Group multiple criteria ABC inventory classification using the TOPSIS approach extended by Gaussian interval type-2 fuzzy sets and optimization programs

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Abstract

The aim of this paper is to extent the technique for order performance by similarity to ideal solution (TOPSIS) approach with Gaussian interval type-2 fuzzy sets (GIT2FSs) as an alternative to the traditional triangular membership functions (MFs) in which GIT2FSs are more suitable for stating curved MFs. For this purpose, a new limit distance (LD)-based on alpha cuts is presented for prioritizing GIT2FSs. The proposed method determines the maximum and minimum reference limits of GIT2FSs as the positive and negative ideal solutions and then calculates distances between assessments and these limits. Also, in order to eliminate the weights from the LD’s calculations, the weights of the quantitative and qualitative criteria are extracted using two linear programming models, separately. In order to show the effectiveness of the proposed method, a case study is exhibited on a real GMCAABCIC problem and the results are then compared with other techniques.

Keywords: Multiple criteria ABC inventory classification; Gaussian interval type-2 fuzzy sets; TOPSIS

1. Introduction

The selection of exact ordering policies such as fixed order size (FOS) for a unimportant item and also inexact ordering policies such as twin bin (TB) for a important item will impose the additional costs as inspection and stock-out penalty costs, respectively. Hence, the determination of optimal policy based on rankings of items is one of popular methods for decreasing these costs. The traditional ABC classification categorizes inventory items into three classes: A, very important; B, moderately important; and C, unimportant. Unfortunately, it only considers the total annual dollar usage’s criterion for classifying items. However, in real world, other important criteria such as average unit cost, annual dollar usage, critical factor, lead time, consumption rate, perishability of items, storing cost of raw materials, stock ability, certainty of supply, number of hits, average value per hit, and payment terms [1, 2] may affect ABC inventory classification. Thus, herein after it is attributed as multiple criteria ABC inventory classification (MCABCIC). Since items in a MCABCIC problem are assessed in relation to a set of qualitative and quantitative criteria and experts may have the different point of views with respect to the qualitative criteria, it can be considered as a group multiple-criteria decision making (GMCDM) problem in which the assessments of items with respect to the qualitative criteria are expressed as linguistic variables (stated with fuzzy sets). There are the different methods for solving the GMCDM problems according to appraising style of criteria or alternatives. The technique for order of preference by similarity to ideal solution (TOPSIS) is one of these techniques. TOPSIS was first developed by Hwang and Yoon [3]. In the classical TOPSIS method, the appraisals and weights of criteria are precise values. However, in real world the crisp data is not suitable because human judgments are vague and imprecise when dealing with decision making issues and cannot be estimated with exact numeric values. To state the ambiguity in real-world problems the fuzzy data instead of crisp data has been incorporated in many MCDM techniques including TOPSIS. In fuzzy TOPSIS (FTOPSIS), all the ratings and weights are defined by means of the fuzzy data. However, a decision maker may have doubts about the measure of membership function (MF). In other words, in type-1 fuzzy set, it is often difficult for an expert to express his/her notions as a specified number in interval [0, 1] related to MF. Hence, the type-2 fuzzy sets were suggested by Zadeh [4] for relieving MF measure’s uniqueness of the type-1 fuzzy sets. Interval type-2 fuzzy sets (IT2FSs) are a particular version of type-2 fuzzy sets characterized by an interval MF. There are the known versions for IT2FSs such as trapezoidal interval type-2 fuzzy sets (TraIT2FSs), triangular interval type-2 fuzzy sets (TriIT2FSs), and Gaussian

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interval type-2 fuzzy sets (GIT2FSs) in literature. Triangular or trapezoidal MFs are the simplest MFs formed using straight lines. MFs of triangular and trapezoidal fuzzy number shave steep slopes in their reference points. In real problems, however, the decision maker may want to consider smoother slope for the MFs in reference points. Hence, “Gaussian MFs are suitable for problems requiring continuously differentiable curves, whereas the triangular and trapezoidal do not possess these abilities” [5]. In this paper, the performance ratings related to the qualitative criteria are expressed as linguistic variables and then GIT2FSs are then defined for them. Generally, the generalization of the TOPSIS method based on GIT2FNs using the proposed ranking method, the aggregation of group decisions presented by experts based on GIT2FNs, and the determination of criteria weights by the linear programs are the principal contributions in this paper.

The rest of this paper is organized as follows: Section 2 presents the literature review related to TOPSIS and IT2FSs and also the MCABCIC techniques. The suggested methodology framework is represented in Section 3. In Section 4, preliminaries (including the type-2 fuzzy sets arithmetic operations) are reviewed. The suggested approach for ranking GIT2FNs is introduced in Section 5. In Section 6, the proposed ranking methodology is incorporated into the TOPSIS framework. Section 7 includes a real case study in which our ranking methodology is used in the TOPSIS method and finally, conclusions are summarized in Section 8.

2. Literature review

In a general classification, most studies implemented in MCABCIC can categorize into the following seven classes: (1) Artificial intelligence techniques; (2) Data envelopment analysis (DEA) approaches (optimization models); (3) Statistical and mathematical approaches; (4) Weighted Euclidean distance-based approaches; (5) MCDM-based techniques; (6) Approaches based on machine learning and (7) Combination approaches.

Several approaches applied the artificial intelligence techniques for the MCABCIC problem. Cherif and Ladhari [6] presented an integrated approach based on the Artificial Bee Colony algorithm and VIKOR method for MCABCIC where the Artificial Bee Colony algorithm was used to learn and optimize the criteria weights as the input parameters for VIKOR and VIKOR was then utilized for ranking items. Isen and Boran [7] generated a hybrid model including genetic algorithm, fuzzy c-means, and adaptive neuro-fuzzy inference system for inventory classification. Their model did not need to be resolved when a new item is arrived in the warehouse and also can consider both quantitative and qualitative criteria. Soto et al. [8] designed a three-layer neural network with discrete activation functions using a multi-start constructive learning procedure to solve efficiently the posteriori MCABCIC problem.

A number of the DEA (optimization)-based methods have also been developed to solve the MCABCIC problem. Ramanathan [1] proposed a weighted linear optimization model (here after the R-model) for MCABCIC where performance score of each item obtains using a DEA-like model. Zhou and Fan [9] extended the R-model by obtaining most favorable and least favorable scores for each item. Then, a composite index was constructed to combine the two scores. Ng [10] proposed a weighted linear model for MCABCIC (here after the Ng-model). Using a proper transformation, Ng obtained the scores of inventory items without any linear optimizer. Since the Ng-model leads to a situation in which the weight of an item may be ignored, Hadi-Venche [11] proposed a simple nonlinear programming model where a common set of weights is determined for all items. Torabi et al. [12] proposed a modified version of an existent common weight DEA-like model that can handle both quantitative and qualitative criteria. Hatefi et al. [13] presented a modified linear optimization method for the MCABCIC problem including both qualitative and quantitative criteria. It transforms data related to each qualitative criterion with the cardinal format using some scales such as Likert. Kaabi and Jabeur [14] combined the Zhou and Fan [9] and Hadi-Venche [11] models for utilizing their advantages. Their hybrid model got better results than the two approaches mentioned above.

Cohen and Ernst [15] introduced a combination of the statistical clustering procedures and operational constraints for the MCABCIC problem. Lei et al. [16] applied the principle component analysis with artificial neural networks (ANNs) and the BP algorithm for the MCABCIC problem. The proposed hybrid approach not only can resolve the shortcomings of input limitation in ANNs, but also can improve the prediction accuracy. Ghorabaee et al. [17] constructed a new approach based on the positive and negative distance from average solution. Raja et al. [18] developed hierarchical clustering procedure for improving inventory policies of spare parts.

