The surgical case scheduling problem with fuzzy duration time: An ant system algorithm

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Abstract

In this paper, we address the surgical case scheduling problem in multi operating theater environment with uncertain service times in order to minimize makespan. In surgical case scheduling, not only the hospital resources are allocated to surgical cases but also the start time of performing surgeries is determined based on sequence of cases in a short-term time horizon. We consider fuzzy numbers for duration times of all stages and hereafter the problem called fuzzy surgical case scheduling. Since the operational environment in the problem is similar to no-wait multi-resource fuzzy flexible job shop problem, we consider constraints of that for formulating and solving problem. This problem is strongly an NP-hard optimization problem, hence we employ ant system algorithm to tackle problem. The proposed approach is illustrated by detailed examples of three test cases, and numerical computational experiments. Therefore, the performance of proposed algorithm is compared with a schedule constructed by first-come-first-service rule on all test instances. Also, a real case is provided from Isfahan’s hospital to evaluate proposed algorithm. Consequently, computational experiments state that algorithm outperforms results obtained by hospital planning as well as fuzzy rule, and these indicate efficiency and capability of our algorithm for optimizing the makespan.

Key words: Surgical case scheduling, Ant System, Operating theater, Fuzzy duration time, Makespan, Mixed integer programming.
1. Introduction

Healthcare industries have been growing in the last decades and as follows costs of this large industry are increasing. Based on statistics, healthcare expenditure of US in 2007 was estimated 16.2% of the gross domestic product [1] and these will reach 19.5% of US GDP by 2017 [2]. On the other hand, operating rooms (ORs) are considered engine of a hospital and the most critical resources that generate highest costs for a hospital so that in many hospitals more than 40% of costs comes from several resources of surgery and ORs [3]. Therefore, it seems essential to improve OR management and patient flow by optimized sequencing and assigning available resources of ORs to patients. As a result, planning and scheduling play a crucial role in OR management and thereby in recent decade, many researchers and practitioners are attracted to study on operating room scheduling problem.

In healthcare, a “surgical schedule” is performed by determining sequence of surgical cases as well as assigning them to operating rooms, surgeons, nurses, etc in order to optimize objectives such as utilization, idle time, overtime, etc. [4]. Operating room scheduling generally deals with strategic, tactical, and operational problems [5-8].

The problem referred in this paper is called surgical case scheduling and is classified in operational level. In the literature of the operating room scheduling problems [9], a problem namely “Surgical Process Scheduling (SPS) problem” is divided into two sub-problems called “advance scheduling” and “allocation scheduling”. First sub-problem solves planning step by determining some future date for surgical cases in medium time horizon. Next level namely “Surgical Case Scheduling (SCS) problem” as second part of SPS solves scheduling step which determines the start time and resource allocation of cases over a short time horizon (typically a day). It must be noted that allocation scheduling is within the scope of SCS problem, whereas advance scheduling is not. In some researches ([10-15]) both sub-problems were addressed, while in other studies [2, 16-22], the researchers reviewed on SCS problem.

Surgical process is divided to three sub-process include of pre-operative/surgery, peri-operative/surgery, and post-operative/surgery, and post-operative/surgery that various parameters are considered as input of each stage [16, 18, 21]. In literature, upstream units comprise pre-operative holding units (PHUs) and wards, while post anesthesia care units (PACUs), and intensive care units (ICUs) are put in downstream units. Multiple operating rooms connect upstream to downstream units [5, 21, 23-25]. Also, patient is divided to elective (inpatient and outpatient) and non-elective (urgent and emergency) case. An elective case is the patient that is scheduled in advance by determining multi-resources as well as the start time of case, while a emergency case that may arrive randomly on the day of surgery, requires to be performed online in the same day. To measure operating room planning and scheduling, eight main performance indicators have been used in literature; waiting time, utilization, leveling, throughput, patient deferrals, makespan, preferences, and financial measures [26]. Also, there are three well-known scheduling strategies/booking systems that dedicate ORs time to surgeon groups: open scheduling strategy, block scheduling strategy, and modified block scheduling strategy. In block scheduling strategy,
a set of time slots is allocated to every surgery in special group typically in cyclic timetable (some weeks). Surgical cases are scheduled in these time slots and these cannot be released. In contrast with block scheduling, surgical cases are scheduled on first-come-first-service (FCFS) and assigned to available ORs basic on surgeon’s convenience by applying open scheduling strategy. In other policy, block scheduling is modified by combining strategies of both block and open scheduling in order to enhance flexibility of strategy [6, 18, 19].

In literature of operating room scheduling, deterministic conditions are considered for scheduling and so some deterministic models are constructed for this problem [21], while some conditions such as unpredicted incidents related to loss of resources, surgical case’s incidents, and etc increase duration of surgeries in operating room in real practice, and these motivate practitioners and researchers [27, 28] to focus on this problem under uncertain conditions and proposing new algorithms to tackle uncertain operating room scheduling problem.

Next sections are described as follows. In section 2, we present various studies on operating room planning and scheduling regarding to scope of our problem. In section 3, problem is stated and then mathematical model for fuzzy surgical case scheduling is built. In section 4, we propose fuzzy ant system algorithm in order to achieve fuzzy ant solution. Section 5, provide illustrative examples and computational experiments, and lastly, we conclude and present our suggestions for future research in section 6.

2. Literature review

Some researches [5, 7, 26, 29] provide the most recent literature reviews on the applying operations research in scope of surgery planning and scheduling problems. Important discussion in some studies relates to uncertain conditions in operating room scheduling and planning for three levels. Therefore, in the first place, some papers, which model this problem using deterministic times, are reviewed and then new researches that tackle problem under uncertainty are reviewed.

