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# A modified variant of grey wolf optimizer

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**KEYWORDS**
Particle Swarm Optimization (PSO);
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**Abstract.** The original version of Grey Wolf Optimization (GWO) algorithm has a few disadvantages such as low solving accuracy, unsatisfactory ability of local searching, and slow convergence rate. In order to compensate these disadvantages of grey wolf optimizer, a new version of grey wolf optimizer algorithm was proposed by modifying the encircling behavior and position update equations of GWO algorithm. The accuracy and convergence performances of the modified variant were tested on several well-known classical, sine datasets, and cantilever beam design functions. For verification, the results were compared with some of the most powerful, well-known algorithms, i.e., particle swarm optimization, grey wolf optimizer, and mean grey wolf optimization. The experimental solutions demonstrated that the modified variant was able to provide very comparable solutions in terms of improved minimum value of objective function, maximum value of objective function, mean, standard deviation, and convergence rate.



## 1. Introduction

Over the last few decades, population-inspired meta-heuristics have received much attention. Several nature-inspired meta-heuristics have been proposed, such as Genetic Algorithm (GA) [1], Particle Swarm Optimization (PSO) [2], and Differential Evolution (DE) [3,4]. Although these meta-heuristics are competent enough to find the solution to complex optimization functions, there is no optimization technique for finding the solutions to all types of functions based on the no free lunch theorem [5]. Therefore, the theorem allows scientists to develop several new nature-inspired techniques. Various recent meta-heuristics include Artificial Bee Colony (ABC) algorithm [6], Cuckoo Search (CS) algorithm [7], Gravitational Search Algorithm (GSA) [8], firefly algorithm [9], cuckoo optimization algorithm [10], adaptive Gbest-guided Gravitational Search Algorithm (GGSA) [11], Grey Wolf Optimization (GWO) [12], Ant Lion Optimizer (ALO) [13], Multiverse Optimizer (MVO) [14], Shuffled Frog-Leaping Algorithm (SFLA) [15], Bacterial Foraging Optimization Algorithm (BFOA) [16], Opposition-based Grey Wolf Optimization (OGWO) [17], one half personal best position particle swarm optimizations [18], half mean particle swarm optimization algorithm [19], personal best position particle swarm optimization [20], Hybrid Particle Swarm Optimization (HPSO) [21], hybrid Mean Gbest Particle Swarm Optimization Gravitational Search Algorithm (MGBPSO-GSA) [22], Mean Grey Wolf Optimization (MGWO) [23], Hybrid Particle Swarm Optimization Grey Wolf Optimization (HPSOGWO) [24], Hybrid Grey Wolf Optimization Sine Cosine Algorithm (HGWOSCA) [25], Hybrid Algorithm Grey Wolf Optimization (HAGWO) [26], and many others.

The biogeography-based optimization algorithm proposed by Simon [27] is a new population-based variant, which studies the geographical distribution of biological organisms. The biogeography-based optimization approach adopts a migration operator to

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share information between solutions. This aspect is the same as that of other nature-inspired variants, i.e., GA and PSO. The performance of the Biogeography-Based Optimization (BBO) variant has been compared with 14 benchmark functions and a real-life sensor selection problem. On the basis of obtaining statistical results, it has been observed that the existing variant produced better quality of solutions that outperformed other recent meta-heuristics.

Bat Algorithm (BA) was proposed by Yang [28]. BA is a bio-inspired variant and has been found very efficient. This variant mimics the echolocation ability of microbat that uses it to navigate and hunt. The position of the bat provides a probable solution to the problem. The fitness of the solution is specified by the best position of a bat to its prey. BA has many advantages over other variants including a number of tunable parameters that provide greater control over the optimization process.

Flower Pollination Algorithm (FPA) was first proposed by Yang [29]. FPA is inspired by the pollination process of flowers. The performance of this variant was tested on ten test functions, and results were compared with those obtained using PSO and GA. On the basis of the simulation results, one can observe that the flower algorithm is more efficient than both PSO and GA are. Furthermore, authors use this variant to solve a nonlinear problem, showing that the convergence rate is almost exponential.

Recently, Mirjalili [30] proposed the GWO algorithm for eight dataset functions, and its performance was compared with other nature-inspired algorithms. On the basis of statistical results, it was proved that the GWO variant provided highly competitive results in terms of improved local optima avoidance.

MGWO was proposed by Singh and Singh [31]. This variant was developed by modifying the position update (encircling behavior) equations of GWO algorithm. MGWO variant was tested on several well-known tests (unimodal, multimodal, and fixed-dimension multimodal functions); moreover, the performance of the modified variant was compared with those of PSO and GWO. In addition, five datasets were classified to assess the accuracy of the modified variant. The obtained results were compared with those obtained by many different meta-heuristic approaches, i.e., GWO, PSO, Population-Based Incremental Learning (PBIL), Ant Colony Optimization (ACO), etc. According to the statistical results, it was observed that the modified variant could find the best solutions in terms of high accuracy level in classification and improved local optima avoidance.

Mittal et al. [32] developed a modified variant of the GWO, called Modified GWO, which focused on proper balance amid exploitation and exploration that led to the optimum accuracy of the variant. The simulations based on standard functions and real-life application demonstrated the verified efficiency and stability of the existing variant based on the basic grey wolf optimizer algorithm and some recent meta-heuristics.

GWO is a newly developed population-based approach inspired by the leadership hierarchy and hunting mechanism of grey wolves in nature and has been effectively applied to solve feature subset selection [33], economic dispatch problems [34], flow shop scheduling problem [35], optimal design of double-layer grids [36], time forecasting [37], optimizing key values in the cryptography algorithms [38], and optimal power flow problem [39]. A number of the nature-inspired algorithms are also developed to improve the performance of basic GWO that include a hybrid version of GWO with PSO [40], binary GWO [41], parallelized GWO [42,43], and integration of DE with GWO [44].

Li et al. [45] proposed a modified discrete GWO variant to realize the multi-level image segmentation and optimize image histograms. Based on the high efficiency of grey wolf optimizer in the course of stability and optimization, this article effectively applied the Modified Discrete Grey Wolf Optimizer (MDGWO) algorithm to the field of Machine Translation (MT) by improving the location of the agents during the hunting and using weights to optimize the final position of prey. The MDGWO approach not only obtains better segmentation quality, but also proves its obvious superiority over ABC, DE, GWO, and Multilevel Thresholding Electromagnetism-like Optimization (MTEMO) in terms of accuracy, multilevel thresholding, and stability.

Liu et al. [46] developed an intelligent grey wolf optimizer variant, called DCS-GWO, by combining q-thresholding with the GWO variant. In this variant, positions of the grey wolves were initialized by the q-thresholding approach and updated by using the idea of GWO. The experimental solutions illustrated that the existing variant enjoyed better recovery accuracy than previous greedy pursuit approaches at the expense of computational complexity.

Mirjalili et al. [47] proposed two novel optimization techniques, Salp Swarm Algorithm (SSA) and Multi-objective Salp Swarm Algorithm (MSSA), for finding the solution of global optimization functions with multiple and single objectives. The main inspiration of SSA and MSSA is the swarming behavior of Salps when navigating and foraging in the ocean. The performance of the existing variant was tested on several standard and real-life applications. Based on the solutions of the existing variant, it was proven that this variant could obtain approximately Pareto optimal results with high convergence and coverage.

Raj and Bhattacharyya [48] applied several recent meta-heuristics to achieve the best possible optimal solution for reactive power planning with FACTS devices. Further, some more recent techniques have been also applied to find the best optimal setting of all control variables. The working performance of the existing variant was illustrated by comparing the solutions obtained with all other recent meta-heuristics. Based on the simulation results, the existing variant showed few generations, which do not get trapped in the local minima, and offered promising convergence characteristics.

This article focuses on grey wolf optimizer, developed by Mirjalili et al. [12] in 2014, based on the simulation of hunting behavior and social leadership of grey wolves in nature. Experimental results proved that the better accuracy of the existing variant was also comparable to that of other meta-heuristics. Since it is easy and simple to implement and has fewer control constants, grey wolf optimizer has received much attention and used to find the solution of practical real-life functions.

PSO, GA, evolutionary algorithm, differential algorithm, and ACO are the most popular meta-heuristic global optimization approaches. These nature-inspired techniques expand the search area dimension, while grey wolf optimizer provides an unsatisfactory convergence behavior regarding exploitation [49,50]. Hence, it is essential to emphasize that our research effort revolves around the increase of the local search ability of grey wolf optimizer technique. In order to improve the local search ability of the GWO algorithm, a newly modified meta-heuristic is proposed in this research, and its performance is compared with the performance results of grey wolf optimizer and some other recent nature-inspired algorithms; ultimately, Modified Variant of Grey Wolf Optimization (MVGWO) performs significantly better.