The approaches based on weighted Euclidean distance have also been adopted for the MCABCIC problem. Chen et al. [19] proposed a case-based distance model to handle the MCABCIC problems in which the criteria weights and sorting thresholds are generated by a quadratic optimization program based on the decision maker’s assessment of a case set. It resolves difficulties related to direct acquisition of preference information. Ma [20] suggested a two-phase classification approach based on the concept of mixed integer programming and case-based distance methods for removing the shortcomings of Chen et al. [19] approach. The proposed approach can decrease the number of misclassifications, improve the problem of multiple solutions, and remove the impact of outliers.

The fifth class is related to application of the MCDM techniques. Bhattacharya et al. [2] adopted the TOPSIS method for the MCABCIC problem and then applied the analysis of variance (ANOVA) technique for studying the suitability, practicability, and effectiveness of the TOPSIS method. Jiang [21] implemented the analytic hierarchy process(AHP) method for classifying the fresh agricultural products. Arikan and Citak [22] proposed AHP-TOPSIS for
ranking the inventory items in an electronics firm. Dhar and Sarkar [23] adopted the multi-objective optimization by ratio analysis (MULTIMOORA) approach for MCABCIC where AHP was handled to weight criteria.

There are also the machine learning-based methods for MCABCIC. For example, Douiss and Jabeur [24] utilized the PROAFTN method as a supervised learning algorithm to classify items into one of the three categories. Lajili et al. [25] utilized and compared five well-known machine learning techniques: (1) decision trees, (2) naive Bayesian networks, (3) ANNs, (4) support vector machines, and (5) K-nearest neighbors for inventory classification. Hu et al. [26] suggested the dominance-based rough set approach where three main phases are: (1) learning, (2) validation, and (3) classifying the spare parts in industrial manufacturing. Lolli et al. [27] applied the exhaustive simulative method on a subset of items for attaining their optimal classes and then utilized decision trees and random forests for specifying the class of the non-simulated items.

And, finally, there are some papers where at least two decision making approaches are integrated to classify items in the last class. Hadi-Venecheh and Mohamadghasemi [28] adopted AHP and DEA for the MCABCIC problem. Kabir and Sumi [29] applied fuzzy Delphi method and fuzzy AHP (FAHP) for the MCABCIC problem. Kabir and Hasin[30] integrated FAHP and ANN for determining the weights of criteria and classifying inventories into different classes, respectively. Lolli et al. [31] integrated AHP with the K-means algorithm to solve the MCABCIC problem where the AHP and K-means techniques are applied for ranking items and sorting to classes, respectively. Douissa and Jabeur [32] used the ELECTRE III method for ranking items in which the continuous variable neighborhood search metaheuristic method was adopted to estimate the indifference, preference, and veto thresholds.

3. The proposed methodology framework

The first stage in the proposed methodology is to define the qualitative criteria, quantitative criteria, and items (as shown in Fig. 1).

<Take in Fig. 1.>

Afterwards the group multiple criteria ABC inventory classification (GMCABCIC) matrix is constructed for the MCABCIC problem in which the assessment measures in relation to the qualitative and quantitative criteria are GIT2FNs and crisp data, respectively. Next, TOPSIS is extended by the proposed method for calculating the distances of qualitative assessments from the positive and negative ideal solutions. At last, since calculating the distances of the positive and negative ideal solutions are carried out using two different methods, the weights of criteria are determined based on two linear programming models.

4. Preliminaries

4.1. Type-2 fuzzy sets and their arithmetic operations

Definition 4.1.1. A type-2 fuzzy set \( \tilde{A} \) in the universe of discourse \( X \) is described by a type-2 MF expressed as follows [33]:

\[
\tilde{A} = \left\{ (x,u) \middle| \forall x \in X, \forall u \in J_s \subseteq [0,1], 0 \leq \mu_{\tilde{A}}(x,u) \leq 1 \right\}
\]  

(1)

where \( \mu_{\tilde{A}} \) refers to the MF (secondary MF) of \( \tilde{A} \) and \( J_s \) is an sub-interval in[0, 1] denoting the primary MF. The type-2 fuzzy set \( \tilde{A} \) can be also represented as follows:

\[
\tilde{A} = \int_{x \in X} \int_{u \in J_s} \mu_{\tilde{A}}(x,u) \delta(x,u),
\]  

(2)

where \( J_s \subseteq [0,1] \) and \( \int \int \) denotes the union overall admissible \( x \) and \( u \).

Definition 4.1.2. For a type-2 fuzzy set \( \tilde{A} \), if all \( \mu_{\tilde{A}}(x,u) = 1 \), \( \tilde{A} \) is named IT2FS. An IT2FS \( \tilde{A} \) can be described as follows [33]:
\[
\tilde{\mathbf{z}} = \int_{x \in X} \int_{u \in J_X} 1/(x, u)
\]

where \( J_X \subseteq [0,1] \).

**Definition 4.1.3.** Footprint of uncertainty (FOU) is derived from the union of all primary memberships:

\[
\text{FOU}(\tilde{\mathbf{z}}) = \int_{x \in X} J_X ,
\]

A FOU can also be represented by the lower and upper MFs [34]:

\[
\text{FOU}(\tilde{\mathbf{z}}) = \bigcap_{x \in X} \left[ \mu^L_X (x), \mu^R_X (x) \right],
\]

where \( \mu^L_X (x) \) and \( \mu^R_X (x) \) are the lower and upper MFs of the type-2 fuzzy set. An IT2FS \( \tilde{\mathbf{z}} \), is said to be normal if \( \mu^L_X (x) = \mu^R_X (x) = 1 \). An IT2FS \( \tilde{\mathbf{z}} \), is said to be subnormal if \( \mu^L_X (x) < 1 \) and \( \mu^R_X (x) = 1 \).

**Definition 4.1.4.** Let \( \bar{X}^L \) and \( \bar{X}^U \) (\( L \) and \( U \) are equal to the lower and upper MF) be two non-negative trapezoidal type-1 fuzzy numbers [35,36]. Also, Let \( H^L_X \) and \( H^U_X \) denote the heights of \( \bar{X}^L \) and \( \bar{X}^U \), respectively. Let \( x_1^L, x_1^U, x_2^L, x_2^U, x_3^L, x_3^U, x_4^L, x_4^U \), and \( x_5^L \) be non-negative real values. A trapezoidal interval type-2 fuzzy numbers (TrafT2FNs) defined on the universe of discourse \( X \) is given by (see Fig. 2):

\[
\tilde{\mathbf{x}} = [\bar{X}^L, \bar{X}^U] = \left[ \left[ x_1^L, x_2^L, x_3^L, x_4^L, x_5^L, H^L_X \right], \left[ x_1^U, x_2^U, x_3^U, x_4^U, x_5^U, H^U_X \right] \right].
\]

Definition 4.1.5. Let \( \tilde{\mathbf{x}}_1 \) and \( \tilde{\mathbf{x}}_2 \) be two non-negative TrafT2FNs, where \( \tilde{\mathbf{x}}_1 = [\bar{X}^L_1, \bar{X}^U_1] = \left[ \left[ x_1^L_1, x_2^L_1, x_3^L_1, x_4^L_1, x_5^L_1, H^L_{X_1} \right], \left[ x_1^U_1, x_2^U_1, x_3^U_1, x_4^U_1, x_5^U_1, H^U_{X_1} \right] \right] \) and \( \tilde{\mathbf{x}}_2 = [\bar{X}^L_2, \bar{X}^U_2] = \left[ \left[ x_1^L_2, x_2^L_2, x_3^L_2, x_4^L_2, x_5^L_2, H^L_{X_2} \right], \left[ x_1^U_2, x_2^U_2, x_3^U_2, x_4^U_2, x_5^U_2, H^U_{X_2} \right] \right] \). The arithmetic operations between \( \tilde{\mathbf{x}}_1 \) and \( \tilde{\mathbf{x}}_2 \) are defined as follows:

