Experimental Study of the Wedge Effects on the Performance of a Hard-chine Planing Craft in Calm Water

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Abstract
In this paper, effects of a wedge on the performance of planing craft in calm water are experimentally investigated. Experiments are carried out on three different cases distinguished by the wedge type. The model, built of fiberglass, is a prismatic planing hull with deadrise angle of 24 degrees. Towing tests are conducted at different Froude numbers ranging from 0.21 to 2.1. The total trim angle, resistance, rise up at the CG as well as stern and bow, keel wetted length, chine wetted length, stagnation angle, and the length of stagnation line are measured. They are used to study the effect of installing a wedge on the performance as well as the effect of height on the hydrodynamic characteristics. Based on the observations made, it is concluded that, when the wedge is applied to the hull, the risk of model exhibiting instability diminishes, while total trim angle largely decreases, keel wetted length is enlarged, wetted surface becomes thinner, CG rise up is lowered, and the resistance is reduced. Moreover, experimental measurements and theoretical 2D+T theory are combined to bring deeper insight about physics of the flow and pressure distribution when a wedge is installed on the bottom of a planing hull.

Keywords: Experimental study; Planing hull, Wedge, Performance, Calm water; Combination of Experimental and Theoretical studies.

Nomenclature

2D+T Theory

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>c</td>
<td>Half beam of spray root</td>
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<tr>
<td>(c)</td>
<td>Time derivative of Half beam of spray root</td>
</tr>
<tr>
<td>(C_r)</td>
<td>Reduction ratio</td>
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<tr>
<td>(f_{2D}^{HD})</td>
<td>2D hydrodynamic force (N/m)</td>
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<td>(f_{3D}^{HD})</td>
<td>3D hydrodynamic force (N/m)</td>
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<td>(p)</td>
<td>Pressure (N/m²)</td>
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<tr>
<td>(t)</td>
<td>Time (s)</td>
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<tr>
<td>(t_e)</td>
<td>Ending time for solving solid body water entry (m)</td>
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<td>(w)</td>
<td>Water entry speed (s)</td>
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<td>(y)</td>
<td>Lateral distance from the wedge apex (m)</td>
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Characteristics of Model

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<tr>
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<td>(D_D)</td>
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<tr>
<td>(D_T)</td>
<td>Draft at transom</td>
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<tr>
<td>(L)</td>
<td>Length (m)</td>
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<tr>
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<td>Length between perpendiculars (m)</td>
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<td>Mass (Kg)</td>
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<td>Volume (m³)</td>
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<tr>
<td>(VCG)</td>
<td>Vertical center of gravity (m)</td>
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<tr>
<td>(x)</td>
<td>Distance from transom</td>
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<tr>
<td>(\beta)</td>
<td>Deadrise angle (deg)</td>
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<tr>
<td>(\Delta)</td>
<td>Weight (N)</td>
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<td>(\tau_S)</td>
<td>Static trim angle (deg)</td>
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Coefficients

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<tr>
<td>(C_F)</td>
<td>Frictional resistance coefficient</td>
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Hydrodynamic Characteristics of Mode During Steady Motion

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<td>Speed Coefficient (C_V = U / \sqrt{gB})</td>
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<td>Weight coefficient (C_A = U / (\rho g B^3))</td>
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<tr>
<td>(Fr)</td>
<td>Froude Number (Fr = U / \sqrt{gL})</td>
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<td>(Re)</td>
<td>Reynolds Number (Re = U L / \nu)</td>
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<td>(L)</td>
<td>Lift force (N)</td>
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<td>(L_C)</td>
<td>Chine wetted length (mm)</td>
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<tr>
<td>(L_K)</td>
<td>Keel wetted length (mm)</td>
</tr>
<tr>
<td>(L_M)</td>
<td>Mean wetted length (mm)</td>
</tr>
<tr>
<td>(L_S)</td>
<td>Stagnation line length (mm)</td>
</tr>
<tr>
<td>(LCG_e)</td>
<td>Effective longitudinal center of gravity (m)</td>
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1. Introduction

Planing hulls are fast, agile, and popular boats that are used in different segments of marine transportations. They are characterized by the hydrodynamic load acting on their bottom which chiefly affects the boat performance and help them reach high speeds. This force results in reduction of the wetted surface, increase of the bow wave length, decrease of the wave making resistance, and trimming the boat bow up. These vessels have empowered the naval engineers to better design high-speed boats. However, the common instabilities observed in these boats, especially at high-speeds [1, 2], has intensified the engineers’ concerns about their appropriate performance at high speeds and avoiding these instabilities is a central focus for the designers. Several methods have been proposed to improve the stability of these boats in calm water and waves. For example, transom flaps are used for increasing the longitudinal stability of planing hulls by De al Cruz et al. [3], Xi and Sung [4], or proactive control of thrust force was used by van Deyzen [5].