Xiang et al [21] formulated a surgical case scheduling problem as multi-resource FJSP by using mixed integer linear programming model in order to minimize makespan during a day because of observing similarities between operating theater scheduling and FJSP. The authors considered sequence of operations (pre-operative, peri-operative, and post-operative) for each elective surgery in FJSP under open scheduling policy, and the deterministic durations were provided in all stages. They elaborated ant colony optimization (ACO) approach with a two-level hierarchical graph to integrate sequencing jobs and allocating resources at the same time to solve their model. It should be noted that they used variation coefficient of working time for all resources in their analysis so as to evaluate their ACO algorithm. Meskens et al [19] addressed a multi-objective surgical case scheduling problem. The researchers applied genetic algorithm to tackle problem by minimizing makespan, minimizing overtime hours, and maximizing desiderata of the surgical team. Moreover, block scheduling system and deterministic surgery’s duration was assumed for operating rooms.
Lamiri et al [30] addressed surgery planning problem that is assumed as SPS in planning level. Assigning elective cases to different periods over a planning horizon was done in order to minimize two objectives consists of the sum of elective patient’s related costs and overtime costs of operating rooms. The authors modeled the problem under uncertain condition. In the first place, a novel stochastic mathematical programming was proposed and then, the Monte-Carlo simulation and MIP model was integrated to tackle the problem. In other advanced study, Lamiri et al [31] addressed same previous problem and then, the authors proposed and compared several optimization approaches in order to minimize expected overtime costs, and patient’s related costs simultaneously under stochastic condition. The authors applied Monte-Carlo simulation, MILP model, and meta-heuristics to solve the problem. Saremi et al [32] proposed simulation-based optimizations in order to tackle outpatient case scheduling or SCS problem by providing stochastic service time data from major Canadian hospital. The researchers modeled the problem to minimize multi-objectives include of patient’s waiting time, patient’s completion time, and the number of surgery cancellations. Their first approach was simulation-based tabu search (STS) that integrates discrete-event simulation and tabu search to schedule surgical cases. The second and third methods were integer programming enhanced tabu search (IPETS) and binary programming enhanced tabu search (BPETS). IPETS and BPETS were improved on STS by combining integer programming and binary programming models, respectively. Lee and Yih [27] introduced a scheduling strategy to find fuzzy the start time of surgical cases operating theaters as a SCS problem with uncertainty. The authors formulated this problem as flexible job shop with fuzzy sets. Resources of downstream such as PACU and service time were applied as constraints of their model. The researchers considered multi-objective consist of patient’s waiting time in the process flow, clinical resource idling, and total completion times in order to evaluate their model. The genetic algorithm was applied to solve the problem in two phases. In the first step, the relative order of surgical case was determined, and then in second stage, definite start time of surgical cases was found. A Monte-Carlo simulation was conducted to assess the schedules obtained and therefore, the results of current method was compared to previous work’s authors i.e. a simulation based scheduling. It was indicated that the new approach outperforms traditional approach. Marques and Captivo [28] focused on the surgical case assignment in Portuguese hospital in order to optimize the use of the available surgical resources, and improve equity and access to operated and waiting elective patients. The researchers conducted problem in two stages; firstly, some patients from a large waiting list are selected to be scheduled for surgery, then, a day, an operating room, and a time block is allocated to patients. To tackle problem, three deterministic MILP models were constructed concerning the administration intention, surgeons, and a halfway between administration and surgeons, and finally a robust approach was proposed concerning the OR and surgeon occupation time for each surgery under uncertainty. The authors developed deterministic models in uncertain conditions using approaches from literature. In other work conducted by Mateus et al [33], elective surgeries scheduling problem was studied in Portuguese public hospital. The authors developed local search heuristics to tackle different versions of problem. In this study, problem was divided in two sub-problems; selecting patients from a waiting list for surgery, and then, a day, an
operating room, and a time block is assigned to patients. The researchers evaluated the proposed algorithms on real case comparing with MILP models.

As it was described nature of surgical operations in problem, especially, duration of service times in all stages are considered certain. According to literature [27, 30, 32], duration times in operating theaters are not precise and due to uncertain conditions of service time for operations in SCS problem, it would be better to insert probabilistic data to this problem. So if uncertainty is taken into account for duration time parameters, the complexity of problem would be increased. Dubois et al [34] stated that we are permitted to use either probabilistic distributions or fuzzy numbers in order to represent uncertainties in setting of possibility theory. Besides, they emphasized on using fuzzy set in scheduling problems because fuzzy scheduling addresses not only scheduling under flexible constraints but also scheduling under incomplete or imprecise information. On the other hand, Gonzalez-Rodríguez et al. [35, 36] argued that it is possible to model ill-known durations in job shop scheduling problems using fuzzy numbers instead of stochastic scheduling, because fuzzy theory and possibility theory can be alternative to the probabilistic models. Furthermore, applying fuzzy set has advantages over probability distributions in scheduling, since fuzzy logic demands less data and so it reduces computational difficulties, and it has expressive capability of uncertain events in order to handle incomplete knowledge of scheduling data. Moreover, Palacios et al [37] declared that in addition of probability distributions, other representation for uncertain processing times would be a human-originated confidence interval while we envisage to incomplete or little knowledge available for flexible job shop problem (FJSP). In some studies on FJSP [38-41], both the processing time of operations on eligible machine and completion time following by those processing times are represented as triangular fuzzy numbers. The researchers underlined using fuzzy data since processing time on machines are not precise enough. Since, our problem concerns to scheduling scope and in some real cases (special in Iran) face to incomplete database of surgery’s duration, modeling SCS with fuzzy data and finding approach to tackle this would be very important.

So, this paper deals with a fuzzy surgical case scheduling problem in which all patients are elective and all processing time parameters are assumed fuzzy. We provide a contribution in the field of operations research technique to employ available resources and make surgical case schedule by minimizing fuzzy makespan of operating theater under open scheduling policy. There are two novelties in this paper; a) extension of surgical case surgery problem using fuzzy processing time in all stages of operating room, b) proposing and constructing a fuzzy ant system according to the model to tackle problem under uncertain condition.

3. Problem statement

In this part, problem statement is described completely. For this propose, details are divided to four subsections; a) fuzzy surgical case scheduling for elective patients, b) structure of no-wait multi-resources fuzzy flexible job shop scheduling in operating theater, c) operations on triangular fuzzy number, d) mathematical programming for fuzzy surgical case scheduling.

3.1. Description of fuzzy surgical case scheduling for elective patients
In this section, we give a brief description of surgical case processing from input to output in operating theater (Fig. 1). Firstly, the patient is transported from either ward or ambulatory surgical unit (ASU) to PHU. While patient is being hold in PHU, nurse checks its documents and prepare him/her for surgery. Patient occupies both nurse and PHU bed. Then, he/she is moved to operating room where anesthetist manages anesthesia process and a specific surgeon performs surgical procedure on case. In this stage, another resources such as nurse, OR, anesthetist, medical technicians, scrubs, and surgeon are allocated to surgical case. In the end of the surgical process, anesthesia is reversed by anesthetist and then patient is transported to PACU where he/she recovers from residual effects of anesthesia under care of PACU nurse. In third stage, nurse and PACU bed are assigned to patient. When effects of anesthesia are being diminished and patient’s condition becomes stable, he/she is moved to several different destinations according to existent conditions; usual inpatient is returned to ward, critical inpatient (e.g. cardiac or thoracic case) is moved directly to ICU where he/she benefits from specially trained nurses and specialized equipment, and outpatient is taken to ASU for going through a second recovery. It must be noted that all processing time include of duration time of pre-surgery stage, duration of surgery, and duration of post surgery or recovery stage are under uncertain condition in operating theater and are considered fuzzy based expert’s information.

