beta}$, and α and β were defined earlier.

$$\mathrm{Var}\left(\bar{y}_{CK2}^{(u)}\right) \cong \bar{Y}^2\psi\Bigg[\left\{q_3^2t^2\gamma^2\lambda^2C_x^2 - 2q_3t\gamma\lambda C_{yx} + C_y^2\right\} - \psi\left\{q_3t\gamma\lambda\left(\frac{t+1}{2}\gamma\lambda C_x^2 - C_{yx}\right)\right\}^2 - \psi q_3t\gamma\lambda\left\{2\frac{C_{yx}}{\rho_{yx}}(q_3t\gamma\lambda C_x\theta_{12x} - C_y\theta_{21x}) - (t+1)\gamma\lambda C_x^2(q_3t\gamma\lambda C_{yx} - C_y^2)\right\}\Bigg]. \quad (15)$$

The optimal value of q_3 is given below:

$$q_{3(opt)} = \frac{\Psi}{\Delta},$$

where:

$$\Psi = t\gamma\lambda\left[C_{yx}\left\{1 - \frac{\psi C_y\theta_{21x}}{\rho_{yx}}\right\} + \frac{(t+1)}{2}\psi\gamma\lambda C_x^2C_y^2\right],$$

$$\Delta = t^2\gamma^2\lambda^2\left[C_x^2 + \psi\left\{C_{yx}\left((t+1)\gamma\lambda C_x^2 - 2\frac{C_x\theta_{12x}}{\rho_{yx}}\right) - \left(\frac{(t+1)}{2}\gamma\lambda C_x^2 - C_{yx}\right)^2\right\}\right].$$

We have the minimum variance by placing the optimal value of q_3 in Eq. (15) as follows:

$$\mathrm{Var}_{\min}\left(\bar{y}_{CK2}^{(u)}\right) \cong \bar{Y}^2\psi\left(C_y^2 - \frac{\Psi^2}{\Delta}\right). \quad (16)$$

- Muneer et al. [10] followed the lines of Gupta and Shabbir [24] and Singh and Singh [30] to propose the following estimators. These estimators utilized the information on two auxiliary variables.

$$\bar{y}_{MU,\gamma} = \left[q_4\bar{y} + q_5\left(\bar{X} - \bar{x}\right)\right]\left[\gamma\left\{2 - \exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right)\right\} + (1-\gamma)\exp\left(\frac{\bar{Z} - \bar{z}}{\bar{Z} + \bar{z}}\right)\right]. \quad (17)$$

By putting $\gamma = 0$ and 1, two estimators $\bar{y}_{MU,0}$ and $\bar{y}_{MU,1}$ are obtained; q_4 and q_5 are the constants to be determined.

The optimal values of q_4 and q_5 are shown in Box III. Using $q_{4(opt)}$ and $q_{5(opt)}$, we get the $MSE_{\min}(\bar{y}_{MU,\gamma})$ as shown in Box IV.

- Recently, Shabbir and Gupta [11] used the linear combination of two auxiliary variables to propose a difference cum exponential ratio-type estimator. This concept was initiated by Gupta and Shabbir [24] and Grover and Kaur [31].

$$\bar{y}_{SG} = \left[q_6\bar{y} + q_7\left(\bar{X} - \bar{x}\right) + q_8\left(\bar{Z} - \bar{z}\right)\right]\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right), \quad (19)$$

where q_6, q_7, and q_8 are the feasible constants defined in Box V.

The minimum MSE is obtained using the above optimal constants as follows:

$$MSE_{\min}\left(\bar{y}_{SG}\right) \cong \bar{Y}^2\left[1 - \frac{\left(1 + \frac{1}{64}\psi^2C_x^4\right) + \frac{1}{4}\psi^2C_y^2C_x^2\left(1 - R_{y.xz}^2\right)}{1 + \psi C_y^2\left(1 - R_{y.xz}^2\right)}\right]. \quad (20)$$