Dynamic instabilities of planing hulls are observed in transverse, horizontal, and vertical planes [1]. Among these instabilities, porpoising is a well-known instability during which boat experiences an oscillatory motion in the vertical plane. Through addition of some appendages like flaps and wedges, this instability may be reduced or diminished [4]. The appendages can produce an extra lift in addition to the pressure lift and allow the vessel to reach a dynamic equilibrium at speeds where the vessel without any appendages would not experience it. At other speeds, this equipment may positively help the vessel to move at smaller trim angle which results in a smaller resistance. Appropriate understanding of the role and effects of these appendages on the performance is pivotal for the designers. In the current work, two wedges are placed at the bottom of a planing boat and their effects on the vessel performance in calm water are investigated.

One of the first methods developed for investigating the planing hulls equipped with flaps in the steady state condition was the work by Savitsky and Brown [6]. They presented empirical relations for computing the lift, moment, and drag force from the trim tabs. Aside from this study, majority of the studies in this realm have focused on experimental and numerical works. Millward [7] installed different wedges at the bottom of planing hulls and showed that in some specific conditions, the wedge may positively help reduce the resistance, increase the trim, and avoid the proposing phenomenon. Karafiat and Fisher [8] used both numerical and experimental methods and showed that equipping a high-speed ship with a wedge can bring about reduction in the trim angle and resistance. In addition, Wang [9] showed that adding wedges, interceptors, and trim tabs can lead to an extra hydrodynamic force that results in the reduction of both trim and resistance. In a work conducted by Tsai and Hwang [10], combination of wedge and flaps was studied and discussions about appropriate condition for installing these appendages were presented. Moreover, Cumming et al. [11] showed that a wedge may also lead to the reduction of cavitation of the propeller in the vessel. Jang et al. [12] numerically investigated the effect of a wedge on the performance of a passenger ship. The experimental work of Steen et al. [13] also presented additional insights about the effects of appendages on the performance of planing hull and reduction of possible instabilities. In recent years, Karimi et al.
[14] conducted a parametric study to investigate the effects of the interceptors on the performance of planing hulls and presented statistical analysis for specifying the depth of this appendage. Performance of the planing hulls with appendages cannot be mathematically modeled easily in both steady and unsteady conditions. For such cases, the previous semi-empirical works [15-17] and analytical works [18-20] cannot be used directly and the available empirical relations [6, 21] might be applicable which are limited to some specified conditions. Therefore, by following the previous works, the best alternatives for studying such phenomena are recommended to be numerical and experimental approaches. Accordingly, during the last decade, wide ranges of experimental studies have been devoted to study different characteristics of the planing hulls. Performance of these hulls [22-27], their seakeeping [28, 29], roll motion [30], and even steady yawed [31, 32] condition have been studied. The satisfactory, promising, and useful findings of these studies signal that experimental works can be considered as a very reliable alternative methodology for investigating the planing hull characteristics.

In the current paper, a planing hull is experimentally modeled in a towing tank with and without a wedge. This hull is not desirably stable without a wedge. Unlike previous works of De la Cruz et al. [3] and Streen et al. [13], who used controllable flaps and interceptors to reduce instability of the boat, a fixed wedge is used in the current paper, in order to reduce the boat instability. Moreover, the model studied in this paper is comparatively larger than those in previous studies. Therefore, unlike the previous studies, the current paper focuses on larger Reynolds numbers. It is shown how a wedge and its height can affect the performance of the model in calm water. The problem is firstly defined and the most important parameters are introduced. The applied test methodology, the model, the facilities, and run conditions are also presented. The main results of the paper include the trim angle, the CG rise up, the stagnation line angle, the keel wetted length, and the resistance, and it is demonstrated how a wedge can affect the performance of planing hulls. Subsequently, the empirical relation by Savitsky and Brown [6] is used and it is assessed how accurate this approach estimates the trim angle of the planing hulls by the added wedges. Later, 2D+T theory and the measured trim angle and keel wetted are used to ascertain how a wedge can affect the longitudinal force distribution for a planing hull. Furthermore, the wedge lift is determined using the measured trim angle and keel wetted length that are implemented in 2D+T theory. Ultimately, the conclusions of the current study are presented and future works are outlined.

2. Material and Methods

2.1 Problem definition

In the current experimental tests, it is aimed to find the running attitude of planing hulls at different speeds and how they are influenced by a wedge. The speed of the model is assumed to be \( U \). Since the model speed may exceed the displacement flow regime, a hydrodynamic force is expected to be produced at the bottom of the hull. This force can push the solid body up which can lead to a trim angle of \( \tau \) (Figure 1). Two wetted length including keel wetted length \( (L_K) \) and chine wetted length \( (L_C) \) are defined. The first length is referred to the length between the transom and intersection of the calm water with keel. The latter represents the length between the transom and longitudinal position where water firstly drenches the chine (by considering the water rise-up). Rise up of the vessel is considered at three different positions. The first position is the transverse section of the boat (transom), the second is the longitudinal position of CG and the third position is the transverse section 10 (bow section). All parameters are found in an equilibrium condition. Also, the boat mat experiences a porpoising instability in vertical direction [33]. For such a condition, no trim, rise up, keel wetted length and resistance would be reported. The boat speed is turned into non-dimensional form using beam Froude number as in

\[
Fr_B = \frac{U}{\sqrt{gB}}
\]

where \( B \) is its beam and \( g \) is the gravity acceleration.