The rest of the paper is structured as follows. Section 2 describes the GWO algorithm. Section 3 presents the newly proposed algorithm, MVGWO. The MVGWO mathematical model and pseudocode are discussed in Section 3. The tested Unimodal, Multimodal, and Fixed-dim Multimodal classical functions are presented in Section 4. Results and discussion are summarized in Sections 5 and 6, respectively. Sine dataset and cantilever beam design functions are briefly described in Sections 7 and 8. Conclusions are drawn on the basis of the results obtained, as will be presented in Section 9.

## 2. Grey wolf optimization algorithm

The grey wolf optimizer algorithm is a new global optimization approach that simulates the grey wolves leadership and hunting in nature. These approaches have been inspired by simple concepts.

Mirjalili, et al. [12] proposed a GWO meta-heuristic approach. The GWO variant mimics the hunting mechanism and leadership hierarchy of grey wolves in nature. In the hierarchy of GWO, alpha is considered as the dominating agent among the group. The rest of the subordinates to alpha include beta and delta that help control the majority of wolves in the hierarchy that are considered as omega.

In addition, three main steps of hunting, searching for prey, encircling prey, and attacking prey, are implemented to perform optimization.

The encircling behavior of each member of the population is represented by the following mathematical equations:

$$d = \left|c.x_{p(t)} - x(t)\right|, \tag{1}$$

$$x(t+1) = x_{p(t)} - a.d, \tag{2}$$

where $x_p$ is the position vector of the prey, $t$ is the time, and $x$ indicates the position vector of a grey wolf.

Vectors $a$ and $c$ are mathematically calculated as follows:

$$a = 2l.r_1 - l, \tag{3}$$

$$c = 2.r_2, \tag{4}$$

where the above components are linearly decreased from 2 to 0 over the course of generations, and $r_1.r_2 \in [0, 1]$ are random vectors.

**Hunting**: In order to mathematically simulate the hunting behavior, it is supposed that alpha ($\alpha$), beta ($\beta$), and delta ($\delta$) have better knowledge about the potential location of prey. The following mathematical equations are developed in this regard:

$$\vec{d}_\alpha = |\vec{c}_1.\vec{x}_\alpha - \vec{x}|, \quad \vec{d}_\beta = |\vec{c}_2.\vec{x}_\beta - \vec{x}|,$$

$$\vec{d}_\delta = |\vec{c}_3.\vec{x}_\delta - \vec{x}|, \tag{5}$$

$$\vec{x}_1 = \vec{x}_\alpha - \vec{a}_1.(\vec{d}_\alpha), \quad \vec{x}_2 = \vec{x}_\beta - \vec{a}_2.(\vec{d}_\beta),$$

$$\vec{x}_3 = \vec{x}_\delta - \vec{a}_3.(\vec{d}_\delta), \tag{6}$$

$$\frac{\vec{x}_1 + \vec{x}_2 + \vec{x}_3}{3}, \tag{7}$$

where $\vec{x}_\alpha$, $\vec{x}_\beta$, and $\vec{x}_\delta$ are the positions of the member of the population in the searching space at the $t$th iteration, $t$ indicates the current iteration, and $\vec{x}(t)$ presents the position of the grey wolf at the $t$th iteration:

$$\vec{a}_{(.)} = 2\vec{l}.\vec{r}_1 - \vec{l}, \tag{8}$$

$$\vec{c}_{(.)} = 2.\vec{r}_2, \tag{9}$$

where components of $\vec{l}$ are linearly decreased from 2 to 0 over the course of generations, and $r_1$, $r_2$ are random vectors in [0,1]. In addition, $\vec{a}_{(.)}$ and $\vec{c}_{(.)}$ are the coefficient vectors of alpha ($\alpha$), beta ($\beta$), and delta ($\delta$) wolfs.

**Searching for prey and attacking prey:** $A$ is a random value in the gap $(-a, a)$. When random value $|A| < 1$, the wolves are forced to attack the prey. Searching for prey is an exploration ability, and attacking the prey is an exploitation ability. Arbitrary values of are utilized to force the search to move away from the prey.

When $|A| > 1$, the members of the population are enforced to diverge from the prey.

## 3. Modified variant of grey wolf optimizer

Mirjalili et al. [14] proposed a new version of population-based algorithms, called GWO. The GWO variant mimics the hunting mechanism and leadership hierarchy of grey wolves in nature. In the hierarchy of GWO, alpha is considered the dominating agent among the group. The rest of the subordinates to alpha include beta and delta that help control the majority of wolves in the hierarchy that are considered as omega. In addition, three main steps of hunting, i.e., searching for prey, encircling prey, and attacking prey, are implemented to perform optimization.

The proposed variant has been developed by modifying encircling behavior and position update equation of GWO algorithm with the aim of improving the performance, convergence speed, and accuracy of the grey wolf optimizer meta-heuristics. In the MVGWO, the population is divided into five different groups, such as alpha, beta, gamma, delta, and omega, which are employed for simulating the leadership hierarchy (see in Figure 1). The rest of all operations are the same as grey wolf optimizer variant [14].

**Social Hierarchy:** In order to develop the proposed mathematical model, the social hierarchy of wolves is considered when designing an MVGWO, where the fittest solution is alpha. Accordingly, the second, third, and fourth best solutions are named beta, gamma, and delta. The rest of the agent solutions are assumed to be omega.

The mathematical model of the encircling behavior is represented by the following equations:

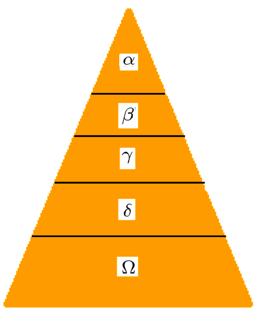


**Figure 1.** Hierarchy of grey wolf (dominance decreases from top to down).

$$d = \left|c.x_{p(t)} - \mu \times x(t)\right|, \tag{10}$$

$$x(t+1) = x_{p(t)} - a.d, \tag{11}$$

where coefficient vectors, $a$ and $c$, are given by:

$$a = 2l.\vec{r}_1, \tag{12}$$

$$c = 2.\vec{r}_2, \tag{13}$$

where the components are as follows: $l \in [2, 0]$ and $\vec{r}_1.\vec{r}_2 \in [0, 1]$.

**Hunting:** In order to simulate the hunting behavior mathematically, it is supposed that alpha ($\alpha$), beta ($\beta$), gamma ($\gamma$), and delta ($\delta$) have better knowledge about the potential location of prey. The following mathematical equations are developed in this regard:

$$\vec{d}_\alpha = |\vec{c}_1.\vec{x}_\alpha - \vec{x}|, \quad \vec{d}_\beta = |\vec{c}_2.\vec{x}_\beta - \vec{x}|,$$

$$\vec{d}_\gamma = |\vec{c}_3.\vec{x}_\gamma - \vec{x}|, \quad \vec{d}_\delta = |\vec{c}_4.\vec{x}_\delta - \vec{x}|, \tag{14}$$

$$\vec{x}_1 = \vec{x}_\alpha - \vec{a}_1.(\vec{d}_\alpha), \quad \vec{x}_2 = \vec{x}_\beta - \vec{a}_2.(\vec{d}_\beta),$$

$$\vec{x}_3 = \vec{x}_\gamma - \vec{a}_3.(\vec{d}_\gamma), \quad \vec{x}_4 = \vec{x}_\delta - \vec{a}_4.(\vec{d}_\delta), \tag{15}$$

$$\frac{\vec{x}_1 + \vec{x}_2 + \vec{x}_3 + \vec{x}_4}{4}, \tag{16}$$

$$\vec{a}_{(.)} = 2\vec{l}.\vec{r}_1 - \vec{l}, \tag{17}$$

$$\vec{c}_{(.)} = 2.\vec{r}_2. \tag{18}$$

***Pseudo Code of MVGWO:***
Initialization of population
Initialize $l$, $a$, and $c$
Evaluate the fitness of each search member
$x_\alpha$, $x_\beta$, $x_\gamma$, and $x_\delta$ as the first, second, third and fourth best search members
while ($t <$ max no. of iter)
for each search member
Update the position of each member of the population by mathematical Equation (1.16)
end for
Update $l$, $a$, and $c$
Evaluate the fitness of all search members
Update $x_\alpha$, $x_\beta$, $x_\gamma$, and $x_\delta$
$t = t + 1$
end while
return $x_\alpha$

**Table 1.** Unimodal benchmark functions.

| Function | Dim | Range | $f_{\min}$ |
|---|---|---|---|
| $F_1(x) = \sum_{i=1}^{n} x_i^2$ | 30 | $[-100, 100]$ | 0 |
| $F_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$ | 30 | $[-10, 10]$ | 0 |
| $F_3(x) = \sum_{i=1}^{n} (\sum_{j-1}^{i} x_j)^2$ | 30 | $[-100, 100]$ | 0 |
| $F_4(x) = \max_i \{\lvert x_i \rvert,\ 1 \leq\ i\ \leq n\}$ | 30 | $[-100, 100]$ | 0 |
| $F_5(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$ | 30 | $[-30, 30]$ | 0 |
| $F_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$ | 30 | $[-100, 100]$ | 0 |
| $F_7(x) = \sum_{i=1}^{n} i x_i^4 + rand[0, 1)$ | 30 | $[-1.28, 1.28]$ | 0 |