- **Addition operation**: 
  \[
  \tilde{\mathbf{x}}_1 \oplus \tilde{\mathbf{x}}_2 = \left[ \left[ x_1^L_1 + x_2^L_1 + x_3^L_1 + x_4^L_1 + x_5^L_1, \min \left\{ H^L_{X_1}, H^L_{X_2} \right\} \right], \left[ x_1^U_1 + x_2^U_1 + x_3^U_1 + x_4^U_1 + x_5^U_1, \min \left\{ H^U_{X_1}, H^U_{X_2} \right\} \right] \right]
  \]

- **Subtraction operation**: 
  \[
  \tilde{\mathbf{x}}_1 \ominus \tilde{\mathbf{x}}_2 = \left[ \left[ x_1^L_1 - x_2^L_2 - x_3^L_2 - x_4^L_2 - x_5^L_2, \min \left\{ H^L_{X_1}, H^L_{X_2} \right\} \right], \left[ x_1^U_1 - x_2^U_2 - x_3^U_2 - x_4^U_2 - x_5^U_2, \min \left\{ H^U_{X_1}, H^U_{X_2} \right\} \right] \right]
  \]

- **Multiplication operation**: 
  \[
  \tilde{\mathbf{x}}_1 \otimes \tilde{\mathbf{x}}_2 = \left[ \left[ x_1^L_1 \cdot x_2^L_1 \cdot x_3^L_1 \cdot x_4^L_1 \cdot x_5^L_1, \min \left\{ H^L_{X_1}, H^L_{X_2} \right\} \right], \left[ x_1^U_1 \cdot x_2^U_1 \cdot x_3^U_1 \cdot x_4^U_1 \cdot x_5^U_1, \min \left\{ H^U_{X_1}, H^U_{X_2} \right\} \right] \right]
  \]

- **Division operation**: 
  \[
  \tilde{\mathbf{x}}_1 \div \tilde{\mathbf{x}}_2 = \left[ \left[ x_1^L_1 \div x_2^L_2 \div x_3^L_2 \div x_4^L_2 \div x_5^L_2, \min \left\{ H^L_{X_1}, H^L_{X_2} \right\} \right], \left[ x_1^U_1 \div x_2^U_2 \div x_3^U_2 \div x_4^U_2 \div x_5^U_2, \min \left\{ H^U_{X_1}, H^U_{X_2} \right\} \right] \right]
  \]
\[ \tilde{X}_2 = \left[ \tilde{x}_2^L, \tilde{x}_2^R \right] = \left[ \frac{x_1^L}{x_2^L}, \frac{x_1^L}{x_2^L}, \frac{x_1^L}{x_2^L}, \frac{x_1^L}{x_2^L} \right] \cup \left[ \frac{x_1^R}{x_2^L}, \frac{x_1^R}{x_2^L}, \frac{x_1^R}{x_2^L}, \frac{x_1^R}{x_2^L} \right] \]

\[ \tilde{X}_2 \alpha \tilde{X}_2 = \left[ \frac{x_1^L}{x_2^L} \alpha \frac{x_1^L}{x_2^L}, \frac{x_1^L}{x_2^L} \alpha \frac{x_1^L}{x_2^L}, \frac{x_1^L}{x_2^L} \alpha \frac{x_1^L}{x_2^L}, \frac{x_1^L}{x_2^L} \alpha \frac{x_1^L}{x_2^L} \right] \cup \left[ \frac{x_1^R}{x_2^L} \alpha \frac{x_1^R}{x_2^L}, \frac{x_1^R}{x_2^L} \alpha \frac{x_1^R}{x_2^L}, \frac{x_1^R}{x_2^L} \alpha \frac{x_1^R}{x_2^L}, \frac{x_1^R}{x_2^L} \alpha \frac{x_1^R}{x_2^L} \right] \]

- Multiplication by an ordinary number:
\[ \tilde{X}_1 \cdot r = r \cdot \tilde{X}_1 = \left[ \frac{r \cdot x_1^L}{x_2^L}, \frac{r \cdot x_1^L}{x_2^L}, \frac{r \cdot x_1^L}{x_2^L}, \frac{r \cdot x_1^L}{x_2^L} \right] \cup \left[ \frac{r \cdot x_1^R}{x_2^L}, \frac{r \cdot x_1^R}{x_2^L}, \frac{r \cdot x_1^R}{x_2^L}, \frac{r \cdot x_1^R}{x_2^L} \right] \]
if \( r \geq 0, \) \[ \tilde{X}_1 \cdot r = r \cdot \tilde{X}_1 = \left[ \frac{r \cdot x_1^L}{x_2^L}, \frac{r \cdot x_1^L}{x_2^L}, \frac{r \cdot x_1^L}{x_2^L}, \frac{r \cdot x_1^L}{x_2^L} \right] \cup \left[ \frac{r \cdot x_1^R}{x_2^L}, \frac{r \cdot x_1^R}{x_2^L}, \frac{r \cdot x_1^R}{x_2^L}, \frac{r \cdot x_1^R}{x_2^L} \right] \]
if \( r \leq 0. \]

**Definition 4.1.6.** Let \( \tilde{G} \) be a normal GIT2FN as follows (see also Fig. 3):
\[ \tilde{G} = \left[ \tilde{G}^L, \tilde{G}^U \right] = \left[ \mu^L : \sigma^L : \mu^U : \sigma^U \right] \]
where \( \mu^L : \sigma^L \) and \( \mu^U : \sigma^U \) are the mean and standard deviation of the lower and upper Gaussian MF, respectively, such that \( \mu^L = \mu^U \) and \( \sigma^L = \sigma^U. \)

<Take in Fig. 3.>

**Definition 4.1.7.** The \( \alpha \)-cut of \( \tilde{A} \) is presented as follows [37]:
\[ A_{\alpha} = \left\{ (x, u) \big| f(x, u) \geq \alpha \right\} \]

**Definition 4.1.8.** The \( \alpha \)-cut of \( \tilde{A} \) may also be represented by the \( \alpha \)-cut of its FOU:
\[ A_{\alpha} = \left\{ \mu^L (x) \geq \alpha, \mu^U (x) \geq \alpha \right\} \]

for a GIT2FN \( \tilde{G} \), the \( \alpha \)-cut may be presented as interval as follows (see Fig. 4):
\[ \tilde{G}_{\alpha} = \left[ \tilde{X}_{\alpha_{10}}, \tilde{X}_{\alpha_{20}} \right] \]
\[ \tilde{X}_{\alpha_{10}} \cup \tilde{X}_{\alpha_{20}} \]

Where \( \alpha \) and \( r \) show the left and right MFs \( \tilde{G} \), respectively.