Figure 1. A pictograph of the considered problem.
2.2 Physical description of the model
In the current study, a V-shape hard-chine planing hull is investigated. A 1:5 scale model made up of fiberglass is built. The model length is 2.6 m and it has length over beam ($L/B$) ratio of 4.78. The deadrise angle of the boat is 24 degrees at stern and is constant from its stern (section A) to amidship (section B), and then increases from 24 to 40 degrees at its bow (section C). The mass of the model is 86.024 Kg and its longitudinal center of gravity is located at 0.7914 m from the transom. The model has no step in its longitudinal and transverse direction. Principal characteristics of the model are displayed in Table 1 and its body profile is depicted in Figure 2.

![Table 1](image1.png)

**Table 1.** Principal characteristic of the investigated model.

2.3 Experimental set up
The experimental set up is based on towing method and recommendation of ITTC [34] on High Speed Marine Vehicle resistance tests. The experiments are carried out in the National Persian Gulf Towing Tank, located in Tehran. The tank length is 400 m, its width is 6 m and the water depth is set to be 4 m. The maximum carriage speed is 18 m/s. The characteristics of the towing tank are displayed in Table 2. The costume-built manned carrier moves on the rails and can measure different hydrodynamic parameters. The model is towed from its CG. During the conducted tests, the model does not exhibit any roll, sway, and yaw motions. It is fixed in these directions and its initial roll and yaw angles are assumed to be zero. The drag force and the trim angle are measured during each test. The position at which drag force is measured, is the intersection of the shaft and LCG. It should be noted that the angle between the shaft line and base line is 6 degrees in all the considered tests. It should be pointed out that, although the current paper explores steady performance of a model, shaft line is also considered in building of the model. This is due to the fact that, this is a model of a real planing hull which is being studied by the current authors in order to reduce the possibility of porpoising phenomenon.

![Table 2](image2.png)

**Table 2.** Characteristics of National Persian Gulf Towing Tank.

The rise up of the three reported sections is found using the installed potentiometer at the sections (as evident in Figure 3a). The trim angle of the boat is determined by computing the tangent of the line connecting the rise up of the transom section ($Z_t$) to the rise up of the bow ($Z_b$) section as

$$
\tau = \tan^{-1}\left(\frac{Z_{t0} - Z_{b1}}{L_{30-1}}\right)
$$

where $L_{0:10}$ is the longitudinal length between these two sections. It should be noted that boat is located at a static trim angle in each of the conducted tests and that static trim angle has significant effects on the final dynamic trim angle [35, 36]. Therefore, if any researcher is interested in modeling according to the current paper, this point should be taken into consideration, as well. Moreover, the reported values of dynamic trim angle are the total trim angle, and not the absolute angle. It is important to state that the keel wetted length and the chine wetted length are also measured in this study. To this end, a camera which is located under the boat and moves with the boat, is used. This camera is a 720*1280 with 30 frame in each second. At each frame, the photos are taken. The value of the wetted lengths are recorded by using the marked numbering on the body of the model. A photograph of the bottom of the model is shown in Fig. 3b. It should be pointed out that, when boat the reaches steady condition, the value of wetted lengths has no variation and becomes fixed.

Regarding the repeatability and uncertainty of the problem, it should be noted that selected tests for the planing hull without a wedge are conducted four times and it is observed that trim angle, resistance, and sinkage differ by about 0.01 to 1%.
2.4 Run Conditions

The targeted tests are carried out in three different conditions. In the first condition, the model is not equipped with any appendages. Based on the experiments, this model undergoes proposing at speeds larger than 7 m/s. In the other conditions, it is attempted to add wedge to the bottom of the boat in order to lower the weight of the boat and change the position of the CG. The wedge height is selected by considering the boundary layer thickness of the boat bottom. This layer is determined using

$$\delta(x) = 0.37 \text{Re}^{-1/5}$$

where Re is the Reynolds number and is found by

$$Re_l = \frac{v l_m}{\nu} \quad L_m = \frac{l_k + l_c}{2}$$

where $v$ is the kinematic viscosity of the fluid. In the current study, Reynolds number varies from $3.73 \times 10^9$ to $2.27 \times 10^{10}$, which yields to boundary layer of 0.36 to 0.3 of $L$ by using the above equations. Therefore, the selected value for the $h/L$ of the wedge (height over length of the wedge ratio) should be smaller than 0.3. The ranges of $h/L$ of the previous research and current study are also shown in Fig. 4. Based on this figure, another point is observed that distinguishes the current study from the previous ones, which is the Reynolds number range of the tests. Unlike the previous studies in which Reynolds Number is mostly smaller than $10^9$, in the current paper, larger range of Reynolds number is considered.

**Figure 4.** Values of $h/L$ ratio of different studies: reproduction of figure 2 of Karimi et al [14] by adding data of current paper.