3.2. Structure of no-wait multi-resources fFJSP in operating theater

The various structures of the shop are taken into account to model and solve SCS problem. For instance, [42] formulated a two-stage hybrid flow shop model in order to minimize total overtime in operating rooms, and [43] modeled SCS problem as a four-stage flow shop under open scheduling policy. In other studies [16, 21, 27], the similarities between operating room scheduling environment and job shop scheduling (JSP) were observed. A FJSP is introduced by [44] as generalization of the job shop and the parallel machine environments. Each order has its own route to follow through the shop. Even though flow shop may be modeled for operating theater in case of identical surgical procedures, in real-world, various surgical cases with several surgical procedures require their own route and this motivated others to apply JSP environment in their studies. [16] developed novel multi-mode blocking JSP to model SCS problem in order to minimize makespan, however [21] considered generalization of job shop and then formulated a multi-resource FJSP in order to minimize makespan by applying novel two-level ACO procedure. The authors assumed the operating sequence of three stages must be followed completely and in sequence, so this assumption makes a constraint that follows rules of no-wait flow shop. [44] defined the no-wait requirement as phenomenon that may occur in flow shops with zero intermediate storage. In no-wait situation, orders are not permitted to wait between two successive machines. By using this constraint in FJSP, the start time of first stage in PHU for surgical case has to be delayed to ensure that the case can go through the FJSP without having to wait for any resource. Therefore, the surgical cases are actually pulled down the line by resources that have become idle. Besides, [21] assumed that three general stages are essential for all cases in FJSP. We can observe flexibility in all stages because of diversification in resources of each stage. Furthermore, there are diverse routes for cases in second stage because of varieties in specific surgical procedure and this confirms the similarities between SCS in operating theater
and FJSP environment. On the other hand, we describe fuzzy FJSP (fFJSP) in operating theater because the fuzzy conditions are considered in problem. There are some works that have focused on fFJSP so that the researchers have combined fuzzy scheduling and flexible scheduling in job shop environment. To solve a fFJSP, a co-evolutionary genetic algorithm was proposed to solve the problem by [38], since this problem has high complexity. The author introduced fuzzy Gantt chart and considered triangular fuzzy start time and completion time. In other work conducted by [39], a teaching-learning based optimization algorithm was introduced to tackle FJSP with fuzzy processing time. [40], proposed a fast estimation distribution algorithm to solve fFJSP. Comparison of their approach with estimation distribution algorithm indicates their algorithm outperforms previous work. Also, [41] applied a hybrid biogeography-based optimization as meta-heuristic for fFJSP. In this paper, we take into account the fuzzy flexible job shop environment introduced by [38] for SCS problem because of similarities between fFJSP and SCS. As it is shown in Fig. 2, there are a set of surgical cases \( SC = \{SC_1, SC_2, \ldots, SC_n\} \) to be operated by combination of available required resources \( R = \{Nurse_1, \ldots, Nurse_m, Bed_1, \ldots, Bed_m, An_1, \ldots, An_m, S_1, \ldots, S_m, OR_1, \ldots, OR_m\} \) in each stage. Since there are three stages for each case, each surgical case \( SC_i \) is formed by a sequence of three operations \( \{O_{i,1}, O_{i,2}, O_{i,3}\} \). Also, multi-resource assigned to i-th surgical case for each stage is presented in blue-boxes over the \( SC_i \). The processing time of the \( O_{ij} \) (pre-surgery, surgery, post-surgery) on required resources is represented as a triangular fuzzy number (TFN) \( \tilde{P}_{ijk} = (P_{ijk}^b, P_{ijk}^m, P_{ijk}^w) \) where \( P_{ijk}^b \) pertains to the best processing time, \( P_{ijk}^m \) is the most probable processing time, and \( P_{ijk}^w \) relates to the worst processing time (Fig. 3). Similarly, the fuzzy completion time of \( O_{ij} \) (here, is notated by \( ET \) that is abbreviated from end time) is displayed as a TFN \( \tilde{ET}_{ijk} = (ET_{ijk}^b, ET_{ijk}^m, ET_{ijk}^w) \), where \( ET_{ijk}^b \) is the best completion time, \( ET_{ijk}^m \) is the most probable completion time, and \( ET_{ijk}^w \) is the worst completion time. Hereafter, we called fuzzy surgical case scheduling (fSCS) instead of fFJSP problem in which both allocation of efficient available resources and sequence of surgical cases on all resources are determined in order to minimize fuzzy makespan (\( C_{max} \)).

According to defined problem in this paper (extension the model presented by Xiang et al 2015 to model using fuzzy duration time), surgeon as main resource plays key role in operating room and is effective on duration of makespan as a key performance for assessing scheduling problem. As it was explained the OR scheduling problem is similar to FJSP environment, and role of special surgeons in operating room is similar to effective human resources in FJS. In real case, some assumptions may be relaxed to construct a complex and real model. For example in real cases, surgeon and her/his assistance are effective on makespan simultaneously and a dual constraint model is required to be constructed to consider this condition that makes more complex model. But in this study, a basic model was extended using fuzzy parameters to be practicable in some real cases.
3.3. Operations on triangular fuzzy number

For providing and using fuzzy duration time in operating room scheduling problem, the definitions of fuzzy number operations for TFN are essentially to build a feasible schedule. The necessary operations in this paper include of operation of adding two fuzzy numbers, the ranking approach of fuzzy numbers, and max operation of two fuzzy numbers. Summation operation is applied to sum the fuzzy start time of the process with fuzzy processing time and to calculate the fuzzy completion time. The ranking operation is used to find the maximum fuzzy makespan value. The max operation is to determine the fuzzy start time of the process.

For two TFN \( \tilde{S} = (s_1, s_2, s_3) \) and \( \tilde{P} = (p_1, p_2, p_3) \), the summation is obtained by \( \tilde{S} + \tilde{P} = (s_1 + p_1, s_2 + p_2, s_3 + p_3) \)

The following criteria are adopted to rank \( \tilde{S} \) and \( \tilde{P} \) in fuzzy scheduling [45].

**Criterion 1:** If \( C_1(\tilde{S}) = \frac{s_1 + 2s_2 + s_3}{4} > (<) C_1(\tilde{P}) = \frac{p_1 + 2p_2 + p_3}{4} \), then \( \tilde{S} > (<) \tilde{P} \) to rank them.

**Criterion 2:** If both TFNs have the identical \( C_1 \), then \( C_2(\tilde{S}) = s_2 \) is compared with \( C_2(\tilde{P}) = p_2 \) to rank them as second criterion.

**Criterion 3:** If both TFNs have the same \( C_1 \) and \( C_2 \), then the difference of the spreads i.e. \( C_3(\tilde{S}) = s_3 - s_1 \) is compared with \( C_3(\tilde{P}) = p_3 - p_1 \) to rank them as last criterion.

Moreover, the approximate max of two TFNs \( \tilde{S} \) and \( \tilde{P} \) is obtained as rule introduced by [38]. That is, if \( \tilde{S} > \tilde{P} \), then \( \tilde{S} \cup \tilde{P} = \tilde{S} \); else \( \tilde{S} \cup \tilde{P} = \tilde{P} \).

It should be noted that above fuzzy equations are used to construct schedule as applied before by [38] and [41] in order to solve fFJSP. As it is shown, this is the first time that TFN processing times are provided in operating room scheduling and then fuzzy results of scheduling are presented according to fuzzy Gantt chart [38] as a robust output.

As it was noted in introduction, there are unpredicted incidents in operating room that increase usual surgery times and hence, some criteria such as makespan are increased. So, using deterministic times under uncertain condition does not give us a near optimal and reliable schedule. When there is incomplete or little knowledge available concerning surgery time in operating room, using TFNs according to expert’s information in model presents a robust schedule so that three makespan (the best, most probable, and the worst) are obtained and real makespan is obtained according to expert’s knowledge considering the worst (under incidents that increase surgery’s duration) or the best (surprising incidents that reduce duration).
3.4. Mathematical programming for fSCS based on no-wait multi-resource fFJSP

Since structure of fFJSP is NP-hard problem, mathematical programming models cannot provide efficient tools to tackle these problems with large size, but these can be considered as first step to develop an effective heuristic. In a study conducted by [46], mathematical models for FJSP were evaluated. The models were divided into three class based on their binary variables: sequence-position variable based model, precedence variable based model, and time-indexed model. The authors concluded that precedence variable based models especially MILP model developed by [47] outperforms other models with least computation time. It is noted that this model is only linear model among others. Therefore, we formulate and develop Ozguven’s MILP model for fSCS problem based on the no-wait multi-resource fFJSP. Several assumptions are adopted in order to define daily fSCS as follows:

1. Only elective patients (inpatients) are involved in this study and all of them are in access before scheduling on given day at zero time, so release/arrival time is assumed static and it equals to zero.
2. Preemption is not allowed because no stages can be interrupted.
3. The required resources are determined before scheduling.
4. During working day and zero time, all resources are always available and there is no resource failure, so failure time is not assumed.
5. Clean-up and setup time are assumed to be included in surgery time and setup time is not sequence dependent.
6. All human as well as equipment resources are assumed to be identical in processing time exception of specific surgeons.
7. The priority of all surgical cases is assumed to be identical.
8. The transportation times between operations (surgery stages) for each case are neglected because the transporters are always assumed to be available.
9. All surgical cases must be operated in sequence of three stages completely and in sequence.
10. All data consist of pre-surgery, surgery, and post-surgery duration are assumed uncertain or fuzzy in problem.
11. Patients are only allowed to be operated on subset of surgeons based on specialty.