$$q_{4(opt)} = \frac{1 + \left(\frac{3}{8} - \frac{\gamma}{4}\right)\psi C_z^2 - \frac{1}{2}\psi C_{yz} - \frac{\psi C_{xz}(C_{xz} - C_{yx})}{2C_x^2}}{1 + \psi C_y^2 + \left(1 - \frac{\gamma}{2}\right)\psi C_z^2 - 2\psi C_{yz} - \frac{\psi(C_{xz} - C_{yx})^2}{C_x^2}},$$

$$q_{5(opt)} = \frac{\bar{Y}}{\bar{X}}\left[\frac{C_{xz}}{2C_x^2} - \frac{1 + \left(\frac{3}{8} - \frac{\gamma}{4}\right)\psi C_z^2 - \frac{1}{2}\psi C_{yz} - \frac{\psi C_{xz}(C_{xz} - C_{yx})}{2C_x^2}}{1 + \psi C_y^2 + \left(1 - \frac{\gamma}{2}\right)\psi C_z^2 - 2\psi C_{yz} - \frac{\psi(C_{xz} - C_{yx})^2}{C_x^2}}\left\{\frac{C_{xz} - C_{yx}}{C_x^2}\right\}\right].$$

Box III

$$MSE_{\min}\left(\bar{y}_{MU,\gamma}\right) \cong \bar{Y}^2\left[1 - \frac{\psi C_{xz}^2}{4C_x^2} - \frac{\left\{1 + \left(\frac{3}{8} - \frac{\gamma}{4}\right)\psi C_z^2 - \frac{1}{2}\psi C_{yz} - \frac{\psi C_{xz}(C_{xz} - C_{yx})}{2C_x^2}\right\}^2}{1 + \psi C_y^2 + \left(1 - \frac{\gamma}{2}\right)\psi C_z^2 - 2\psi C_{yz} - \frac{\psi(C_{xz} - C_{yx})^2}{C_x^2}}\right]. \quad (18)$$

Box IV

$$q_{6(opt)}=\frac{1-\frac{1}{8}\psi C_x^2}{1+\psi C_y^2(1-R_{y.xz}^2)},$$

$$q_{7(opt)}=\frac{\bar{Y}}{\bar{X}}\left[\frac{\frac{1}{2}C_x(1-\rho_{xz}^2)\left\{\psi C_y^2\left(1-R_{y.xz}^2\right)-\left(1-\frac{1}{4}\psi C_x^2\right)\right\}+C_y(\rho_{yx}-\rho_{yz}\rho_{xz})\left(1-\frac{1}{8}\psi C_x^2\right)}{C_x(1-\rho_{xz}^2)\left\{1+\psi C_y^2(1-R_{y.xz}^2)\right\}}\right],$$

$$q_{8(opt)}=\frac{\bar{Y}}{\bar{Z}}\left[\frac{C_y\left(\rho_{yz}-\rho_{yx}\rho_{xz}\right)\left(1-\frac{1}{8}\psi C_x^2\right)}{C_x\left(1-\rho_{xz}^2\right)\left\{1+\psi C_y^2(1-R_{y.xz}^2)\right\}}\right],$$

where $R_{y.xz}^2=\frac{\rho_{yx}^2+\rho_{yz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}}{1-\rho_{xz}^2}$.

Box V

3. Proposed family of estimators

It is valuable to mention that the estimator $\bar{y}_{SG}$ has a limited application due to the restricted transformation of the auxiliary variable. This section considers a general linear transformation of auxiliary information and uses a special version of Shabbir and Gupta [11] estimator to present a family of Hartley-Ross-type unbiased estimators. Minimum variance of the new class is also derived up to the first order of approximation.