Based on the reported values by in Fig. 3, two different depths of 10 and 5 mm ($h/L = 0.108$ and 0.054) are considered for the intended experiments. The length of the wedge is also assumed be 92 mm. A schematic of the wedges are shown in Figure 5. It should be noted that the wedge of height 10mm is named wedge-1 and the other is named as wedge-2 in the current study.

**Figure 5.** The installed wedge at the stern of the model.

For each case, 10 different speeds ranging from 1 to 10 are considered. Speeds of 1 and 2 m/s are recognized as the displacement regimes, speeds 3 and 4 m/s are categorized as semi-planing condition, and speeds larger than 4 are classified as planing mode. During each run, the following parameters are determined: (1) Trim angle (in degree), (2) Rise up at stern, CG and bow (in mm), (3) Keel wetted length and chine wetted length (in mm), and (4) Resistance in (kgf).

3. Results and Discussion

3.1 Measured parameters for each case

The data produced by the conducted tests are reported in this sub-section. In addition to the earlier mentioned parameters, the stagnation line angle $\alpha$ and the stagnation line length ($L_{st}$) are also determined for each test. The measured parameters for the model of no wedge are displayed in Table 3. The tests are conducted at different speeds ranging from 1 to 10 m/s. As observed in this table, at speeds larger than 8m/s, the model experiences vertical
instability and no fixed trim angle is recorded. The instability is denoted by PORP as an abbreviation of porpoising. Also, some photographs of the model at beam Froude numbers 0.86, 1.72, 3.01, and 3.87 are illustrated in Figure 6. Moreover, it is checked whether the previous empirical equations predict the porpoising in the current model. Celano [37] suggested that porpoising of the a planing hull can be determined by

\[ t_{\text{Critical}} = 0.1197 \beta_{\text{deg}}^{0.7561} e^{15.7132 \sqrt{\frac{C_L}{2}} \sqrt{\frac{\Delta}{C_v}}} \]  

(6)

where

\[ \sqrt{\frac{C}{L}} = \sqrt{\frac{C\Delta}{2C_v}}. \]  

(6)

It should be mentioned that the above empirical equation shows that the critical total trim angle of the boat at a speed 7 m/s is 4.8 degrees which is smaller than the measured total trim angle. This indicates that the equation correctly demonstrates that the model undergoes instability.

Table 3. Measured parameters for the case of no wedge.

Figure 6. Photographs of the tests in the case of no wedge for different beam Froude numbers.

The recorded results for the case with wedge-1 are shown in Table 4. The results indicate that by installing wedge-1, the model experiences a steady movement and no instability occurs. It can also be seen that at the beam Froude numbers larger than 1.71, the chine is dry and only the keel is wetted. Maximum value of total trim angle in this condition is 4.93 degrees which is smaller in comparison with the case of no wedge. Once again, some photographs of the model while being towed, is depicted in Figure 7 at beam Froude numbers 0.86, 1.72, 3.01, and 3.87.

Table 4. Measured parameters for the case with wedge 1.

Figure 7. Photographs of the tests in the case with wedge-1 for different beam Froude numbers.

Finally, the results of the case with wedge-2 are illustrated in Table 5. A close scrutiny of the reported parameters in this table reveals that the installed wedge also prevents possible instability and improves the vertical stability of the model. The maximum total trim angle resulted by this wedge is 5.27 degrees, and the wetted length of chine is again zero at the two largest speeds. Four photographs taken during the tests are shown in Figure 8 that correspond to beam Froude numbers 0.86, 1.72, 3.01, and 3.87.

Table 5. Measured parameters for the case with wedge-2.

Figure 8. Photographs of the tests in the case with wedge-2 for different beam Froude Numbers.

As observed in the displayed Tables, wedges 1 and 2 eliminate the porpoising instabilities. In order to provide a better insight regarding the effects of these wedges, time history of the total trim angle of the model with and without a wedge at 10 m/s speed is shown in Figure 9. As evident in this figure, when the boat is advances forward
without any wedge, it experiences oscillations in the direction of total trim angle. However, in the presence of wedges 1 and 2, the total trim angle shows a steady behavior and does not vary in time.

**Figure 9.** Time history of the trim angle of the model at speed of 10 m/s with and without wedge.

### 3.2 Comparison of different parameters for different models

Through comparison of the obtained results for different conducted test cases, one may better understand the effects of installing a wedge on a planing model and also the influence of height of this appendage at the same time. Figure 10 illustrates the measured trim angles of each test model at different beam Froude numbers. As evident in this figure, by installing a wedge at the bottom of the model, the model trim angle decreases on top of preventing the vertical instability. The results also show that the case with wege-1 has smaller trim angles in comparison with the case with wedge-2. This goes to show that for a wedge with larger depth, the trim angle reduces more significantly. It should be noted that when the wedge depth increases, it lowers the efficient weight of the model further. As a result, the trim angle should be further reduced. It should be noted that, in the previous research by Millward [7], such phenomenon was also observed. His results showed that boat trim angle decreases by 25-33%, when a wedge is added to the bottom of the boat. At larger speeds, the reduction was more significant, and this behavior is like what is observed in the current study. However, in the current study, these reductions are about 13-49%, when wedge 1 is used. Meanwhile, wedge 2 reduces the trim angle of the vessel by 7 to 37%. As stated earlier, the current research explores the effects of the wedge height on the performance of a planing hull, while the work by Millward [7] focused on the inclination angle of the wedge. Moreover, the length of the model in the current study is large, hence, the Reynolds number is different.

