A Modified Variant of Grey Wolf Optimizer

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Abstract: The original version of Grey Wolf Optimization (GWO) algorithm has small number of disadvantages of low solving accuracy, bad local searching ability and slow convergence rate. In order to overcome these disadvantages of Grey Wolf Optimizer, a new version of Grey Wolf Optimizer algorithm has been proposed by modifying the encircling behavior and position update equations of Grey Wolf Optimization Algorithm. The accuracy and convergence performance of modified variant has been tested on several well known classical further more like sine dataset and cantilever beam design functions. For verification, the results are compared with some of the most powerful well known algorithms i.e. Particle Swarm Optimization, Grey Wolf Optimizer and Mean Grey Wolf Optimization. The experimental solutions demonstrate that the modified variant is able to provide very competitive solutions in terms of improved minimum objective function value, maximum objective function value, mean, standard deviation and convergence rate.

Keywords: Particle Swarm Optimization (PSO), Grey Wolf Optimization (GWO), Mean Grey Wolf Optimization and Meta-heuristics.

1. Introduction

In last few decades, Population inspired meta-heuristics has received much attention. Several nature inspired meta-heuristics have been proposed, such as genetic algorithm (GA) [1], particle swarm optimization (PSO) [2] and
Differential Evolution (DE) [3, 4]. Although these meta-heuristics are competent to find the solution of the complex optimization functions, it has been demonstrated by the familiar No Free Lunch theorem that there is no optimization technique finding the solutions of all types of functions [5]. Therefore the theorem allows the scientists to develop several newly nature inspired techniques. Various of the recent meta-heuristics are artificial bee colony (ABC) algorithm [6], Cuckoo search (CS) algorithm [7], gravitational search algorithm (GSA) [8], Firefly algorithm (FA) [9], Cuckoo Optimization Algorithm (COA) [10], Adaptive Gbest-guided Gravitational Search Algorithm (GGSA) [11], Grey Wolf Optimizer (GWO) [12], Ant Lion Optimizer (ALO) [13] and Multiverse Optimizer (MVO) [14], Shuffled frog-leaping algorithm (SFLA) [15], Bacterial foraging optimization algorithm (BFOA) [16], Opposition-based grey wolf optimization (OGWO) [17], One Half Personal Best Position Particle Swarm Optimizations (OHGBPSSO) [18], Half Mean Particle Swarm Optimization Algorithm (HMPSO) [19], Personal Best Position Particle Swarm Optimization (PBPPSO) [20], Hybrid Particle Swarm Optimization (HPSO) [21], Hybrid MGBPSSO-GSA [22], MGWO [23], HPSOGWO [24], HGWOSCA [25] and HAGWO [26] and many others.

Biogeography based optimization algorithm proposed by D. Simon [27] is a newly population based variant which is, a study of the geographical distribution of biological organisms. Biogeography based optimization approach adopts the migration operator to share information between solutions. This aspect is same to other nature inspired variant i.e. Genetic algorithm and Particle Swarm Optimization. The performance of BBO variant has been compared with 14 benchmark functions and a real life sensor selection problem. On the basis of obtaining statistical results, it was observed that existing variant gives better quality of solutions outperform than other recent meta-heuristics.

Bat algorithm (BA) was proposed by X. She Yang [28]. BA is a bio-inspired variant and has been found to be very efficient. This variant mimics the echolocation ability of micro bat which use it for navigating and hunting. The position of the bat provides the probable solution of the problem. Fitness of
the solution is specified by the best position of a bat to its prey. Bat Algorithm has big advantage over other variants that it has a number of tunable parameters giving a greater control over the optimization process.

Flower Pollination Algorithm (FPA) was firstly proposed by X. She Yang [29]. FPA is inspired by the pollination process of flowers. The performance of this variant was tested on ten test functions and results were compared with those obtained using Particle Swarm Optimization and Genetic Algorithms. On the basis of simulation results, one can observe that the flower algorithm is more efficient than both PSO and GA. Furthermore, authors also use this variant to solve a nonlinear problems, which shows the convergence rate is almost exponential.

S. Mirjalili [30] is recently proposed GWO algorithm on eight dataset functions and its performance was compared with other nature inspired algorithms. On the basis of statistical results, it proved that GWO variant provide very competitive results in terms of improved local optima avoidance.

Mean Grey Wolf Optimizer (MGWO) was proposed by N. Singh and S.B.Singh [31]. This variant has been developed by modifying the position update (encircling behavior) equations of Grey Wolf Optimization Algorithm. MGWO variant has been tested on several well known test (Unimodal, Multimodal and Fixed dimension multimodal) functions, and the performance of modified variant has also been compared with Particle Swarm Optimization and Grey Wolf Optimization. In addition, authors have also considered five datasets classification are utilized to standard benchmark the accuracy of the modified variant. The obtained results was compared with the results using many different meta heuristic approaches i.e. Grey Wolf Optimization, Particle Swarm Optimization, Population based Incremental Learning (PBIL), Ant Colony Optimization (ACO) etc. On the basis of statistical results it has been observed that the modified variant is able to find best solutions in terms of high level of accuracy in classification and improved local optima avoidance.