**Table 2.** Multimodal benchmark functions.

| Function | Dim | Range | $f_{\min}$ |
|---|---|---|---|
| $F_8(x) = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$ | 30 | $[-500, 500]$ | $-418.9829 \times 5$ |
| $F_9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$ | 30 | $[-5.12, 5.12]$ | 0 |
| $F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$ | 30 | $[-32, 32]$ | 0 |
| $F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$ | 30 | $[-600, 600]$ | 0 |
| $F_{12}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1}) + (y_{n-1})^2\right]\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$; $y_i = 1 + \frac{x_i + 1}{4}$; $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$ | 30 | $[-50, 50]$ | 0 |
| $F_{13}(x) = 0.1\left\{\sin^2(3\pi x_i) + \sum_{i=1}^{n}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2 + \left[1 + \sin^2(2\pi x_n)\right]\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$ | 30 | $[-50, 50]$ | 0 |

## 4. Testing functions

In this section, twenty-three classical functions are used to verify the performance of the MVGWO. These test functions can be divided into three different groups: unimodal, multimodal, and fixed-dimension multimodal functions. Specific details of these functions are represented by Tables 1, 2, and 3, respectively.

## 5. The convergence performance graphs of MVGWO algorithm

The performances of several population-based meta-heuristics have been verified with the MVGWO variant in order to test the convergence rate, stability, and computational accuracy in a number of iterations in Figure 2. Similar parameter values have been considered for the entire algorithms to make a fair comparison. The results illustrate that, in convergence Figure 2, by plotting the best optimal values of the functions, values have been compared to a number of generations for the simplified model of the molecule with different sizes from 1000 to 5000 dimensions.

The graphs show that the standard test function values quickly decrease as the number of generations increases for newly existing variant solutions as compared

**Table 3.** Fixed-dimension multimodal benchmark functions.

| Function | Dim | Range | $f_{\min}$ |
|---|---|---|---|
| $F_{14}(x)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}\left(x_i-a_{ij}\right)^6}\right)^{-1}$ | 2 | $[-65,65]$ | 1 |
| $F_{15}(x)=\sum_{i=1}^{11}\left[a_i-\frac{x_1\left(b_i^2+b_ix_2\right)}{b_i^2+b_ix_i+x_4}\right]^2$ | 4 | $[-5,5]$ | 0.00030 |
| $F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$ | 2 | $[-5,5]$ | $-1.0316$ |
| $F_{17}(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$ | 2 | $[-5,5]$ | 0.398 |
| $F_{18}(x)=\left[1+\left(x_1+x_2+1\right)^2\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]\times\left[\begin{array}{l}30+\left(2x_1-3x_2\right)^2\\ \times\left(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2\right)\end{array}\right]$ | 2 | $[-2,2]$ | 3 |
| $F_{19}(x)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{3}a_{ij}\left(x_j-p_{ij}\right)^2\right)$ | 3 | $[1,3]$ | $-3.86$ |
| $F_{20}(x)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{6}a_{ij}\left(x_j-p_{ij}\right)^2\right)$ | 6 | $[0,1]$ | $-3.32$ |
| $F_{21}(x)=-\sum_{i=1}^{5}\left[\left(X-a_i\right)\left(X-a_i\right)^T+c_i\right]^{-1}$ | 4 | $[0,10]$ | $-10.1532$ |
| $F_{22}(x)=-\sum_{i=1}^{7}\left[\left(X-a_i\right)\left(X-a_i\right)^T+c_i\right]^{-1}$ | 4 | $[0,10]$ | $-10.4028$ |
| $F_{23}(x)=-\sum_{i=1}^{10}\left[\left(X-a_i\right)\left(X-a_i\right)^T+c_i\right]^{-1}$ | 4 | $[0,10]$ | $-10.5363$ |

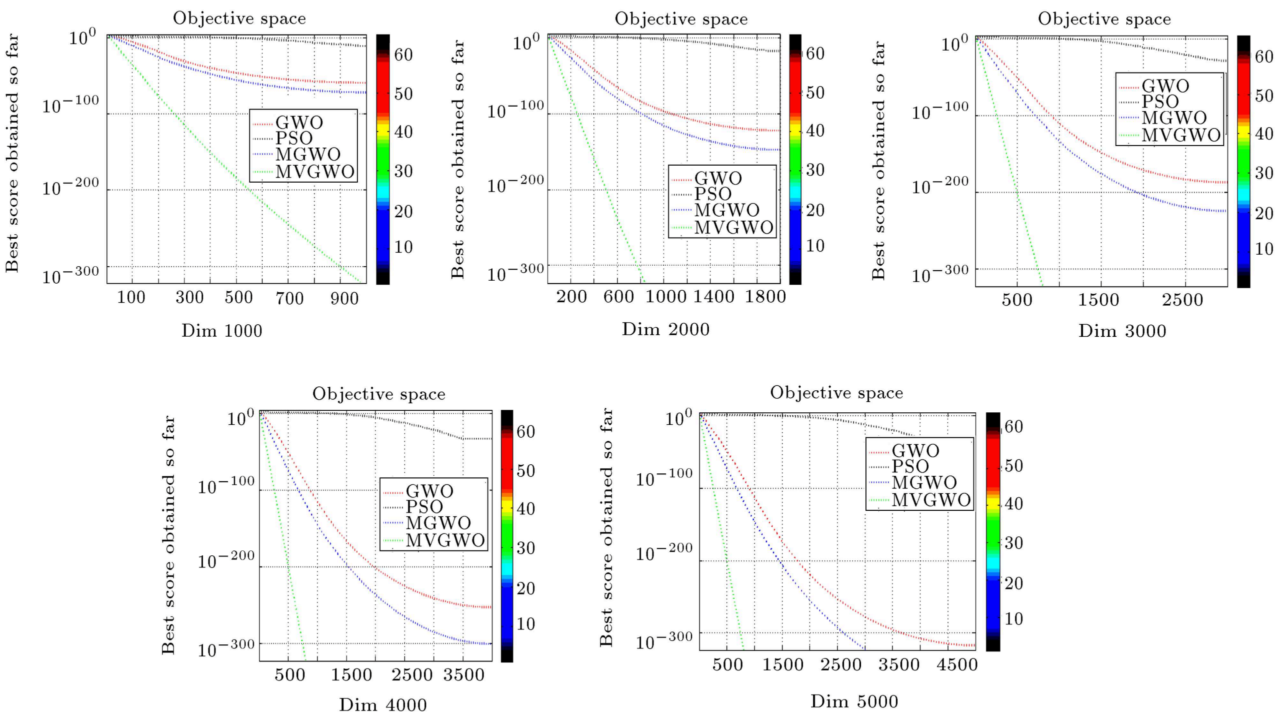


**Figure 2.** Convergence graphs of algorithms.

to those of the other metaheuristics. In Figure 2, PSO, GWO, HGWO, and MVGWO variants suffer from slow convergence and are stalled in the partitioning procedure; nevertheless, the mean grey wolf variant plays a role for the existing hybrid algorithm to avoid trapping in local minima and accelerate the search.

## 6. Results and discussion

The MVGWO, MGWO, GWO, and PSO algorithms are coded by MATLAB R2013a and implemented by Intel HD Graphics, Pentium-Intel Core (TM), i5 Processor 430 M , 15.6" 16.9 HD LCD, 3GB Memory, and 320 GB HDD. Parameters including the number of search agents (30), the maximum number of iterations (1000), and $l \in [2, 0]$ are used to confirm the quality of modified meta-heuristics.

Generally, any nature-inspired technique is tested by computing its results with those obtained through other meta-heuristics. In addition, this study follows the same procedure and employs twenty-three classical functions for judgment. These test functions are divided into three parts: unimodal, multimodal, and fixed-dimension multimodal functions. The mathematical formulation of classical functions is presented in Tables 1–3. Thirty variables of multimodal and unimodal classical functions are considered to further improve their difficulties.

The accuracy of the newly modified algorithm has been confirmed; thus, this algorithm is applied to the classical, sine dataset, and cantilever beam design functions in terms of minimum objective function values, maximum objective function values, mean, and standard deviation (Tables 4–9).