Some assumptions in this paper are considered to simplify model under uncertainty. If some assumptions such as (5), (6) and (7) are relaxed, the complexity of model and algorithm will be increased. So, assumptions (5) and (7) are considered in MILP modeling based on [2, 21, 27] and (6) is according to ([16, 21]). So, this paper focused on simple model with these assumptions under uncertain conditions. The model with these assumptions is solved by simulated data and suitable real cases in hospital. Some assumption such as (1), (3), (4), (8), (9), and (11) are case dependant and a case is selected that is consistent to all assumptions for assessing the model. Finally, assumption (2) is consistent to real cases because the surgical operating cannot be interrupted [19].
The sets/indices are described as shown in Table 1. In the problem, some elective patients in set $I$, and 8 resource types in set $R$, which some types are involved in each surgical stage are defined. In first stage, there are two resource types including nurses and PHU bed, in second stage, there are four types including surgeon, OR, nurse, and anesthesia. Also, in last stage, there are two types including PACU bed and nurse. This structure is based on [21]. Also some parameters are defined according to Table 2.

Applied variables in this mathematical model are divided to decision and auxiliary variables that are described by notations as Table 3. In following table, auxiliary variable $ET_i$ is employed to calculate makespan as objective function.

A general model of MILP is formulated for the fSCS problem with $n$ patients as below:

\[
\begin{align*}
\min & \quad C_{\text{max}} \\
\text{s.t.} & \quad ET_i \leq C_{\text{max}} \quad \forall i \in I \\
& \quad ET_i \geq \sum_{k \in K_{ij}} ET_{ijk} \quad \forall i \in I, j = 3, r = 8 \\
& \quad ST_{ijk} + ET_{ijk} \leq Mv_{ijk} \quad \forall i \in I / I_s, j \in \{1, 2, 3\}, r \in R_{ij} \{1, 2, 3, 7, 8\}, k \in K_{r_q} \\
& \quad ST_{ijk} + \tilde{P}_{ijk} - M \left(1 - v_{ijk}\right) \leq ET_{ijk} \quad \forall i \in I / I_s, j \in \{1, 2, 3\}, r \in R_{ij} \{1, 2, 3, 7, 8\}, k \in K_{r_q} \\
& \quad ST_{ijk} + ET_{ijk} \leq Mv_{ijk} \quad \forall i \in I, j = 2, r \in R_{ij} \{4, 5, 6\}, k \in K_{r_q} \\
& \quad ST_{ijk} + \sum_{k \in K_{ij}} \tilde{P}_{ijk} g_{ijk} - M \left(1 - v_{ijk}\right) \leq ET_{ijk} \quad \forall i \in I, j = 2, r \in R_{ij} \{4, 5, 6\}, k \in K_{r_q} \\
& \quad ET_{hgrk} - Mz_{ijhgrk} \leq ST_{ijk} \quad \forall i, h \in I / I_s, i \ll h, j, g \in J, r \in R_{ij} \bigcap R_{hg}, k \in K_{r_q} \bigcap K_{r_{hs}} \\
& \quad ET_{ijk} - M \left(1 - z_{ijhgrk}\right) \leq ST_{hgrk} \quad \forall i, h \in I / I_s, i \ll h, j, g \in J, r \in R_{ij} \bigcap R_{hg}, k \in K_{r_q} \bigcap K_{r_{hs}} \\
& \quad \sum_{k \in K_{ij}} ST_{ijk} = \sum_{k \in K_{r_{j(i-1)}}} ET_{i(j-1)k} \quad \forall i \in I / I_s, j \in \{2, 3\}, r \in R_{ij} \\
& \quad \sum_{k \in K_{ij}} ST_{ijk} = \sum_{k \in K_{ij}} ST_{ijr'k} \quad \forall i \in I / I_s, j \in J, r, r' \in R_{ij} \\
& \quad \sum_{k \in K_{ij}} ET_{ijk} = \sum_{k \in K_{ij}} ET_{ijr'k} \quad \forall i \in I / I_s, j \in J, r, r' \in R_{ij} \\
& \quad \sum_{k \in K_{ij}} v_{ijk} = 1 \quad \forall i \in I / I_s, j \in J, r \in R_{ij}
\end{align*}
\]
\[
\sum_{k \in K_i} g_{ijk} = 1 \forall i \in I, j = 2, r = 3
\]

(14)

\[
ST_{ijk} \geq 0 \forall i \in I, j \in J, r \in R_{ij}, k \in K_{rj}
\]

(15)

\[
ET_{ijk} \geq 0 \forall i \in I, j \in J, r \in R_{ij}, k \in K_{rj}
\]

(16)

\[
ET_i \geq 0 \forall i \in I
\]

(17)

\[
C_{max} \geq 0
\]

(18)

\[
v_{ijk} \in \{0,1\} \forall i \in I, j \in J, r \in R_{ij}, k \in K_{rj}
\]

(19)

\[
g_{ijk} \in \{0,1\} \forall i \in I, j = 2, r = 3, k \in K_{rj}
\]

(20)

\[
z_{ijhk} \in \{0,1\} \forall i, h \in I / I_i, i \leq h, j, g \in J, r \in R_{ij} \cap R_{hg}, k \in K_{rj} \cap K_{rg}
\]

(21)

In the above model, Equation (1) states minimum objective functions i.e. makespan. Constraint (2) determines makespan according to completion time of surgical cases. Equation (3) reflects completion times of surgical cases in the end of third stage. Constraints (4) and (5) make sure that the difference between the start time and the end time of the operation of the surgical cases in first to third stages (only surgeon in second stage) is equal to the processing time of these stages on related resources. Constraints (6) and (7) guarantee same requirements of equations (4) and (5) but for other involved resources in second stage exception of surgeon i.e. (resource types #4, #5, #6). Constraints (8) and (9) are used to specify that two different operations of \(O_{ij}\) and \(O_{hg}\) cannot be performed at the same time on any resource in set \(R_{ij} \cap R_{hg}\). Equation (10) ensures that \(j\)th operation of surgical case must be exactly started after the completion time of \((j-1)\)th of the operation of same surgical case. Constraints (11) and (12) specify all required resources for each surgery stage must have a same start time and completion time respectively. Equation (13) demands that one and only one resource from each resource type can be allocated to an operation of surgical case. Finally, constraint (14) enforces that one and only one surgeon from surgery specialty group can perform surgical procedure and it is possible that others are idle or are allocated to another case. Constraints (15-21) are positive and binary decision variables.