N. Mittal et al. [32] is developed a modified variant of the GWO called Modified GWO (mGWO), which focuses on proper balance amid exploitation
and exploration that leads to an optimal accuracy of the variant. Simulations based on standard functions and real life application demonstrate the efficiency, stability and effectiveness of existing variant verified with the basic grey wolf optimizer algorithm and some recent meta-heuristics.

Grey Wolf Optimizer (GWO) is a newly developed population based approach inspired from the leadership hierarchy and hunting mechanism of grey wolves in nature and has been effectively applied for solving feature subset selection [33], economic dispatch problems [34], flow shop scheduling problem [35], optimal design of double later grids [36], time forecasting [37], optimizing key values in the cryptography algorithms [38] and optimal power flow problem [39]. A number of nature inspired algorithms are also developed to improve the performance of basic GWO that include a hybrid version of GWO with PSO [40], binary GWO [41], parallelized GWO [42, 43] and integration of DE with GWO [44].

L. Li et al. [45] proposes a modified discrete GWO variant, which is used to realize the multilevel image segmentation and optimize the image histograms. Based on the high efficiency of grey wolf optimizer in the course of stability and optimization, this article effectively applies the MDGWO to the field of MT by improving the location of the agents during the hunting and using weight to optimize the final position of prey. The MDGWO approach not only obtains better segmentation quality but also proves obvious superiority over ABC, DE, GWO, MTEMO and in accuracy, multilevel thresholding and stability.

H. Liu et al. [46], an intelligent grey wolf optimizer variant called DCS-GWO is developed by combining q-thresholding and GWO variant. In this variant, the grey wolves’ positions are initialized by using the q-thresholding approach and updated by using the idea of GWO. The experimental solutions illustrate that the existing variant has better recovery accuracy than previous greedy pursuit approaches at the expense of computational complexity.

S. Mirjalili et al. [47] proposed two novel optimization techniques called Salp Swarm Algorithm (SSA) and Multi-objective Salp Swarm Algorithm (MSSA) for finding the solution of global optimization functions with multiple and
single objectives. The main inspiration of Salp Swarm Algorithm and Multi-objective Salp Swarm Algorithm is the swarming behavior of salps when navigating and foraging in oceans. The performance of existing variant has been tested on several standard and real life applications. On the basis of solutions of existing variant prove that this variant can approximate pareto optimal results with high convergence and coverage.

S. Raj et al. [48], several recent meta-heuristics applied for the best possible optimal solution of reactive power planning with FACTS devices. Further some more recent techniques have been also applied to find the best optimal setting of all control variables. The working performance of the existing variant has been illustrated by comparing the solutions obtained with all other recent meta-heuristics. On the basis of simulation results proven that the existing variant shows lesser number of generations which does not gets trapped in the local minima and offers promising convergence characteristics.

In this article, we focus on grey wolf optimizer, developed by Mirjalili [12] in 2014 based on simulating hunting behavior and social leadership of grey wolves in nature. Experimental results proved that the better accuracy of the existing variant is also competitive to that of other meta-heuristics. Since it is easy and simple to implement and has fewer control constants, grey wolf optimizer has caused a lot attention and has been used to find the solution of practical real life functions.

Particle Swarm Optimization, Genetic algorithm, Evolutionary algorithm, Differential algorithm and Ant Colony Optimization are most well-liked meta-heuristic global Optimization approaches. These nature inspired techniques, as the growth of the search area dimension, Grey Wolf Optimizer provides a bad convergence behavior at exploitation [49, 50]. Hence, it is essential to emphasize that our research effort falls in increasing the local search ability of Grey Wolf Optimizer technique. In order to improve the local search ability of GWO algorithm, we propose a newly modified meta-heuristic in this research and it's performance has been compare with the results of grey wolf optimizer and some other recent nature inspired algorithms, MVGWO performs significantly better.
The rest of the paper is structured as follows.

In section 2, describes the Grey Wolf Optimizer (GWO) Algorithm. The newly proposed algorithm Modified Variant of Grey Wolf Optimization Algorithm (MVGWO) is presented in section 3. The MVGWO mathematical model and pseudo code has been discussed in section 3. The tested Unimodal, Multimodal and Fixed-dim Multimodal classical functions are presented in section 4. Results and discussion is summarized in 5-6 respectively. Sine dataset function and Cantilever Beam Design function are briefly describes in section 7-8. Conclusions drawn on the basis of results obtained have been given in section 9 of this paper.

2. Grey Wolf Optimizer (GWO) Algorithm

The Grey Wolf Optimizer algorithm is a newly global optimization approach which simulate the grey wolves leadership and hunting in nature. These approaches have been inspired by simple concepts.

Mirjalili, S. et al. [12] proposed a Grey Wolf Optimization meta-heuristic approach. The Grey Wolf Optimization variant mimics the hunting mechanism and leadership hierarchy of grey wolves in nature. In the hierarchy of Grey Wolf Optimization, alpha is considered the dominating agent among the group. The rest of the subordinates to alpha are beta, delta which helps to control the majority of wolves in the hierarchy that are considered as omega.