Herein, the maximum and minimum values of the

**Table 4.** The optimal solutions obtained by the algorithms on unimodal benchmark functions.

| Problem no. | PSO | | GWO | | MGWO | | MVGWO | |
|---|---|---|---|---|---|---|---|---|
| | Min | Max | Min | Max | Min | Max | Min | Max |
| 1 | 2.1532e-09 | 6.6032e+04 | 6.1668e-61 | 5.8943e+04 | 2.7170e-73 | 6.1504e+04 | **0.00** | **7.1072e+04** |
| 2 | 5.3156e-06 | 1.5784e+09 | 1.9775e-35 | 1.4305e+12 | 7.4227e-43 | 3.2669e+09 | **1.6053e-175** | **3.7809e+13** |
| 3 | 12.0227 | 1.9267e+05 | 3.3073e-15 | 1.8876e+05 | 2.5227e-25 | 1.1651e+05 | **1.2105e-282** | **6.1749e+04** |
| 4 | 2.6791 | 0.6791 | 2.6647e-14 | 89.0706 | 2.4202e-20 | 88.0388 | **1.7751e-152** | **91.6933** |
| 5 | 107.7967 | 107.7967 | 27.1178 | 2.1621e+08 | 27.1631 | 2.3050e+08 | **27.1003** | **3.1495e+08** |
| 6 | 2.6127e-11 | 6.8467e+04 | 1.2547 | 6.5240e+04 | 1.2505 | 7.0794e+04 | **3.8767** | **7.4148e+04** |
| 7 | 0.0367 | 133.3527 | 3.6149e-04 | 92.2196 | 5.7914e-04 | 80.6522 | **1.4141e-05** | **153.5907** |

**Table 5.** The statistical results obtained by the algorithms on unimodal benchmark functions.

| Problem no. | PSO | | GWO | | MGWO | | MVGWO | |
|---|---|---|---|---|---|---|---|---|
| | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ |
| 1 | 408.5162 | 3.9524e+03 | 215.8357 | 2.5750e+03 | 173.1716 | 2.5391e+03 | **125.5010** | **2.4662e+03** |
| 2 | 9.5933e+06 | 3.0290e+08 | 1.4316e+09 | 4.5236e+10 | 3.2708e+06 | 1.0331e+08 | **1.2715e+10** | **1.1956e+12** |
| 3 | 1.8412e+03 | 1.3960e+04 | 1.7745e+03 | 1.0505e+04 | 674.7361 | 5.8039e+03 | **1.2536e+03** | **1.0787e+04** |
| 4 | 3.5511 | 7.3907 | 2.6146 | 12.1387 | 0.9451 | 6.7658 | **0.3492** | **4.3699** |
| 5 | 5.9195e+05 | 1.0872e+07 | 7.4518e+05 | 1.0897e+07 | 6.7132e+05 | 9.9844e+06 | **3.5409e+05** | **1.0001e+07** |
| 6 | 516.2407 | 4.6252e+03 | 335.9674 | 3.4849e+03 | 197.7060 | 2.8166e+03 | **123.8129** | **2.5129e+03** |
| 7 | 53.3529 | 56.2679 | 0.4942 | 5.8041 | 0.1335 | 2.6681 | **0.2020** | **4.9747** |

**Table 6.** The optimal solutions obtained by the algorithms on multimodal benchmark functions.

| Problem no. | PSO | | GWO | | MGWO | | MVGWO | |
|---|---|---|---|---|---|---|---|---|
| | Min | Max | Min | Max | Min | Max | Min | Max |
| 1 | −6.6067e+03 | −1.4706e+03 | −5.8056e+03 | −2.4436e+03 | −4.8344e+03 | −2.5399e+03 | **−2.2540e+03** | **−2.2373e+03** |
| 2 | 39.7987 | 422.6854 | 5.6843e−14 | 458.7865 | 0 | 438.1148 | **0** | **488.0757** |
| 3 | 1.5846e−05 | 20.5268 | 1.5099e−14 | 20.7623 | 1.5099e−14 | 20.5150 | **1.4409e−15** | **20.8472** |
| 4 | 2.0755e−12 | 667.1103 | 0.0092 | 665.7767 | 0 | 527.3462 | **0** | **555.0353** |
| 5 | 2.0193e−12 | 6.1692e+08 | 0.0304 | 5.5204e+08 | 0.0538 | 6.1414e+08 | **0.5589** | **8.1057e+08** |
| 6 | 6.5797e−08 | 1.0597e+09 | 0.6975 | 8.0560e+08 | 1.1284 | 9.0172e+08 | **0.1930** | **9.2376e+08** |

**Table 7.** The statistical results obtained by the algorithms on multimodal benchmark functions.

| Problem No. | PSO | | GWO | | MGWO | | MVGWO | |
|---|---|---|---|---|---|---|---|---|
| | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ |
| 8 | −6.0956e+03 | 1.1171e+03 | −4.0393e+03 | 967.7908 | −3.3352e+03 | 731.9012 | **−2.2511e+03** | **6.3372** |
| 9 | 161.3787 | 124.3555 | 10.7854 | 48.0111 | 4.9465 | 32.6783 | **2.3389** | **26.4361** |
| 10 | 2.9657 | 3.4941 | 0.4125 | 2.2762 | 0.2497 | 1.7307 | **0.1045** | **1.1726** |
| 11 | 25.7028 | 100.2551 | 3.1497 | 32.9323 | 1.4174 | 20.4885 | **0.9590** | **18.9328** |
| 12 | 1.1219e+06 | 2.2349e+07 | 1.5736e+06 | 2.4904e+07 | 1.1003e+06 | 2.1982e+07 | **1.0443e+06** | **2.1584e+07** |
| 13 | 2.5209e+06 | 4.4037e+07 | 3.2041e+06 | 4.3667e+07 | 1.5261e+06 | 3.2837e++07 | **1.1030e+06** | **2.9672e+07** |

**Table 8.** The optimal solutions obtained by the algorithms on fixed-dimension multimodal benchmark functions.

| Problem no. | PSO | | GWO | | MGWO | | MVGWO | |
|---|---|---|---|---|---|---|---|---|
| | Min | Max | Min | Max | Min | Max | Min | Max |
| 7 | 2.9920 | 12.6709 | 10.7632 | 86.5835 | 12.6705 | 76.5329 | **2.9821** | **35.7641** |
| 8 | 9.8869e–04 | 0.3069 | 3.0750e–04 | 0.1331 | 3.0749e–04 | 0.2142 | **3.749e–04** | **0.6090** |
| 9 | –1.0316 | 0.0804 | –1.0316 | –0.1653 | –1.0316 | –0.8485 | **–1.0316** | **–0.8485** |
| 10 | 0.3979 | 2.2225 | 0.3979 | 0.4187 | 0.3979 | 0.4716 | **0.3979** | **2.5346** |
| 11 | 3 | 73.9801 | 3 | 44.4885 | 3 | 58.2138 | **3** | **171.6938** |
| 12 | –3.8628 | –3.6393 | –3.8596 | –3.6339 | –3.8627 | –2.9834 | **–3.8628** | **–3** |
| 13 | –3.3220 | –0.7636 | –2.8404 | –2.1889 | –3.1421 | –1.7246 | **–3.2450** | **–1.2158** |
| 14 | –10.1532 | –10.1532 | –5.0552 | –0.3997 | –5.0999 | –0.3774 | **–5.0999** | **–0.3282** |
| 15 | –5.1288 | –1.2345 | –10.4024 | –0.6547 | –10.4022 | –0.5490 | **–10.6466** | **–0.4647** |
| 16 | –10.5364 | –0.6398 | –10.5360 | –0.7648 | –10.5357 | –0.7656 | **–10.9786** | **–0.6271** |

**Table 9.** The statistical results obtained by the algorithms on fixed-dimension multimodal benchmark functions.

| Problem no. | PSO | | GWO | | MGWO | | MVGWO | |
|---|---|---|---|---|---|---|---|---|
| | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ |
| 14 | 2.1505 | 0.9110 | 11.2055 | 4.2682 | 12.9194 | 2.8310 | **3.0536** | **1.4674** |
| 15 | 0.0016 | 0.0107 | 8.6397e–04 | 0.0053 | 7.8362e–04 | 0.0070 | **0.0013** | **0.0201** |
| 16 | –1.0299 | 0.0363 | –1.0275 | 0.1167 | –1.0303 | 0.0287 | **–1.0312** | **0.0061** |
| 17 | 0.4004 | 0.0584 | 0.3992 | 0.0041 | 0.4004 | 0.0074 | **0.4012** | **0.0707** |
| 18 | 3.2555 | 2.4028 | 3.1319 | 2.0225 | 3.1918 | 2.5571 | **3.0323** | **0.6594** |
| 19 | –3..8614 | 0.0077 | –3.8555 | 0.0147 | –3.8586 | 0.0282 | **–3.8565** | **0.0129** |
| 20 | –3.1235 | 0.2311 | –2.8158 | 0.0565 | –3.1068 | 0.0769 | **–3.2015** | **0.1366** |
| 21 | –4.2193 | 2.2680 | –4.7394 | 0.7968 | –2.9990 | 1.2537 | **–4.9287** | **0.3697** |
| 22 | –4.8911 | 0.6718 | –7.7375 | 2.6723 | –6.6836 | 3.4455 | **–8.6227** | **0.4368** |
| 23 | –9.3903 | 2.5222 | –8.2760 | 1.8966 | –7.7390 | 2.8551 | **–4.8748** | **0.3978** |

objective functions produce the best suitable cost of the classical problems in the least number of iterations. On the other hand, the mean and standard deviation of statistical values are used to evaluate the reliability. Further, the convergence graphs of the classical problems represent the convergence performance of the variants.