Since our mathematical programming model represents a possibilistic model, in the first place we transform this to a crisp equivalent model according to the weighted average method (WAM) introduced by [48]. We consider WAM for defuzzification of processing time in left hand side of constraints (5) and (7) as follow:

\[
ST_{ijk} + \left(w_{1}^b P_{ijk} + w_{2} P_{ijk} + w_{3} P_{ijk}^w\right) - M \left(1 - v_{ijk}\right) = ET_{ijk} \forall i \in I / I_i, j \in \{1, 2, 3\}, r \in R_{ij} \{1, 2, 3, 7, 8\}, k \in K_{rj}
\]

(22)

\[
ST_{ijk} + \sum_{k \in K_{rj}} \left(w_{1}^b P_{ijk} + w_{2} P_{ijk} + w_{3} P_{ijk}^w\right) g_{ijk} - M \left(1 - v_{ijk}\right) = ET_{ijk} \forall i \in I, j = 2, r \in R_{ij} \{4, 5, 6\}, k \in K_{rj}
\]

(23)
[39] defined \( \frac{x_1 + 2x_2 + x_3}{4} \) as average response variable value (ARV) for defuzzification of TFN \( X = (x_1, x_2, x_3) \), and then we set \( w_1 = 0.25, w_2 = 0.5, w_3 = 0.25 \) for processing time. After that all positive decision variables such as start time, end time, and makespan are obtained as crisp number.

In our model, the start and end time of each surgery, the start and end time for involved resources in each stage for surgery, and assigning required resources in each stage of surgery are as variables in order to minimize makespan. Also, some parameters are needed as inputs of model such as available resources in each type, fuzzy processing time of each stage for each surgical case, special surgeon groups, and determining surgeons inside each special group. Finally, the output of the model includes of variables that present the start and end time of surgeries to determine sequencing surgical cases as a part of problem, and the start and end time of involved resources to determine resource assigning to each surgical case as another part of problem.

4. Fuzzy ant system for solving fSCS

In this paper, we suggest meta-heuristic approach in order to tackle combinatorial nature of fSCS problem. In many researches in field of crisp SCS problem, some heuristic or meta-heuristic procedures such as genetic algorithm [12, 13], column generation based heuristic [10], tabu search [31, 32], and ant colony optimization [21] were developed to achieve near optimal solutions. As it was described, structure of problem is similar to fFJSP and thereby Ant system (AS) algorithm was extended to solve optimization problem such as fFJSP. [21] proposed an ACO algorithm with two-level hierarchical graph (outer and inner graph) to solve SCS. The authors considered outer and inner graphs in order to combine sequencing surgical cases and allocating resources simultaneously. First ACO algorithm namely AS was introduced by [49] as optimizer, learning, and natural algorithm; and also it was applied to tackle Traveling Salesman Problem (TSP) by [50]. To the best of our knowledge among the area of fSCS in literature, no research is available that employs AS algorithm, so we developed that for operating theater problem with fuzzy parameters. Then we compare our proposed procedure with surgical case scheduling solved by using FCFS rule, and hence the efficiency of the method is determined according the quality of solutions obtained. Our elementary AS algorithm is based upon proposed algorithm by [21] and then we extend it so as to solve fSCS.

4.1. Description of fuzzy AS algorithm

Two-level ACO algorithm is tailored by mapping cities to surgical cases and thereby, a nodes tour turns to be the sequence of surgical cases. In Xiang’s procedure, surgical cases are sequenced in outer graph and required multi-resources types of every stage are allocated to surgical cases in inner graph. Available resources in the same resource type are represented by nodes inside the inner graph. The resource assigning to the surgical case in each stage is determined according to path that ant forages in inner graph. A mix pheromone update strategy is defined for algorithm and it comprises one local and two global. In outer level, best ant updates
the trails according to global iteration best strategy in order to search best sequence. In inner level, surgery-related pheromone is defined to save the information that connects surgical case with required resource based on global strategy, while an inner resource-related is defined to record information related to resource utilization based on local strategy. It must be noted that local updating is effective until ant forages path of inner graph and it is invalid after going out the inner. Following algorithm shows framework of fuzzy ant system.

**Algorithm. Two-level fuzzy Ant system**

1. **Input:** an instance SCS of a combinatorial problem Pm
2. Initialize Pheromone Values \( (\tau_0, \infty_0, \infty_0, \lambda_0, q_0) \) and other parameters \((i, m), \beta, \alpha, \rho\)
3. While iteration termination condition not met \((i < ii)\) do
4. Put \(m\) ants on arbitrary node (surgical case)
5. Construct an ant solution with fuzzy start time of resources \((0, 0, 0)\)
6. While ant termination condition not met, \(k < m\) do
7. Initialize tabu:= \(\varnothing\); surgical cases (SC):= \(I\)
8. Construct an ant solution by visiting a node \(i\) in outer graph according to the transition rule Eqs(28-29)
9. \(tabu = tabu \cup \{i\}\)
10. \(I = I \setminus \{i\}\)
11. Determine fuzzy start time (ST) and end time (ET) of each SC based on the no-wait
12. Determine available resources (set AR) for surgical case
13. Ant enters into the inner graph and constructs resource set G
14. Construct a resource allocation ant solution
15. For each resource type \(r\) do
16. Construct an ant solution by visiting a node \(tm\) in inner graph
17. Local update inner pheromone trial based on Eq(32)
18. End for
19. Update fuzzy time window of occupied resources
20. End while
21. Calculate single objective (fuzzy makespan) for an ant solution and defuzzification
22. Compare iteration based best ant solution, record it’s tabu as global best solution
23. Global update in both outer and inner pheromone trial based on Eqs(24-25) and (26-27)
24. End while
25. Plot related graphs (Gantt chart, convergence)
26. Return the best solution found

It must be noted that the start (available) time of all resources before planning in algorithm are set on \((0, 0, 0)\).

Pheromone updates procedure related to algorithm fuzzy AS (fAS), and transition rules are described in this section. Since objective value in our problem is fuzzy makespan, we employ same defuzzification method used in MILP to transform the fuzzy \(\tilde{C}_{\max}\) into a crisp \(C_{\max}\). This approach is applied in order to calculate crisp \(\Delta \tau_0^k\) in above equation as [51] discussed in their fuzzy ACO method. In fAS algorithm, all computations are according to operations of triangular
fuzzy numbers while pheromone update computation is based on ordinary deterministic operations. Pheromone update strategy in outer graph is according to follow equation:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^{m}\Delta\tau_{ij}^k$$

(24)

Where $$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{C_{\text{max}}} & \text{if ant k gous through (i, j) in this iteration} \\ 0, & \text{otherwise} \end{cases}$$

(25)

and $$\Delta\tau_{ij}^k$$ is increasing value of pheromone from patient $$i$$ to patient $$j$$ in iteration related to route of $$k$$th ant and crisp $$C_{\text{max}}$$ is makespan of $$k$$th agent. Beside, Pheromone update strategy in inner graph is according to following equation:

$$\text{in} (\tau_{im}^*(t+1)) = (1-\rho)\text{in} (\tau_{im}^*(t)) + \sum_{k=1}^{m}\text{in} (\tau_{im}^{i,k})$$

(26)

Where $$\text{in} (\tau_{im}^{i*}) = \begin{cases} \frac{Q}{C_{\text{max}}} & \text{if ant k gous through surgery (i) with resource graph (t, m)} \\ 0, & \text{otherwise} \end{cases}$$

(27)

and $$\text{in} (\tau_{im}^{i,k})$$ is increasing value of pheromone for patient $$i$$ with choosing resource $$m$$ from resource type $$t$$ in iteration related to route of of $$k$$th ant and crisp $$C_{\text{max}}$$ is makespan of $$k$$th agent.