In addition, three main steps of hunting, searching for prey, encircling prey, and attacking prey, are implemented to perform optimization.

The encircling behavior of each member of the population is represented by the following mathematical equations:

\[ d = \| c \cdot x_{p(t)} - x(t) \| \]  \hspace{1cm} (1)

\[ x(t + 1) = x_{p(t)} - a.d \]  \hspace{1cm} (2)
Where, $x_p$ is the position vector the prey, $t$ indicates the time and $x$, indicate the position vector of a grey wolf

The vectors $a$ and $c$ are mathematically calculated as:

$$a = 2t, r_1 - t$$  \hspace{1cm} (3)

$$c = 2, r_2$$  \hspace{1cm} (4)

Where components of $a$ are linearly decreased from 2 to 0 over the course of generations and $r_1, r_2 \in [0,1]$ are random vectors.

Hunting: In order to mathematically simulate the hunting behavior, we suppose that the alpha ($\alpha$), beta ($\beta$) and delta ($\delta$) have better knowledge about the potential location of prey. The following mathematically equations are developed in this regard.

$$\ddot{d}_\alpha = \dddot{c}_1 \dddot{x}_\alpha \dddot{x}_\alpha \dddot{x}_\alpha, \ddot{d}_\beta = \dddot{c}_2 \dddot{x}_\beta \dddot{x}_\beta \dddot{x}_\beta, \ddot{d}_\delta = \dddot{c}_3 \dddot{x}_\delta \dddot{x}_\delta \dddot{x}_\delta$$  \hspace{1cm} (5)

$$\dddot{x}_1 = \dddot{x}_\alpha - \dddot{a}_1(\dddot{d}_\alpha), \dddot{x}_2 = \dddot{x}_\beta - \dddot{a}_2(\dddot{d}_\beta), \dddot{x}_3 = \dddot{x}_\delta - \dddot{a}_3(\dddot{d}_\delta)$$  \hspace{1cm} (6)

$$\frac{\dddot{x}_1 + \dddot{x}_2 + \dddot{x}_3}{3}$$  \hspace{1cm} (7)

Where $\dddot{x}_\alpha$, $\dddot{x}_\beta$ and $\dddot{x}_\delta$ are the position of the member of the population in the searching space at $t$th iteration, $t$ indicate the current iteration and $\dddot{x}(t)$ presents the position of the grey wolf at $t$th iteration

$$\ddot{a}_\alpha = 2t, \ddot{r}_1 - t$$  \hspace{1cm} (8)

$$\ddot{c}_\alpha = 2, \ddot{r}_2$$  \hspace{1cm} (9)

Where components of $\ddot{t}$ are linearly decreased from 2 to 0 over the course of generations and $r_1, r_2$ are random vector in[0,1]. The $\ddot{a}(\alpha)$ and $\ddot{c}(\alpha)$ are coefficient vector of alpha ($\alpha$), beta ($\beta$) and delta ($\delta$) wolves.
Search for prey and attacking prey: The $A$ is random value in the gap $(-a, a)$. When random value $|A| < 1$ the wolves are forced to attack the prey. Searching for prey is the exploration ability and attacking the prey is the exploitation ability. The arbitrary values of $A$ are utilized to force the search to move away from the prey.

When $|A| > 1$, the members of the population are enforced to diverge from the prey.

3. Modified Variant of Grey Wolf Optimizer

Mirjalili, S. et al. [14] proposed a new version of population based algorithms, called Grey Wolf Optimizer (GWO). The Grey Wolf Optimization variant mimics the hunting mechanism and leadership hierarchy of grey wolves in nature. In the hierarchy of Grey Wolf Optimization, alpha is considered the dominating agent among the group. The rest of the subordinates to alpha are beta, delta which helps to control the majority of wolves in the hierarchy that are considered as omega. In addition, three main steps of hunting, searching for prey, encircling prey, and attacking prey, are implemented to perform optimization.

The proposed variant has been developed by modifying encircling behavior and position update equation of Grey Wolf Optimization algorithm with an idea to improve the performance, convergence speed and accuracy of the Grey Wolf Optimizer meta-heuristics. In modified variant of Grey Wolf Optimization, the population is divided into five different groups such as alpha, beta, gamma, delta, and omega which are employed for simulating the leadership hierarchy [see in Figure 1]. The all rest operations are same as Grey Wolf Optimizer variant [14].

Social Hierarchy: In order to develop mathematical model the social hierarchy of wolves when designing modified variant of GWO (MVGWO), we consider the fittest solution as the alpha. Accordingly, the second, third and fourth best
solutions are named beta, gamma and delta. The rest of the agent solutions are
assumed to be omega.

The mathematical model of the encircling behavior is represented by the
equations:

\[
d = \left| c \cdot p(t) - \mu \times x(t) \right| \tag{10}
\]

\[
x(t + 1) = x_{p(t)} - a \cdot d \tag{11}
\]

Here \( a \) and \( c \) coefficient vectors are given by:

\[
a = 2l \bar{r}_1 \tag{12}
\]

\[
c = 2r_2 \tag{13}
\]

Where components of \( l \in [2, 0] \) and \( \bar{r}_1, r_2 \in [0, 1] \).