Tables 4, 6, and 8 show that the newly modified algorithm produces the best optimal values of the classical problems in terms of minimum and maximum values of the functions as compared to other meta-heuristics. Tables 5, 7, and 9 illustrate that the modified algorithm produces the superior quality of standard and mean values on the maximum classical functions in the form of the least values as compared to other meta-heuristics. In the end, the convergence graphs (Figures 3–25) prove that the existing approach finds the best possible optimal values of the standard functions in the least number of iterations as compared to others.

Based on the results given in Tables 4 and 5, it is clear that the proposed variant outperforms other

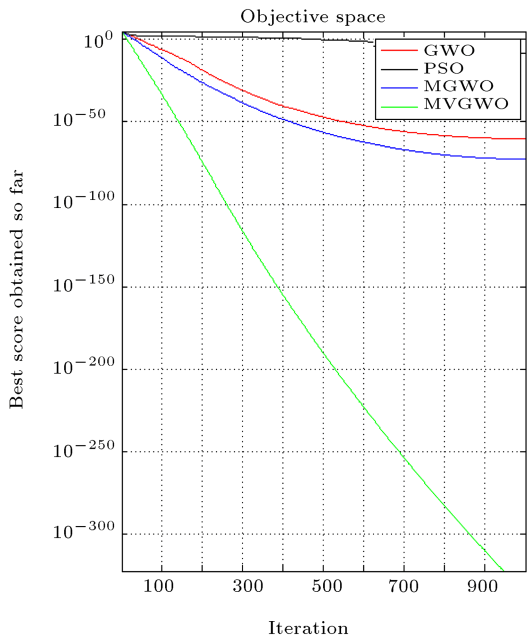


**Figure 3.** Convergence graph of fixed-dimension multimodal benchmark function ($F_1$).

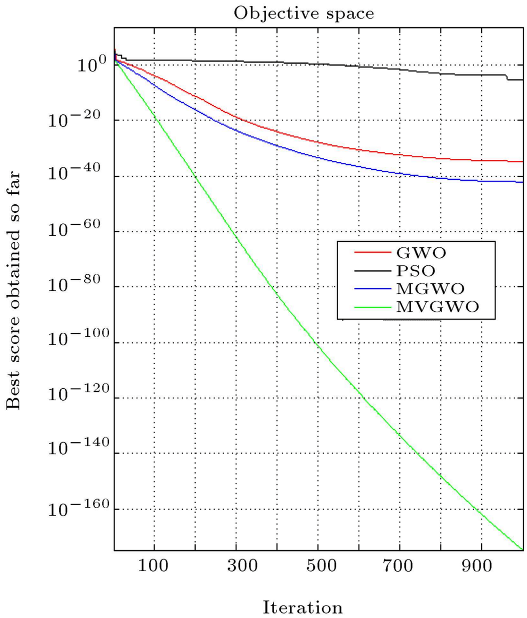


**Figure 4.** Convergence graph of fixed-dimension multimodal benchmark function ($F_2$).

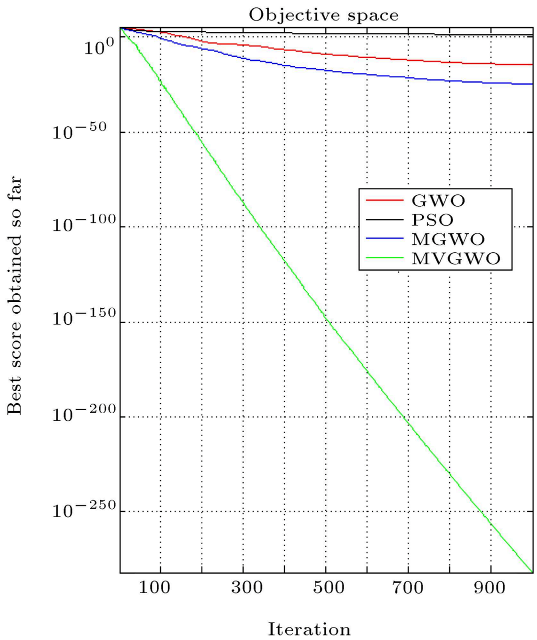


**Figure 5.** Convergence graph of fixed-dimension multimodal benchmark function ($F_3$).

meta-heuristics, including PSO, GWO, and MGWO, in terms of mean, standard deviation, and min/max cost function, and exploits the optimum. Accordingly, the proposed variant is highly comparable to other meta-heuristics.

Further, the convergence behaviors of the proposed variant, PSO, MGWO, and GWO algorithms have been investigated, and convergence curve is plotted in Figures 3–25. In order to examine the convergence behavior of the modified variant of GWO,

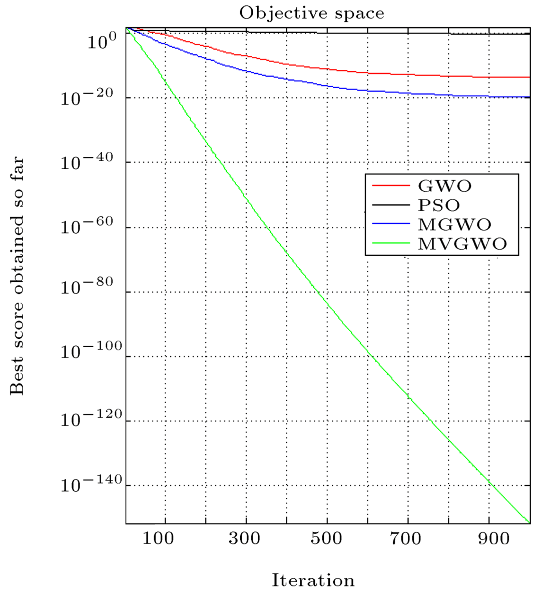


**Figure 6.** Convergence graph of fixed-dimension multimodal benchmark function ($F_4$).

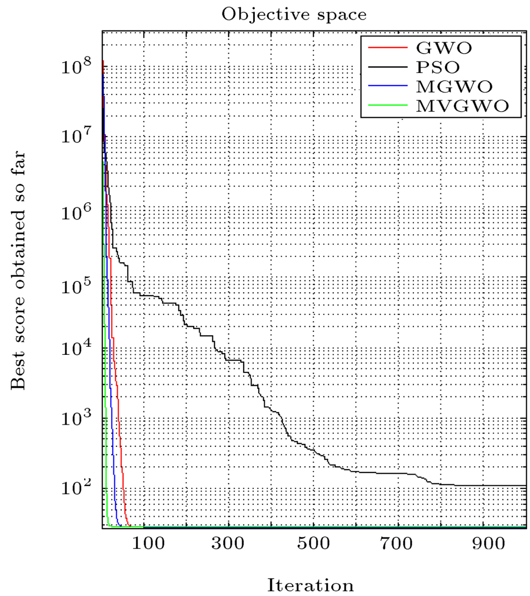


**Figure 7.** Convergence graph of fixed-dimension multimodal benchmark function ($F_5$).

PSO, MGWO, and grey wolf optimizer algorithms, the search history and path of the first search member of the population in its first dimension are illustrated in Figures 3–25. Based on the convergence curve, it is observed that the modified variant produces better convergence points as compared to others.

Furthermore, the appropriate results of multi-modal and fixed-dimension multimodal functions are illustrated in Tables 6–9. The multimodal and fixed-dimension functions have many local optima whose number and dimension grow exponentially. This makes them fit for benchmarking the exploration capacity