Using a general linear transformation of auxiliary information in the Shabbir and Gupta [11] estimator, we get:

$$(\bar{y}_{SG,Gen})=\left[q_9\bar{y}+q_{10}\left(\bar{X}-\bar{x}\right)+q_{11}\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\alpha\left(\bar{X}-\bar{x}\right)}{\alpha\left(\bar{X}+\bar{x}\right)+2\beta}\right]. \tag{21}$$

Bias, up to the first degree of approximation, of the generalized family presented in Eq. (21) is obtained as follows:

$$\mathrm{Bias}\left(\bar{y}_{SG,Gen}\right)\cong(q_9-1)\bar{Y}+q_9\psi\bar{Y}\left(\frac{3}{8}\lambda^2C_x^2-\frac{1}{2}\lambda C_xC_y\rho_{yx}\right)+\frac{1}{2}q_{10}\psi\lambda\bar{X}C_x^2+\frac{1}{2}q_{11}\psi\lambda\bar{Z}C_xC_z\rho_{xz}.$$

Subtracting Bias($\bar{y}_{SG,Gen}$) from Eq. (21), we get:

$$\left[q_9\bar{y}+q_{10}\left(\bar{X}-\bar{x}\right)+q_{11}\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\alpha\left(\bar{X}-\bar{x}\right)}{\alpha\left(\bar{X}+\bar{x}\right)+2\beta}\right]-\left[\left(q_9-1\right)\bar{Y}+q_9\psi\bar{Y}\left(\frac{3}{8}\lambda^2C_x^2-\frac{1}{2}\lambda\frac{S_{yx}}{\bar{X}\bar{Y}}\right)+\frac{1}{2}q_{10}\psi\lambda\bar{X}C_x^2+\frac{1}{2}q_{11}\psi\lambda\bar{Z}C_xC_z\rho_{xz}\right]. \tag{22}$$

After doing some simplifications and replacing the parameters $\bar{Y}$ and S_{yx} by their unbiased estimators $\bar{y}$ and s_{yx} in Eq. (22), a new family of Hartley-Ross type unbiased estimators is proposed as follows:

$$\bar{y}_P^{(u)}=\left[q_9\bar{y}+q_{10}\left(\bar{X}-\bar{x}\right)+q_{11}\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\alpha\left(\bar{X}-\bar{x}\right)}{\alpha\left(\bar{X}+\bar{x}\right)+2\beta}\right]+\bar{y}-\frac{1}{2}q_{10}\psi\lambda\bar{X}C_x^2-\frac{1}{2}q_{11}\psi\lambda C_{xz}\bar{Z}-q_9\bar{y}-\frac{3}{8}q_9\psi\lambda^2\bar{y}C_x^2+\frac{1}{2}q_9\psi\lambda\frac{s_{yx}}{\bar{X}}, \tag{23}$$

where q_9, q_{10}, and q_{11} are the suitable weights to be chosen. $\alpha(\neq 0)$ and β are either known constants or functions of any known population parameters, including coefficient of variation C_x or C_z, coefficient of skewness $\beta_{1(x)}$, coefficient of Kurtosis $\beta_{2(x)}$, coefficient of correlation ρ_{yx} or ρ_{yz}, etc., and s_{yx} is an unbiased estimator of S_{yx}.

To express Eq. (23) in terms of $\varepsilon's$, the relative error terms defined in Eq. (1) are used. Expression is expanded up to the first order of approximation.

$$\bar{y}_P^{(u)}\cong\left[q_9\bar{Y}(1+\varepsilon_0)-q_{10}\bar{X}\varepsilon_1-q_{11}\bar{Z}\varepsilon_2\right]\left[1-\frac{1}{2}\lambda\varepsilon_1+\frac{3}{8}\lambda^2\varepsilon_1^2\right]+\bar{Y}(1+\varepsilon_0)-\frac{1}{2}q_{10}\psi\lambda\bar{X}C_x^2-\frac{1}{2}q_{11}\psi\lambda C_{xz}\bar{Z}-q_9\bar{Y}(1+\varepsilon_0)-\frac{3}{8}q_9\psi\lambda^2C_x^2\bar{Y}(1+\varepsilon_0)+\frac{1}{2}q_9\psi\lambda\frac{S_{yx}(1+\varepsilon_3)}{\bar{X}}.$$