Hunting: In order to simulate the hunting behavior mathematically, we
suppose that the alpha (\( \alpha \)), beta (\( \beta \)), gamma (\( \gamma \)) and delta (\( \delta \)) have better
knowledge about the potential location of prey. The following mathematically
equations are developed in this regard.

\[
\begin{align*}
\bar{d}_\alpha &= \bar{c}_1 \cdot \bar{x}_1 - \bar{x}, \quad \bar{d}_\beta &= \bar{c}_2 \cdot \bar{x}_2 - \bar{x}, \quad \bar{d}_\gamma &= \bar{c}_3 \cdot \bar{x}_3 - \bar{x}, \quad \bar{d}_\delta &= \bar{c}_4 \cdot \bar{x}_4 - \bar{x} \\
\bar{x}_1 &= \bar{x}_\alpha - \bar{a}_1 \cdot \bar{d}_\alpha, \quad \bar{x}_2 = \bar{x}_\beta - \bar{a}_2 \cdot \bar{d}_\beta, \quad \bar{x}_3 = \bar{x}_\gamma - \bar{a}_3 \cdot \bar{d}_\gamma, \quad \bar{x}_4 = \bar{x}_\delta - \bar{a}_4 \cdot \bar{d}_\delta \\
\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4 &= \frac{4}{4} \tag{14}
\end{align*}
\]

\[
\bar{a}_i = 2l_i \bar{r}_i - \bar{I} \tag{15}
\]

\[
\bar{c}_i = 2r_i \tag{16}
\]

Pseudo Code of Modified Variant of GWO (MVGWO):

- Initialization of population
- Initialize \( l, a \) and \( c \)
- Evaluate the fitness of each search member
\[ x_{\alpha}, x_{\beta}, x_{\gamma}, \text{and } x_{\delta} \] are the first, second, third and fourth best search member

while \((t < \text{max no. of iter})\)

for each search member

Update the position of each member of the population by mathematical equation (1.16)

end for

Update \(l, a\) and \(c\)

Evaluate the fitness of all search members

Update \(x_{\alpha}, x_{\beta}, x_{\gamma}, \text{and } x_{\delta}\)

\(t = t + 1\)

end while

return \(x_{\alpha}\)

4. Testing Functions

In this section, twenty three classical functions are used to verify the performance of modified variant of GWO (MVGWO). These test functions can be divided into three different groups: Unimodal, Multimodal and Fixed dimension multimodal functions. The specific details of these functions are represented by Table 1, Table 2 and Table 3 respectively.

5. The convergence performance graphs of MVGWO algorithm

The performance of several population based meta-heuristics has been verified with the MVGWO variant in order to test the convergence rate, stability and computational accuracy on the number of iterations in figure 2. The similar parameter values have been taken for the entire algorithms to make fair comparison. The results illustrates that in convergence figure 2, by plotting the best optimal values of function, values against the number of generations for simplified model of the molecule with different size from 1000 to 5000 dimensions.
The graphs show that the standard test function values quickly decrease as the number of generations increases for newly existing variant solutions than those of the other metaheuristics. In figure 2, PSO, GWO, HGWO and MVGWO variants suffer from the slow convergence, gets stuck in the partitioning procedure, nevertheless and many local minima and invoking the Mean Grey Wolf variant in the newly existing hybrid algorithm avoid trapping in local minima and accelerate the search.

6. Results and Discussion

The Modified Variant of GWO, Mean GWO, GWO and Particle Swarm Optimization algorithms are coded in MATLAB R2013a and implemented on Intel HD Graphics, Pentium-Intel Core (TM), i5 Processor 430 M , 15.6” 16.9 HD LCD, 3GB Memory and 320 GB HDD. Number of search agents (30), maximum number of iterations (1000) and $l \in [2, 0]$ all these parameter are used to confirm the quality of modified meta-heuristics.

Generally any nature inspired technique is tested by computing its results with those obtained through other meta-heuristics. We also follow the same procedure and employ twenty three classical functions for judgment. These test functions are divided to three parts: unimodal, multimodal and fixed-dimension multimodal functions. The mathematical formulation of classical functions is presented in Tables 1–3. We consider thirty variables for multimodal and unimodal classical function for further improving their difficulties.

The accuracy of newly modified algorithm have been applied on the classical, sine dataset and cantilever beam design functions in the terms of minimum objective function values, maximum objective function values, mean and standard deviation [Table 4 - Table 9].

Here, the maximum and minimum value of the objective functions gives the best suitable cost of the classical problems in the least number of iterations. On the other side, the mean and standard deviation statistical values are used
to evaluate the reliability. Further, the convergence graphs of the classical problems represented the convergence performance of the variants.

The tables 4, 6 and 8 are represented that the newly modified algorithm gives the best global optimal values of the classical problems in the terms of minimum and maximum values of the functions as comparison to others meta-heuristics and tables 5, 7 and 9 illustrates that, the modified algorithm also gives the superior quality of standard and mean values on the maximum classical functions in form of least values as comparison to the other meta-heuristics. At the end, the convergence of graphs [Figures 3-25] proven that, the existing approach finds the best possible optimal values of the standard functions in the least number of iterations as comparison to others.