**Definition 4.1.9.** Let \( X = [x_1, x_2] \) and \( Y = [y_1, y_2] \) be two positive interval numbers such that \( x_1 \leq x \leq x_2 \) and \( y_1 \leq y \leq y_2 \). \( x_1, y_1 \) and \( x_2, y_2 \) are the infimum and the supremum, respectively. Interval arithmetic operations of addition, subtraction, multiplication, and division are defined as follows, respectively [38]:

- Addition operation:
\[ X + Y = [x_1 + y_1, x_2 + y_2] \]

- Subtraction operation:
\[ X - Y = [x_1 - y_2, x_2 - y_1] \]

- Multiplication operation:
\[ X, Y = \{ \min (x_1, y_1, x_2, y_2), \max (x_1, y_1, x_2, y_2) \} \]

- Division operation:
  \[
  \frac{X}{Y} = \left\{ \frac{1}{y_1, y_2} \right\}, \quad \frac{1}{y_1, y_2} = \left\{ \frac{1}{y_2, y_1} \right\} \quad \text{if} \quad 0 \not\in \{ y_1, y_2 \},
  \]

- Distance between \( X \) and \( Y \):
  \[
  \Delta_{X-Y} = \frac{1}{2} [x_1 - y_2 + (x_2 - y_1)].
  \]

5. A new limit distance (LD) for ranking GIT2FNs

5.1. The normal GIT2FNs case

This paper presents an approach based on \( \alpha \)-cut for comparing and ranking GIT2FNs. The proposed approach is able for calculating the distances on different levels and concurrently ranking GIT2FNs in interval \([0, 1]\).

The proposed methodology first selects the left and right reference limits. For this purpose, let the MF of \( \mu_{G}(x,u) \) for a GIT2FN is splitted into two curves \( \mu_{l}(x,u) \) and \( \mu_{r}(x,u) \), the left and right MF of \( \bar{G} \), respectively (as shown in Fig. 4).

\[
\mu_{\bar{G}}(x,u) = \left\{ \begin{array}{ll}
\mu_{l}(x,u) & \text{for } x \not\in \mu \\
\mu_{r}(x,u) & \text{for } x \in \mu
\end{array} \right.,
\]

<Take in Fig. 4.>

Also, the minimum reference limit \( \mu_{\min}(x,u) \) and the maximum reference limit \( \mu_{\max}(x,u) \) are \( \\{ \min \{ \mu_{l}(x,u) \} \} \) \( i \in \) all GIT2FNs \( \} \) and \( \{ \max \{ \mu_{r}(x,u) \} \} \) \( i \in \) all GIT2FNs \( \} \), respectively. In order to show the left and right reference limits and \( \alpha \)-cut of a GIT2FN, consider GIT2FNs shown in Fig. 4.

Note that the \( \alpha \)-cut of a GIT2FN creates the interval numbers, thus, one can apply the interval arithmetic operations on them. As illustrated in Fig. 4, suppose that the \( \alpha \)-cut of the minimum and maximum reference limits \( \mu_{\max}(x,u) \) and \( \mu_{\min}(x,u) \) (intersection level \( \alpha \) with the MFs of \( \mu_{\min}(x,u) \) and \( \mu_{\max}(x,u) \) ) makes intervals \([ \bar{X}_{\min}^{\alpha}, \bar{X}_{\max}^{\alpha} ]_{\alpha} \) and \([ X_{\min}^{\alpha}, X_{\max}^{\alpha} ]_{\alpha} \), respectively on \( X \), where \( \bar{X}_{\min}^{\alpha} \) and \( \bar{X}_{\max}^{\alpha} \) are related to the upper and lower MFs of \( \mu_{\min}(x,u) \), respectively and \( X_{\min}^{\alpha} \) and \( X_{\max}^{\alpha} \) are equal to the lower and upper MFs of \( \mu_{\max}(x,u) \), respectively. Also, let \( \alpha \)-cut of the left and right MFs of a GIT2FN such as \( \bar{G}_{2}, \mu_{l}(x,u) \) and \( \mu_{r}(x,u) \) (intersection points of level \( \alpha \) with the MFs of \( \mu_{l}(x,u) \) and \( \mu_{r}(x,u) \) ) generates the intervals \([ \bar{X}_{l}^{\alpha}, \bar{X}_{r}^{\alpha} ]_{\alpha} \) and \([ X_{l}^{\alpha}, X_{r}^{\alpha} ]_{\alpha} \), respectively, where \( \bar{X}_{l}^{\alpha} \) and \( \bar{X}_{r}^{\alpha} \) are related to upper and lower MFs of \( \mu_{l}(x,u) \) and \( \mu_{r}(x,u) \) are equal to lower and upper MFs of \( \mu_{r}(x,u) \). With these assumptions, for a GIT2FN, the LD can be calculated for the positive ideal (PI) solution with respect to cost (C) criteria as follows:

\[
LD_{PI,C}(\bar{A}) = \frac{\sum_{\alpha=0.1}^{1} \{ \mu_{l}(x,u) - \mu_{\min}(x,u) \} \alpha}{\sum_{\alpha=0.1}^{1} \{ \mu_{l}(x,u) - \mu_{\min}(x,u) \} - \sum_{\alpha=0.1}^{1} \{ \mu_{r}(x,u) - \mu_{\max}(x,u) \}},
\]
where $\alpha = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$, and $0.9$. In Eq. (22), $\sum_{i=0}^{l}(\mu_{i0}(x,u) - \mu_{i\alpha}^{\min}(x,u))$ is a positive value and $\sum_{i=0}^{l}(\mu_{i\alpha}(x,u) - \mu_{i\alpha}^{\max}(x,u))$ is a negative value. Therefore, the negative sign is considered in denominator. To simplify the calculations, Eq. (22) can be converted into the following equation:

$$LD_{PI,C}(\bar{A}) = \sum_{x=0.1}^{l} \frac{[x_{1}, x_{2}]_{\alpha} - [x_{1}^{\min}, x_{2}^{\min}]_{\alpha}}{[x_{1}, x_{2}]_{\alpha} - [x_{1}^{\max}, x_{2}^{\max}]_{\alpha}}$$

(23)

Obviously, in the situations $[x_{1}, x_{2}]_{\alpha} - [x_{1}^{\max}, x_{2}^{\max}]_{\alpha}$ and $[x_{1}, x_{2}]_{\alpha} - [x_{1}^{\max}, x_{2}^{\max}]_{\alpha}$, one always obtains a negative measure while $(x_{1} - x_{2})^{\max}$ and $(x_{1} - x_{2})^{\min}$, instead, using Eq. (20) the distance between two interval numbers is calculated as follows:

$$LD_{PI,C}(\bar{A}) = \sum_{x=0.1}^{l} \frac{1}{2} \frac{[x_{1}^{\min} - x_{2}^{\min}]_{\alpha} + (x_{2}^{\min} - x_{1}^{\min})_{\alpha}}{[x_{1}^{\max} - x_{2}^{\max}]_{\alpha}}$$

Similarly, the PI solution for the set of benefit (B) criteria, the negative ideal (NI) solution for the set of C criteria, and the NI solution for the set of B criteria are as follows, respectively:

$$LD_{PI,B}(\bar{A}) = \sum_{x=0.1}^{l} \frac{[x_{1}^{\min} - x_{2}^{\min}]_{\alpha} + (x_{2}^{\min} - x_{1}^{\min})_{\alpha}}{[x_{1}^{\max} - x_{2}^{\max}]_{\alpha}}$$

(25)

$$LD_{NI,C}(\bar{A}) = \sum_{x=0.1}^{l} \frac{[x_{1}^{\min} - x_{2}^{\min}]_{\alpha} + (x_{2}^{\min} - x_{1}^{\min})_{\alpha}}{[x_{1}^{\max} - x_{2}^{\max}]_{\alpha}}$$

(26)

$$LD_{NI,B}(\bar{A}) = \sum_{x=0.1}^{l} \frac{[x_{1}^{\min} - x_{2}^{\min}]_{\alpha} + (x_{2}^{\min} - x_{1}^{\min})_{\alpha}}{[x_{1}^{\max} - x_{2}^{\max}]_{\alpha}}$$

(27)

Obviously, the measures obtained from the above equations include in interval $[0, 1]$. Since measures $(\bar{x}_{1} - x_{2}^{\max}) + (x_{2} - x_{1}^{\max})$ and $(\bar{x}_{1} - x_{2}^{\min}) + (x_{2} - x_{1}^{\min})$ are equal to zero while $[\bar{x}_{1}, x_{2}]$ matches $\mu_{\alpha}^{\max}(x,u)$ and $[\bar{x}_{1}, x_{2}]$ matches $\mu_{\alpha}^{\min}(x,u)$, respectively, or while distances of the reference limits $\mu_{\alpha}^{\max}(x,u)$ and $\mu_{\alpha}^{\min}(x,u)$ obtain from themselves, the measures $\sum_{x=0.1}^{l} [x_{2}^{\max} - x_{1}^{\max}]_{\alpha}$ and $\sum_{x=0.1}^{l} [x_{2}^{\min} - x_{1}^{\min}]_{\alpha}$ are used for calculating LDs, respectively.

