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Group multiple criteria ABC inventory classification using the TOPSIS approach extended by Gaussian interval type-2 fuzzy sets and optimization programs

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Abstract. The aim of this paper is to extend the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) approach with Gaussian Interval Type-2 Fuzzy Sets (GIT2FSs) as an alternative to the traditional triangular Membership Functions (MFs) in which GIT2FSs are more suitable for stating curved MFs. For this purpose, a new Limit Distance (LD) based on alpha cut is presented for prioritizing GIT2FSs. The proposed method determines the maximum and minimum reference limits of GIT2FSs as the positive and negative ideal solutions and, then, calculates distances between assessments and these limits. In addition, in order to eliminate the weights derived from the LD calculations, the weights of the quantitative and qualitative criteria are extracted using two linear programming models, separately. In order to show the effectiveness of the proposed method, a case study is exhibited on a real GMCABCIC problem, and the results are then compared with those obtained by other techniques.

1. Introduction

The selection of exact ordering policies, such as Fixed Order Size (FOS), for an unimportant item and, also, inexact ordering policies, such as Twin Bin (TB), for an important item will impose additional costs such as inspection and stock-out penalty costs, respectively. Hence, the determination of ordering policy based on rankings of items is one of the popular methods for decreasing the costs. The traditional ABC classification categorizes inventory items into three classes: (A) very important; (B) moderately important; (C) unimportant. Unfortunately, it only considers the criterion of the total annual dollar usage for classifying items. However, in the real world, other important criteria such as average unit cost, annual dollar usage, critical factor, lead time, consumption rate, perishability of items, storing cost of raw materials, stock ability, certainty of supply, number of hits, average value per hit, and payment terms [1,2] may affect ABC inventory classification. Thus, herein, it is attributed as Multiple Criteria ABC Inventory Classification (MCABCIC). Since items in an MCABCIC problem are assessed with respect to a set of qualitative and quantitative criteria and experts may have different points of view regarding

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the qualitative criteria, it can be considered as a Group Multiple-Criteria Decision-Making (GMCDM) problem in which the assessments of items with respect to the qualitative criteria are expressed as linguistic variables (stated with fuzzy sets). There are different methods for solving GMCDM problems according to the appraising style of criteria or alternatives. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is one of these techniques. TOPSIS was first developed by Hwang and Yoon [3]. In the classical TOPSIS method, the appraisals and weights of criteria are precise values. However, in the real world, the crisp data are not suitable, because human judgments are vague and imprecise when dealing with decision-making issues and cannot be estimated with exact numeric values. To state the ambiguity in real-world problems, the fuzzy data instead of crisp data have been incorporated in many MCDM techniques including TOPSIS. In Fuzzy TOPSIS (FTOPSIS), all the ratings and weights are defined by means of the fuzzy data. However, a decision-maker may have doubt about the measure of Membership Function (MF). In other words, in a type-1 fuzzy set, it is often difficult for an expert to express his/her notions as a specified number at an interval [0, 1] related to MF. Hence, the type-2 fuzzy sets were suggested by Zadeh [4] for relieving the uniqueness of MF measure of the type-1 fuzzy sets. Interval Type-2 Fuzzy Sets (IT2FSs) represent a particular version of type-2 fuzzy sets characterized by an interval MF. There are known versions for IT2FSs such as Trapezoidal Interval Type-2 Fuzzy Sets (TraIT2FSs), Triangular Interval Type-2 Fuzzy Sets (TriIT2FSs), and Gaussian Interval Type-2 Fuzzy Sets (GIT2FSs) in the literature. Triangular or trapezoidal MFs are the simplest MFs formed using straight lines. MFs of triangular and trapezoidal fuzzy numbers have steep slopes in their reference points. In real problems, however, the decision-maker may consider a smoother slope for the MFs in reference points. Hence, "Gaussian MFs are suitable for problems requiring continuously differentiable curves, whereas the triangular and trapezoidal fuzzy numbers do not possess these abilities" [5].

In this paper, the performance ratings related to the qualitative criteria are expressed as linguistic variables; then, GIT2FSs are then defined for them. Generally, the generalization of the TOPSIS method based on GIT2FNs using the proposed ranking method, the aggregation of group decisions presented by experts based on GIT2FNs, and the determination of criteria weights by the linear programs are the principal contributions in this paper.

The rest of this paper is organized as follows: Section 2 presents the literature review related to TOPSIS and IT2FSs and, also, the MCABCIC techniques. The suggested methodology framework is represented in Section 3. In Section 4, preliminaries (including arithmetic operations of the type-2 fuzzy sets) are reviewed. The suggested approach to ranking GIT2FNs is introduced in Section 5. In Section 6, the proposed ranking methodology is incorporated into the TOPSIS framework. Section 7 includes a real case study in which the proposed ranking methodology is used in the TOPSIS method and, finally, conclusions are summarized in Section 8.

2. Literature review

In a general classification, most studies implemented in MCABCIC can be categorized into the following seven classes:

1. Artificial intelligence techniques;
2. Data Envelopment Analysis (DEA) approaches (optimization models);
3. Statistical and mathematical approaches;
4. Weighted Euclidean distance-based approaches;
5. MCDM-based techniques;
6. Approaches based on machine learning;
7. Combination approaches.

Several approaches have applied artificial intelligence techniques to the MCABCIC problem. Cherif and Ladhari [6] presented an integrated approach based on the artificial bee colony algorithm and VIKOR method for MCABCIC where the artificial bee colony algorithm was used to learn and optimize the criteria weights as the input parameters for VIKOR, which was then utilized for ranking items. Isen and Boran [7] generated a hybrid model including genetic algorithm, fuzzy c-means, and adaptive neuro-fuzzy inference system for inventory classification. Their model does not need to be resolved when a new item arrives at the warehouse and, also, can consider both quantitative and qualitative criteria. Lopes-Soto et al. [8] designed a three-layer neural network with discrete activation functions using a multi-start constructive learning procedure to solve the posteriori MCABCIC problem efficiently.

A number of the DEA-based (optimization) methods have also been developed to solve the MCABCIC problem. Ramanathan [1] proposed a weighted linear optimization model (after the R-model) for MCABCIC where the performance score of each item was obtained by a DEA-like model. Zhou and Fan [9] extended the R-model by obtaining the most and the least favorable scores of each item. Then, a composite index was constructed to combine the two scores. Ng [10] proposed a weighted linear model for MCABCIC (here, after the Ng-model). By using proper transformation, Ng obtained the scores of inventory items without any linear optimizer. Since the Ng-model leads to a situation in which the weight of an item may be ignored, Hadi-Vencheh [11] proposed a simple nonlinear programming model where a common set of weights was determined for all items. Torabi et al. [12] proposed a modified version of an existing common weight DEA-like model that can handle both quantitative and qualitative criteria. Hatefi et al. [13] presented a modified linear optimization method for the MCABCIC problem including both qualitative and quantitative criteria. It transforms data relating to each qualitative criterion with the cardinal format using some scales such as Likert. Kaabi and Jabeur [14] combined Zhou and Fan [9] and Hadi-Vencheh [11] models for utilizing their advantages. Their hybrid model obtained better results than the two approaches mentioned above.

Cohen and Ernst [15] introduced a combination of the statistical clustering procedures and operational constraints for the MCABCIC problem. Lei et al. [16] applied the principal component analysis with Artificial Neural Networks (ANNs) and the BP algorithm to the MCABCIC problem. The proposed hybrid approach can not only resolve the shortcomings of input limitation in ANNs, but also improve the prediction accuracy. Ghorabaee et al. [17] constructed a new approach based on the positive and negative distances from the average solution. Raja et al. [18] developed a hierarchical clustering procedure for improving inventory policies of spare parts.

The approaches based on weighted Euclidean distance have also been adopted for the MCABCIC problem. Chen et al. [19] proposed a case-based distance model to handle the MCABCIC problems in which the criteria weights and sorting thresholds were generated by a quadratic optimization program based on the decision-maker's assessment of a case set. It resolves difficulties related to the direct acquisition of preference information. Ma [20] suggested a two-phase classification approach based on the concept of mixed integer programming and case-based distance methods for removing the shortcomings of Chen et al. [19] approach. The proposed approach can decrease the number of misclassifications, improve the problem of multiple solutions, and remove the impact of outliers.

The fifth class is related to the application of MCDM techniques. Bhattacharya et al. [2] adopted the TOPSIS method for the MCABCIC problem and, then, applied the analysis of variance (ANOVA) technique for studying the suitability, practicability, and effectiveness of the TOPSIS method. Jiang [21] implemented the Analytic Hierarchy Process (AHP) method for classifying fresh agricultural products. Arikan and Citak [22] proposed AHP-TOPSIS for ranking the inventory items in an electronics firm. Dhar and Sarkar [23] adopted the multi-objective optimization by ratio analysis (MULTIMOORA) approach for MCABCIC where AHP handled the weights of criteria.

There are also machine learning-based methods for MCABCIC. For example, Douiss and Jabeur [24] utilized the PROAFTN method as a supervised learning algorithm to classify items into one of the three categories. Lajili et al. [25] utilized and compared five well-known machine learning techniques: (1) decision trees, (2) naive Bayesian networks, (3) ANNs, (4) support vector machines, and (5) K-nearest neighbors for inventory classification. Hu et al. [26] suggested the dominance-based rough set approach where the three main phases are: (1) learning, (2) validation, and (3) classification of the spare parts in industrial manufacturing. Lolli et al. [27] applied the exhaustive simulation method to a subset of items for attaining their optimal classes and, then, utilized decision trees and random forests for specifying the class of the non-simulated items.

Finally, there are some papers that have, at least, integrated two decision-making approaches to classify items in the last class. Hadi-Vencheh and Mohamadghasemi [28] adopted AHP and DEA for the MCABCIC problem. Kabir and Sumi [29] applied the fuzzy Delphi method and Fuzzy AHP (FAHP) for the MCABCIC problem. Kabir and Hasin [30] integrated FAHP and ANN for determining the weights of criteria and classifying inventories into different classes, respectively. Lolli et al. [31] integrated AHP with the K-means algorithm to solve the MCABCIC problem where the AHP and K-means techniques were applied for ranking items and sorting classes, respectively. Douissa and Jabeur [32] used the ELECTRE III method for ranking items in which the continuous variable neighborhood search metaheuristic method was adopted to estimate the indifference, preference, and veto thresholds.

3. The proposed methodology framework

The first stage in the proposed methodology is to define the qualitative criteria, quantitative criteria, and items (as shown in Figure 1).

Then, the Group Multiple Criteria ABC Inventory Classification (GMCABCIC) matrix is constructed for the MCABCIC problem in which the assessment measures with respect to the qualitative and quantitative criteria are GIT2FNs and crisp data, respectively. Next, TOPSIS is extended by the proposed method to calculate the distances of qualitative assessments from the positive and negative ideal solutions. At last, since the two different methods have been used for calculating the distances of the positive and negative ideal solutions, the weights of criteria are determined based on two linear programming models.