It is clear from the results given in tables 4-5, that the proposed variant outperforms other meta-heuristics like PSO, GWO and MGWO in terms of mean, standard deviation, min/max cost function and exploiting the optimum. As such the proposed variant is highly competitive with other meta-heuristics.

Further the convergence behavior of modified variant of Grey Wolf Optimizer, Particle Swarm Optimization, Mean Grey Wolf Optimization and Grey Wolf Optimizer algorithms has been investigated and convergence curve has been plotted by Figures 3-25. In order to examine the convergence behavior of the modified variant of GWO, Particle Swarm Optimization, Mean Grey Wolf Optimization and Grey Wolf Optimizer algorithms, the search history and path of the first search member of the population in its first dimension are illustrated in Figures 3-25. On the basis of convergence curve we observe that modified variant give better convergence points as comparison to other.

Furthermore, the suitable results of multimodal and fixed-dimension multimodal functions are illustrated in Tables 6-9. The multimodal and fixed dimensional functions have many local optima with the number growing exponentially with dimension. This makes them fit for benchmarking the exploration capacity of a variant. On the basis of results of Tables 6-9, modified variant is able to present better quality of solutions on the maximum
number of multimodal and fixed dimensional multi-modal functions as compared to PSO, GWO and MGWO algorithm. These solutions demonstrate that the modified variant (MVGWO) has merit in terms of exploration.

A number of criteria have been used to find out the accuracy of Grey Wolf Optimizer (GWO), Particle Swarm Optimization (PSO), Mean Grey Wolf Optimizer (MGWO) and the newly modified algorithm of Grey Wolf Optimizer GWO (MVGWO). The mean and standard deviation statistical values are used to evaluate the reliability in Tables 5, 7 and 9. The average computational time of the successful runs and the average number of function evaluations of successful runs, are applied to estimate the cost of the standard function.

In Figures 3-25, the convergence performance of GWO, PSO, MGWO and MVGWO algorithms in solving classical problems have been compared, obtained convergence solutions prove that the MVGWO algorithm is more able to find the best optimal solution in minimum number of iterations. Hence MVGWO algorithm avoids premature convergence of the search process to local optimal point and provides superior exploration of the search course.

To sum up, all simulation solutions assert that the newly existing algorithm is very helpful in increasing the efficiency of the Grey Wolf Optimizer Algorithm in the terms of result quality as well as computational efforts.

7. Sine dataset function

This dataset has number of attributes 01, structure 1-15-1 chosen to be trained and solve this dataset (S. Mirjalili, 2015). This function has four peaks that make it extremely difficult to be approximated. Sine dataset function has been testing on different nature inspired meta-heuristics. On the basis of obtaining results, we observe that modified variant of grey wolf optimizer provides extremely accurate solutions on this dataset as can be inferred from test error in Table 10 and convergence and best solution performance of MVGWO is plotted by Figures 26-27.

8. Cantilever Beam Design function
This function is related to the design variables are the width of the different beam elements and weight optimization, the thickness is constant [51]. The briefly description of the cantilever beam function is as follows:

$$\text{Min}(X) = 0.0624(l + m + n + o + p)$$

(19)

Subject to

$$g(X) = \frac{61}{l^3} + \frac{37}{m^3} + \frac{19}{n^3} + \frac{7}{o^3} + \frac{1}{p^3} - 1 \leq 0,$$

(20)

where \(0.01 \leq l, m, n, o, p \leq 100\). The global optimal results for the given function are listed in Table 11.

During last few decades several researchers have used different types of meta-heuristics to find the best possible optimal solutions of the cantilever beam design function in the literature such as Convex Linearization method (CONLIN) [51], cuckoo search (CS) [51], method of moving asymptotes (MMA) [51], Grid based clustering algorithm –I and II (GCA-I and GCA-II) [52] and Symbiotic Organisms Search (SOS) [53].

The experimental results of the different variants on given function are illustrated in Table 11. That experiment has been tested on the following parameter settings; search agents (30) and maximum number of iterations (500).

It can be seen that the best optimal value of the cantilever beam design function on MVGWO is 1.33966. Hence MVGWO variant gives the better quality of the solutions as compared to other recent algorithms.

9. Conclusion

This paper presents a modified variant of GWO called MVGWO. This modified variant has been developed by modifying encircling behavior and position update equation of Grey Wolf Optimization algorithm with an idea to improve the performance, convergence speed and accuracy of the Grey Wolf Optimizer meta-heuristics. These modifications are used to balance
exploration and exploration over the path of generations. The performance of proposed variant has been tested using several benchmark functions. It is observed that the modified variant has an edge of high exploration over other meta-heuristics like Particle Swarm Optimization (PSO), Grey Wolf Optimizer (GWO) and Mean Grey Wolf Optimization (MGWO). Further the performance of modified variant has been tested on sine dataset function and cantilever beam design function and the experimental results have been compared with several recent nature inspired algorithms. The result shows that the modified variant has proved to give very effective solutions of sine dataset and cantilever beam design function as compared to other meta-heuristics.