The measured rise ups of CG for all three cases are illustrated in Figure 11. Based on the presented results, when the wedge is added to the hull, the rise up of CG decreases at beam Froude numbers larger than 2.15 and the model tends to be pushed down at beam Froude numbers larger than 0.86. Also, the case with wedge-1 has lower rise up in comparison with the case with wege-2. It can thus be concluded that larger depth of the wedge leads to a decrease in the CG rise up. It is again noteworthy that, in the previous work by Millward [7], it was observed that adding a wedge to the bottom of a planing boat has no significant effects on the CG rise-up, and such phenomenon is also observed in the current paper. The reduction of CG is much smaller, compared to the reductions observed in trim angle.

**Figure 10.** Comparison of the measured trim angle for different test cases.

**Figure 11.** Comparison of the measured CG rise up for different test cases.

The measured stagnation angle of different test cases is presented in Figure 12. Based on the reported results, when the wedge is installed on the model, the stagnation line angle significantly increases. This implies that installing a wedge may yield a thin wetted surface. However, this may negatively affect the transverse stability of the model [38]. A comparison between both of the test cases involving a wedge shows that at Froude numbers 2.15 to 3.01, the case with larger depth has larger stagnation line, but at larger speeds, this difference reduces.

Figure 13 displays the measured keel wetted length for all of three test conditions. Through comparison of $L_K$ versus Froude numbers (Fr.) plots, it can be concluded that when a wedge is added to the model, the keel wetted length of the model increases. The observed increase associated with wedge-1 is larger than that of wedge-2 which shows larger depth of the wedge leads to a larger keel wetted length. Overall, the keel wetted length is decreased by 2 to 33%, when wedge 1 is used, and by 2 to 28%, when wedge 2 is used.
In order to provide a better understanding about the wetted surface and wedge effects on it, the top view of the measured wetted surfaces are displayed in Figure 14. Based on the presented results in this figure, it can be concluded that at first three speeds that are not categorized as planing regime, the wedge does not have significant effect on the wetted surface and for all three test cases, a similar top view of the wetted surface is observed. However, as the speed increases and beam Froude number approaches 1.72, the wedge effect on the wetted surface becomes considerable. Meanwhile, the keel wetted length becomes larger, the chine wetted length becomes smaller, and the wetted surface gets thinner. The obtained results in the case with wedge-1 and 2 prove that for the larger wedge depth, the wetted surface becomes thinner.

The computed resistances of the considered cases are displayed in Figure 15. The results reveal that the resistance of all three cases are almost similar at Froude numbers smaller than 1.29. Beyond this Froude number, the case of no wedge produces larger resistance in comparison with the cases with wedge. Based on the results in Figure 16, resistance in the case with wedge-1 is smaller at Froude numbers lower than 3.44. However, beyond this specific Froude number, the resistance of the case with wedge-2 becomes smaller. This is indicative of the fact that for the case with larger wedge depth, the resistance finally becomes smaller. Overall, it may be concluded that the larger wedge depth may lower the trim angle, but it can lead to a large resistance. Therefore, for selection of an appropriate wedge, an optimization procedure must be applied. In the current paper, resistance of the planing hull is reduced by 6-15%, when wedge 1 is used, and it is reduced by 2-11%, when wedge 2 is used. In comparison with previous studies, it should be mentioned that Karimi et al. [14] reported a reduction of 3.3 to 11%, when interceptors with height to length ratio of 0.4 is used. Also, the results of Karimi et al. [14] showed that, resistance is reduced by 8-19%, when an interceptor with height to length ratio of 0.6 is used. The results of the current study show that, the installed wedges yield promising results, especially in the case of wedge 1. It should be stated that, the height to length ratio of wedges 1 and 2 are respectively 0.1 and 0.05.

As proposed by ITTC, the resistance of a planing hull can be written in the form of

$$R_r = R_{pa} + R_{ws} = R_f + R_n + R_{ws}$$

$$R_f = \frac{1}{2} \rho U^2 S_{pa} C_f + \frac{1}{2} \rho U^2 S_{ws} C_f$$

where subscripts PA and WS refer to pressure area and whisker spray area, respectively. On the other hand, $F$ and $R$ denote the friction and residual forces. $S$ represents the wetted surface and $C$ represent the force coefficient. It should be pointed out that there might be frictional resistance over whisker spray area, as well [39, 40]. The coefficients related to the resistance components may be determined using
\[ C_T = C_r + C_s + C_{r, as} \frac{S_{as}}{S_{pa}} \]  

(6)

as proposed by Begovic and Bertorello [27]. It would be interesting to find \( C_R \) and \( C_T \) of each case and effects of the wedge and its height on these coefficients. Total resistance coefficients of all test models are illustrated in Figure 16. Based on the plots shown in this figure, a wedge may only lead to larger total resistance coefficient at first two Froude numbers, but it lowers the peak value. Beyond this Froude number, the case equipped with wedge-1 produces the smallest resistance coefficient among all test conditions. However, the case with wedge-2 has larger value of \( C_T \) larger than the case with wedge-1. Meanwhile, it should be noted that as Froude number increases, the difference between \( C_T \) of the case with wedge-1 and wedge-2 diminishes and their values approximately become similar at Froude number of 1.92. Measured values of \( C_R \) are displayed in Figure 17. Again, what occurred for \( C_T \) is also observed for \( C_R \).