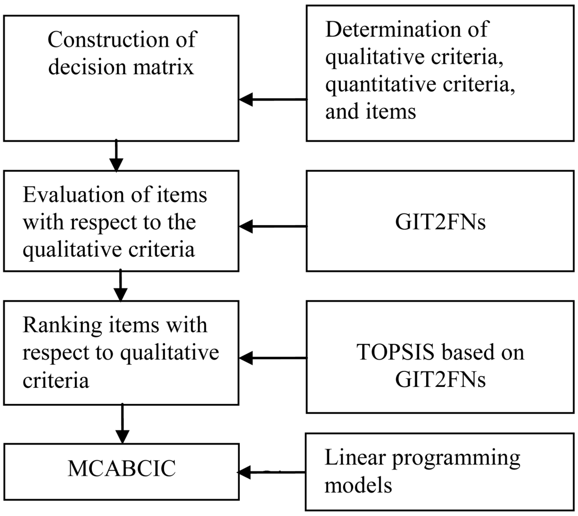

Figure 1. The framework of the proposed methodology.

4. Preliminaries

4.1. Type-2 fuzzy sets and their arithmetic operations

Definition 1. A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X is described by a type-2 MF expressed as follows [33]:

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u) \right) \middle| \forall x \in X,\ \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1 \right\}, \quad (1)$$

where $\mu_{\tilde{\tilde{A}}}$ refers to the MF (secondary MF) of $\tilde{\tilde{A}}$, and J_x is a sub-interval in [0, 1] denoting the primary MF. The type-2 fuzzy set $\tilde{\tilde{A}}$ can be also represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_X} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u), \quad (2)$$

where $J_x \subseteq [0,1]$, and $\int\int$ denotes the overall admissible union of x and u.

Definition 2. For the type-2 fuzzy set $\tilde{\tilde{A}}$, if all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, $\tilde{\tilde{A}}$ is named IT2FS. An IT2FS $\tilde{\tilde{A}}$ can be described as follows [33]:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_X} 1/(x,u), \quad (3)$$

where $J_x \subseteq [0,1]$.

Definition 3. Footprint Of Uncertainty (FOU) is derived from the union of all primary memberships:

$$FOU(\tilde{\tilde{A}}) = \int_{x\in X} J_X. \quad (4)$$

The FOU can also be represented by the lower and upper MFs [34]:

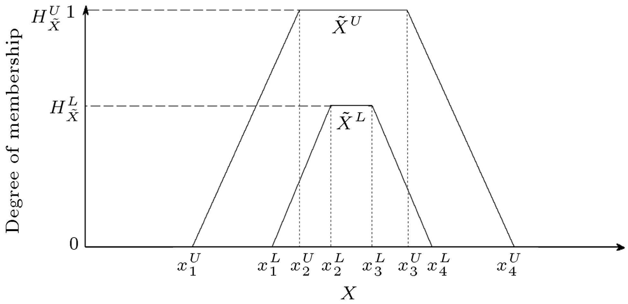

Figure 2. A subnormal TraIT2FN.

$$FOU(\tilde{\tilde{A}}) = \int_{x\in X} \left[\underline{\mu}_{\tilde{\tilde{A}}}(x), \bar{\mu}_{\tilde{\tilde{A}}}(x) \right], \quad (5)$$

where $\underline{\mu}_{\tilde{\tilde{A}}}(x)$ and $\bar{\mu}_{\tilde{\tilde{A}}}(x)$ are the lower and upper MFs of the type-2 fuzzy set. An IT2FS $\tilde{\tilde{A}}$ is said to be normal if $\underline{\mu}_{\tilde{\tilde{A}}}(x) = \bar{\mu}_{\tilde{\tilde{A}}}(x) = 1$. An IT2FS $\tilde{\tilde{A}}$ is said to be subnormal if $\underline{\mu}_{\tilde{\tilde{A}}}(x) < 1$ and $\bar{\mu}_{\tilde{\tilde{A}}}(x) = 1$.

Definition 4. Let $\tilde{X}^L$ and $\tilde{X}^U$ (L and U are equal to the lower and upper MFs) be two non-negative trapezoidal type-1 fuzzy numbers [35,36]. In addition, let $H^L_{\tilde{A}}$ and $H^U_{\tilde{A}}$ denote the heights of $\tilde{X}^L$ and $\tilde{X}^U$, respectively. Let x^L_1, x^L_2, x^L_3, x^L_4, x^U_1, x^U_2, x^U_3, and x^U_4 be non-negative real values. Trapezoidal Interval Type-2 Fuzzy Numbers (TraIT2FNs) defined on the universe of discourse X are given by (see Figure 2):

$$\tilde{\tilde{X}} = [\tilde{X}^L, \tilde{X}^U] = \left[\left(x^L_1, x^L_2, x^L_3, x^L_4; H^L_{\tilde{X}} \right), \left(x^U_1, x^U_2, x^U_3, x^U_4; H^U_{\tilde{X}} \right) \right], \quad (6)$$

Definition 5. Let $\tilde{\tilde{X}}_1$ and $\tilde{\tilde{X}}_2$ be two non-negative TraIT2FNs, where:

$$\tilde{\tilde{X}}_1 = [\tilde{X}^L_1, \tilde{X}^U_1] = \left[\left(x^L_{11}, x^L_{12}, x^L_{13}, x^L_{14}; H^L_{\tilde{X}_1} \right), \left(x^U_{11}, x^U_{12}, x^U_{13}, x^U_{14}; H^U_{\tilde{X}_1} \right) \right],$$

and:

$$\tilde{\tilde{X}}_2 = [\tilde{X}^L_2, \tilde{X}^U_2] = \left[\left(x^L_{21}, x^L_{22}, x^L_{23}, x^L_{24}; H^L_{\tilde{X}_2} \right), \left(x^U_{21}, x^U_{22}, x^U_{23}, x^U_{24}; H^U_{\tilde{X}_2} \right) \right].$$

The arithmetic operations between $\tilde{\tilde{X}}_1$ and $\tilde{\tilde{X}}_2$ are defined as follows:

Addition operation:

$$\tilde{\tilde{X}}_1 \oplus \tilde{\tilde{X}}_2$$
$$= \Bigg[\Bigg(x_{11}^L + x_{21}^L, x_{12}^L + x_{22}^L, x_{13}^L + x_{23}^L, x_{14}^L + x_{24}^L;$$
$$\min \left\{ H_{\tilde{X}1}{}^L, H_{\tilde{X}2}{}^L \right\} \Bigg), \Bigg],$$
$$\Bigg[\Bigg(x_{11}^U + x_{21}^U, x_{12}^U + x_{22}^U, x_{13}^U + x_{23}^U, x_{14}^U + x_{24}^U;$$
$$\min \left\{ H_{\tilde{X}1}{}^U, H_{\tilde{X}2}{}^U \right\} \Bigg) \Bigg]. \tag{7}$$

Subtraction operation:

$$\tilde{\tilde{X}}_1 \Theta \tilde{\tilde{X}}_2$$
$$= \Bigg[\Bigg(x_{11}^L - x_{24}^L, x_{12}^L - x_{23}^L, x_{13}^L - x_{22}^L, x_{14}^L - x_{21}^L;$$
$$\min \left\{ H_{\tilde{X}1}{}^L, H_{\tilde{X}2}{}^L \right\} \Bigg), \Bigg],$$
$$\Bigg[\Bigg(x_{11}^U - x_{24}^U, x_{12}^U - x_{23}^U, x_{13}^U - x_{22}^U, x_{14}^U - x_{21}^U;$$
$$\min \left\{ H_{\tilde{X}1}{}^U, H_{\tilde{X}2}{}^U \right\} \Bigg) \Bigg]. \tag{8}$$

Multiplication operation:

$$\tilde{\tilde{X}}_1 \oplus \tilde{\tilde{X}}_2$$
$$= \Bigg[\Bigg(x_{11}^L.x_{21}^L, x_{12}^L.x_{22}^L, x_{13}^L.x_{23}^L, x_{14}^L.x_{24}^L;$$
$$\min \left\{ H_{\tilde{X}1}{}^L, H_{\tilde{X}2}{}^L \right\} \Bigg),$$
$$\Bigg(x_{11}^U.x_{21}^U, x_{12}^U.x_{22}^U, x_{13}^U.x_{23}^U, x_{14}^U.x_{24}^U;$$
$$\min \left\{ H_{\tilde{X}1}{}^U, H_{\tilde{X}2}{}^U \right\} \Bigg) \Bigg]. \tag{9}$$

Division operation:

$$\tilde{\tilde{X}}_1 \varphi \tilde{\tilde{X}}_2$$
$$= \Bigg[\Bigg(\frac{x_{11}^L}{x_{24}^L}, \frac{x_{12}^L}{x_{23}^L}, \frac{x_{13}^L}{x_{22}^L}, \frac{x_{14}^L}{x_{21}^L}; \min \left\{ H_{\tilde{X}1}{}^L, H_{\tilde{X}2}{}^L \right\} \Bigg),$$
$$\Bigg(\frac{x_{11}^U}{x_{24}^U}, \frac{x_{12}^U}{x_{23}^U}, \frac{x_{13}^U}{x_{22}^U}, \frac{x_{14}^U}{x_{21}^U}; \min \left\{ H_{\tilde{X}1}{}^U, H_{\tilde{X}2}{}^U \right\} \Bigg) \Bigg]. \tag{10}$$

Multiplication by an ordinary number:

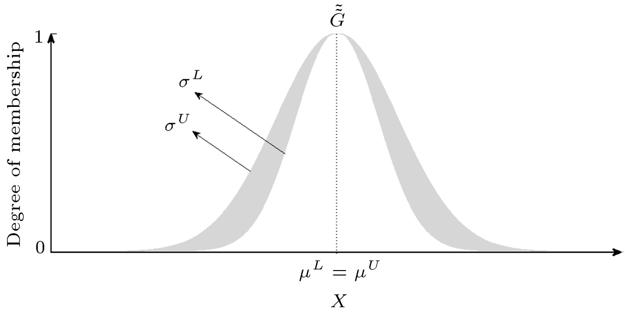

Figure 3. A normal GIT2FN.

$$\tilde{\tilde{X}}_1.r = r.\tilde{\tilde{X}}_1$$
$$= \begin{cases} \Bigg[\Bigg(r.x_{11}^L, r.x_{12}^L, r.x_{13}^L, r.x_{14}^L; H_{\tilde{X}1}{}^L \Bigg), \\ \Bigg(r.x_{11}^U, r.x_{12}^U, r.x_{13}^U, r.x_{14}^U; H_{\tilde{X}1}{}^U, \Bigg) \Bigg] & \text{if } r \geq 0, \\ \Bigg[\Bigg(r.x_{14}^L, r.x_{13}^L, r.x_{12}^L, r.x_{11}^L; H_{\tilde{X}1}{}^L \Bigg), \\ \Bigg(r.x_{14}^U, r.x_{13}^U, r.x_{12}^U, r.x_{11}^U; H_{\tilde{X}1}{}^U, \Bigg) \Bigg] & \text{if } r \leq 0. \end{cases} \tag{11}$$

Definition 6. Let $\tilde{\tilde{G}}$ be a normal GIT2FN as follows (see also Figure 3):

$$\tilde{\tilde{G}} = [\tilde{G}^L, \tilde{G}^U] = \Bigg[\Bigg(\mu^L; \sigma^L; H_{\tilde{G}}^L \Bigg), \Bigg(\mu^U; \sigma^U; H_{\tilde{G}}^U \Bigg) \Bigg], \tag{12}$$

where μ^L; σ^L and μ^U; σ^U are the mean and standard deviation of the lower and upper Gaussian MFs, respectively, such that $\mu^L = \mu^U$ and $\sigma^L < \sigma^U$.