Solving the above, we have:

$$\left(\bar{y}_P^{(u)}-\bar{Y}\right)\cong\bar{Y}\left(1-\frac{3}{8}q_9\psi\lambda^2C_x^2\right)\varepsilon_0 - \left(\frac{1}{2}q_9\bar{Y}\lambda+q_{10}\bar{X}\right)\varepsilon_1-q_{11}\bar{Z}\varepsilon_2 + \frac{1}{2}q_9\psi\lambda\frac{S_{yx}}{\bar{X}}\varepsilon_3-\frac{1}{2}q_9\bar{Y}\lambda\varepsilon_0\varepsilon_1 + \frac{1}{2}q_{11}\lambda\bar{Z}\varepsilon_1\varepsilon_2+\left(\frac{3}{8}q_9\bar{Y}\lambda^2+\frac{1}{2}q_{10}\bar{X}\lambda\right)\varepsilon_1^2 - \frac{1}{2}\psi\lambda\Big(q_{10}\bar{X}C_x^2+q_{11}C_{xz}\bar{Z}+\frac{3}{4}q_9\bar{Y}\lambda C_x^2 - q_9\bar{Y}C_{yx}\Big). \tag{24}$$

We get approximately zero bias by taking expectation on both sides of Eq. (24). It is indicated that the proposed class generates Hartley-Ross type unbiased estimators.

$$\text{Bias}\left(\bar{y}_P^{(u)}\right)=E\left(\bar{y}_P^{(u)}-\bar{Y}\right)\cong 0.$$

To obtain the variance of the proposed estimators up to the first degree of approximation, both sides of Eq. (24) are squared and the expectation is taken as follows:

$$\text{Var}\left(\bar{y}_P^{(u)}\right)\cong\bar{Y}^2\psi C_y^2-\bar{Y}^2\psi\lambda V_1q_9-2\bar{X}\bar{Y}\psi C_{yx}q_{10} - 2\bar{Y}\bar{Z}\psi C_{yz}q_{11}+\bar{Y}^2\lambda^2\psi V_2q_9^2 + \bar{X}^2\psi C_x^2V_3q_{10}^2+\bar{Z}^2\psi C_z^2V_4q_{11}^2 + \bar{X}\bar{Y}\psi\lambda C_xV_5q_9q_{10}+\bar{Y}\bar{Z}\psi\lambda V_6q_9q_{11} + 2\bar{X}\bar{Z}\psi C_{xz}V_3q_{10}q_{11}, \tag{25}$$

where:

$$V_1=\frac{3}{4}\psi\lambda C_y^2C_x^2+C_{yx}-\frac{\psi C_{yx}C_y\theta_{21x}}{\rho_{yx}},$$

$$V_2=\frac{1}{4}C_x^2-\frac{9}{64}\psi\lambda^2C_x^4-\frac{1}{4}\psi C_{yx}^2-\frac{\psi C_{yx}C_x\theta_{12x}}{2\rho_{yx}} + \frac{3}{4}\psi\lambda C_x^2C_{yx},$$

$$V_3=1-\frac{1}{4}\psi\lambda^2C_x^2,$$

$$V_4=1-\frac{1}{4}\psi\lambda^2C_x^2\rho_{xz}^2,$$

$$V_5=C_x+\frac{5}{4}\psi\lambda C_xC_{yx}-\frac{\psi C_{yx}\theta_{12x}}{\rho_{yx}}-\frac{3}{8}\psi\lambda^2C_x^3,$$

$$V_6=\frac{3}{4}\psi\lambda C_x^2C_{yz}+C_{xz}-\frac{\psi C_{yx}C_z\theta_{12z}}{\rho_{yz}} + \frac{1}{2}\psi\lambda C_{xz}C_{yx}-\frac{3}{8}\psi\lambda^2C_x^2C_{xz}.$$

In order to get the optimal values of $q_i's$, we differentiate Eq. (25) with respect to q_i, $i=9,10,11$, and then equate them to zero. Therefore, we get:

$$q_{9(\text{opt})}=\frac{2B_1}{\lambda B_2},$$

$$q_{10(\text{opt})}=\frac{\bar{Y}}{\bar{X}}\left[\frac{V_1A_2B_2-4V_2A_2B_1-V_6(A_3B_2-A_6B_1)}{C_xV_5A_2B_2}\right],$$

$$q_{11(\text{opt})}=\frac{\bar{Y}}{\bar{Z}}\left[\frac{A_3B_2-A_6B_1}{A_2B_2}\right].$$