**Figure 16.** Comparison of the measured \( C_T \) for different test cases.

**Figure 17.** Comparison of the measured \( C_R \) for different test cases.

### 4. Mathematical analyses

#### 4.1 Examining Savitsky’s method in determining the trim angle

After presenting the measurements, an assessment of Savitsky and Brown [6] relation in predicting the trim angle is presented. This is done to find how this method and presented equations by Savitsky and Brown [6] can predict the trim angle of a planing hull equipped with a wedge, in comparison with the obtained experimental data in the current study. Also, it can be determined how the assumption of static trim angle can affect the results of this method. It is important to comment that, fast predictions of performance of a planning hull in early stage design are always important for the engineers. Therefore, authors would like to take advantageous of the current experimental data to examine Savitsky’s method in estimating the performance of planning hulls with a wedge and possible sources of error. Savitsky and Brown [6] proposed that the lift of a flap may be computed by

\[ \Delta_r = 0.14025L_F \sigma B \left( \frac{\rho}{2} U^2 \right) \]  

(7)

where \( L_F \) is the flap chord which is hereby is set to be the wedge length. \( \sigma \) is the flap span-beam ratio which, in the current study, is set to be the beam to span-beam ratio of the wedge, while \( \delta \) is the flap angle with the direction of base line that is set to be the wedge angle in the current research. Based on the suggestion made by Savitsky and Brown [6], the appendage may lead to the reduction of the hull mass as in

\[ m_e = m - \frac{\Delta_r}{g} \]  

(8)

and shifting of the longitudinal center of gravity as in

\[ LCG_e = \frac{(mg \times LCG - 0.6 \times \Delta_r \times B)}{m_g} \]  

(9)
Savitsky and Brown [6] proposed that the values found by empirical equations of (8) to (10) be implemented in Savitsky’s [15] relation in order to find the equilibrium condition. Based on Savitsky’s method [15], the lift force coefficient \( C_L \) is determined by

\[
C_{L,0} = r^{11} (0.012 \lambda^{0.5} + \frac{0.0055 \lambda^{2.5}}{C_v^2})
\]

\[
C_L = C_{L,0} - 0.0065 C_v^{0.6}
\]

where \( \lambda \) is the normalized mean wetted length and can be found by

\[
\lambda = \frac{L_k + L_c}{2B}
\]

and \( C_{L,0} \) is the lift force coefficient of planing plate. The center of pressure is found by

\[
c_p = 0.75 - \frac{1}{\frac{5.21 C_v^2}{\lambda^2} + 2.39}
\]

Using Savitsky’s method [15], the trim angle for all of the considered conditions are computed and compared with the current experimental results. In the current paper, a previous computer program developed and validated by Ghadimi et al. [21] is utilized for this purpose. Comparison of the experimental data with those of Savitsky and Brown [6] relations is displayed in Figure 18. Based on the presented plots in the case of no wedge, the empirical relations estimate the trim angle with relatively good ability. This shows that, when Reynolds number is high, like the current case, the h/L ratio of the wedge is near 5\%, and there is no source of error in Savitsky’s method. For the case with wedge 1, Savitsky’s method [16] with input from the current experiments leads to larger error. In this case, Reynolds number is high, static trim angle is also relatively large (2.43 degrees), and moreover, the h/L ratio of the wedge is about 0.1\%. Therefore, it can be concluded that, when Savitsky’s method [15] and empirical relation by Savitsky and Brown [6] is used for estimation of the trim angle of the hulls with a wedge, and Reynolds number is higher than \( 10^9 \), the static trim angle is large and wedge depth is high, sources of error are augmented and yield larger values of trim angle.

**Figure 18.** Comparison of the predicted trim angles by Savitsky’s method [15] against experimental data: (a) no wedge, (b) wedge 1, and (c) wedge 2.

### 4.2 Longitudinal force distribution

It would also be interesting to find how a wedge can affect the hydrodynamic force distribution in longitudinal direction of a planing hull, because when a wedge is installed on the bottom of a vessel, large value of pressure is produced by the wedge. Therefore, distribution of the vertical force highly changes, when a wedge is added. In the current subsection, this phenomenon is explored. For this purpose, 2D+T theory is used. This theory has been accepted as an appropriate theoretical model for hydrodynamic simulation of planing hulls in calm water [41-43], and in waves [44-47], as well as roll motion [48-51]. It is considered that the model passes through a transverse plane and the three dimensional problem can be reduced to a water entry of a solid body with wedge section, as shown in Figure 19.