Definition 7. The α-cut of $\tilde{\tilde{A}}$ is presented as follows [37]:

$$A_\alpha = \left\{ (x, u) \middle| f_x(u) \geq \alpha \right\}. \tag{13}$$

Definition 8. The α-cut of $\tilde{\tilde{A}}$ may also be represented by the α-cut of its FOU:

$$A_\alpha = \left\{ x \middle| \underline{\mu}_{\tilde{\tilde{A}}}(x) \geq \alpha, \bar{\mu}_{\tilde{\tilde{A}}}(x) \geq \alpha \right\}. \tag{14}$$

For a GIT2FN $\tilde{\tilde{G}}$, the α-cut may be presented as an interval as follows (see Figure 4):

$$\hat{G}_\alpha = \left[\left[\bar{x}_{1\alpha}^l, \underline{x}_{2\alpha}^l \right], \left[\underline{x}_{1\alpha}^r, \bar{x}_{2\alpha}^r \right] \right], \tag{15}$$

where l and r show the left and right MFs of $\tilde{\tilde{G}}$, respectively.

Definition 9. Let $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ be two positive interval numbers such that $x_1 \leq x \leq x_2$

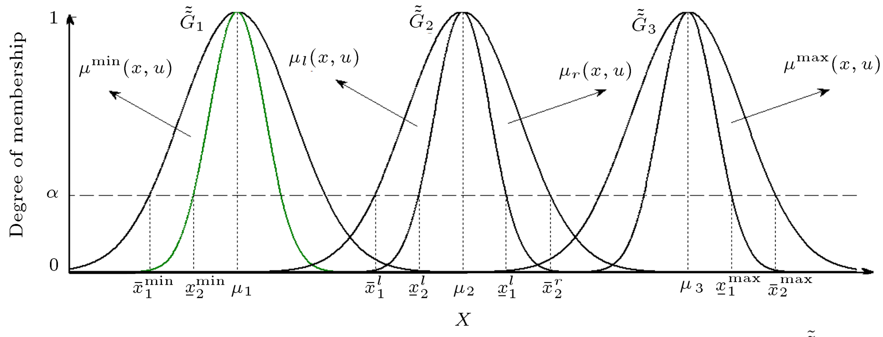

Figure 4. The left, right, minimum, and maximum reference limits $\tilde{\tilde{G}}$.

and $y_1 \leq y \leq y_2$ (x_1, y_1 and x_2, y_2 are the infima and the suprema, respectively). Interval arithmetic operations of addition, subtraction, multiplication, and division are defined, respectively, as follows [38]:

Addition operation:

$$X + Y = [x_1 + y_1, x_2 + y_2]. \tag{16}$$

Subtraction operation:

$$X - Y = [x_1 - y_2, x_2 - y_1]. \tag{17}$$

Multiplication operation:

$$\begin{aligned} X.Y = &[\min(x_1.y_1, x_1.y_2, x_2.y_1, x_2.y_2), \\ &\max(x_1.y_1, x_1.y_2, x_2.y_1, x_2.y_2)]. \end{aligned} \tag{18}$$

Division operation:

$$\frac{X}{Y} = [x_1, x_2].\left(\frac{1}{[y_1, y_2]}\right), \quad \text{where}$$

$$\frac{1}{[y_1, y_2]} = \left[\frac{1}{y_2}, \frac{1}{y_1}\right] \quad \text{if} \quad 0 \notin [y_1, y_2]. \tag{19}$$

Distance between X and Y:

$$\Delta_{X-Y} = \frac{1}{2}\left|(x_1 - y_2) + (x_2 - y_1)\right|. \tag{20}$$

5. A new Limit Distance (LD) for ranking GIT2FNs

5.1. The normal GIT2FNs case

This paper presents an approach based on α-cut for comparing and ranking GIT2FNs. The proposed approach is able to calculate the distances at different levels and concurrently rank GIT2FNs at the interval [0, 1].

The proposed methodology first selects the left and right reference limits. For this purpose, let the MF of $\mu_{\tilde{\tilde{G}}}(x,u)$ for a GIT2FN split into two curves $\mu_l(x,u)$ and $\mu_r(x,u)$, the left and right MFs of $\tilde{\tilde{G}}$, respectively (as shown in Figure 4).

$$\mu_{\tilde{\tilde{G}}}(x,u) = \begin{Bmatrix} \mu_l(x,u) & \text{for} \quad x\langle\mu \\ \mu_r(x,u) & \text{for} \quad x\rangle\mu \end{Bmatrix}. \tag{21}$$

In addition, the minimum reference limit, $\mu^{\min}(x,u)$, and the maximum reference limit, $\mu^{\max}(x,u)$, are $\{\min\{\mu_{li}(x,u)\} i \in \text{all GIT2FNs}\}$, and $\{\max\{\mu_{ri}(x,u)\}\ i \in \text{all GIT2FNs}\}$, respectively. In order to show the left and right reference limits and α-cut of a GIT2FN, GIT2FNs are considered, as shown in Figure 4.

Note that the α-cut of a GIT2FN creates interval numbers; thus, one can apply the interval arithmetic operations to them. As illustrated in Figure 4, suppose that the α-cut of the minimum and maximum reference limits $\mu_\alpha^{\min}(x,u)$ and $\mu_\alpha^{\max}(x,u)$ (intersection points of level α with the MFs of $\mu^{\min}(x,u)$ and $\mu^{\max}(x,u)$) makes intervals $[\bar{x}_1^{\min}, \underline{x}_2^{\min}]_\alpha$ and , $[\underline{x}_1^{\max}, \bar{x}_2^{\max}]_\alpha$ respectively, on X, where $\bar{x}_1^{\max}$ and $\underline{x}_2^{\min}$ are related to the upper and lower MFs of $\mu^{\min}(x,u)$, respectively, and $\underline{x}_{\max 1}$ and $\bar{x}_{\max 2}$ are equal to the lower and upper MFs of $\mu^{\max}(x,u)$, respectively. Moreover, let α-cut of the left and right MFs of a GIT2FN such as $\tilde{\tilde{G}}_2$, $\mu_{l\alpha}(x,u)$ and $\mu_{r\alpha}(x,u)$ (intersection points of level α with the MFs of $\mu_l(x,u)$ and $\mu_r(x,u)$) generate the intervals $[\bar{x}_1^l, \underline{x}_2^l]_\alpha$ and $[\underline{x}_1^r, \bar{x}_2^r]_\alpha$, respectively, where $\bar{x}_1^l$ and $\underline{x}_2^l$ are related to upper and lower MFs of $\mu_l(x,u)$, and $\underline{x}_1^r$ and $\bar{x}_2^r$ are equal to lower and upper MFs of $\mu_r(x,u)$. With these assumptions in mind, for a GIT2FN, the LD can be calculated for the Positive Ideal (PI) solution with respect to cost (C) criterion (Eq. (22) shown in Box I), where α = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. In Eq. (22) shown in Box I, $\sum_{\alpha=0.1}^{1}(\mu_{l\alpha}(x,u) - \mu_\alpha^{\min}(x,u))$ is a positive value, and $\sum_{\alpha=0.1}^{1}(\mu_{r\alpha}(x,u) - \mu_\alpha^{\max}(x,u))$ is a negative value. Therefore, the negative sign is considered in the denominator. To simplify the calculations, Eq. (22) can be converted into Eq. (23) shown in Box II. Obviously, in the situations such as $[\bar{x}_1^l, \underline{x}_2^l]_\alpha - [\underline{x}_1^{\max}, \bar{x}_2^{\max}]_\alpha$ and $[\underline{x}_1^r, \bar{x}_2^r]_\alpha - [\underline{x}_1^{\max}, \bar{x}_2^{\max}]_\alpha$, one always obtains a negative measure, while ($\bar{x}_1^l < \bar{x}_2^{\max}; \underline{x}_2^l < \underline{x}_1^{\max}$) or ($\underline{x}_1^r < \bar{x}_2^{\max}; \bar{x}_2^r < \underline{x}_1^{\max}$). Instead, by using Eq. (20), the distance between two interval numbers is calculated by Eq. (24) as shown in Box III.

Similarly, the PI solution for the set of benefit

$$LD_{PI,C}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{1}\left(\mu_{l\alpha}(x,u)-\mu_{\alpha}^{\min}(x,u)\right)\alpha}{\sum_{\alpha=0.1}^{1}\left(\mu_{l\alpha}(x,u)-\mu_{\alpha}^{\min}(x,u)\right)-\sum_{\alpha=0.1}^{1}\left(\mu_{r\alpha}(x,u)-\mu_{\alpha}^{\max}(x,u)\right)}. \tag{22}$$

Box I

$$LD_{PI,C}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{1}[\bar{x}_1^l,\underline{x}_2^l]_\alpha-[\bar{x}_1^{\min},\underline{x}_2^{\min}]_\alpha}{\sum_{\alpha=0.1}^{1}[\bar{x}_1^l,\underline{x}_2^l]_\alpha-[\bar{x}_1^{\min},\underline{x}_2^{\min}]_\alpha-\sum_{\alpha=0.1}^{1}[\underline{x}_1^r,\bar{x}_2^r]_\alpha-[\underline{x}_1^{\max},\bar{x}_2^{\max}]_\alpha}. \tag{23}$$

Box II

(B) criteria, the Negative Ideal (NI) solution for the set of C criteria, and the NI solution for the set of B criteria are calculated, respectively, by Eqs. (25), (26), and (27), as shown in Box IV. Obviously, the measures obtained through the above equations are included at the interval [0, 1]. Since measures $(\underline{x}_1^r-\bar{x}_2^{\max})+(\bar{x}_2^r-\underline{x}_1^{\max})$ and $(\bar{x}_1^l-\underline{x}_2^{\min})+(\underline{x}_2^l-\bar{x}_1^{\min})$ are equal to zero, while $[\underline{x}_1^r,\bar{x}_2^r]$ matches $\mu^{\max}(x,u)$ and $[\bar{x}_1^l,\underline{x}_2^l]$ matches $\mu^{\min}(x,u)$, or while distances of reference limits $\mu^{\max}(x,u)$ and $\mu^{\min}(x,u)$ are obtained from themselves, measures $\frac{\sum_{\alpha=0.1}^{1}\left|(\bar{x}_2^{\max}-\underline{x}_1^{\max})_\alpha\right|}{\alpha}$ and $\frac{\sum_{\alpha=0.1}^{1}\left|(\underline{x}_2^{\min}-\bar{x}_1^{\min})_\alpha\right|}{\alpha}$ are used for calculating LDs.