References


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Table 1.Unimodal benchmark functions
<table>
<thead>
<tr>
<th>Function</th>
<th>Dim</th>
<th>Range</th>
<th>$f_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1(x) = \sum_{i=1}^{n} x_i^2$</td>
<td>30</td>
<td>[-100,100]</td>
<td>0</td>
</tr>
<tr>
<td>$F_2(x) = \sum_{i=1}^{n}</td>
<td>x_i</td>
<td>+ \prod_{i=1}^{n}</td>
<td>y_i</td>
</tr>
<tr>
<td>$F_3(x) = \sum_{i=1}^{n} \left( \sum_{j=1}^{i} x_j \right)^2$</td>
<td>30</td>
<td>[-100,100]</td>
<td>0</td>
</tr>
<tr>
<td>$F_4(x) = \max_i \left{</td>
<td>x_i</td>
<td>, 1 \leq i \leq n \right}$</td>
<td>30</td>
</tr>
<tr>
<td>$F_5(x) = \sum_{i=1}^{n-1} \left[ 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$</td>
<td>30</td>
<td>[-30,30]</td>
<td>0</td>
</tr>
<tr>
<td>$F_6(x) = \sum_{i=1}^{n-1} \left[ (x_i + 0.5)^2 \right]$</td>
<td>30</td>
<td>[-100,100]</td>
<td>0</td>
</tr>
<tr>
<td>$F_7(x) = \sum_{i=1}^{n} x_i^4 + \text{rand}[0,1)$</td>
<td>30</td>
<td>[-]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Multimodal benchmark functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Dim</th>
<th>Range</th>
<th>$f_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{</td>
<td>x_i</td>
<td>})$</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>500,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\times$ 5</td>
</tr>
<tr>
<td>$F_9(x) = \sum_{i=1}^{n} x_i^2 -10\cos(2\pi x_i)+10$</td>
<td>30</td>
<td>[-]</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.12,5.12</td>
</tr>
<tr>
<td>$F_{10}(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^{n} \cos(2\pi x_i)\right) + 20 + e$</td>
<td>30</td>
<td>[-32,32]</td>
<td>0</td>
</tr>
</tbody>
</table>
\[ F_{11}(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1 \quad 30 \quad [-, 0] \quad 600, 600 \]

\[ F_{12}(x) = \frac{n}{\pi} \left[ 10\sin(\pi y_j) + \sum_{i=1}^{n-1} (y_{i+1} - 1)^2 \left[ 1 + 10\sin^2 \left( \frac{\pi y_{i+1}}{30} \right) \right] \right] + \sum_{i=1}^{n} u(x_i, 10, 100, 4) \]

\[ y_j = 1 + \frac{x_j + 1}{4} \]

\[ u(x_j, a, k, m) = \begin{cases} k(x_j - a)^m & x_j > a \\ 0 & -a < x_j < a \\ k(-x_j - a)^m & x_j < -a \end{cases} \]

\[ F_{13}(x) = 0.1 \left[ \sin^2(3\pi x_j) + \sum_{i=1}^{n} (y_{i+1} - 1)^2 \left[ 1 + \sin^2 \left( \frac{3\pi x_j}{30} \right) \right] \right] + \sum_{i=1}^{n} u(x_i, 5, 100, 4) \]

---

**Table 3. Fixed-dimension multimodal benchmark functions**

<table>
<thead>
<tr>
<th>Function (F_i)</th>
<th>Dim</th>
<th>Range</th>
<th>(f_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_{14}(x) = \left( \frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{10}(x_i - a_{ij})} \right)^{-1} )</td>
<td>2</td>
<td>[-, 1]</td>
<td>65, 65</td>
</tr>
<tr>
<td>(F_{15}(x) = \sum_{i=1}^{11} a_i \left( \frac{x_i (b_i^2 + b_i x_i + x_i)}{b_i^2 + b_i x_i + x_i^2} \right)^2 )</td>
<td>4</td>
<td>[-5, 5]</td>
<td>0.0003</td>
</tr>
<tr>
<td>(F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4 )</td>
<td>2</td>
<td>[-5, 5]</td>
<td>-1.0316</td>
</tr>
</tbody>
</table>
\[
F_{17}(x) = \left( x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left( 1 - \frac{1}{8\pi} \cos x_1 + 10 \right)
\]

\[
F_{18}(x) = \left[ 1 + (x_1 + x_2 + 1)^2 \left( 19 - 14x_1 + 3x_1^2 \right) \right] \left( -14x_2 + 6x_1x_2 + 3x_2^2 \right) \times \left[ 30 + (2x_1 - 3x_2)^2 \right] \times \left[ 18 - 32x_1 + 12x_1^2 \right] \times \left[ +48x_2 - 36x_1x_2 + 27x_2^2 \right]
\]

\[
F_{19}(x) = -\sum_{i=1}^{4} c_i \exp \left( -\sum_{j=1}^{3} a_i (x_j - p_{ij})^2 \right)
\]

\[
F_{20}(x) = -\sum_{i=1}^{4} c_i \exp \left( -\sum_{j=1}^{6} a_i (x_j - p_{ij})^2 \right)
\]

\[
F_{21}(x) = -\sum_{i=1}^{5} \left[ (X - a_i)(X - a_i)^T + c_i \right]^{-1}
\]

\[
F_{22}(x) = -\sum_{i=1}^{7} \left[ (X - a_i)(X - a_i)^T + c_i \right]^{-1}
\]

\[
F_{23}(x) = -\sum_{i=1}^{10} \left[ (X - a_i)(X - a_i)^T + c_i \right]^{-1}
\]