**Figure 19.** 2D+T theory for the steady state problem of the current planning model.

The water entry problem can be solved from time zero to an ending time determined by
\[ t_c = \frac{L_K}{U \cos \tau} \]  

and the solid body enters the fluid with a speed \( w = U \sin \tau \)  

Dynamic pressure distribution over the wall of the wedge can be computed using analytical scheme, numerical methods [52-67] or experimental measurements [68-74]. Here, the Wagner solution [75] is used as in  

\[ p = \rho \left[ \frac{wc \dot{c}}{\sqrt{c^2 - y^2}} - \frac{w^2}{2} \frac{y^2}{c^2 - y^2} \right]. \]  

Where \( c \) is the half beam of spray root at each section, \( \dot{c} \) is the time derivative of \( c \), and \( y \) is the lateral distance from the wedge apex. Parameter \( c \) and its time derivative may be found using  

\[ c = \frac{\pi}{2} \frac{w t}{\tan \beta}, \]  
\[ \dot{c} = \frac{\pi}{2} \frac{w}{\tan \beta}, \]  

when the water has not drenched the chine. As this happens, the boundary condition \( P = 0 \) is applied at chine. The 2D hydrodynamic force can then be determined by integrating of the pressure over the wedge body as in  

\[ f_{HD}^{2D} = \int_s p n_z dy \]  

Also, force is reduced at each section in order to correlate the 2D force to the 3D force. Accordingly, the reduction function introduced by Garne [76] is applied which is  

\[ C_v = \tanh \left( \frac{2.5}{0.34BC_v} x \right) \]  

where \( x \) is the longitudinal distance from the transom. Therefore, force at each section is calculated by  

\[ f_{HD}^{2D} = C_v f_{HD}^{2D} \]  

2D hydrodynamic forces for each test case are computed by implementing the measured trim angle and keel wetted length as inputs. These values are implemented in the above mentioned equations and the 2D forces are determined. The estimated 2D force distribution is computed using a previously developed and validated computer program by Ghadimi [41] which is shown in Figure 20. Based on the obtained results, installing a wedge on the bottom of the test model at each speed causes a significant reduction in the sectional forces. It is obvious that maximum value of the force produced at the position where chine gets wet is larger for the cases of no wedge. Also, at all other locations, this case displays larger values. A comparison between sectional force of the case with wedge-1 and the case with wedge-2 indicates that for the case equipped with wedge-2, the forces are larger. This shows that wedge-2 has smaller contribution than wedge-1 in supporting the boat weight. At Froude numbers 1.79, 1.92 and 2.1, no sectional force is reported for the case of no wedge, since the model undergoes porpoising phenomenon at these speeds. It should be noted that at Froude numbers 1.92 and 2.1, the value of sectional forces for wedge-2 are very small. This may be attributed to the nature of 2D+T theory. This method may not have accurate results for trim angles smaller than 1 degree [77, 78], while for the case with wedge-1, the trim angle is smaller than 1 at these speeds.

**Figure 20.** Distributions of 2D normal force in longitudinal direction at (a) Fr=0.43, (b) Fr=0.86, (c) Fr=0.129, (d) Fr=1.72, (e) Fr=2.5, (f) Fr=2.58, (g) Fr=3.01, (h) Fr=3.44, (i) Fr=3.87, (g) Fr=4.3.
4.3 Wedge lift computation

Finally, the produced lift by the wedge is estimated in this subsection using the measured trim angle and keel wetted length. Here, the 2D+T theory is used and the measured values are implemented to find the sectional forces. Subsequently, these forces are extended in the longitudinal direction. Integrating the sectional hydrodynamic force leads to

\[ F_{HD}^{3D} = \left( \int_{L_k} C_{pf} f^{2D} dx \right) \cos \tau \]  

(21)

The sectional hydrostatic force is computed using the submerged area of each section as in

\[ F_{HS}^{3D} = \int_{L_k} C_{pr} \rho g A dx \]  

(22)

The total lift due to hydrostatic and hydrodynamic sectional forces is computed by summation of these components as in

\[ L = F_{HD}^{3D} + F_{HS}^{3D} \]  

(23)

The lift force produced by the wedge is computed by

\[ \Delta F = mg - L \]  

(24)

The sectional forces and the 3D values are again found using the developed program by Ghadimi et al. [20]. The estimated wedge lift computed by 2D+T theory and by implementing the measurements are displayed and compared against the results of Savitsky and Brown [6] empirical relation (equation (8)) in Figure 21. It can be observed that for the wedge-1, equation (8) yields larger lift compared to the estimated values, but for wedge-2, the predicted values by equation (8) and estimated values agree with each other. As observed again, for the wedge 1, where h/L of the wedge is slightly large, the resulting error increases.

Figure 21. Comparison of the estimated lift of the wedge by implementing the measurements in 2D+T theory against the results of Savitsky and Brown’s empirical equation [6]: (a) wedge-1 and (b) wedge-2.