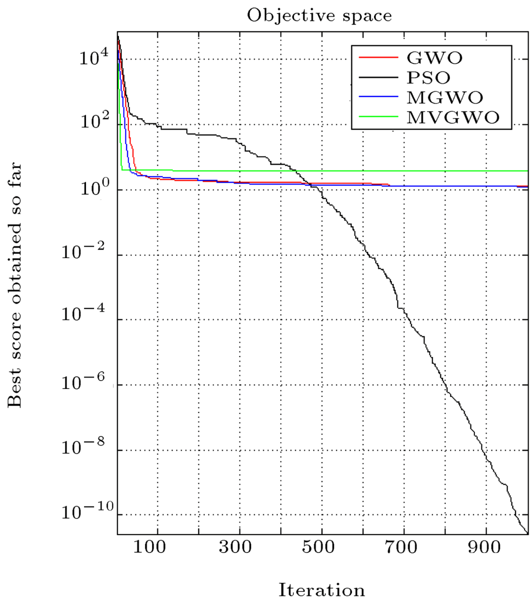


**Figure 8.** Convergence graph of fixed-dimension multimodal benchmark function ($F_6$).

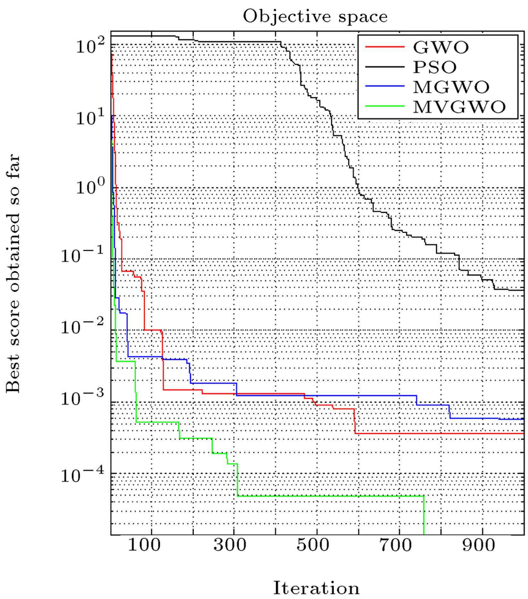


**Figure 9.** Convergence graph of fixed-dimension multimodal benchmark function ($F_7$).

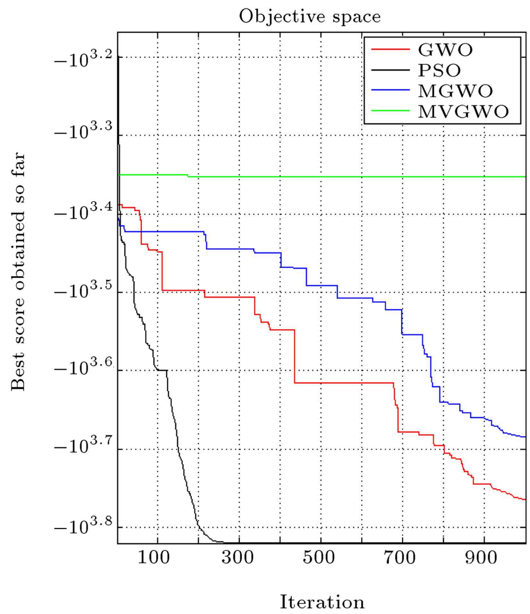


**Figure 10.** Convergence graph of fixed-dimension multimodal benchmark function ($F_8$).

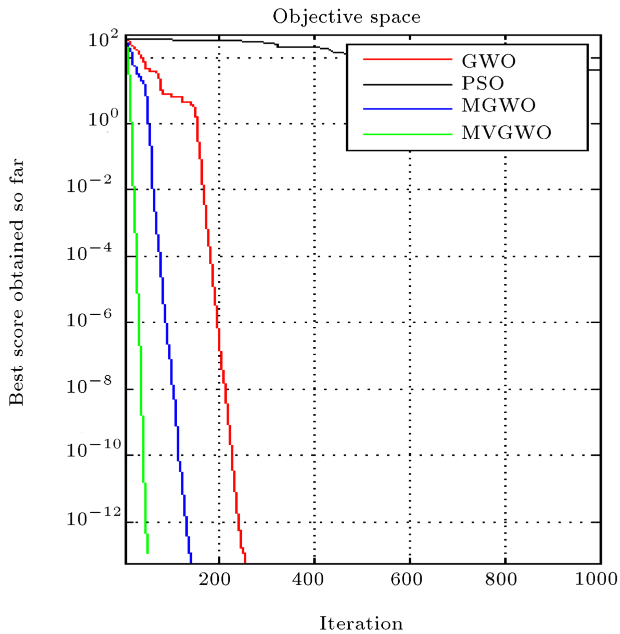


**Figure 11.** Convergence graph of fixed-dimension multimodal benchmark function ($F_9$).

of a variant. Based on the results of Tables 6–9, the modified variant is able to present better solution quality with respect to the maximum number of multimodal and fixed-dimension multi-modal functions as compared to PSO, GWO, and MGWO algorithm. These solutions demonstrate that the MVGWO has advantages in terms of exploration.

A number of criteria have been used to determine the accuracy of the proposed algorithm, GWO, PSO, and MGWO. The mean and standard deviation of statistical values are used to evaluate reliability in Tables 5, 7, and 9. The average computation time of the successful runs and the average number of function evaluations of successful runs are applied to estimate the cost of the standard function.

In Figures 3–25, the convergence performances of GWO, PSO, MGWO, and MVGWO algorithms in solving classical problems are compared. The obtained convergence solutions prove that the MVGWO algorithm is more capable to find the best optimal solution in the minimum number of iterations. Hence, MVGWO algorithm avoids premature convergence of the search

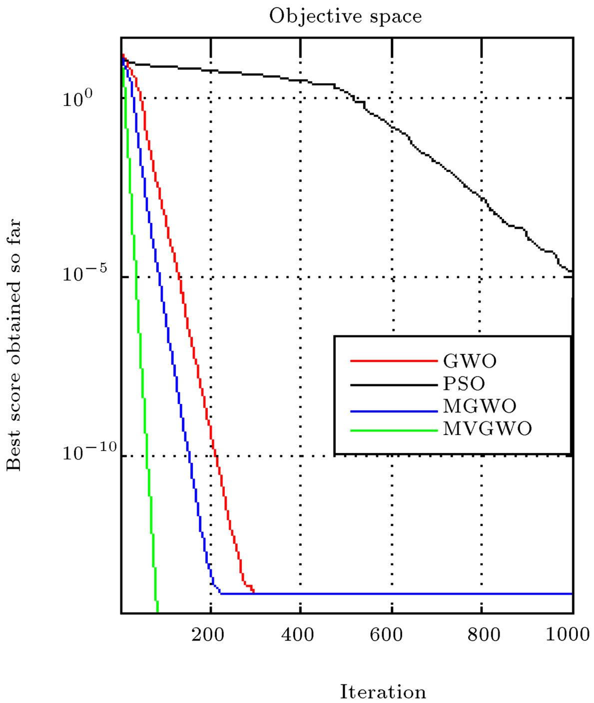


**Figure 12.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{10}$)

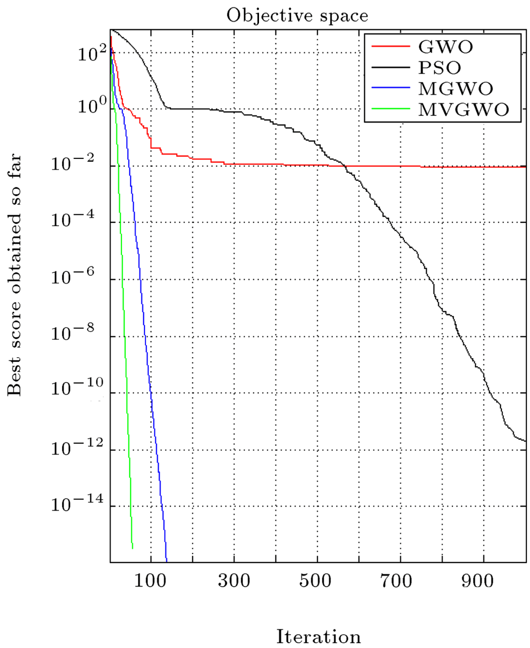


**Figure 13.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{11}$).

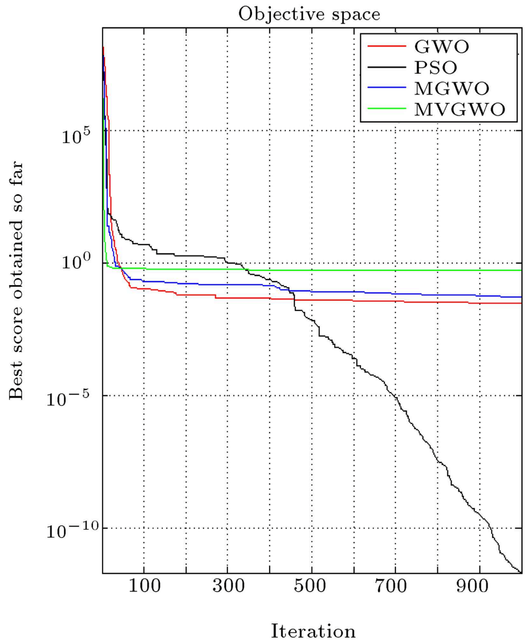


**Figure 14.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{12}$).

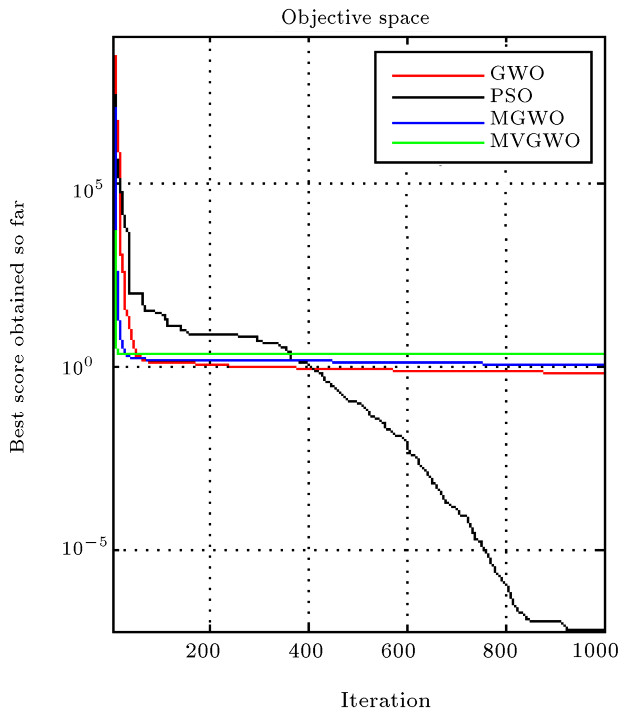


**Figure 15.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{13}$).

process to local optimal point and provides a superior exploration of the search course.