5.2. The subnormal GIT2FNs case

If GIT2FN is subnormal (see Fig. 5), then the LDs are based on Eqs. (24-27) (for $\alpha \leq H_{G}^{b}$) and Eqs. (28-31) for ( $H_{G}^{b} \leq \alpha \leq H_{G}^{u}$).

7
\[
LD_{M,C}(A) = \frac{\sum_{\alpha=r}^{H_n^{\alpha}} |(x'_1 - x'^{\min})_\alpha|}{\sum_{\alpha=r}^{H_n^{\alpha}} |(x'_1 - x'^{\max})_\alpha| + \sum_{\alpha=r}^{H_n^{\alpha}} |(x'_2 - x'^{\min})_\alpha|},
\]

(28)

\[
LD_{M,B}(A) = \frac{\sum_{\alpha=0,1}^{H_n^{\alpha}} |(x'_r - x'^{\min})_\alpha|}{\sum_{\alpha=0,1}^{H_n^{\alpha}} |(x'_r - x'^{\max})_\alpha| + \sum_{\alpha=0,1}^{H_n^{\alpha}} |(x''_2 - x'^{\min})_\alpha|},
\]

(29)

\[
LD_{N,C}(A) = \frac{\sum_{\alpha=r}^{H_n^{\alpha}} |(x'_1 - x'^{\max})_\alpha|}{\sum_{\alpha=r}^{H_n^{\alpha}} |(x'_1 - x'^{\min})_\alpha| + \sum_{\alpha=r}^{H_n^{\alpha}} |(x'_2 - x'^{\max})_\alpha|},
\]

(30)

\[
LD_{N,B}(A) = \frac{\sum_{\alpha=0,1}^{H_n^{\alpha}} |(x'_r - x'^{\min})_\alpha|}{\sum_{\alpha=0,1}^{H_n^{\alpha}} |(x'_r - x'^{\max})_\alpha| + \sum_{\alpha=0,1}^{H_n^{\alpha}} |(x''_2 - x'^{\max})_\alpha|},
\]

(31)

<Take in Fig. 5.>

6. Using the new LD in TOPSIS with GIT2FNs

In this section, the TOPSIS approach is generalized for GIT2FNs using LDs stated in Section 5. The interested readers can refer to Hwang and Yoon [3] for studying the steps of the classical TOPSIS. Although, the method is explained for GIT2FNs, one can apply it for TraIT2FNs or TriIT2FNs. The following stages show the proposed approach for normal GIT2FNs:

1. Let a decision maker wants to evaluate \( m \) alternatives \( A_i \) \((i=1, \ldots, m)\) under \( n \) criteria \( C_j \) \((j=1, \ldots, n', n'+1, \ldots,n)\) via the MCDM matrix \((D, \tilde{D})=[x_{ij}, \tilde{x}_{ij}]_{mn}\) where \( C_j \) \((j=1, \ldots, n')\), \( C_j \) \((j=n'+1, \ldots,n)\), \( D=[x_{ij}]_{mn}\), and \( \tilde{D}=[\tilde{x}_{ij}]_{mn} \) represent the quantitative criteria, the qualitative criteria, the crisp values (with respect to the quantitative criteria), and GIT2FNs (with respect to the qualitative criteria), respectively, as follows:

\[
A_i = \begin{bmatrix}
C_1 & C_2 & \ldots & C_i & \ldots & C_n
\end{bmatrix},
\]

\[
D, \tilde{D} = A_1 \begin{bmatrix}
x_{11} & x_{12} & x_{13} & \ldots & x_{1n'} & \tilde{x}_{1n'+1} & \tilde{x}_{1n'+2} & \ldots & \tilde{x}_{1n}
\end{bmatrix},
\]

\[
A_2 \begin{bmatrix}
x_{21} & x_{22} & x_{23} & \ldots & x_{2n'} & \tilde{x}_{2n'+1} & \tilde{x}_{2n'+2} & \ldots & \tilde{x}_{2n}
\end{bmatrix},
\]

\[
\vdots \begin{bmatrix}
\vdots & \vdots & \vdots & \ldots & \vdots & \tilde{x}_{m'n'+1} & \tilde{x}_{m'n'+2} & \ldots & \tilde{x}_{m'n}
\end{bmatrix},
\]

\[
A_m \begin{bmatrix}
x_{m1} & x_{m2} & x_{m3} & \ldots & x_{mn'} & \tilde{x}_{mn'+1} & \tilde{x}_{mn'+2} & \ldots & \tilde{x}_{mn}
\end{bmatrix},
\]

(32)

where \( \tilde{x}_{ij} \) is the synthetic Gaussian interval type-2 fuzzy rating aggregated by \( L \) experts. It is calculated as follows:

\[
\tilde{x}_{ij} = (L/L) \ominus (\tilde{x}_{ij}^{L} \ominus \tilde{x}_{ij}^{L} \ominus \ldots \ominus \tilde{x}_{ij}^{L}),
\]

\[
i = 1, \ldots, m; \quad j = n'+1, \ldots,n,
\]

(33)

2. Suppose that \( x_{ija}^r, x_{ija}^l, x_{ija}^r, x_{ija}^l, \tilde{x}_{ija}^r, \tilde{x}_{ija}^l \) be the projection of \( \alpha \)-cut’s intersection points with the left and right MFS of GIT2FN \( \tilde{G} = \{x, \mu^L; \sigma^L\} \), \( \{x, \mu^U; \sigma^U\} \) when evaluating alternative \( i \) under criterion \( j \). Then, GIT2FNs \( \tilde{x}_{ija} \) in level \( \alpha \) can be represented as follows::

8
\[ \hat{x}_{ija} = \left[ x_{ija}^{L}, x_{ija}^{R}, \Delta_{2ija} \right], \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n, \]  

(34)

Similarly, GIT2FN \( \tilde{x}_{ija}^l \) selected by \( l \)th expert in level \( \alpha \) is given by:

\[ \tilde{x}_{ija}^l = \left[ x_{ija}^{L}, x_{ija}^{R}, \Delta_{2ija}^l \right], \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n; \quad l = 1, \ldots, L, \]  

(35)

Three reference points of the triangular fuzzy numbers selected by \( L \) experts, namely \( \tilde{x}_y = (Lb_y, Mb_y, Ub_y) \) according to Buckley [39] are given by:

\[ Lb_y = \left( \frac{\sum_{l=1}^{L} Lb_y^l}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n, \]  

(36)

\[ Mb_y = \left( \frac{\sum_{l=1}^{L} Mb_y^l}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n, \]  