5.2. The subnormal GIT2FNs case

If GIT2FN is subnormal (see Figure 5), then the LDs are based on Eqs. (24)-(27) for $\alpha\le H_{\tilde{G}}^L$ and Eqs. (28)-(31) for $H_{\tilde{G}}^L<\alpha\le H_{\tilde{G}}^U$.

$$LD_{PI,C}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=H_{\tilde{G}}^L}^{H_{\tilde{G}}^U}\left|(\bar{x}_1^l-\bar{x}_1^{\min})_\alpha\right|}{\sum_{\alpha=H_{\tilde{G}}^L}^{H_{\tilde{G}}^U}\left|(\bar{x}_1^l-\bar{x}_1^{\min})_\alpha\right|+\sum_{\alpha=H_{\tilde{G}}^L}^{H_{\tilde{G}}^U}\left|(\bar{x}_2^r-\bar{x}_2^{\max})_\alpha\right|}, \tag{28}$$

$$LD_{PI,B}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{H_{\tilde{G}}^U}\left|(\bar{x}_2^r-\bar{x}_2^{\max})_\alpha\right|}{\sum_{\alpha=H_{\tilde{G}}^L}^{H_{\tilde{G}}^U}\left|(\bar{x}_2^r-\bar{x}_2^{\max})_\alpha\right|+\sum_{\alpha=H_{\tilde{G}}^L}^{H_{\tilde{G}}^U}\left|(\bar{x}_1^l-\bar{x}_1^{\min})_\alpha\right|}, \tag{29}$$

$LD_{NI,C}(\tilde{\tilde{A}})=$

$$LD_{PI,C}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{1}\frac{1}{2}\left|(\bar{x}_1^l-\underline{x}_2^{\min})_\alpha+(\underline{x}_2^l-\bar{x}_1^{\min})_\alpha\right|}{\sum_{\alpha=0.1}^{1}\frac{1}{2}\left|(\bar{x}_1^l-\underline{x}_2^{\min})_\alpha+(\underline{x}_2^l-\bar{x}_1^{\min})_\alpha\right|+\sum_{\alpha=0.1}^{1}\frac{1}{2}\left|(\underline{x}_1^r-\bar{x}_2^{\max})_\alpha+(\bar{x}_2^r-\underline{x}_1^{\max})_\alpha\right|},$$
$$\frac{\sum_{\alpha=0.1}^{1}\left|(\bar{x}_1^l-\underline{x}_2^{\min})_\alpha+(\underline{x}_2^l-\bar{x}_1^{\min})_\alpha\right|}{\sum_{\alpha=0.1}^{1}\left|(\bar{x}_1^l-\underline{x}_2^{\min})_\alpha+(\underline{x}_2^l-\bar{x}_1^{\min})_\alpha\right|+\sum_{\alpha=0.1}^{1}\left|(\underline{x}_1^r-\bar{x}_2^{\max})_\alpha+(\bar{x}_2^r-\underline{x}_1^{\max})_\alpha\right|}. \tag{24}$$

Box III

$$LD_{PI,B}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{1}\left|(\underline{x}_1^r-\bar{x}_2^{\max})_\alpha+(\bar{x}_2^r-\underline{x}_1^{\max})_\alpha\right|}{\sum_{\alpha=0.1}^{1}\left|(\underline{x}_1^r-\bar{x}_{\max 2})_\alpha+(\bar{x}_2^r-\underline{x}_1^{\max})_\alpha\right|+\sum_{\alpha=0.1}^{1}\left|(\bar{x}_1^l-\underline{x}_2^{\min})_\alpha+(\underline{x}_2^l-\bar{x}_1^{\min})_\alpha\right|}, \tag{25}$$

$$LD_{NI,C}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{1}\left|(\bar{x}_1^l-\bar{x}_2^{\max})_\alpha+(\underline{x}_2^l-\underline{x}_1^{\max})_\alpha\right|}{\sum_{\alpha=0.1}^{1}\left|(\bar{x}_1^l-\bar{x}_2^{\max})_\alpha+(\underline{x}_2^l-\underline{x}_1^{\max})_\alpha\right|+\sum_{\alpha=0.1}^{1}\left|(\underline{x}_1^r-\underline{x}_2^{\min})_\alpha+(\bar{x}_2^r-\bar{x}_1^{\min})_\alpha\right|}, \tag{26}$$

$$LD_{NI,B}(\tilde{\tilde{A}})=\frac{\sum_{\alpha=0.1}^{1}\left|(\underline{x}_1^r-\underline{x}_2^{\min})_\alpha+(\bar{x}_2^r-\bar{x}_1^{\min})_\alpha\right|}{\sum_{\alpha=0.1}^{1}\left|(\underline{x}_1^r-\underline{x}_2^{\min})_\alpha+(\bar{x}_2^r-\bar{x}_1^{\min})_\alpha\right|+\sum_{\alpha=0.1}^{1}\left|(\bar{x}_1^l-\bar{x}_2^{\max})_\alpha+(\underline{x}_2^l-\underline{x}_1^{\max})_\alpha\right|}. \tag{27}$$

Box IV

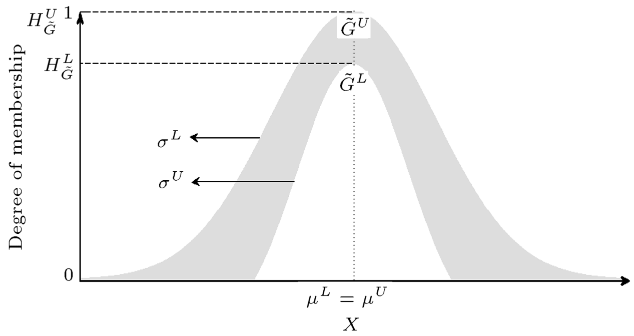

Figure 5. A subnormal GIT2FN.

$$\frac{\sum_{\alpha=H^L_{\tilde{G}}}^{H^U_{\tilde{G}}} \left|(\bar{x}^l_1 - \bar{x}^{\max}_2)_\alpha\right|}{\sum_{\alpha=H^L_{\tilde{G}}}^{H^U_{\tilde{G}}} \left|(\bar{x}^l_1 - \bar{x}^{\max}_2)_\alpha\right| + \sum_{\alpha=H^L_{\tilde{G}}}^{H^U_{\tilde{G}}} \left|(\bar{x}^r_2 - \bar{x}^{\min}_1)_\alpha\right|}, \quad (30)$$

$$LD_{NI,B}(\tilde{\tilde{A}}) =$$

$$\frac{\sum_{\alpha=0.1}^{H^U_{\tilde{G}}} \left|(\bar{x}^r_2 - \bar{x}^{\min}_1)_\alpha\right|}{\sum_{\alpha=H^L_{\tilde{G}}}^{H^U_{\tilde{G}}} \left|(\bar{x}^r_2 - \bar{x}^{\min}_1)_\alpha\right| + \sum_{\alpha=H^L_{\tilde{G}}}^{H^U_{\tilde{G}}} \left|(\bar{x}^l_1 - \bar{x}^{\max}_2)_\alpha\right|}. \quad (31)$$

6. Application of a new LD in TOPSIS with GIT2FNs

In this section, the TOPSIS approach is generalized for GIT2FNs using LDs, as stated in Section 5. The interested readers can refer to [3] for studying the steps of the classical TOPSIS. Although the method is explained for GIT2FNs, one can apply it to TraIT2FNs or TriIT2FNs. The following stages show the proposed approach to normal GIT2FNs:

1. Let a decision-maker evaluate m alternatives A_i $(i = 1, ..., m)$ under n criteria C_j $(j = 1, ..., n', n' + 1, ..., n)$ via the MCDM matrix $(D, \tilde{\tilde{D}}) = [x_{ij}, \tilde{\tilde{x}}_{ij}]_{m\times n}$ where $C_j(j = 1, ..., n')$, $C_j(j = n' + 1, ..., n)$, $D = [x_{ij}]_{m\times(1,...,n')}$, and $\tilde{\tilde{D}} = [\tilde{\tilde{x}}_{ij}]_{m\times(n'+1,...,n)}$ represent the quantitative criteria, the qualitative criteria, the crisp values (with respect to the quantitative criteria), and GIT2FNs (with respect to the qualitative criteria), respectively (Eq. (32), shown in Box V), where $\tilde{\tilde{x}}_{ij}$ is the synthetic Gaussian interval type-2 fuzzy rating aggregated by L experts. It is calculated as follows:

$$\tilde{\tilde{x}}_{ij} = (1/L) \otimes (\tilde{\tilde{x}}^1_{ij} \oplus \tilde{\tilde{x}}^2_{ij} \oplus \cdots \oplus \tilde{\tilde{x}}^L_{ij}),$$

$$i = 1, ..., m; \qquad j = n' + 1, ..., n. \qquad (33)$$

2. Suppose that $\bar{x}^l_{1ij\alpha}$, $\underline{x}^l_{2ij\alpha}$, $\underline{x}^r_{1ij\alpha}$, and $\bar{x}^r_{2ij\alpha}$ is the projection of α-cut's intersection points with the left and right MFs of GIT2FNs $\tilde{\tilde{G}} = \left[\left(x, \mu^L; \sigma^L\right), \left(x, \mu^U; \sigma^U\right)\right]$ when evaluating alternative i under criterion j. Then, GIT2FN $\tilde{\tilde{x}}_{ij\alpha}$ at level α can be represented as follows:

$$\hat{x}_{ij\alpha} = \left\{\left[\bar{x}^l_{1ij\alpha}, \underline{x}^l_{2ij\alpha}\right], \left[\underline{x}^r_{1ij\alpha}, \bar{x}^r_{2ij\alpha}\right]\right\},$$

$$i = 1, ..., m; \;\; j = n' + 1, ..., n. \qquad (34)$$

Similarly, GIT2FN $\tilde{\tilde{x}}^l_{ij\alpha}$ selected by the lth expert at level α is given by:

$$\hat{x}^l_{ij\alpha} = \left\{\left[\bar{x}^{l\,l}_{1ij\alpha}, \underline{x}^{l\,l}_{2ij\alpha}\right], \left[\underline{x}^{r\,l}_{1iij\alpha}, \bar{x}^{r\,l}_{2iij\alpha}\right]\right\},$$