To sum up, based on all of the simulation solutions, the recent existing algorithm is very helpful in increasing the efficiency of the GWO algorithm in terms of result quality and computational efforts.

## 7. Sine dataset function

This dataset has a number of attributes 01 and structures 1–15–1 chosen to train and solve this dataset [30]. This function has four peaks that make it extremely difficult to approximate. Sine dataset function has been tested on different nature-inspired meta-heuristics. Ac-

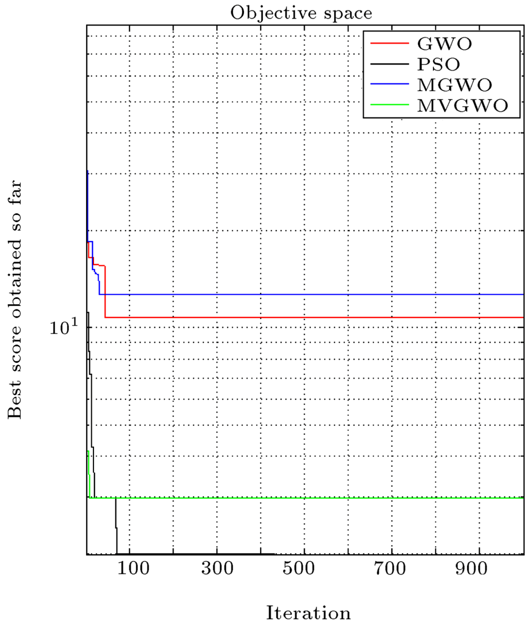


**Figure 16.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{14}$).

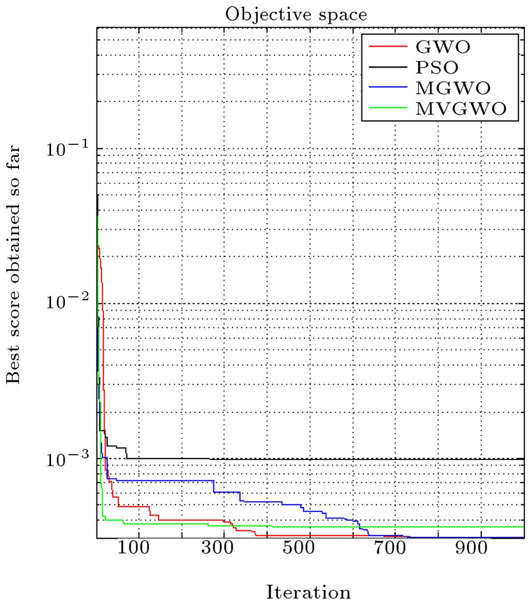


**Figure 17.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{15}$).

cording to the obtained results, it is observed that the modified variant of grey wolf optimizer provides extremely accurate solutions in this dataset as can be inferred from test error in Table 10, and convergence and best solution performance of MVGWO are plotted in Figures 26 and 27.

## 8. Cantilever beam design function

This function is associated with the design variables including the width of different beam elements, weight

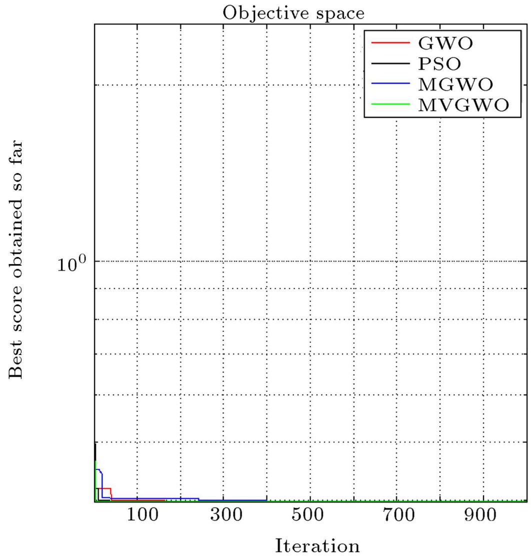


**Figure 18.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{16}$).

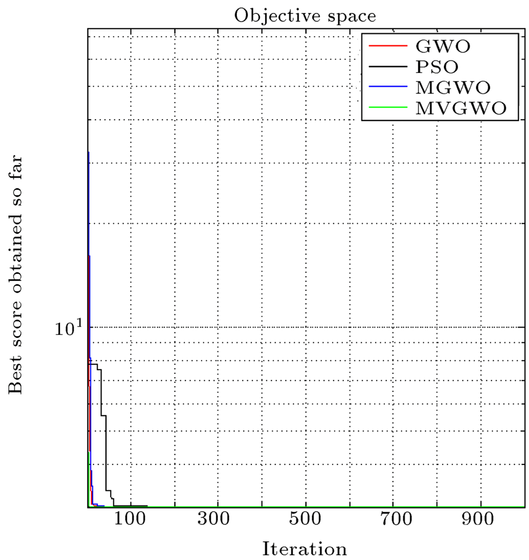


**Figure 19.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{17}$).

**Table 10.** Experimental results for the sine datasets.

| Algorithm | $\mu$ | $\sigma$ | Test error |
|---|---|---|---|
| MVGWO | 0.2549 | 0.0018 | 2.7221e+04 |
| GWO | 0.261970 | 0.115080 | 43.754 |
| PSO | 0.526530 | 0.072876 | 124.89 |
| GA | 0.421070 | 0.061206 | 111.25 |
| ACO | 0.529830 | 0.053305 | 117.71 |
| ES | 0.706980 | 0.077409 | 142.31 |
| PBIL | 0.483340 | 0.007935 | 149.60 |

optimization, and constant thickness [51]. A quick description of the cantilever beam function is presented as follows:

$$\min\ (X) = 0.0624(l + m + n + o + p), \qquad (19)$$

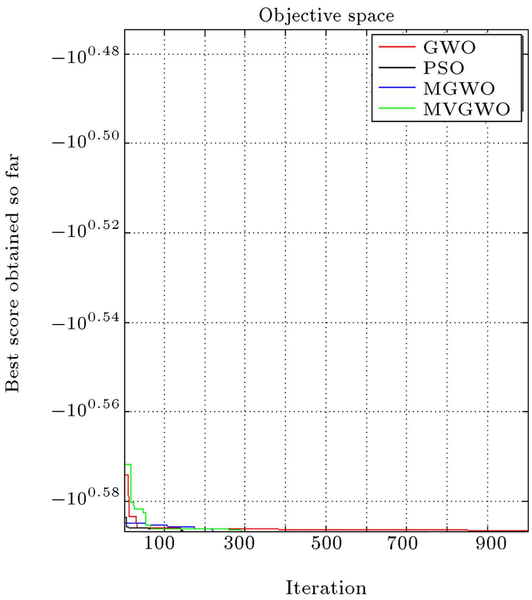


**Figure 20.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{18}$).

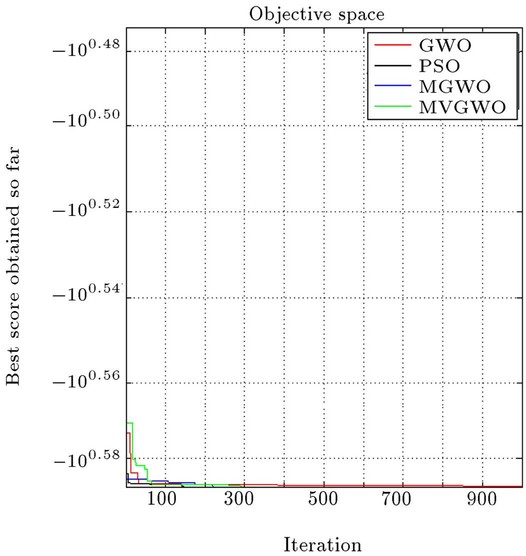


**Figure 21.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{19}$).

subject to:

$$g(X) = \frac{61}{l^3} + \frac{37}{m^3} + \frac{19}{n^3} + \frac{7}{o^3} + \frac{1}{p^3} - 1 \leq 0, \qquad (20)$$

where $0.01 \leq l, m, n, o, p \leq 100$. The global optimal results of the given function are listed in Table 11.