(37)

\[ Ub_y = \left( \frac{\sum_{l=1}^{L} Ub_y^l}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n, \]  

(38)

Obtain the four reference points for GIT2FNs chosen by \( L \) experts as \( \hat{x}_{ija} = \left[ x_{ija}^{L}, x_{ija}^{R}, \Delta_{2ija}^l, \Delta_{1ija}^{\alpha} \right] \) in level \( \alpha \) for \( \alpha = \alpha_1, \ldots, \alpha_N \) (\( N \) is the number of alpha cuts); \( i = 1, \ldots, m; \) and \( j = n'+1, \ldots, n \), using extension of the technique explained above as follows:

\[ \hat{x}_{ija}^l = \left( \frac{\sum_{l=1}^{L} x_{ija}^{l}}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n; \alpha = \alpha_1, \ldots, \alpha_N, \]  

(39)

\[ \Delta_{2ija}^l = \left( \frac{\sum_{l=1}^{L} \Delta_{2ija}^l}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n; \alpha = \alpha_1, \ldots, \alpha_N, \]  

(40)

\[ \Delta_{1ija}^{\alpha} = \left( \frac{\sum_{l=1}^{L} \Delta_{1ija}^{\alpha}}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n; \alpha = \alpha_1, \ldots, \alpha_N, \]  

(41)

\[ \Delta_{1ija}^l = \left( \frac{\sum_{l=1}^{L} \Delta_{1ija}^{l}}{L} \right), \quad i = 1, \ldots, m; \quad j = n'+1, \ldots, n; \alpha = \alpha_1, \ldots, \alpha_N, \]  

(42)

Also, suppose that \( w_j \) (\( j = 1, \ldots, n' \)), \( w_j (n'+1, \ldots, n) \), and \( w_j \in \left[ w_j^l, w_j^r \right] \) are the weights of the quantitative criteria, the weights of the qualitative criteria, and the admissible range for \( j \)-th criterion \( w_j \).

3. Let \( \tilde{X} = [\tilde{X}^L, \tilde{X}^U] \) \( J = [x_{i1}^L, x_{i2}^L, x_{i3}^L; H_A] \left( x_{i1}^U, x_{i2}^U, x_{i3}^U; H_A \right) \) be a TraT2FN. The normalized performance measures can be calculated by Rashid et al. [40] for benefit criteria (BC) and cost criteria (CC), respectively:

\[ \tilde{z}_{ij} = \left[ \left( x_{ij}^L + x_{ij}^R + \Delta_{2ija}^l + \Delta_{1ija}^{\alpha} \right) \left( x_{ij}^U + x_{ij}^R + \Delta_{2ija}^l + \Delta_{1ija}^{\alpha} \right) \left( x_{ij}^U + x_{ij}^R + \Delta_{2ija}^l + \Delta_{1ija}^{\alpha} \right) \right], \quad \text{for} \quad i = 1, \ldots, m; \quad x_{ij}^+ = \max x_{ij}^l \text{ where } j \in BC, \]  

(43)

and

\[ \tilde{z}_{ij} = \left[ \left( x_{ij}^L + x_{ij}^R + \Delta_{2ija}^l + \Delta_{1ija}^{\alpha} \right) \left( x_{ij}^U + x_{ij}^R + \Delta_{2ija}^l + \Delta_{1ija}^{\alpha} \right) \left( x_{ij}^U + x_{ij}^R + \Delta_{2ija}^l + \Delta_{1ija}^{\alpha} \right) \right], \quad \text{for} \quad i = 1, \ldots, m; \quad x_{ij}^- = \min x_{ij}^l \text{ where } j \in CC, \]  

(44)

Create the normalized decision matrix \( \tilde{N} \) for cuts of \( G \) for \( i = 1, \ldots, m \) and \( j = n'+1, \ldots, n \), using extension of the above normalization methodology as follows:
\begin{align}
\hat{n}_{ij} &= \left[ \frac{x_{ij}^1, x_{ij}^2}{x_j^+}, \frac{x_{ij}^\alpha, x_{ij}^\beta}{x_j^-} \right], \quad \text{for } i = 1, \ldots, m; \quad \alpha = \alpha_1, \ldots, \alpha_n; \quad x_j^+ = \max_i \check{x}_{ij}^l, \quad \text{where } j \in BC,
\end{align}

and
\begin{align}
\tilde{n}_{ij} &= \left[ \frac{x_j^+, x_j^-}{x_j^+}, \frac{x_j^+, x_j^-}{x_j^+} \right], \quad \text{for } i = 1, \ldots, m; \quad \alpha = \alpha_1, \ldots, \alpha_n; \quad x_j^- = \min_i \check{x}_{ij}^l, \quad \text{where } j \in CC,
\end{align}

Where $N$ is number of $\alpha$-cuts. Also, obtain the normalized decision matrix $\hat{D}$ for the crisp values as follows:
\begin{align}
n_{ij} &= \frac{x_{ij}}{\sqrt{\sum^m_{i=1} x_{ij}^2}}, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n',
\end{align}

Obtain the positive ideal solution $\hat{A}^+$ and the negative ideal solution $\hat{A}^-$ for the qualitative criteria, respectively, using:
\begin{align}
\hat{A}^+ &= \left[ v_1^+, v_2^+, \ldots, v_n^+ \right] = \left[ \mu_{\alpha}^{\max}(x,u) = \max \left\{ \mu_{\alpha}(x,u) \right\} j \in BC \right] \left[ \mu_{\alpha}^{\min}(x,u) = \min \left\{ \mu_{\alpha}(x,u) \right\} j \in CC \right],
\end{align}

\begin{align}
\hat{A}^- &= \left[ v_1^-, v_2^-, \ldots, v_n^- \right] = \left[ \mu_{\alpha}^{\min}(x,u) = \min \left\{ \mu_{\alpha}(x,u) \right\} j \in BC \right] \left[ \mu_{\alpha}^{\max}(x,u) = \max \left\{ \mu_{\alpha}(x,u) \right\} j \in CC \right],
\end{align}

For the crisp values, calculate the positive ideal solution $A^+$ and the negative ideal solution $A^-$ for the quantitative criteria as follows:
\begin{align}
A^+ &= \left[ \left( \max, n_j \right) j \in BC \right] \left[ \left( \min, n_j \right) j \in CC \right] = \left[ v_1^+, v_2^+, \ldots, v_n^+ \right],
\end{align}

\begin{align}
A^- &= \left[ \left( \min, n_j \right) j \in BC \right] \left[ \left( \max, n_j \right) j \in CC \right] = \left[ v_1^-, v_2^-, \ldots, v_n^- \right],
\end{align}

4. Calculate $\check{S}_i^+$ and $\check{S}_i^-$ for each $j = n' + 1, \ldots, n$ based on the measures (distances) $LD_M$ and $LD_N$, respectively, between alternatives and the positive and negative ideal solutions for $\check{G}$ using Eqs. (24-27) or Eqs. (28-31), and $S^+_i$ and $S^-_i$ for each $j = 1, \ldots, n'$ using:
\begin{align}
S_i^+ &= \sqrt{\sum^{n'}_{j=1} (n_{ij} - n^+_j)^2}, \quad i = 1, \ldots, m,
\end{align}

\begin{align}
S_i^- &= \sqrt{\sum^{n'}_{j=1} (n_{ij} - n^-_j)^2}, \quad i = 1, \ldots, m,
\end{align}