Then, transition rules in outer and inner graph are shown as following equations. In outer graph, choosing probability of patient $$j$$ after patient $$i$$ is presented as following:

$$p_{ij}^k(t) = \frac{[\tau_{ij}(t)]^\alpha \cdot \eta_{ij}^\beta}{\sum_{l=1}^{k} [\tau_{il}(t)]^\alpha \cdot \eta_{il}^\beta} \text{if } j \in I_i$$

(28)

where, $$\tau_{ij}(t)$$ is pheromone value of current iteration for arc patient $$i$$ to $$j$$, and $$\alpha$$, $$\beta$$ are pheromone factor and heuristic factor. Heuristic information of problem in outer graph for arc patient is $$i$$ to $$j$$ is notated $$\eta_{ij}$$ that is presented according to Xiang’s equation as following:

$$\eta_{ij} = \frac{(T_{j_1} + T_{j_3} + \max(T_{j_2}^{SGm})) / (T_{j_1} + T_{j_3} + \max(T_{j_2}^{SGm}) + A)}{(T_{j_1} + T_{j_3} + \max(T_{j_2}^{SGm}) + A)}$$

(29)

where, $$T_{j_1}, T_{j_2}^{SGm}, T_{j_3}$$ are crisp processing times (pre-surgery, surgery, and post-surgery) those are defuzzified based on Xu’s equation, also parameter A is constant and is determined according to level of initial pheromone. In inner graph, choosing probability of resource $$m$$ from resource type $$t$$ is shown as following:
\[ P_{mk}^{ij}(t) = \frac{[in(\tau_{mj}^i(t))]^{\alpha} [in(\lambda_{m})]^{\alpha} [in(\eta_{mj})]^{\beta}}{\sum_{g \in G_i^j} [in(\tau_{rg}^i(t))]^{\alpha} [in(\lambda_{r})]^{\alpha} [in(\eta_{rg})]^{\beta}} \text{ if } j \in G_i^k \]  

(30)

where, \( in(\tau_{mj}^i(t)) \) is pheromone value of current iteration for edge resource \( m \) from resource type \( t \), and \( in(\eta_{mj}) \) is heuristic information in inner graph for edge resource \( m \) from resource type \( t \) that is presented as following:

\[ in(\eta_{mj}) = B \left( ES_{im}^m + T_{im}^m \right) \]

(31)

where, \( ES_{im}^m \) is crisp earliest time for available resource \( m \) from resource type \( t \) for patient \( i \) in stage \( l \) and \( T_{im}^m \) is crisp processing time of patient \( i \) in stage \( l \) when resource \( m \) from resource type \( t \) is involved. These parameters are defuzzified based on Xu’s equation. Also parameter \( B \) is constant and is determined according to level of initial pheromone On the other hand, \( in(\lambda_{m}) \) is resource-related pheromone in inner graph that is described in equation (32). Finally, there is new pheromone update strategy namely resource-related pheromone \( (in(\lambda_{m})) \) that is shown in array of Fig. 4 and is very effective for resource utilization (red boxes are less probable to be selected by ant in inner graph). Because, this strategy decreases pheromone value of each resource node in inner graph so that selection probability of same resource by ant is diminished and other resources can have opportunity to be selected. This strategy is performed locally. When an agent trails pheromone in inner graph and selects a resource for a case in each stage, the value of the resource-related pheromone is updated and then it will be reset to preliminary pheromone \( (in(\lambda_0)) \) after obtaining a feasible sequence by agent.

\[ in(\lambda_{m}) = in(\lambda_{m}) - q_0 \]

(32)

where, \( q_0 \) states decremented pheromone value.

5. **Computational experiments**

5.1. **Illustrative examples**

In order to evaluate the proposed approaches, we took three test surgery cases. These cases are classified into small, medium, and large those are different in surgery’s duration, the number of the surgery cases, and the allocated resources. It must be noted that each case consists of three various instances or problems. Cases category and their specifications are shown in Table 4. Surgeries are categorized to five types of small, medium, large, extra large, and special based on their duration (Table 5), which deterministic surgery’s duration is according to simulation model constructed by [21] and each problem in each case can be generated based on different structure of surgery’s duration types. Since processing durations in this paper are fuzzy numbers, we provided fuzzy processing times \( (t - u, t, t + v) \) in which \( u \) and \( v \) were approximated between intervals of 1% to 30% of deterministic duration \( (t) \), randomly. As it is observed, three problems
of each case are different in size of surgeries (column 3), size of resources (column 4-9), and surgery type structure (column 10). The fAS algorithm was coded in MATLAB language and ran on an Intel Core (TM) Duo CPU T2450, 2.00 GHz computer with 1 GB of RAM. Moreover, MILP model was coded in GAMS software and run by CPLEX solver.

5.2. Setting Parameters of proposed algorithm

Various parameters in general ACO algorithms are effective on their performance specially solution quality and computational time. For instance, some parameters like the number of the ants ($m$), the number of the iteration ($Max-It$), evaporation rate ($\rho$), weighted importance of pheromone ($\alpha$), and weighted importance of heuristic information ($\beta$) are considered as the elementary. However, two novel parameters are introduced according to Xiang’s two-level ACO that are inner resource-related pheromone ($\lambda_0$), and decremented pheromone value ($q_0$). We designed experiments according orthogonal Taguchi design of experiment (DOE) in order to examine the effect of parameter settings on each ARV obtained by algorithm for three test case groups. Final setting parameters of algorithm for three cases are displayed in Table 6.

5.3. Evaluation of algorithm’s performance on all considered instances

Assessing the proposed algorithm is done in this subsection and is divided to two parts; in the first place, algorithm is validated on small simulated case in comparison with MILP model and then is evaluated on small to large simulated cases in comparison with FCFS rule. In second place, a real case in accordance with our model (with considering all assumptions) in a private hospital of Isfahan province is provided to evaluate the proposed algorithm on real data.

Firstly, we ran fFCFS method and MILP model on very small case as presented in Table 7 along with small data of Table 4 to validate our approach. The algorithm was repeated 10 times and mean of makespan found by fAS was compared with MILP. It must be noted that fAS algorithm is validated in comparison with MILP model on five small cases as shown in Table 8. Columns (2-3) show results of methods, and columns (4-5) display gap between result of two methods, and computational time of those.

After determination of the best setting parameter for solving small to large instance tested, we ran fAS algorithm in order to measure algorithm’s performance. Table 9 indicates the comparison between average performance of fAS and results of fFCFS on all instances. The first and second columns display considered test problems and approaches, respectively. The next three columns represent the average, best and worst of fuzzy makespan for solutions obtained. As table describes, fAS outperforms fFCFS for solving all instances from small to large size.

The ANOVA test results for ARV as a response variable are presented in Table 10. According to the $P$-value for sample problem’s main effect and that for algorithm’s main effect it is seen that aforementioned factor’s effect is significant. It means that there is significant difference between the mean makespan obtained for two methods and also there is significant difference between the mean makespan obtained for nine sample problems. Moreover, interaction effect of
problems and algorithms is significant. Fig. 5 shows that proposed fAS algorithm obtains lower ARV in comparison with f/FCFS for each instance.