<table>
<thead>
<tr>
<th>Problem</th>
<th>PSO</th>
<th>GWO</th>
<th>MGWO</th>
<th>MVGWO</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>1.</td>
<td>2.1532e-6</td>
<td>6.6032e+04</td>
<td>6.1668e-5</td>
<td>5.8943e+04</td>
</tr>
</tbody>
</table>

Table 4. (Max and Min) results of Unimodal benchmark functions
<table>
<thead>
<tr>
<th>Problem No.</th>
<th>PSO</th>
<th>GWO</th>
<th>MGWO</th>
<th>MVGWO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>$\mu$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>1.</td>
<td>408.5162</td>
<td>3.9524e+03</td>
<td>215.8357</td>
<td>2.5750e+03</td>
</tr>
<tr>
<td>2.</td>
<td>9.5933e+06</td>
<td>3.0290e+08</td>
<td>1.4316e+09</td>
<td>4.5236e+10</td>
</tr>
<tr>
<td>3.</td>
<td>1.8412e+03</td>
<td>1.3960e+04</td>
<td>1.7745e+03</td>
<td>1.0505e+04</td>
</tr>
<tr>
<td>4.</td>
<td>3.5511</td>
<td>7.3907</td>
<td>2.6146</td>
<td>12.1387</td>
</tr>
<tr>
<td>5.</td>
<td>5.9195e+05</td>
<td>1.0872e+07</td>
<td>7.4518e+05</td>
<td>1.0897e+07</td>
</tr>
<tr>
<td>6.</td>
<td>516.2407</td>
<td>4.6252e+03</td>
<td>335.9674</td>
<td>3.4849e+03</td>
</tr>
<tr>
<td>7.</td>
<td>53.3529</td>
<td>56.2679</td>
<td>0.4942</td>
<td>5.8041</td>
</tr>
</tbody>
</table>

Table 5. (Mean and S.D.) results of Unimodal benchmark functions

Figure 1. Hierarchy of grey wolf (dominance decreases from top down)

Figure 2. Convergence graphs of algorithms

Figure 3. Convergence graph of fixed-dimension multimodal benchmark function ($f_1$)
Figure 4. Convergence graph of fixed-dimension multimodal benchmark function ($F_1$)

Figure 5. Convergence graph of fixed-dimension multimodal benchmark function ($F_3$)

Figure 6. Convergence graph of fixed-dimension multimodal benchmark function ($F_4$)

Figure 7. Convergence graph of fixed-dimension multimodal benchmark function ($F_5$)

Figure 8. Convergence graph of fixed-dimension multimodal benchmark function ($F_6$)

Figure 9. Convergence graph of fixed-dimension multimodal benchmark function ($F_7$)

Figure 10. Convergence graph of fixed-dimension multimodal benchmark function ($F_8$)

Figure 11. Convergence graph of fixed-dimension multimodal benchmark function ($F_9$)

Figure 12. Convergence graph of fixed-dimension multimodal benchmark function ($F_{10}$)

Figure 13. Convergence graph of fixed-dimension multimodal benchmark function ($F_{11}$)

Figure 14. Convergence graph of fixed-dimension multimodal benchmark function ($F_{12}$)

Figure 15. Convergence graph of fixed-dimension multimodal benchmark function ($F_{13}$)
Figure 16. Convergence graph of fixed-dimension multimodal benchmark function ($F_{14}$)

Figure 17. Convergence graph of fixed-dimension multimodal benchmark function ($F_{15}$)

Figure 18. Convergence graph of fixed-dimension multimodal benchmark function ($F_{16}$)

Figure 19. Convergence graph of fixed-dimension multimodal benchmark function ($F_{17}$)

<table>
<thead>
<tr>
<th>Figure 20. Convergence graph of fixed-dimension multimodal benchmark function ($F_{18}$)</th>
<th>Figure 21. Convergence graph of fixed-dimension multimodal benchmark function ($F_{19}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 22. Convergence graph of fixed-dimension multimodal benchmark function ($F_{20}$)</td>
<td>Figure 23. Convergence graph of fixed-dimension multimodal benchmark function ($F_{21}$)</td>
</tr>
<tr>
<td>Figure 24. Convergence graph of fixed-dimension multimodal benchmark function ($F_{22}$)</td>
<td>Figure 25. Convergence graph of fixed-dimension multimodal benchmark function ($F_{23}$)</td>
</tr>
</tbody>
</table>

Figure 26. Sine graph of MVGWO

Figure 27. Convergence graph of MVGWO
### Table 6. (Max and Min) results of multimodal benchmark functions