$$i=1, ..., m; \quad j=n' + 1, ..., n; \quad l=1, ..., L. \qquad (35)$$

Three reference points of the triangular fuzzy numbers selected by L experts, namely $\tilde{x}_{ij} = (Lb_{ij}, Mb_{ij}, Ub_{ij})$ according to Buckley [39], are given by:

$$Lb_{ij} = \left(\sum_{l=1}^{L} Lb^l_{ij}\right) \Big/ L,$$

$$i = 1, ..., m; \quad j = n' + 1, \cdots, n, \qquad (36)$$

$$Mb_{ij} = \left(\sum_{l=1}^{L} Mb^l_{ij}\right) \Big/ L,$$

$$i = 1, ..., m; \quad j = n' + 1, \cdots, n, \qquad (37)$$

$$Ub_{ij} = \left(\sum_{l=1}^{L} Ub^l_{ij}\right) \Big/ L,$$

$$i = 1, ..., m; \quad j = n' + 1, \cdots, n. \qquad (38)$$

$$D, \tilde{\tilde{D}} = \begin{matrix} & C_1{}^{w_1} & C_2{}^{w_2} & C_3{}^{w_3} & \ldots & C^{w_{n'}}_{n'} & C^{w_{n'+1}}_{n'+1} & C^{w_{n'+2}}_{n'+2} & \cdots & C^{w_n}_n \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \multicolumn{9}{c}{\begin{bmatrix} x_{11} & x_{12} & x_{13} & \ldots & x_{1n'} & \tilde{\tilde{x}}_{1n'+1} & \tilde{\tilde{x}}_{1n'+2} & \ldots & \tilde{\tilde{x}}_{1n} \\ x_{21} & x_{22} & x_{23} & \ldots & x_{2n'} & \tilde{\tilde{x}}_{2n'+1} & \tilde{\tilde{x}}_{2n'+2} & \ldots & \tilde{\tilde{x}}_{2n} \\ x_{31} & x_{32} & x_{33} & \ldots & x_{3n'} & \tilde{\tilde{x}}_{3n'+1} & \tilde{\tilde{x}}_{3n'+3} & \ldots & \tilde{\tilde{x}}_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn'} & \tilde{\tilde{x}}_{mn'+1} & \tilde{\tilde{x}}_{mn'+2} & \cdots & \tilde{\tilde{x}}_{mn} \end{bmatrix}} \end{matrix}. \qquad (32)$$

Box V

The four reference points for GIT2FNs chosen by L experts as $\hat{x}_{ij\alpha} = \{\lfloor \bar{x}^l_{1ij\alpha}, \underline{x}^l_{2ij\alpha} \rfloor, \lfloor \underline{x}^r_{1ij\alpha}, \bar{x}^r_{2ij\alpha} \rfloor\}$ at level α for $\alpha = \alpha_1, ..., \alpha_N$ (N is the number of alpha cuts), $i = 1, ..., m$, and $j = n'+1, ..., n$, are obtained in the following by using the extended technique explained above:

$$\bar{x}^l_{1ij\alpha} = \left(\sum_{l=1}^{L} \bar{x}^{ll}_{1ij\alpha} \right) \Big/ L, \quad i = 1, ..., m;$$

$$j = n' + 1, \cdots, n; \quad \alpha = \alpha_1, \cdots, \alpha_N, \tag{39}$$

$$\underline{x}^l_{2ij\alpha} = \left(\sum_{l=1}^{L} \underline{x}^{ll}_{2ij\alpha} \right) \Big/ L, \quad i = 1, ..., m;$$

$$j = n' + 1, \cdots, n; \quad \alpha = \alpha_1, \cdots, \alpha_N, \tag{40}$$

$$\underline{x}^r_{1ij\alpha} = \left(\sum_{l=1}^{L} \underline{x}^{rl}_{1ij\alpha} \right) \Big/ L, \quad i = 1, ..., m;$$

$$j = n' + 1, \cdots, n; \quad \alpha = \alpha_1, \cdots, \alpha_N, \tag{41}$$

$$\bar{x}^r_{2ij\alpha} = \left(\sum_{l=1}^{L} \bar{x}^{rl}_{2ij\alpha} \right) \Big/ L, \quad i = 1, ..., m;$$

$$j = n' + 1, \cdots, n; \quad \alpha = \alpha_1, \cdots, \alpha_N. \tag{42}$$

In addition, suppose that $w_j (j = 1, ..., n')$, $w_j (n' + 1, ..., n)$, and $w_j \in \{w^l_j, w^u_j\}$ are the weights of the quantitative criteria, the weights of the qualitative criteria, and the admissible range for the jth criterion, w_j.

3. Let $\tilde{\tilde{X}} = [\tilde{X}^L, \tilde{X}^U] = [(x^L_1, x^L_2, x^L_3, x^L_4; H^L_{\tilde{A}}), (x^U_1, x^U_2, x^U_3, x^U_4; H^U_{\tilde{A}})]$ be a TraIT2FN. The normalized performance measures can be calculated by Rashid et al. [40] for Benefit Criteria (BC) and Cost Criteria (CC), respectively:

$$\tilde{\tilde{n}}_{ij} = \left[\left(\frac{x^L_{1ij}}{x^+_{4j}}, \frac{x^L_{2ij}}{x^+_{4j}}, \frac{x^L_{3ij}}{x^+_{4j}}, \frac{x^L_{4ij}}{x^+_{4j}}; H^L_{\tilde{\tilde{x}}_{ij}} \right), \left(\frac{x^U_{1ij}}{x^+_{4j}}, \frac{x^U_{2ij}}{x^+_{4j}}, \frac{x^U_{3ij}}{x^+_{4j}}, \frac{x^U_{4ij}}{x^+_{4j}}; H^U_{\tilde{\tilde{x}}_{ij}} \right) \right],$$

$$\text{for} \quad i = 1, ...m; \quad x^+_{4j} = \max_i x^U_{4ij}$$

$$\text{where } j \in BC, \tag{43}$$

and:

$$\tilde{\tilde{n}}_{ij} = \left[\left(\frac{x^-_{1j}}{x^L_{4ij}}, \frac{x^-_{1j}}{x^L_{3ij}}, \frac{x^-_{1j}}{x^L_{2ij}}, \frac{x^-_{1j}}{x^L_{1ij}}; H^L_{\tilde{\tilde{x}}_{ij}} \right), \left(\frac{x^-_{1j}}{x^U_{41j}}, \frac{x^-_{1j}}{x^U_{3ij}}, \frac{x^-_{1j}}{x^U_{2ij}}, \frac{x^-_{1j}}{x^U_{1ij}}; H^U_{\tilde{\tilde{x}}_{ij}} \right) \right],$$

$$\text{for} \quad i = 1, ...m; \quad x^-_{1j} = \min_i x^L_{1ij}$$

$$\text{where} \quad j \in CC. \tag{44}$$

The normalized decision matrix, $\hat{N}$, is created for α-cuts of $\tilde{\tilde{G}}$ for $i = 1, ..., m$ and $j = n' + 1, ..., n$, using the extension of the above normalization methodology as follows:

$$\widehat{n}_{ij\alpha} = \left\{ \left[\frac{\bar{x}^l_{1ij\alpha}}{x^+_j}, \frac{\underline{x}^l_{2ij\alpha}}{x^+_j} \right], \left[\frac{\underline{x}^r_{1ij\alpha}}{x^+_j}, \frac{\bar{x}^r_{2ij\alpha}}{x^+_j} \right] \right\}$$

$$\text{for} \quad i = 1, ...m; \quad \alpha = \alpha_1, ..., \alpha_N;$$

$$x^+_j = \max_i \bar{x}^r_{2ij\alpha} \quad \text{where} \quad j \in BC, \tag{45}$$

and:

$$\widehat{n}_{ij\alpha} = \left\{ \left[\frac{x^-_j}{\bar{x}^r_{2ij\alpha}}, \frac{x^-_j}{\underline{x}^r_{1ij\alpha}} \right], \left[\frac{x^-_j}{\underline{x}^l_{2ij\alpha}}, \frac{x^-_j}{\bar{x}^l_{1ij\alpha}} \right] \right\}$$

$$\text{for} \quad i = 1, ...m; \quad \alpha = \alpha_1, ..., \alpha_N;$$

$$x^-_j = \min_i \bar{x}^l_{1ij\alpha} \quad \text{where} \quad j \in CC, \tag{46}$$

where N is the number of α-cuts. In addition, the normalized decision matrix, $\hat{D}$, for the crisp values is obtained as follows:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x^2_{ij}}}, \quad i = 1, ...m; \quad j = 1, ...n'. \tag{47}$$

The positive ideal solution, $\hat{A}^+$, and the negative ideal solution, $\hat{A}^-$, respectively, are for the qualitative criteria, using:

$$\hat{A}^+_\alpha = \left\{ \nu^+_{n'+1\alpha}, ..., \nu^+_{n\alpha} \right\} = \left\{ \left(\mu^{\max}_\alpha(x, u) = \max\{\mu_{ri}(x, u)\} | j \in BC \right), \left(\mu^{\min}_\alpha(x, u) = \min\{\mu_{li}(x, u)\} = | j \in CC \right) \right\}, \tag{48}$$

$$\hat{A}^-_\alpha = \left\{ \nu^-_{1\alpha}, \nu^-_{2\alpha}, ..., \nu^-_{n\alpha} \right\} = \left\{ \left(\mu^{\min}_\alpha(x, u) = \min\{\mu_{li}(x, u)\} | j \in BC \right), \left(\mu^{\max}_\alpha(x, u) = \max\{\mu_{ri}(x, u)\} | j \in CC \right) \right\}. \tag{49}$$

For the crisp values, the positive ideal solution, A^+, and the negative ideal solution, A^-, for the quantitative criteria are calculated as follows:

$$A^+ = \left\{ \left(\max_i n_{ij} | j \in BC \right), \left(\min_i n_{ij} | j \in CC \right) \right\}$$

$$= \{\nu_1^+, \nu_2^+, ..., \nu_{n'}^+\}, \tag{50}$$

$$A^- = \left\{ \left(\min_i n_{ij} | j \in BC \right), \left(\max_i n_{ij} | j \in CC \right) \right\}$$

$$= \{\nu_1^-, \nu_2^-, ..., \nu_{n'}^-\}. \tag{51}$$

4. Calculate $\hat{S}_i^+$ and $\hat{S}_i^-$ for each $i = n'+1, ..., n$ based on the measures (distances) LD_{PI} and LD_{NI}, respectively, between alternatives and the positive and negative ideal solutions for $\tilde{\tilde{G}}$ using Eqs. (24)-(27) or Eqs. (28)-(31) and, then, calculate S_i^+ and S_i^- for each $j = 1, ..., n'$ using:

$$S_i^+ = \sqrt{\sum_{j=1}^{n'} (n_{ij} - n_j^+)^2} \qquad i = 1, ..., m, \tag{52}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n'} (n_{ij} - n_j^-)^2} \qquad i = 1, ..., m. \tag{53}$$