5. Calculate the relative closeness $RC_i$ to the ideal alternatives with respect to the quantitative criteria ($j = 1, \ldots, n'$) and the qualitative criteria ($j = n' + 1, \ldots, n$), respectively, as follows:
\begin{align}
RC_i &= \frac{S^-_i \check{S}_i^+ + S^+_i \check{S}_i^-}{S_i^+ + S_i^-}, \quad i = 1, \ldots, m,
\end{align}

and
\begin{align}
RC_i &= \frac{S_i^- \check{S}_i^+ + S_i^- \check{S}_i^-}{S^+_i + S^-_i}, \quad i = 1, \ldots, m,
\end{align}

6. According to Shipley et al. [41], the separation of each alternative from $S_i^+$ and $S_i^-$ is dependent to the criteria weights and the criteria weights are incorporated in the distances measurements. Therefore, in order to eliminate
the criteria weights from \( S_i^+ \) and \( S_i^- \), solve the following linear programming model (1) for prioritizing alternatives (the bigger measure of objective function, the better alternative) related to the quantitative criteria:

Model (1):

\[
S_i = \max \sum_{j=1}^{n'} \left( w_j \frac{n_{ij} - v_j^-}{\sqrt{(n_{ij} - v_j^-)^2 + (n_{ij} - v_j^+)^2}} \right), \quad \text{for } i = 1\ldots m,
\]

s.t. \[
\sum_{j=1}^{n'} w_j \leq 1,
\]

\[
w_j' \leq w_j \leq w_j^n, \quad j = 1\ldots n',
\]

\[
w_j = w_j', \quad j \neq j', \quad j, j' \in 1\ldots n',
\]

\[
w_j \geq w_j' \text{ or } w_j \leq w_j', \quad j \neq j', \quad j, j' \in 1\ldots n',
\]

\[
w_j \geq 0, \quad j = 1\ldots n',
\]

where the constraints (57-59) show the lower (\( w_j' \)) and the upper (\( w_j^n \)) limits of weights, the equal importance of criteria, and the ranking order of criteria, respectively. Obviously since the weights of criteria are interval, it can be said that in the case of overlapping intervals, the constraints (58) and (59) are applied for showing the preferences level of decision maker.

Similarly, obtain weights of the qualitative criteria using the following linear programming model (2):

Model (2):

\[
S_i' = \max \sum_{j=n'+1}^{n} \left( w_j' \frac{LD_{NI}}{LD_{PL} + LD_{NI}} \right), \quad \text{for } i = 1\ldots m,
\]

s.t. \[
\sum_{j=n'+1}^{n} w_j' \leq 1,
\]

\[
w_j'^{l} \leq w_j' \leq w_j'^{u}, \quad j = n' + 1\ldots n',
\]

\[
w_j' = w_j', \quad j \neq j', \quad j, j' \in n' + 1\ldots n',
\]

\[
w_j' \geq w_j'^{l} \text{ or } w_j' \leq w_j'^{u}, \quad j \neq j', \quad j, j' \in n' + 1\ldots n',
\]

\[
w_j' \geq w_j' \text{ or } w_j' \leq w_j', \quad j \neq j'', \quad j', j'' \in n' + 1\ldots n',
\]

\[
w_j' \geq 0, \quad j = n' + 1\ldots n',
\]

where the constraint \( w_j' \geq w_j' \text{ or } w_j' \leq w_j' \) is applied for showing the preferences level between the weights of the quantitative and the qualitative criteria.

7. Case study

To show effectiveness of the proposed approach, it is implemented in the material warehouse of a soft-drink factory, Zahedan, Iran. This warehouse consists of the 35 items. The authors used the suggestions and point of views of three inventory managers.

7.1. Determination of criteria
Inventory managers have chosen the five criteria (annual dollar usage, lead time, average lot cost, limitation of warehouse space, and availability of the substitute raw material) as the most important criteria affecting the ranking items.

Here, it is worth stating that the first four criteria are benefit-type criteria and the fifth criterion is a cost-type criterion (the smaller measure, the more important). On the other hand, the first three criteria and the last two criteria are the quantitative and qualitative criteria, respectively.

7.2. Construction of GMCABCIC matrix

The linguistic variables with their GIT2FNs (as represented in Table 1) are applied to evaluate the items with respect to qualitative criteria. Three experts are then asked to select one of them for determining their preference degree. Also, the representation of GIT2FNs for these linguistic variables has been showed in Fig. 6.

Table 2 represents the measures of items related to the quantitative criteria and also the assessments of items with respect to the qualitative criteria, respectively, in which the linguistic variables are selected by inventory managers based on Table 1 and are aggregated using Eqs. (39-42). In fact, this table is the GMCABCIC matrix.

Afterwards the GMCABCIC matrix is normalized for the quantitative criteria using Eq. (47) and for the qualitative criteria using Eqs. (45-46). Table 3 presents the normalized GMCABCIC matrix where, in order to summarize calculations, the normalized measures of the qualitative criteria have been entered only for \( \alpha = 0.1 \).

Table 4 shows the Euclidean distances \( S^+_i \) and \( S^-_i \) between items and the ideal solutions for criteria \( C_1, C_2, \) and \( C_3 \) using Eqs. (52-53) separately and also the \( LD_{pf} \) and \( LD_{Nf} \) (for \( \alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9 \)) using Eq. (25) and Eq. (27) for criteria \( C_4 \) and \( C_5 \), respectively.

In order to construct the objective functions of programs (1) and (2), the measures \( RC \) obtain for each criterion using Eq. (54) based on data in Table 4 and are represented in Table 5.

Now, the linear programs (1) and (2) are solved for obtaining the scores of items with respect to the quantitative and qualitative criteria (\( SE \) and \( SE' \)), respectively:

Model (1):
\[ SE_i = \max \sum_{j=1}^{3} w_j \left( \frac{v_j - v_i^{-}}{\sqrt{(v_j - v_i^{-})^2 + (v_j - v_i^{+})^2}} \right) , \text{ for } i = 1, \ldots, 5, \]
\[
\text{s.t. } \sum_{j=1}^{3} w_j \leq 1, \\
0.2 \leq w_1 \leq 0.45, \\
0.1 \leq w_2, \\
w_3 \leq 0.15, \\
w_1 \geq w_3, \\
w_3 \geq w_2, \\
w_1 \geq 0, w_2 \geq 0, w_3 \geq 0. \\
\]

Model (2):
\[ SE'_i = \max \sum_{j=1}^{5} w'_j \left( \frac{LD_{Nj}}{LD_{Pl} + LD_{Nl}} \right) , \text{ for } i = 1, \ldots, 5, \]
\[
\text{s.t. } \sum_{j=1}^{5} w'_j \leq 1, \\
0.2 \leq w'_1 \leq 0.55, \\
0.1 \leq w'_2 \leq 0.2, \\
w'_3 \geq w'_5, \\
w'_4 \geq w'_1 \text{ (obtained from Model(1))}, \\
w'_4 \geq 0, w'_5 \geq 0.
\]

Where the importance order of criteria has been adjusted based on the inventory managers’ point of views and experiences.

Table 6 shows measures \( SE, SE' \), total score (the larger score, the greater preference), and ABC classification based on the proposed method and other techniques in literature.

<Take in Table 6>

In order to compare the results of the proposed model with other approaches, the data in Table 2 together with some settings is applied.

First, the authors compared the obtained results from their method with those of the VIKOR technique. From Table 6, 33 items remained in the same classes. For example, the 2 items with the changed classes are items 1 and 16 where item 1 and item 16 have been moved to class B and class C, respectively. Although, order of rankings obtained in each class is somewhat different from our approach (for example, item 6 is more important than item 10), the similarity of the classes obtained from the two models can be a good reason for the effectiveness of our approach.