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>PSO</th>
<th>GWO</th>
<th>MGWO</th>
<th>MVGWO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>1.</td>
<td>-</td>
<td>6.6067e+03</td>
<td>-</td>
<td>1.4706e+03</td>
</tr>
<tr>
<td>2.</td>
<td>39.7987</td>
<td>422.6854</td>
<td>5.6843e-14</td>
<td>458.7865</td>
</tr>
<tr>
<td>3.</td>
<td>1.5846e-05</td>
<td>20.5268</td>
<td>1.5099e-14</td>
<td>20.7623</td>
</tr>
<tr>
<td>4.</td>
<td>2.0755e-12</td>
<td>667.1103</td>
<td>0.0092</td>
<td>665.7767</td>
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<tr>
<td>5.</td>
<td>2.0193e-12</td>
<td>6.1692e+08</td>
<td>0.0304</td>
<td>5.5204e+08</td>
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<tr>
<td>6.</td>
<td>6.5797e-08</td>
<td>1.0597e+09</td>
<td>0.6975</td>
<td>8.0560e+08</td>
</tr>
</tbody>
</table>

### Table 7. (Mean and S.D.) results of multimodal benchmark functions

<table>
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<tr>
<th>Problem No.</th>
<th>PSO</th>
<th>GWO</th>
<th>MGWO</th>
<th>MVGWO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μ</td>
<td>σ</td>
<td>μ</td>
<td>σ</td>
</tr>
<tr>
<td>8.</td>
<td>-</td>
<td>6.0956e+03</td>
<td>1.1171e+03</td>
<td>-</td>
</tr>
<tr>
<td>10.</td>
<td>2.9657</td>
<td>3.4941</td>
<td>0.4125</td>
<td>2.2762</td>
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<tr>
<td>11.</td>
<td>25.7028</td>
<td>100.2551</td>
<td>3.1497</td>
<td>32.9323</td>
</tr>
<tr>
<td>12.</td>
<td>1.1219e+06</td>
<td>2.2349e+07</td>
<td>1.5736e+06</td>
<td>2.4904e+07</td>
</tr>
<tr>
<td>13.</td>
<td>2.5209e+06</td>
<td>4.4037e+07</td>
<td>3.2041e+06</td>
<td>4.3667e+07</td>
</tr>
</tbody>
</table>
Table 8. (Max and Min) results of fixed-dimension multimodal benchmark functions

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>PSO</th>
<th>GWO</th>
<th>MGWO</th>
<th>MVGWO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>7.</td>
<td>2.9920</td>
<td>12.6709</td>
<td>10.7632</td>
<td>86.5835</td>
</tr>
<tr>
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<td>9.8869e-04</td>
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<td>3.0750e-04</td>
<td>0.1331</td>
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<tr>
<td>9.</td>
<td>-1.0316</td>
<td>0.0804</td>
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<td>-0.1653</td>
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Table 9. (Mean and S.D.) results of fixed-dimension multimodal benchmark functions

<table>
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<tr>
<th>Problem No.</th>
<th>PSO $\mu$</th>
<th>PSO $\sigma$</th>
<th>GWO $\mu$</th>
<th>GWO $\sigma$</th>
<th>MGWO $\mu$</th>
<th>MGWO $\sigma$</th>
<th>MVGWO $\mu$</th>
<th>MVGWO $\sigma$</th>
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<tbody>
<tr>
<td>14.</td>
<td>2.1505</td>
<td>0.9110</td>
<td>11.2055</td>
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<td>12.9194</td>
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<tr>
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<td>0.0107</td>
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<td>0.0070</td>
<td>0.0013</td>
<td>0.0201</td>
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<tr>
<td>16.</td>
<td>-</td>
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<td>-1.0275</td>
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<td>-1.0303</td>
<td>0.0287</td>
<td>-</td>
<td>1.0312</td>
</tr>
<tr>
<td>17.</td>
<td>0.4004</td>
<td>0.0584</td>
<td>0.3992</td>
<td>0.0041</td>
<td>0.4004</td>
<td>0.0074</td>
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<tr>
<td>18.</td>
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<td>3.1319</td>
<td>2.0225</td>
<td>3.1918</td>
<td>2.5571</td>
<td>3.0323</td>
<td>0.6594</td>
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<tr>
<td>19.</td>
<td>-</td>
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<td>-</td>
<td>3.8565</td>
</tr>
<tr>
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<td>-3.1068</td>
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<tr>
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<td>-</td>
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</table>
Table 10. Experimental results for the sine datasets

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Test Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVGW O</td>
<td>0.2549</td>
<td>0.0018</td>
<td>2.7221e+04</td>
</tr>
<tr>
<td>Algorithms</td>
<td>$l$</td>
<td>$m$</td>
<td>$n$</td>
</tr>
<tr>
<td>-----------</td>
<td>-------</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>CONLIN</td>
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<td>GCA-II</td>
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<td>4.4900</td>
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<td>[18]</td>
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<td>4.4900</td>
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<td>4.49386</td>
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</table>

Table 11. Best Optimal solutions of the cantilever beam design function by different meta-heuristics.