5. Calculate the relative closeness, RC_i, to the ideal alternatives with respect to the quantitative criteria ($j = 1, ..., n'$) and the qualitative criteria ($j = n' + 1, ..., n'$), respectively, as follows:

$$RC_i = \frac{S_i^-}{S_i^- + S_i^+} \qquad i = 1, ..., m,$$

and :

$$RC_i = \frac{(\hat{S}_i^-)}{(\hat{S}_i^-) + (\hat{S}_i^+)}, \qquad i = 1, ..., m. \tag{54}$$

6. According to Shipley et al. [41], the separation of each alternative from S_i^+ and S_i^- is dependent on the criteria weights, and the criteria weights are incorporated in the distances measurements. Therefore, in order to eliminate the criteria weights from S_i^+ and S_i^-, the following linear programming model (1) is solved for prioritizing alternatives (the bigger the measure of objective function, the better the alternative) related to the quantitative criteria:

Model (1):

$$S_i = \max \sum_{j=1}^{n'} \left(w_j \frac{\sqrt{(n_{ij} - \nu_j^-)^2}}{\sqrt{(n_{ij} - \nu_j^-)^2} + \sqrt{(n_{ij} - \nu_j^+)^2}} \right)$$

$$\text{for} \quad i = 1, ...m, \tag{55}$$

s.t.:

$$\sum_{j=1}^{n'} w_j \leq 1, \tag{56}$$

$$w_j^l \leq w_j \leq w_j^u, \qquad j = 1, ..., n', \tag{57}$$

$$w_j = w_{j'}, \qquad j \neq j', \quad j; j' \in 1, ..., n', \tag{58}$$

$$w_j \geq w_{j'} \quad \text{or} \quad w_j \leq w_{j'},$$

$$j \neq j', \quad j; j' \in 1, ..., n', \tag{59}$$

$$w_j \geq 0, \qquad j = 1, ..., n', \tag{60}$$

where Constraints (57)-(59) show the lower (w_j^l) and the upper (w_j^u) limits of weights, the equal importance of criteria, and the ranking order of criteria, respectively. Obviously, since the interval weights of criteria are considered, it can be mentioned that, in the case of overlapping intervals, Constraints (58) and (59) are applied to show the preference level of a decision-maker.

Similarly, weights of the qualitative criteria are obtained by the following linear programming model (2):

Model (2):

$$S_i' = \max \sum_{j=n'+1}^{n} \left(w_j' \frac{LD_{NI}}{LD_{PI} + LD_{NI}} \right)$$

$$\text{for} \quad i = 1, ...m, \tag{61}$$

s.t.:

$$\sum_{j=n'+1}^{n'} w_j' \leq 1, \tag{62}$$

$$w'^l_j \leq w'_j \leq w'^u_j, \qquad j = n' + 1, ..., n', \tag{63}$$

$$w'_j = w'_{j'}, \qquad j \neq j', \qquad j; j' \in n' + 1, ..., n', \tag{64}$$

$$w'_j \geq w'_{j'} \quad \text{or} \quad w'_j \leq w'_{j'},$$

$$j \neq j', \qquad j; j' \in n' + 1, ..., n', \tag{65}$$

$$w'_j \geq w_{j''}, \quad \text{or} \quad w'_j \leq w_{j''},$$

$$j \neq j'' \qquad j'' \in 1, ..., n'; \qquad j' \in n' + 1, ..., n', \tag{66}$$

$$w'_j \geq 0, \qquad j = n' + 1, ..., n', \tag{67}$$

where the constraint $w_j' \geq w_{j''}$ or $w_j' \leq w_{j''}$ is applied for showing the preference level between the weights of the quantitative and the qualitative criteria.

7. Case study

To show the effectiveness of the proposed approach, it is implemented in the material warehouse of a soft-drink factory, Zahedan, Iran. This warehouse consists of 35 items. The authors used the suggestions and points of view of three inventory managers.

7.1. Determination of criteria

Inventory managers have chosen five criteria (annual dollar usage, lead time, average lot cost, limitation of warehouse space, and availability of the substitute raw material) as the most important criteria that affect the ranked items.

Here, it is worth stating that the first four criteria are of benefit type, and the fifth criterion is a cost-type criterion (the smaller the measure, the more important it will be). On the other hand, the first three criteria and the last two criteria are the quantitative and qualitative criteria, respectively.

7.2. Construction of GMCABCIC matrix

The linguistic variables with their GIT2FNs (as represented in Table 1) are applied to evaluate the items with respect to qualitative criteria. Three experts are then asked to select one of them for determining their preference degree. In addition, the representation of GIT2FNs for these linguistic variables is shown in Figure 6.

Table 2 represents the measures of items related to the quantitative criteria and, also, the assessment of items with respect to the qualitative criteria, respectively, in which the linguistic variables are selected by inventory managers based on Table 1 and are aggregated using Eqs. (39)-(42). In fact, this table is the GMCABCIC matrix.

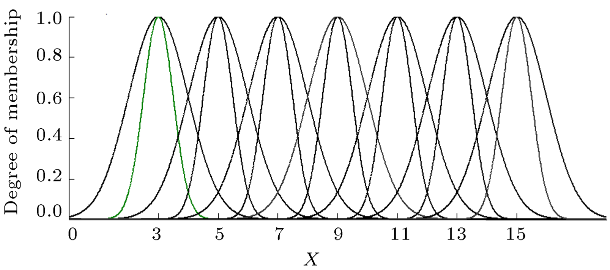

Figure 6. The representation of GIT2FNs defined in Table 1.

Afterwards, the GMCABCIC matrix is normalized for the quantitative criteria using Eq. (47) and for the qualitative criteria using Eqs. (45)-(46). Table 3 presents the normalized GMCABCIC matrix, where in order to summarize calculations, the normalized measures of the qualitative criteria have been incorporated only for $\alpha = 0.01$.

Table 4 shows Euclidean distances S_i^+ and S_i^- between items and the ideal solutions for criteria C_1, C_2, and C_3 using Eqs. (52)-(53) separately and, also, LD_{PI} and LD_{NI} (for $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 0.9$ using Eq. (25) and Eq. (27) for criteria C_4 and C_5, respectively.

In order to construct the objective functions of programs (1) and (2), the measures RC are obtained for each criterion using Eq. (54) based on the data in Table 4 and are represented in Table 5.

Now, the linear programs (1) and (2) are solved for obtaining the scores of items with respect to the quantitative and qualitative criteria (SE and SE′), respectively:

Model (1):

$$SE_i = \max \sum_{j=1}^{3} \left(w_j \frac{\sqrt{(\nu_{ij} - \nu_j^-)^2}}{\sqrt{(\nu_{ij} - \nu_j^-)^2} + \sqrt{(\nu_{ij} - \nu_j^+)^2}} \right)$$

for $i = 1, ..., 5$,

s.t.

$$\sum_{j=1}^{3} w_j \leq 1,$$

$$0.2 \leq w_1 \leq 0.45,$$

Table 1. Definitions of linguistic variables for evaluating items with respect to the qualitative criteria.

Linguistic variables	$\lfloor(\mu^L, \sigma^L; H_{\tilde{G}}^L), (\mu^U, \sigma^U; H_{\tilde{G}}^U)\rfloor$
Absolutely Unimportant (AU)	$[(3, 0.5; 1), (3, 1; 1)]$
Very Unimportant (VU)	$[(5, 0.5; 1), (5, 1; 1)]$
Unimportant (U)	$[(7, 0.5; 1), (7, 1; 1)]$
Medium (M)	$[(9, 0.5; 1), (9, 1; 1)]$
Important (I)	$[(11, 0.5; 1), (11, 1; 1)]$
Very Important (VI)	$[(13, 0.5; 1), (13, 1; 1)]$
Absolutely Important (AI)	$[(15, 0.5; 1), (15, 1; 1)]$

Table 2. Evaluated values of items with respect to different criteria.

Item no.	Annual dollar usage (thousand $)	Lead time (day)	Average lot cost ($)	Limitation of warehouse space			Availability of the substitute raw material		
				Experts			Experts		
				1	2	3	1	2	3
1	9.8	8	50.3	AU	U	VU	AU	VU	U
2	21.88	19	40.5	U	VU	VU	VU	U	VU
3	35.84	17	68.4	M	I	I	U	VU	U
4	32.32	17	66.34	I	M	M	I	VI	VI
5	100.73	16	42.32	VI	I	VI	M	M	I
6	726.86	13	26.5	M	M	I	U	VU	M
7	53.09	13	16.54	M	U	M	U	U	U
8	71.47	13	19.8	I	I	M	U	U	U
9	14.77	13	16.53	U	U	VU	VU	U	VU
10	11.82	32	90.4	VI	I	I	VU	U	VU
11	10.98	32	85.32	VI	I	I	U	VU	U
12	3.2	17	71.8	I	U	VU	I	I	I
13	2.07	17	68.5	U	U	U	I	U	VI
14	3.18	8	13.64	AU	VU	VU	AU	VU	U
15	2.09	8	8.84	VU	VU	VU	AU	VU	U
16	38.34	9	6.39	VU	U	U	VU	U	VU
17	619.39	7	38.9	AI	VI	VI	U	I	M
18	26.8	18	34.5	VU	U	VU	U	M	VU
19	28.11	6	28.87	VU	VU	M	I	U	U
20	55.12	15	37.5	M	U	VU	AI	AI	AI
21	6.29	6	24.67	U	U	U	VU	U	VU
22	26.79	6	23.54	VU	AU	U	VU	U	U
23	38.95	27	33.12	VU	U	VU	U	U	M
24	30.15	27	15.4	VU	AU	VU	U	VU	VU
25	22.58	27	17.21	VU	VU	VU	I	M	M
26	1.02	27	15.87	VU	AU	AU	VU	U	U
27	3.12	27	16.81	VU	VU	AU	AU	VU	U
28	12.1	27	20.9	AU	U	AU	AU	U	U
29	4.06	27	21.4	U	VU	VU	U	U	U
30	13.58	27	19.3	U	AU	VU	I	U	VU
31	1.01	27	18.65	U	U	AU	VU	U	VU
32	15.49	9	11.45	U	VU	VU	U	U	AU
33	8.53	27	9.4	U	U	VU	M	M	U
34	7.35	17	64.17	VU	U	VU	U	M	U
35	19.02	8	43.07	U	VU	U	M	I	M

Table 3. The normalized GMCABCIC matrix.