We obtained the minimum variance at optimal values of q_i, $i=9,10,11$, by inserting them in Eq. (25):

$$\text{Var}_{\min}\left(\bar{y}_P^{(u)}\right)\cong\frac{1}{C_xV_5^2A_2^2B_2^2}\bar{Y}^2\psi\Big[C_xV_5^2A_2^2\Big\{C_y^2B_2^2 - 2V_1B_1B_2+4V_2B_1^2\Big\}+C_1\Big\{C_xV_3C_1 - 2C_{yx}V_5A_2B_2+2C_xV_5^2A_2B_1\Big\} + C_xV_5^2C_2\Big\{C_z^2V_4C_2-2C_{yz}A_2B_2 + 2V_6A_2B_1\Big\}+2C_{xz}V_3V_5C_1C_2\Big], \tag{26}$$

where:

$$A_1=C_xV_1V_3-C_{yx}V_5,$$

$$A_2=C_{xz}V_3V_6-C_z^2C_xV_4V_5,$$

$$A_3=C_{xz}V_1V_3-C_{yz}C_xV_5,$$

$$A_4=C_xV_3V_6-C_{xz}V_3V_5,$$

$$A_5=C_x(4V_2V_3-V_5^2),$$

$$A_6=4C_{xz}V_2V_3-C_xV_5V_6,$$

$$B_1=A_1A_2-A_3A_4,\qquad B_2=A_2A_5-A_4A_6,$$

$$C_1=V_1A_2B_2-4V_2A_2B_1-V_6(A_3B_2-A_6B_1),$$

$$C_2=(A_3B_2-A_6B_1).$$

4. Empirical illustration

In this section, three natural populations are used to present the empirical performance of the suggested almost unbiased estimators as compared to other estimators. Table 3 contains helpful information regarding datasets. Percent Relative Efficiency (PRE) is calculated through the following expression:

$$PRE = \frac{MSE\left(\bar{y}_0^{(u)}\right)}{MSE\left(\bullet\right)} \times 100, \tag{27}$$

where:

- $\bullet = \bar{y}_0^{(u)}, \bar{y}_{HR}^{(u)}, \bar{y}_{GK}, \bar{y}_{S1}^{(u)}, \bar{y}_{S2}^{(u)}, \bar{y}_{CK1}^{(u)}, \bar{y}_{CK2}^{(u)}, \bar{y}_{MU,0},$

  $\bar{y}_{MU,1}, \bar{y}_{SG}, \bar{y}_P^{(u)}$.

We calculated the PREs of all the estimators taken from the literature and for the proposed family of estimators with respect to $\bar{y}_0^{(u)}$. Empirical findings are reported in Tables 4, 5, and 6 for Populations 1, 2, and 3, respectively. From Tables 4, 5, and 6, the following observations are made:

- A comparison between all unbiased/Hartley-Ross-type unbiased estimators, i.e., $\bar{y}_0^{(u)}$, $\bar{y}_{HR}^{(u)}$, $\bar{y}_{S1}^{(u)}$, $\bar{y}_{S2}^{(u)}$, $\bar{y}_{CK1}^{(u)}$, $\bar{y}_{CK2}^{(u)}$, and $\bar{y}_P^{(u)}$ reveals that $\bar{y}_P^{(u)}$ provides maximum gain in PREs as compared to others. This gain in PREs is based on the fact that the proposed estimators use the information on two auxiliary variables, while the others use the information on only one auxiliary variable;
- The proposed family of estimators also provides greater PREs than the well-known Grover and Kaur [23] class of biased estimators, i.e., $\bar{y}_{GK}$. This class considers the information on only one auxiliary variable;
- When the proposed estimators $\bar{y}_P^{(u)}$ are contrasted against the biased estimators using the auxiliary information on two variables, i.e., $\bar{y}_{MU,0}$, $\bar{y}_{MU,1}$, $\bar{y}_{SG}$, we again observe the gain in PREs of the suggested estimators.
- Finally, it is quite obvious that the proposed family gives maximum PREs as compared to all other estimators under study.