5. Conclusions

In the current study, the effects of an installed wedge on the performance of a planing hull are investigated by using towing test models. Three different test cases of no wedge, with wedge 1, and with wedge 2 are considered. This research is different from the previous research in so many ways. First, a wedge is added to the bottom of the vessel in order to reduce the possibility of porpoising in a planing hull. In addition, the length of the model, is considerably larger than the other models. Moreover, the range of the Reynolds number in the current study is much larger than those in the previous work.

The trim angle, the resistance, the rise up, and wetted length are measured during the conducted tests. The reported results indicate that the model of no wedge experiences an instable unsteady motion at high-speeds. As a wedge installed on this model, the porpoising is diminished. The presented time history of the trim angle plots support this assessment. The comparison of the results in different test cases of no wedge and the case with a wedge shows that:

1- Installing a wedge on a planing hull leads to a lower trim angle. Trim angle is reduced by 13 to 49%, when wedge 1 is used, and this parameter is reduced by 7 to 39%, when wedge 2 is used.

2- As a wedge added to the body, the resistance decreases. The investigations show that this force is reduced by 6 to 15%, when wedge 1 is used. Moreover, wedge 2 reduces the resistance of the vessel by 2 to 11%.

3- The keel wetted length increases, when a wedge is applied. Wedge 1 increases the keel wetted length by 2 to 33%, and wedge 2 does the same by 2 to 28%.

4- Total resistance and residual resistance coefficients decreases, when the wedge is added to the body.
5- The wetted surface of a planing hull becomes narrower, when the wedge is installed. The stagnation angle is increased by 48 to 99% m when wedge 1 is used. Moreover, when wedge 2 is used, stagnation angle grows by 44 to 97%.

It is also found that for the case with larger wedge height, the trim angle decreases further. It is also observed that the case with smaller wedge height has smaller resistance at high-speeds and its keel wetted length is also smaller. Thicker wetted surface is observed for the case with larger wedge height. The total resistance and residual resistance coefficients of the case with smaller wedge height is larger at all speeds except the last two which have Froude number of 1.92 and 2.21.

The method by Savitsky and Brown [6] is used to predict the trim angle of the boat with a wedge. It is concluded that for the case with wedge-2, errors are very small. However, for the case with wedge-1, the error is large. The 2D+T theory is also used to find the sectional force distribution and it is observed that the wedge chiefly lowers the sectional force peak and its value. The measured keel wetted lengths and trim angle are implemented in 2D+T theory in order to determine the lift produced by the wedge. The obtained results are compared with predicted results by previous empirical relations. Based on the conducted comparison, the estimated value for the wedge with smaller height and the value computed by empirical relation are similar.

The current work has shown the detailed effects of an installed wedge on the performance of a planing hull in calm water. It can help the naval engineers understand how to improve the stability of a planing boat in calm water. However, effects of the wedge on motions of a planing hull in waves still need to be understood more deeply. Future studies may focus on experimental works on the effect of a wedge on the vertical motion amplitudes and the bow acceleration in regular waves.

**Compliance with Ethical Standards:**

The authors express their sincere gratitude for the cooperation they have received from “National Persian Gulf Towing Tank” during the experiments. Authors of this study received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The authors also declare that they have no conflict of interest.

**References**


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**Figure captions**

**Figure 1.** A pictograph of the considered problem.

**Figure 2.** Body profile of the investigated hull.

**Figure 3.** a) Experimental set up of the model, b) A view of the vessel’s bottom which is used for determining the wetted lengths.

**Figure 4.** Values of h/L ratio of different studies: reproduction of figure 2 of Karimi et al [14] by adding data of current paper.

**Figure 5.** The installed wedge at the stern of the model.

**Figure 6.** Photographs of the tests in the case of no wedge for different Froude numbers.

**Figure 7.** Photographs of the tests in the case with wedge-1 for different Froude numbers.

**Figure 8.** Photographs of the tests in the case with wedge-2 for different Froude Numbers.

**Figure 9.** Time history of the trim angle of the model at speed of 10 m/s with and without wedge.

**Figure 10.** Comparison of the measured trim angle for different test cases.

**Figure 11.** Comparison of the measured CG rise up for different test cases.

**Figure 12.** Comparison of the measured stagnation line angle for different test cases.

**Figure 13.** Comparison of the measured Keel wetted length for different test cases.
**Figure 14.** Top view of the measured wetted surface of the models at different speeds: (a) Fr=0.43, (b) Fr=0.86, (c) Fr=0.1.29, (d) Fr=1.72, (e) Fr=2.5, (f) Fr=2.58, (g) Fr=3.01, (h) Fr=3.44, (i) Fr=3.87, (g) Fr=4.3.

**Figure 15.** Comparison of the measured resistance for different test cases.

**Figure 16.** Comparison of the measured $C_T$ for different test cases.

**Figure 17.** Comparison of the measured $C_R$ for different test cases.

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**Table 2.** Characteristics of National Persian Gulf Towing Tank.

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**Table 4.** Measured parameters for the case with wedge 1.

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**Table 2.** Characteristics of National Persian Gulf Towing Tank.

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### Table 3. Measured parameters for the case of no wedge.

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### Table 4. Measured parameters for the case with wedge 1.

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### Table 5. Measured parameters for the case with wedge 2.

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