Item no.	Annual dollar usage (thousand $)	Lead time (day)	Average lot cost ($)	Limitation of warehouse space	Availability of the substitute raw material
1	0.0100	0.0696	0.2082	[(0.180, 0.248), (0.383, 0.451)]	[(0.166, 0.229), (0.354, 0.416)]
2	0.0225	0.1653	0.1676	[(0.222, 0.291), (0.426, 0.493)]	[(0.205, 0.268), (0.392, 0.455)]
3	0.0368	0.1479	0.2831	[(0.518, 0.586), (0.721, 0.789)]	[(0.245, 0.307), (0.575, 0.581)]
4	0.0332	0.1479	0.2746	[(0.476, 0.544), (0.679, 0.747)]	[(0.612, 0.619), (0.625, 0.631)]
5	0.1036	0.1392	0.1751	[(0.645, 0.713), (0.848, 0.916)]	[(0.663, 0.669), (0.675, 0.682)]
6	0.7480	0.1131	0.1096	[(0.476, 0.544), (0.679, 0.747)]	[(0.283, 0.346), (0.471, 0.533)]
7	0.0546	0.1131	0.0684	[(0.392, 0.460), (0.595, 0.663)]	[(0.284, 0.346), (0.471, 0.533)]
8	0.0735	0.1131	0.0819	[(0.518, 0.586), (0.721, 0.789)]	[(0.284, 0.346), (0.471, 0.533)]
9	0.0152	0.1131	0.0684	[(0.265, 0.333), (0.468, 0.536)]	[(0.206, 0.268), (0.393, 0.456)]
10	0.0121	0.2785	0.3741	[(0.603, 0.670), (0.806, 0.873)]	[(0.206, 0.268), (0.393, 0.456)]
11	0.0112	0.2785	0.3531	[(0.603, 0.670), (0.806, 0.873)]	[(0.245, 0.307), (0.432, 0.494)]
12	0.0032	0.1479	0.2972	[(0.350, 0.418), (0.553, 0.620)]	[(0.517, 0.579), (0.704, 0.767)]
13	0.0021	0.1479	0.2835	[(0.308, 0.375), (0.510, 0.578)]	[(0.478, 0.541), (0.665, 0.728)]
14	0.0032	0.0696	0.0564	[(0.138, 0.207), (0.342, 0.410)]	[(0.167, 0.229), (0.354, 0.417)]
15	0.0021	0.0696	0.0365	[(0.181, 0.249), (0.384, 0.452)]	[(0.167, 0.229), (0.354, 0.417)]
16	0.0394	0.0783	0.0264	[(0.265, 0.333), (0.468, 0.536)]	[(0.206, 0.268), (0.393, 0.456)]
17	0.6374	0.0609	0.1610	[(0.730, 0.797), (0.932, 1.000)]	[(0.400, 0.463), (0.587, 0.650)]
18	0.0275	0.1566	0.1428	[(0.223, 0.291), (0.426, 0.494)]	[(0.283, 0.346), (0.471, 0.533)]
19	0.0289	0.0522	0.1195	[(0.265, 0.333), (0.468, 0.536)]	[(0.361, 0.424), (0.549, 0.611)]
20	0.0567	0.1305	0.1552	[(0.307, 0.375), (0.510, 0.578)]	[(0.751, 0.813), (0.937, 1.000)]
21	0.0064	0.0522	0.1021	[(0.308, 0.375), (0.510, 0.578)]	[(0.206, 0.268), (0.393, 0.456)]
22	0.0275	0.0522	0.0974	[(0.181, 0.249), (0.384, 0.452)]	[(0.245, 0.307), (0.432, 0.494)]
23	0.0400	0.2350	0.1370	[(0.223, 0.291), (0.426, 0.494)]	[(0.323, 0.385), (0.510, 0.572)]
24	0.0310	0.2350	0.0637	[(0.138, 0.207), (0.342, 0.410)]	[(0.206, 0.268), (0.393, 0.456)]
25	0.0232	0.2350	0.0712	[(0.181, 0.249), (0.384, 0.452)]	[(0.439, 0.502), (0.626, 0.689)]
26	0.0010	0.2350	0.0656	[(0.096, 0.164), (0.300, 0.367)]	[(0.245, 0.307), (0.432, 0.494)]
27	0.0032	0.2350	0.0695	[(0.138, 0.207), (0.342, 0.410)]	[(0.167, 0.229), (0.354, 0.417)]
28	0.0124	0.2350	0.0865	[(0.139, 0.206), (0.342, 0.410)]	[(0.206, 0.268), (0.393, 0.456)]
29	0.0041	0.2350	0.0885	[(0.223, 0.291), (0.426, 0.494)]	[(0.284, 0.346), (0.471, 0.533)]
30	0.0139	0.2350	0.0798	[(0.181, 0.249), (0.384, 0.452)]	[(0.322, 0.385), (0.510, 0.572)]
31	0.0010	0.2350	0.0771	[(0.223, 0.291), (0.426, 0.494)]	[(0.206, 0.268), (0.393, 0.456)]
32	0.0159	0.0783	0.0473	[(0.223, 0.291), (0.426, 0.494)]	[(0.206, 0.268), (0.393, 0.456)]
33	0.0087	0.2350	0.0389	[(0.265, 0.333), (0.468, 0.536)]	[(0.361, 0.424), (0.549, 0.611)]
34	0.0075	0.1479	0.2656	[(0.223, 0.291), (0.426, 0.494)]	[(0.323, 0.385), (0.510, 0.572)]
35	0.0195	0.0696	0.1782	[(0.265, 0.333), (0.468, 0.536)]	[(0.439, 0.502), (0.626, 0.689)]

Table 4. The Euclidean and limit distances between items and the ideal solutions.

Item no.	Annual dollar usage (thousand $)		Lead time (day)		Average lot cost ($)		Limitation of warehouse space		Availability of the substitute raw material	
	$\sqrt{(n_{ij}-\nu_j^+)^2}$	$\sqrt{(n_{ij}-\nu_j^-)^2}$	$\sqrt{(n_{ij}-\nu_j^+)^2}$	$\sqrt{(n_{ij}-\nu_j^-)^2}$	$\sqrt{(n_{ij}-\nu_j^+)^2}$	$\sqrt{(n_{ij}-\nu_j^-)^2}$	LD_{PI}	LD_{NI}	LD_{PI}	LD_{NI}
1	0.7379	0.0090	0.2088	0.0174	0.1659	0.1817	0.866	0.231	0.996	0.133
2	0.7254	0.0214	0.1131	0.1131	0.2065	0.1411	0.799	0.280	0.933	0.182
3	0.7111	0.0358	0.1305	0.0957	0.0910	0.2566	0.333	0.622	0.866	0.231
4	0.7147	0.0322	0.1305	0.0957	0.0995	0.2481	0.399	0.573	0.266	0.670
5	0.6443	0.1026	0.1392	0.0870	0.1990	0.1487	0.133	0.768	0.533	0.475
6	0.0000	0.7469	0.1653	0.0609	0.2645	0.0832	0.399	0.573	0.799	0.280
7	0.6933	0.0535	0.1653	0.0609	0.3057	0.0420	0.533	0.475	0.799	0.280
8	0.6744	0.0725	0.1653	0.0609	0.2922	0.0555	0.333	0.622	0.799	0.280
9	0.7328	0.0141	0.1653	0.0609	0.3057	0.0419	0.733	0.329	0.933	0.182
10	0.7358	0.0111	0.0000	0.2263	0.0000	0.3477	0.200	0.719	0.933	0.182
11	0.7367	0.0102	0.0000	0.2263	0.0210	0.3267	0.200	0.719	0.866	0.231
12	0.7447	0.0022	0.1305	0.0957	0.0769	0.2707	0.599	0.426	0.400	0.573
13	0.7458	0.0010	0.1305	0.0957	0.0906	0.2570	0.666	0.378	0.466	0.524
14	0.7447	0.0022	0.2088	0.0174	0.3177	0.0300	0.933	0.182	0.996	0.133
15	0.7458	0.0011	0.2088	0.0174	0.3376	0.0101	0.866	0.231	0.996	0.133
16	0.7085	0.0384	0.2001	0.0261	0.3477	0.0000	0.733	0.329	0.933	0.182
17	0.1105	0.6363	0.2175	0.0087	0.2131	0.1345	0.029	0.866	0.599	0.426
18	0.7204	0.0265	0.1218	0.1044	0.2313	0.1163	0.799	0.280	0.799	0.280
19	0.7190	0.0278	0.2263	0.0000	0.2546	0.0930	0.733	0.329	0.666	0.378
20	0.6912	0.0556	0.1479	0.0783	0.2189	0.1287	0.666	0.378	0.029	0.866
21	0.7415	0.0054	0.2263	0.0000	0.2720	0.0756	0.666	0.378	0.933	0.182
22	0.7204	0.0265	0.2263	0.0000	0.2767	0.0709	0.866	0.231	0.866	0.231
23	0.7079	0.0390	0.0435	0.1827	0.2370	0.1106	0.799	0.280	0.733	0.329
24	0.7169	0.0299	0.0435	0.1827	0.3104	0.0372	0.933	0.182	0.933	0.182
25	0.7247	0.0221	0.0435	0.1827	0.3029	0.0447	0.866	0.231	0.533	0.475
26	0.7469	0.0000	0.0435	0.1827	0.3085	0.0392	0.996	0.133	0.866	0.231
27	0.7447	0.0021	0.0435	0.1827	0.3046	0.0431	0.933	0.182	0.996	0.133
28	0.7355	0.0114	0.0435	0.1827	0.2876	0.0600	0.933	0.182	0.933	0.182
29	0.7438	0.0031	0.0435	0.1827	0.2856	0.0621	0.799	0.280	0.799	0.280
30	0.7340	0.0129	0.0435	0.1827	0.2943	0.0534	0.866	0.231	0.733	0.329
31	0.7469	0.0000	0.0435	0.1827	0.2969	0.0507	0.799	0.280	0.933	0.182
32	0.7320	0.0149	0.2001	0.0261	0.3267	0.0209	0.799	0.280	0.933	0.182
33	0.7392	0.0077	0.0435	0.1827	0.3352	0.0124	0.733	0.329	0.666	0.378
34	0.7404	0.0065	0.1305	0.0957	0.1085	0.2391	0.799	0.280	0.733	0.329
35	0.7284	0.0185	0.2088	0.0174	0.1959	0.1518	0.733	0.329	0.533	0.475

$0.1 \leq w_2, \quad w_2 \leq 0.15,$

$w_3 \leq 0.35, \quad w_1 \geq w_3, \quad w_3 \geq w_2,$

$w_1 \geq 0, \quad w_2 \geq 0, \quad w_3 \geq 0.$

Model (2):

$$SE'_i = \max \sum_{j=4}^{5} \left(w'_j \frac{LD_{NI}}{LD_{PI} + LD_{NI}} \right)$$

for $\quad i = 1, \ldots 5,$

s.t.:

$$\sum_{j=41}^{5} w'_j \leq 1,$$

$0.2 \leq w'_4 \leq 0.55,$

$0.1 \leq w'_5 \leq 0.2,$

$w'_4 \geq w'_5$

$w'_4 \geq w_1$ (obtained from Model (1)),

Table 5. Measures RC with respect to the different criteria.