During the last few decades, several researchers have used different types of meta-heuristics to find the best possible optimal solutions for the cantilever beam design function in the literature such as convex linearization method (CONLIN) [51], CS [51], method of moving asymptotes (MMA) [51], Grid-based Clustering

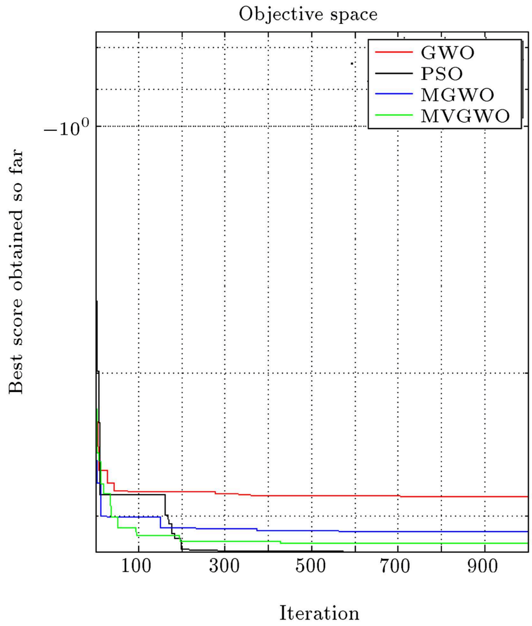


**Figure 22.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{20}$).

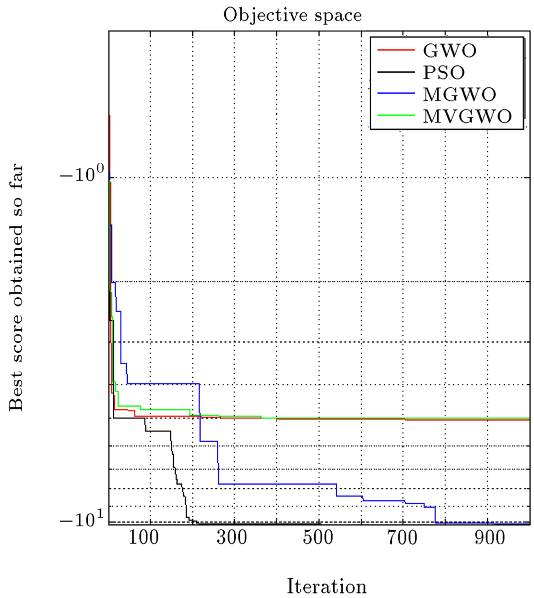


**Figure 23.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{21}$).

Algorithms-I and II (GCA-I and GCA-II) [52], and Symbiotic Organisms Search (SOS) [53].

The experimental results of different variants for the given function are illustrated in Table 11. That experiment has been tested on the following parameter settings: search agents (30) and the maximum number of iterations (500).

It can be seen that the best optimal value of the cantilever beam design function on MVGWO is 1.33966. Hence, MVGWO variant gives better quality of the solutions as compared to other recent algorithms.

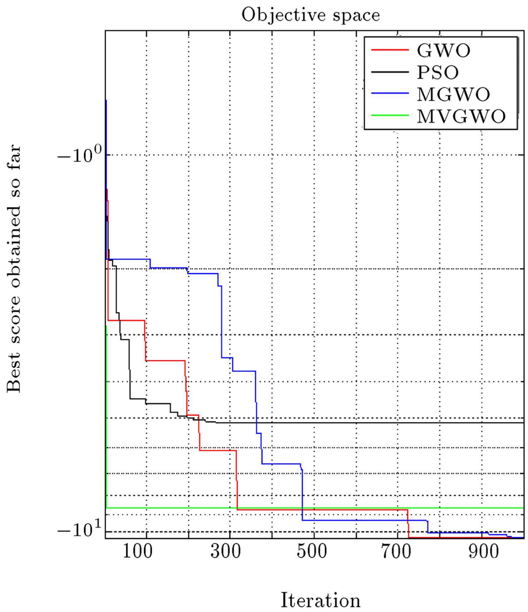


**Figure 24.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{22}$).

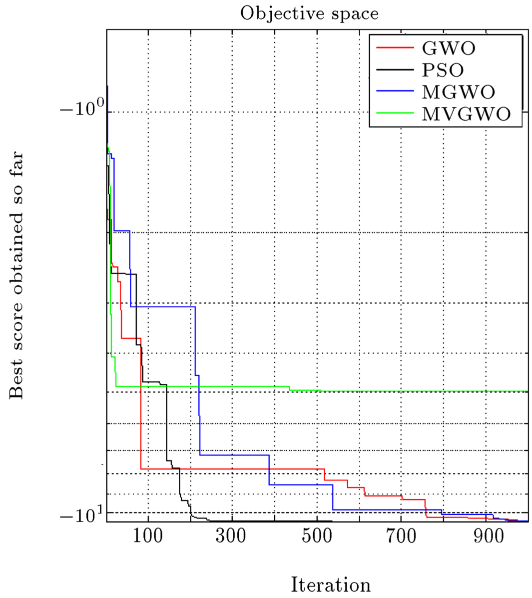


**Figure 25.** Convergence graph of fixed-dimension multimodal benchmark function ($F_{23}$).

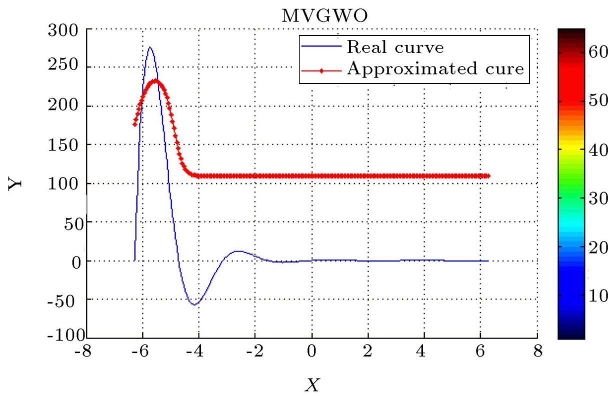


**Figure 26.** Sine graph of MVGWO.

## 9. Conclusion

This paper presented a Modified Variant of Grey Wolf Optimization called MVGWO. This modified variant was developed by modifying the encircling behavior and the position update equation of Grey Wolf Optimization (GWO) algorithm with the aim of improving the performance, convergence speed, and accuracy of the GWO meta-heuristics. These modifications were used to make a balance between exploration and exploration over the path of generations. The performance of the proposed variant was tested using several benchmark functions. It was observed that the modified variant had the edge of high exploration over other meta-heuristics such as Particle Swarm Optimization (PSO), GWO, and Mean Grey Wolf Optimization (MGWO).

Further, the performance of the modified variant was tested on sine dataset and cantilever beam design functions. Moreover, the experimental results were compared with several recent nature-inspired algorithms. The results showed that the modified variant proved to be producing effective solutions of sine dataset and cantilever beam design functions as compared to other meta-heuristics.

**Table 11.** Best optimal solutions of the cantilever beam design function by different meta-heuristics.

| **Algorithms** | $l$ | $m$ | $n$ | $o$ | $p$ | **Min (x)** |
|---|---|---|---|---|---|---|
| CONLIN [17] | 6.0100 | 5.3000 | 4.4900 | 3.4900 | 2.1500 | NC |
| CS [17] | 6.0089 | 5.3049 | 4.5023 | 3.5077 | 2.1504 | 1.33999 |
| MMA [17] | 6.0100 | 5.3000 | 4.4900 | 3.4900 | 2.1500 | 1.3400 |
| GCA-II [18] | 6.0100 | 5.3000 | 4.4900 | 3.4900 | 2.1500 | 1.3400 |
| GCA-I [18] | 6.0100 | 5.3000 | 4.4900 | 3.4900 | 2.1500 | 1.3400 |
| SOS [19] | 6.01878 | 5.30344 | 4.49587 | 3.49896 | 2.15564 | 1.33996 |
| **MVGWO** | **6.01554** | **5.30256** | **4.49386** | **3.49797** | **2.15896** | **1.33966** |

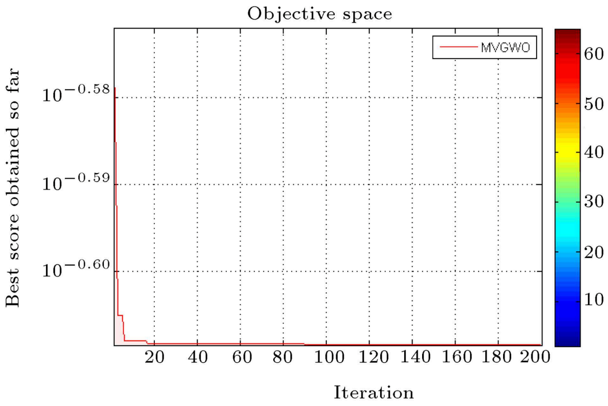


**Figure 27.** Convergence graph of MVGWO.

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