Hence, the suggested family outperforms and provides almost unbiased and efficient estimators for estimating population mean in case of SRSWOR.

5. Concluding remarks

In this article, a new family of Hartley-Ross type unbiased estimators for estimating population mean under SRSWOR was suggested. In this family, the information on two auxiliary variables was utilized to generate

Table 3. Data statistics.

	Population	**1**	**2**	**3**
	Source	**Koyuncu and Kadilar [32]**		**Singh and Mangat [22], page 369**
		Mediterranean region	**Marmara region**	
Variables	**Study variable y**	**Number of teachers**	**Number of teachers**	**Number of tube wells**
	Auxiliary variable x	**Number of classes**	**Number of classes**	**Number of tractors**
	Auxiliary variable z	**Number of students**	**Number of students**	**Net irrigated area**
Population parameters	N	103	127	69
	n	29	31	10
	$\bar{Y}$	573.1748	703.74	135.2608
	$\bar{X}$	431.36	498.28	21.232
	$\bar{Z}$	14309.3	20804.59	345.7536
	C_y	1.8030	1.2559	0.8422
	C_x	1.4209	1.1150	0.7969
	C_z	1.9253	1.4654	0.8478
	ρ_{yx}	0.9835	0.9789	0.9119
	ρ_{yz}	0.9937	0.9366	0.9224
	ρ_{xz}	0.9765	0.9396	0.9007
	$\beta_{1(x)}$	3.1112	1.7205	1.8551
	$\beta_{2(x)}$	10.7864	2.3149	3.7653

Table 4. PRE's of different estimators for Population 1.

Estimator	PREs	Families of estimators					
		α	β	$\bar{y}_{CK1}^{(u)}$	$\bar{y}_{CK2}^{(u)}$	$\bar{y}_{GK}$	$\bar{y}_{P}^{(u)}$
$\bar{y}_0^{(u)}$	100	1	C_x	659.866	2443.415	3150.869	**59413.655**
$\bar{y}_{HR}^{(u)}$	1410.541	1	$\beta_{2(x)}$	561.315	2491.637	3145.299	**51541.452**
$\bar{y}_{S2}^{(u)}$	589.228	$\beta_{2(x)}$	C_x	678.997	2437.056	3151.680	**60736.110**
$\bar{y}_{S1}^{(u)}$	670.420	ρ_{yx}	C_x	659.532	2443.533	3150.854	**59389.838**
$\bar{y}_{MU,0}$	11417.693	$\beta_{2(x)}$	ρ_{yx}	679.634	2436.858	3151.705	**60778.786**
$\bar{y}_{MU,1}$	11673.080	ρ_{yx}	$\beta_{2(x)}$	559.860	2492.604	3145.197	**51413.962**
$\bar{y}_{SG}$	12273.554	1	ρ_{yx}	666.124	2441.249	3151.143	**59854.589**
		C_x	$\beta_{2(x)}$	589.228	2474.781	3147.139	**53930.541**
		C_x	ρ_{yx}	670.420	2439.813	3151.326	**60152.664**
		1	$\beta_{1(x)}$	637.582	2451.854	3149.823	**57780.486**
		S_x	$\beta_{1(x)}$	680.996	2436.436	3151.760	**60869.822**
		1	C_z	652.919	2445.920	3150.555	**58914.809**
		1	ρ_{yz}	665.975	2441.301	3151.137	**59844.192**
		ρ_{yz}	C_z	652.756	2445.981	3150.548	**58902.993**

Note: Bold values indicate maximum PREs.

Table 5. PREs of different estimators for Population 2.