In this part, a hospital from Isfahan, Iran was selected that is consistent to all assumptions of model under open strategy. For instance, in this real case operating theatre, priority of patients are assumed identical in some days and are not important for planning management because surgical cases were categorized in same age-group and there was not cardiac, thoracic, and some urgent cases. To implement proposed algorithm in real case, special surgeon groups divided to groups for inpatients and groups for outpatients. The special group for inpatients consists only a surgeon that is assigned to a surgical case and this is clinical decision. Whereas, general group for outpatients include at least two general surgeons with different processing time that is assigned to case and it is not necessary to be clinical decision, because there are more resources for allocating. Therefore, assigning general surgeon by model can optimize makespan and the schedule is improved.

Real data including fuzzy duration for all stages of each surgical case were collected in 8 different days with getting data from experts such as anesthesia, nurses, and surgeons. Therefore, each instance includes of elective cases to be operated during day, fuzzy durations for cases, and available required resources for all surgeries in same day. The proposed algorithm was repeated 10 times and mean of makespan found by fAS was compared with result of real hospital planning. It must be noted that global solutions found by fAS for all cases tested outperform real planning as shown in Table 11. Also, the ANOVA test results for ARV obtained methods (algorithm and real planning) are displayed in Table 12. According to the $P$-value for method’s main effect it is seen that that factor’s effect is significant. It means that there is significant difference between the mean makespan obtained for proposed algorithm and real planning. Moreover, Fig. 6 shows interval plot (at 95% of confidence interval) of makespan for methods that proposed fAS algorithm obtains lower ARV in comparison with real planning for all instances.

6. Conclusion

In this paper we address surgical case scheduling problem in operating theater under uncertain condition of service time and then a new approach is proposed so as to tackle NP-Hard problem. The unique contribution of this paper is to provide fuzzy processing time in all stages of surgery and to introduce meta-heuristic approach from ACO family so as to solve f/SCS problem for the first time. The criterion of the problem is to minimize fuzzy makespan. Our methodology is based on ACO algorithm so that we developed fuzzy two-level AS, and new algorithm namely fAS was introduced. To illustrate our methodology, we provided three case test data that each case comprises of three problems with different size. Size of problems is changed by increasing surgery’s duration, surgical case numbers, and the number of required resources for all stages of surgery. In the first place, we model f/SCS by using mathematical programming model and our approach was compared with this model on small size problems. On the other hand, we constructed fuzzy schedules with FCFS rule and then both methods were
evaluated by AVR and then their performance was compared. Moreover, a real data from private hospital in Isfahan were provided to evaluate our proposed algorithm. Consequently, it can be observed that our proposed fAS algorithm outperform fFCFS as well as hospital planning in accordance with results and discussions. At last, we suggest some directions as opportunities for future research in this area. Emergency cases as patients can be taken into account in problem and a new ACO algorithm is constructed in order to solve online surgical case scheduling in real world. On the other hand, building a new fuzzy ACO algorithm for bi-objective surgical case scheduling problem can be a novel work in future research. For this purpose, our future research will be to extend a fuzzy two-level ant system in order to solve fuzzy bi-objective surgical case scheduling problem.

Acknowledgements

The authors sincerely appreciate the anonymous referees for their helpful and useful comments and constructive suggestions.

References


Tables

**Table 1. Indices and sets for MILP model**

<table>
<thead>
<tr>
<th>Indices and sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Set of all the surgical cases</td>
</tr>
<tr>
<td>$SG$</td>
<td>Set of surgery’s specialty</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Subset of surgical cases based of surgery’s specialty $s$</td>
</tr>
<tr>
<td>$J_i$</td>
<td>Set of operations of case $i \in I$</td>
</tr>
<tr>
<td>$R$</td>
<td>Set of all resource types</td>
</tr>
<tr>
<td>$O_{ij}$</td>
<td>Surgical case $i \in I$ in stage $j \in J$</td>
</tr>
<tr>
<td>$R_{ij}$</td>
<td>Set of all the resources in resource type for operation $O_{ij}$</td>
</tr>
<tr>
<td>$K_{r_i}$</td>
<td>Set of all the resources in resource type $r$ (exception of surgeon group) $r \in R - {3}$, $r_i \in R_{ij}$</td>
</tr>
<tr>
<td>$K_{r_s}$</td>
<td>Subset of all special surgeon based of surgery’s specialty $s \in SG$ in resource type $r = 3$</td>
</tr>
</tbody>
</table>

**Table 2. Parameters for MILP model**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{P}<em>{ijk} = (P</em>{ijk}^b, P_{ijk}^m, P_{ijk}^w)$</td>
<td>Fuzzy processing time (pre-surgery, surgery, and post-surgery duration) of operation $O_{ij}$ if performed on resource $k$ of type $r$</td>
</tr>
<tr>
<td>$M$</td>
<td>A large positive number</td>
</tr>
<tr>
<td>$n$</td>
<td>Total number of surgical cases</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Total number of resources for each resource type (8 types$^*$)</td>
</tr>
</tbody>
</table>

$^*$ In this study, 8 resource types are introduced

**Table 3. Decision and auxiliary variables for MILP model**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ST_{ijk}$</td>
<td>The start time of operation $O_{ij}$ by resource $k$ of type $r$</td>
</tr>
<tr>
<td>$ET_{ijk}$</td>
<td>The end time of operation $O_{ij}$ by resource $k$ of type $r$</td>
</tr>
<tr>
<td>$C_{\text{max}}$</td>
<td>Makespan</td>
</tr>
<tr>
<td>$V_{ijk}$</td>
<td>Equals to 1 if operation $O_{ij}$ performed on resource $k$ of type $r$, equals 0 otherwise</td>
</tr>
<tr>
<td>$z_{ijhgkr}$</td>
<td>Equals to 1 if operation $O_{ij}$ precedes operation $O_{hg}$ on resource $k$ of type $r$, equals 0 otherwise</td>
</tr>
</tbody>
</table>
\( g_{ijr} \) \( ijk \) \( O_y \) \( r \) \( 3 \) 1 if operation performed by surgeon \( k \) of surgery specialty group \( r \) = 3, equals 0 otherwise. This variable is used for all multi-resources that involved in second stage.

**Auxiliary variables**

\( ET_i \) \( \) The completion time of surgical case \( i \)

<table>
<thead>
<tr>
<th>Cases</th>
<th>problem</th>
<th>Surgical case</th>
<th>PHU bed</th>
<th>Nurse</th>
<th>Surgeons</th>
<th>ORs</th>
<th>PACU bed</th>
<th>Anesthesia</th>
<th>Surgery type (S:M:L:EL:S)</th>
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**Table 4. Test cases and structure**

<table>
<thead>
<tr>
<th>Cases</th>
<th>problem</th>
<th>Surgical case</th>
<th>PHU bed</th>
<th>Nurse</th>
<th>Surgeons</th>
<th>ORs</th>
<th>PACU bed</th>
<th>Anesthesia</th>
<th>Surgery type (S:M:L:EL:S)</th>
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<tr>
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</tbody>
</table>

**Table 5. Duration of surgery stages in different surgery type**

<table>
<thead>
<tr>
<th>Pre-surgery</th>
<th>Surgery Case</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>E-large</th>
<th>Special</th>
</tr>
</thead>
<tbody>
<tr>
<td>(min)</td>
<td>(8,2)</td>
<td>(33,15)</td>
<td>(86,17)</td>
<td>(153,17)</td>
<td>(213,17)</td>
<td>(316,62)</td>
</tr>
</tbody>
</table>