Item no.	Annual dollar usage (thousand $)	Lead time (day)	Average lot cost ($)	Limitation of warehouse space	Availability of the substitute raw material
1	0.0121	0.0769	0.5226	0.2105	0.1178
2	0.0287	0.5000	0.4060	0.2595	0.1632
3	0.0479	0.4230	0.7381	0.6513	0.2105
4	0.0431	0.4230	0.7136	0.5895	0.7158
5	0.1373	0.3846	0.4276	0.8523	0.4712
6	1.0000	0.2692	0.2393	0.5895	0.2595
7	0.0717	0.2692	0.1208	0.4712	0.2595
8	0.0970	0.2692	0.1596	0.6513	0.2595
9	0.0189	0.2692	0.1207	0.3097	0.1632
10	0.0148	0.9999	0.9999	0.7823	0.1632
11	0.0137	0.9999	0.9395	0.7823	0.2105
12	0.0030	0.4230	0.7785	0.4156	0.5889
13	0.0014	0.4230	0.7393	0.3620	0.5292
14	0.0029	0.0769	0.0862	0.1632	0.1178
15	0.0014	0.0769	0.0291	0.2105	0.1178
16	0.0514	0.1153	0.0000	0.3097	0.1632
17	0.8519	0.0384	0.3869	0.9675	0.4156
18	0.0355	0.4615	0.3346	0.2595	0.2595
19	0.0373	0.0000	0.2675	0.3097	0.3620
20	0.0745	0.3461	0.3703	0.3620	0.9675
21	0.0072	0.0000	0.2175	0.3620	0.1632
22	0.0355	0.0000	0.2041	0.2105	0.2105
23	0.0522	0.8076	0.3181	0.2595	0.3097
24	0.0401	0.8076	0.1072	0.1632	0.1632
25	0.0297	0.8076	0.1287	0.2105	0.4712
26	0.0000	0.8076	0.1128	0.1178	0.2105
27	0.0029	0.8076	0.1240	0.1632	0.1178
28	0.0152	0.8076	0.1727	0.1632	0.1632
29	0.0042	0.8076	0.1786	0.2595	0.2595
30	0.0173	0.8076	0.1536	0.2105	0.3097
31	0.0000	0.8076	0.1459	0.2595	0.1632
32	0.0199	0.1153	0.0602	0.2595	0.1632
33	0.0103	0.8076	0.0358	0.3097	0.3620
34	0.0087	0.4230	0.6877	0.2595	0.3097
35	0.0248	0.0769	0.4366	0.3097	0.4712

$$w'_4 \geq 0, \quad w'_5 \geq 0,$$

where the importance order of criteria has been adjusted based on the inventory managers' points of view and experiences.

Table 6 shows measures SE, SE$'$, total score (the larger score, the greater preference), and ABC classification based on the proposed method and other techniques in the literature.

In order to compare the results of the proposed model with those of other approaches, the data in Table 2 together with some settings are applied.

First, the authors compared the obtained results of their method with those of the VIKOR technique. According to Table 6, 33 items remained in the same classes. For example, two items characterized by the changed classes are Items 1 and 16 that have been moved to classes B and C, respectively. Although the order of rankings obtained in each class is somewhat different from our approach (for example, Item 6 is more important than Item 10), the similarity of the classes obtained from the two models can be a good reason for the effectiveness of our approach.

In the second status, the authors requested

Table 6. Measures SE, SE', total score, and MCABCIC based on the different methods.

Item no.	SE	SE'	Total score	The proposed model	VIKOR	The classical TOPSIS	The R-model	The Ng-model	The traditional ABC classification
17	0.5238	0.6149	1.1387	A	A (17)	A	A	A	A
10	0.5065	0.4627	0.9692	A	A (6)	A	A	A	C
11	0.4848	0.4721	0.9569	A	A (10)	A	A	A	C
6	0.5741	0.3758	0.9499	A	A (11)	A	A	A	A
5	0.2691	0.5628	0.8319	A	A (5)	A	A	A	A
4	0.3326	0.4669	0.7995	A	A (3)	B	A	B	B
3	0.3433	0.4000	0.7433	A	A (4)	B	B	A	B
12	0.3371	0.3458	0.6829	B	B (12)	A	A	B	C
13	0.3227	0.3049	0.6276	B	B (13)	A	B	B	C
16	0.4040	0.2025	0.6065	B	C	C	C	C	B
20	0.1624	0.3925	0.5549	B	B (20)	B	B	B	A
8	0.1399	0.4099	0.5498	B	B (34)	B	B	A	A
34	0.3081	0.2043	0.5124	B	B (8)	B	B	B	C
23	0.2560	0.2043	0.4603	B	B (23)	B	B	B	A
35	0.1755	0.2641	0.4396	B	B (2)	B	B	B	B
2	0.2300	0.1754	0.4054	B	B (18)	C	C	C	B
7	0.1149	0.2837	0.3986	B	B (35)	C	C	B	A
18	0.2023	0.1943	0.3966	B	B (7)	C	C	C	B
25	0.1796	0.2097	0.3893	C	C (25)	B	B	B	B
33	0.1383	0.2423	0.3806	C	C (24)	B	B	B	C
29	0.1855	0.1943	0.3798	C	C (30)	B	B	C	C
30	0.1827	0.1773	0.3600	C	C (29)	C	B	C	C
19	0.1104	0.2423	0.3527	C	C (19)	B	C	C	B
31	0.1722	0.1751	0.3473	C	C (28)	C	C	C	C
1	0.1997	0.1393	0.339	C	B	C	C	C	C
21	0.0792	0.2317	0.3109	C	C (31)	C	C	B	C
28	0.1882	0.1223	0.3105	C	C (33)	C	C	C	C
24	0.1767	0.1233	0.3000	C	C (27)	C	C	C	B
9	0.0911	0.2025	0.2936	C	C (22)	C	C	C	C
27	0.1658	0.1131	0.2789	C	C (9)	C	C	C	C
26	0.1606	0.1063	0.2669	C	C (21)	C	C	C	C
22	0.0784	0.1575	0.2359	C	C (26)	C	C	C	B
32	0.0473	0.1751	0.2224	C	C (32)	C	C	C	B
15	0.0224	0.1389	0.1613	C	C (14)	C	C	C	C
14	0.0431	0.1133	0.1564	C	C (15)	C	C	C	C

experts to apply the crisp numbers (1 to 7) instead of the language variables in Table 1 for assessing the qualitative variables and, then, implement the classical TOPSIS method for ranking items. According to the obtained results, only 23 out of the 35 items remained in the same classes. In the real world, experts may want to choose the middle numbers such as 1.5 with interval MF when evaluating the qualitative criteria. It cannot be satisfied by the crisp values. Thus, the linguistic variables, such as IT2FSs, are more suitable for such situations, resulting definitely in different results. This is the reason for the different classifications mentioned above.

Moreover, the results of our approach were compared to those of the R-model [1], in which the following constraint and scale transformation:

$w_4 \geq w_1 \geq w_3 \geq w_5 \geq w_2$,

$$\frac{x_{ij} - \min_{i=1,\dots,m}[x_{ij}]}{\max_{i=1,\dots,m}[x_{ij}] - \min_{i=1,\dots,m}[x_{ij}]},$$

and Eq. (27) were used to show the sequence of weights importance, converted into a 0-1 scale, and to determine the crisp measures of items with respect to the qualitative criteria, respectively. Only 25 items were reclassified into the same classes. The R-model is a compensatory approach, i.e., a significantly weak criterion value of an item could be directly compensated by other good criteria values. On the other hand, the weights of criteria for low measures may be zero when solving the model. These will lead to inappropriate rankings. For example, consider Items 18 and 25. Although Item 18 has higher measures than Item 25 with respect to the first, third, and fourth criteria that have higher weights in sequence $w_4 \geq w_1 \geq w_3 \geq w_5 \geq w_2$, it is in class B and moved to class C using R-model based on our approach.

As a further comparison of the crisp models, the authors utilized the transformed data in Ng-model [10]. According to the obtained results, 27 items remained unchanged in their classes. The drawbacks of the Ng-model are similar to those of the R-model. For example, consider Item 8. Since Item 8 has a higher partial average in relation to the fourth criterion, it was selected as class A without taking into account other partial averages. However, it was chosen as class B in the first four methods.

On the other hand, by comparing the results of the proposed model with traditional ABC classification, only 19 of the 35 items remained in the same classes. Obviously, the results obtained from our model differ from the traditional ABC classification due to the presence of the other four criteria.

8. Conclusions

In the real world, the selection of the best alternative with respect to conflicting criteria is a difficult and complex task when data are vague and inexact. Although type-1 fuzzy sets could greatly resolve ambiguities in decision problems, only a specified measure for MF was taken into account. Thus, type-2 fuzzy sets were applied to consider an interval in [0, 1] for MF when a decision-maker is uncertain about the value of MF. The MFs of the type-2 fuzzy sets can take different versions such as triangular, trapezoidal, and Gaussian. Since GIT2FNs have the smoother MF, the authors adopted them for evaluating the alternatives in relation to the qualitative criteria. On the other hand, since the MCABCIC problem is subject to the qualitative criteria that can be stated with type-2 fuzzy sets, inventory managers have ambiguity in relation to the value of the MF and cannot determine certain measure for it. Hence, this paper presented the TOPSIS method based on GIT2FNs in which a new LD was introduced to prioritize them. The proposed method first calculated $\hat{S}_i^+$ and $\hat{S}_i^-$ by depicting α-cuts and, then, measured distances from reference limits. It is also able to rank TriIT2FN, TraIT2FN, and other curved forms for both normal and subnormal cases. In order to show the effectiveness of the proposed methodology, it was implemented in a real case study. It included both the qualitative and quantitative criteria in an MCABCIC problem. The quantitative data were extracted from the inventory section, whereas the qualitative data were obtained from appraisals of experts. Because of the proposed non-compensatory approach and the usage of type-2 fuzzy sets, the results obtained by our methodology showed more logical results when comparing the crisp methods (the classical TOPSIS, the Ng-model, and the R-model) with the traditional ABC classification, as described above.

Some important directions for further researches are as follows:

1. Managers can carry out this approach to other manufacturing factories or service organizations;
2. Other criteria or sub-criteria may be taken into account in other MCABCIC problems;
3. The distance between alpha cuts was determined 0.1 when LDs were calculated. In order to obtain more accurate results, one can adopt the smaller values for the distance between alpha cuts such as 0.05 or 0.01;
4. The proposed ranking approach is applicable to other areas of mathematics such as statistics (such as normal distribution) and DEA;
5. The proposed approach is also applicable to other MCDM techniques including VIKOR and ELECTRE, in addition to TOPSIS.

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