Estimator	PREs	Families of estimators					
		α	β	$\bar{y}_{CK1}^{(u)}$	$\bar{y}_{CK2}^{(u)}$	$\bar{y}_{GK}$	$\bar{y}_{P}^{(u)}$
$\bar{y}_0^{(u)}$	100	1	C_x	1075.153	2489.092	2440.404	**2657.317**
$\bar{y}_{HR}^{(u)}$	1998.897	1	$\beta_{2(x)}$	1042.644	2489.789	2440.104	**2656.755**
$\bar{y}_{S2}^{(u)}$	1048.831	$\beta_{2(x)}$	C_x	1093.903	2488.732	2440.563	**2657.615**
$\bar{y}_{S1}^{(u)}$	1082.030	ρ_{yx}	C_x	1074.465	2489.106	2440.398	**2657.305**
$\bar{y}_{MU,0}$	1620.000	$\beta_{2(x)}$	ρ_{yx}	1095.707	2488.699	2440.578	**2657.643**
$\bar{y}_{MU,1}$	1685.198	ρ_{yx}	$\beta_{2(x)}$	1041.368	2489.819	2440.092	**2656.731**
$\bar{y}_{SG}$	2591.975	1	ρ_{yx}	1079.081	2489.014	2440.438	**2657.381**
		C_x	$\beta_{2(x)}$	1048.831	2489.649	2440.163	**2656.866**
		C_x	ρ_{yx}	1082.030	2488.957	2440.463	**2657.429**
		1	$\beta_{1(x)}$	1058.295	2489.442	2440.252	**2657.033**
		S_x	$\beta_{1(x)}$	1108.930	2488.463	2440.684	**2657.842**
		1	C_z	1065.279	2489.294	2440.316	**2657.152**
		1	ρ_{yz}	1080.312	2488.990	2440.449	**2657.401**
		ρ_{yz}	C_z	1062.544	2489.351	2440.291	**2657.106**

Note: Bold values indicate maximum PREs.

new estimators. The mathematical expressions were derived for the Bias and the minimum variance of new family up to the first order of approximation. To evaluate the potentiality of the new family, three natural populations were used. Numerical findings confirmed the efficiency of new estimators as compared to:

i) The unbiased/Hartley-Ross-type unbiased estimators, for instance, traditional estimator, Hartley and Ross [12], Singh et al. [19] and Cekim and Kadilar [20];

ii) The biased estimators using the information of one or two auxiliary variable(s) such as Grover and Kaur [23], Muneer et al. [10], and Shabbir and Gupta [11].

Table 6. PREs of different estimators for Population 3.

Estimator	PREs	Families of estimators					
		$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\bar{y}_{CK1}^{(u)}$	$\bar{y}_{CK2}^{(u)}$	$\bar{y}_{GK}$	$\bar{y}_{P}^{(u)}$
$\bar{y}_0^{(u)}$	100	1	C_x	576.732	533.718	609.601	**924.428**
$\bar{y}_{HR}^{(u)}$	589.345	1	$\beta_{2(x)}$	548.203	532.040	606.910	**910.843**
$\bar{y}_{S2}^{(u)}$	515.877	$\beta_{2(x)}$	C_x	539.836	534.694	610.309	**927.952**
$\bar{y}_{S1}^{(u)}$	587.853	ρ_{yx}	C_x	579.800	533.609	609.513	**923.989**
$\bar{y}_{MU,0}$	872.685	$\beta_{2(x)}$	ρ_{yx}	587.853	533.261	609.214	**922.493**
$\bar{y}_{MU,1}$	899.339	ρ_{yx}	$\beta_{2(x)}$	536.079	532.110	606.657	**909.553**
$\bar{y}_{SG}$	898.269	1	ρ_{yx}	581.180	533.557	609.471	**923.775**
		C_x	$\beta_{2(x)}$	515.877	532.319	606.272	**907.579**
		C_x	ρ_{yx}	587.852	533.261	609.214	**922.493**
		1	$\beta_{1(x)}$	592.953	532.574	608.490	**918.857**
		S_x	$\beta_{1(x)}$	530.559	534.893	610.440	**928.604**
		1	C_z	578.804	533.646	609.543	**924.137**
		1	ρ_{yz}	581.547	533.543	609.459	**923.716**
		ρ_{yz}	C_z	581.437	533.547	609.462	**923.733**

Note: Bold values indicate maximum PREs.

On the basis of these findings, it can be recommended using the new family for future applications.

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