**Table 6. Setting parameters of AS algorithm**

<table>
<thead>
<tr>
<th>Case no.</th>
<th>(max-itr, m)</th>
<th>( q_0 )</th>
<th>( \lambda_0 )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25-40</td>
<td>0.1</td>
<td>4</td>
<td>0.9</td>
<td>5</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>30-40</td>
<td>0.1</td>
<td>5</td>
<td>0.9</td>
<td>12</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>60-50</td>
<td>45</td>
<td>9</td>
<td>0.9</td>
<td>2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table 7. Test cases for comparing MILP and fuzzy ACO algorithm**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Surgical case</th>
<th>PHU bed</th>
<th>Nurse</th>
<th>Surgeons</th>
<th>ORs</th>
<th>PACU bed</th>
<th>Anesthesia</th>
<th>Surgery type (S:M:L:EL:S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0:2:1:0:0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>2:2:1:0:0</td>
</tr>
</tbody>
</table>

**Table 8. Comparison of the performance of the fuzzy AS with MILP**

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>MILP</th>
<th>fAS</th>
<th>GAP (%)</th>
<th>CT(MILP/fAS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(very small)</td>
<td>160.25</td>
<td>(144,160,177)=160.25</td>
<td>0.00%</td>
<td>8/15</td>
</tr>
<tr>
<td>2(very small)</td>
<td>249.5</td>
<td>(234,250,264)=249.5</td>
<td>0.00%</td>
<td>180/25</td>
</tr>
<tr>
<td>3(small)</td>
<td>421.75</td>
<td>(391,437,484)=437.25</td>
<td>3.67%</td>
<td>1000/50</td>
</tr>
</tbody>
</table>
23

4(small)  270.5  (248,293,336)=292.5  8.13%  1200/80
5(small)  283.75 (253,297,347)=298.5  5.19%  1200/80
6(medium) ---  (336,381,433)=382.75 ---  ---/120

Table 9. Comparison of the performance of the algorithms on all considered test case problems

<table>
<thead>
<tr>
<th>Problems</th>
<th>Method</th>
<th>Makespan</th>
<th>Average</th>
<th>Best</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instance1</td>
<td>fAS</td>
<td>(397.8,443.6,492.4)</td>
<td>(391,437,484)</td>
<td>(404,451,501)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(500,547,599)</td>
<td>(500,547,599)</td>
<td>(500,547,599)</td>
<td></td>
</tr>
<tr>
<td>Instance2</td>
<td>fAS</td>
<td>(247.6,299.8,353.7)</td>
<td>(248,293,336)</td>
<td>(220,310,401)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(328,389,450)</td>
<td>(328,389,450)</td>
<td>(328,389,450)</td>
<td></td>
</tr>
<tr>
<td>Instance3</td>
<td>fAS</td>
<td>(253.1,302.2,356.7)</td>
<td>(253,297,347)</td>
<td>(284,312,339)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(359,415,479)</td>
<td>(359,415,479)</td>
<td>(359,415,479)</td>
<td></td>
</tr>
<tr>
<td>Instance4</td>
<td>fAS</td>
<td>(339.1,388.3,438)</td>
<td>(336,381,433)</td>
<td>(334,394,448)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(458,496,542)</td>
<td>(458,496,542)</td>
<td>(458,496,542)</td>
<td></td>
</tr>
<tr>
<td>Instance5</td>
<td>fAS</td>
<td>(359.6,403.2,449.3)</td>
<td>(358,398,444)</td>
<td>(365,406,454)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(558,603,651)</td>
<td>(558,603,651)</td>
<td>(558,603,651)</td>
<td></td>
</tr>
<tr>
<td>Instance6</td>
<td>fAS</td>
<td>(419.2,470.1,528.3)</td>
<td>(417,468,520)</td>
<td>(407,476,555)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(615,693,790)</td>
<td>(615,693,790)</td>
<td>(615,693,790)</td>
<td></td>
</tr>
<tr>
<td>Instance7</td>
<td>fAS</td>
<td>(502.4,577.3,652.6)</td>
<td>(485,570,653)</td>
<td>(532,583,633)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(703,770,834)</td>
<td>(703,770,834)</td>
<td>(703,770,834)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(741,832,923)</td>
<td>(741,832,923)</td>
<td>(741,832,923)</td>
<td></td>
</tr>
<tr>
<td>Instance9</td>
<td>fAS</td>
<td>(727.7,801.5,890)</td>
<td>(737,799,873)</td>
<td>(724,801,909)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fFCFS</td>
<td>(815,907,1012)</td>
<td>(815,907,1012)</td>
<td>(815,907,1012)</td>
<td></td>
</tr>
</tbody>
</table>

Table 10. ANOVA result for ARV (crisp makespan)

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>1</td>
<td>924787</td>
<td>924787</td>
<td>130247.40</td>
<td>0.000</td>
</tr>
<tr>
<td>Sample problem</td>
<td>8</td>
<td>4951180</td>
<td>618897</td>
<td>87165.82</td>
<td>0.000</td>
</tr>
<tr>
<td>Interaction</td>
<td>8</td>
<td>102208</td>
<td>12776</td>
<td>1799.37</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>162</td>
<td>1150</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>179</td>
<td>5979324</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11. Comparison of the performance of the algorithm with real planning on all real case problems

<table>
<thead>
<tr>
<th>Problems</th>
<th>Method</th>
<th>Objectives</th>
<th>Mean of fuzzy makespan</th>
<th>ARV(Crisp Makespan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instance1</td>
<td>fAS</td>
<td>HOSPITAL</td>
<td>(406.8,505.4,613.9)</td>
<td>507.875</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>569</td>
</tr>
<tr>
<td>Instance2</td>
<td>fAS</td>
<td>HOSPITAL</td>
<td>(404.2,458.1,503.5)</td>
<td>455.975</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>537</td>
</tr>
<tr>
<td>Instance3</td>
<td>fAS</td>
<td>HOSPITAL</td>
<td>(623.7,708.9,791.5)</td>
<td>708.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>714</td>
</tr>
<tr>
<td>Instance4</td>
<td>fAS</td>
<td>HOSPITAL</td>
<td>(603.2,681.3,753.7)</td>
<td>679.875</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>761</td>
</tr>
<tr>
<td>Instance5</td>
<td>fAS</td>
<td>HOSPITAL</td>
<td>(601.9,670.7,744)</td>
<td>671.825</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>823</td>
</tr>
<tr>
<td>Instance6</td>
<td>fAS</td>
<td>HOSPITAL</td>
<td>(479,530,586)</td>
<td>531.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>666</td>
</tr>
</tbody>
</table>

23
Table 12. ANOVA result for ARV (crisp makespan)

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>1</td>
<td>254303</td>
<td>254303</td>
<td>18.75</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>158</td>
<td>2142369</td>
<td>13559</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>159</td>
<td>2396672</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figures

**Fig. 1.** Patient flow in operating theater under uncertainty (blue-box shows resource)

**Fig. 2.** Fuzzy surgical case scheduling in operating theater
Fig. 3. Triangular possibility distribution of fuzzy parameter ($P_{ijk}$)

![Triangular possibility distribution of fuzzy parameter](image)

Fig. 4. Array: inner resource related pheromone

![Array: inner resource related pheromone](image)

Fig. 5. Interaction between the algorithms and test problems for the ARV (crisp makespan)

![Interaction between the algorithms and test problems for the ARV (crisp makespan)](image)
Fig. 6. Main effect plot and LSD intervals (at 95% of confidence interval) of makespan for algorithms

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