In the second status, the authors requested experts to apply the crisp numbers 1 to 7 instead of the language variables in Table 1 for assessing the qualitative variables and then implement the classical TOPSIS method for ranking items. According to the obtained results, only 23 out of the 35 items stayed in the same classes. In the real world, experts may want to choose the middle numbers such as 1.5 with interval MF when evaluating the qualitative criteria. It cannot be satisfied by the crisp values. Thus, the linguistic variables as IT2FSs are more suitable for such situations and it will definitely give the different results. As a result, it is the reason of the different classifications mentioned above.
Moreover, the results of our approach were compared to the R-model [1], in which constraint
\[ w_4 \geq w_1 \geq w_3 \geq w_5 \geq w_2, \]
\[ x_{ij} - \min_{i=1, \ldots, m} x_{ij} \leq \frac{1}{m} \max_{i=1, \ldots, m} x_{ij} - \min_{i=1, \ldots, m} x_{ij}, \]
and Eq. (27) were used for showing the sequence of weights importance, converting into a 0–1 scale, and determining the crisp measures of items with respect to the qualitative criteria, respectively. Only the 25 items were reclassified into the same classes. The R-model is a compensatory approach, i.e., a significantly weak criterion value for an item could be directly compensated by other good criteria values. On the other hand, the weights of criteria for low measures may be zero when solving the model. These will lead to inappropriate rankings. For example, consider the items 18 and 25. Although the item 18 has the higher measures relative to the item 25 with respect to the first, third and fourth criterion which have the higher weights in sequence \( w_4 \geq w_1 \geq w_3 \geq w_5 \geq w_2 \) and based on our approach is in class B, It has been moved up to class C using R-model.

As a further comparison with the crisp models, the authors utilized the transformed data in the Ng-model [10]. According to the obtained results, the 27 items kept unchanged in their classes. The drawbacks of the Ng-model are similar to the R-model. For example, consider the item 8. Since the item 8 has the higher partial average in relation to the fourth criterion, it was selected as class A without taking into account other partial averages. However, it was chosen as class B in the first four methods.

On the other hand, by comparing results of the proposed model with traditional ABC classification, only 19 of the 35 items remained in the same classes. Obviously, the results obtained from our model differ with the traditional ABC classification, due to the presence of other four criteria.

8. Conclusions

In real world, the selection of the best alternative with respect to conflicting criteria is a difficult and complex work when data is vague and inexact. Although, type-1 fuzzy sets could greatly resolve ambiguities in decision problems, it takes into account only a specified measure for MF. Thus, type-2 fuzzy sets are applied to consider an interval in [0, 1] for MF when decision maker is uncertain about the value of MF. The MFs of the type-2 fuzzy sets can take different versions such as triangular, trapezoidal, and Gaussian. Since GIT2FNs have the smoother MF, the authors adopted them for evaluating the alternatives in relation to the qualitative criteria. On the other hand, since the MCABCIC problem is subject to the qualitative criteria that can be stated with type-2 fuzzy sets, inventory managers have ambiguity in relation to the value of the MF and cannot determine certain measure for it. Hence, this paper presents the TOPSIS method based on GIT2FNs in which a new LD is introduced to prioritize them. The proposed method first calculates \( \hat{S}_i^+ \) and \( \hat{S}_i^- \) by depicting \( \alpha \)-cuts and then measures distances from reference limits. It is also able to rank TriIT2FN, TraIT2FN, and other curved forms for both normal and subnormal case. In order to show the effectiveness of the proposed methodology, it was implemented in a real case study. It included both the qualitative and quantitative criteria in a NCABCIC problem. The quantitative data was extracted from the inventory section, whereas the qualitative data was obtained from appraisals of experts. Because of non-compensatory of the proposed approach and the usage of type-2 fuzzy sets, the results obtained from our methodology show more logical results when comparing with the crisp methods (the classical TOPSIS, the Ng-model, and the R-model) and the traditional ABC classification as described above.

Some important directions for further researches are: (1) managers can carry out this approach for other manufacturing factories or service organizations, (2) other criteria or sub-criteria may be taken into account in other MCABCIC problems, (3) the distance between alpha cuts was determined 0.1 when LDs were calculated. In order to obtain more accurate results, one can adopt the smaller values for the distance between alpha cuts such as 0.05 or 0.01, (4) the proposed ranking approach is applicable in other areas of mathematics such as statistics (such as normal distribution) and DEA, and (5) the proposed approach is also applicable for other MCDM techniques including VIKOR and ELECTRE, in addition to TOPSIS.

References


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Mohammad Khalilzadeh received his Ph.D. degree in Industrial Engineering from Sharif University of Technology. He is assistant professor at the Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran. He has several publications in various reputed journals published by Elsevier, Emerald Insight, Springer, and Hindawi as well as different international conferences. Also, he is the member of the scientific committee of ProjMAN conference, and used to be the member of scientific committee of various international conferences. Mohammad Khalilzadeh does research on Project Management, Applied Mathematics, Industrial Engineering, and Human Resource Management. He used to be the head of department from 2014 until 2017. He supervised a considerable number of Ph.D. and Master Theses.
Determination of qualitative criteria, quantitative criteria, and items

Ranking items with respect to qualitative criteria

Evaluation of items with respect to the qualitative criteria

Construction of decision matrix

GIT2FNs

TOPSIS based on GIT2FNs

Linear programming models

MCABCIC

Fig. 1. The framework of the proposed methodology
Fig. 2. A subnormal TraIT2FN.
Fig. 3. A normal GIT2FN.
Fig. 4. The left, right, minimum, and maximum reference limits $\tilde{G}$. 
Fig. 5. A subnormal GIT2FN.
Fig. 6. The representation of GIT2FNs defined in Table 1.
### Table 1

<table>
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<tr>
<th>Definitions of linguistic variables for evaluating items with respect to the qualitative criteria</th>
<th>([\mu^L, \sigma^L; H^L_0](\mu^U, \sigma^U; H^U_0))</th>
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\([5,0.5;1], (5,1;1)\) |
| Unimportant (U) | 
\([7,0.5;1], (7,1;1)\) |
| Medium (M) | 
\([9,0.5;1], (9,1;1)\) |
| Important (I) | 
\([11,0.5;1], (11,1;1)\) |
| Very important (VI) | 
\([13,0.5;1], (13,1;1)\) |
| Absolutely important (AI) | 
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Table 2
Evaluation values of items with respect to the different criteria

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<th>Annual dollar Usage (Thousand $)</th>
<th>Lead time (day)</th>
<th>Average lot cost ($)</th>
<th>Limitation of warehouse space</th>
<th>Availability of the substitute raw material</th>
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</table>
Table 3

The normalized GMCABCIC matrix

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<th>Number of Item</th>
<th>Annual dollar Usage (Thousand $)</th>
<th>Lead time(day)</th>
<th>Average lot cost($)</th>
<th>Limitation of warehouse space</th>
<th>Availability of the substitute raw material</th>
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<td>0.0696</td>
<td>0.2082</td>
<td>[0.180, 0.248), (0.383, 0.451)]</td>
<td>[0.166, 0.229), (0.354, 0.416)]</td>
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<tr>
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<td>0.1653</td>
<td>0.1676</td>
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<td>[0.205, 0.268), (0.392, 0.455)]</td>
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<tr>
<td>3</td>
<td>0.0368</td>
<td>0.1479</td>
<td>0.2831</td>
<td>[0.518, 0.586), (0.721, 0.789)]</td>
<td>[0.245, 0.307), (0.575, 0.681)]</td>
</tr>
<tr>
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<td>0.1479</td>
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Table 5
Measures $RC$ with respect to the different criteria

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Table 6: Measures $SE$, $SE'$, total score, and MCABCIC based on the different methods.