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Full versus partial coordination in serial N-echelon supply chains and a new profit-sharing contract

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Abstract. Despite the significance of full coordination of N-echelon supply chains in real-world decision-making situations, the relevant literature has rarely addressed this issue. Furthermore, there is a scarcity of mathematical models in the supply chain management literature for partially coordinated cases. To address these shortcomings, this study concerns both the full and partial coordination in serial N-echelon supply chains facing stochastic demand. In particular, three general cases including decentralized (no coordination), sub-supply chain coordination (partial coordination), and centralized (full coordination) cases are examined to support decisions on ordering and pricing. In addition, this study adds to the literature by investigating how to coordinate a serial N-echelon supply chain through a new spanning profit-sharing contract, which can coordinate the entire supply chain through only one contract. Furthermore, this study analytically proves the occurrence of externality benefit in partially coordinated cases, which is a paradoxical phenomenon suggesting that small coalitions are unstable. Two numerical examples extracted from the literature are given to verify the effectiveness and validity of the proposed contracts and models. The results show that the proposed contracts can be applied in a rather simple and convenient way and are reliable enough for use in real-world applications.

1. Introduction

Coordination and integration are widely regarded as two important building blocks of a Supply Chain (SC) to improve the competitiveness of its members [1] and to help them increase profitability. Due to the crucial importance of three well-known flows of information, goods and fund components [2] in order to fulfill customer needs and expectations at minimum cost, it becomes essential for SC members to adopt effective strategies to confront demand uncertainties. In addition, the pressures of competition today require SC members to cooperate and collaborate with each other [3]. Despite the independency of SC agents, they can benefit from different coordination mechanisms including information technology, information sharing, coordination contracts, and joint decision-making [4]. In order to eliminate the potential mismatch between demand and supply, various coordination contracts have been proposed in the literature, including, but not limited to, revenue-sharing [5-9], buy-back or return

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policies [10-12], downward direct discounts [13], option contracts [14] and even a combination of two or more contracts (e.g. [11,15]). The supply chain coordination can be attained by specifying the trade parameters such as pricing and order quantity [5,16].

For a coordination contract to be effective in coordinating SC members, two desirable features must be presented simultaneously [6,11]. First, the contract should achieve the same performance measure (i.e., profit) as in the centralized case. Second, it should lead to a win-win situation in the sense that cooperating is more beneficial than acting alone for each member. The first feature, namely coordination situation, implies that the decentralized case under a coordination contract behaves like a centralized case in which the entire SC is virtually governed by the decision-maker, whereas the second feature, namely a win-win situation, means that each member's expected profit in a coordinated situation is higher than that in a non-coordinated situation. In fact, as is emphasized by van der Rhee et al. [6], these two desirable features do not imply each other. Therefore, when implementing a coordination contract, it is important to ensure that both full coordination and win-win situations are satisfied.

While many researchers deal with full coordination through contracts, there seems to be, relatively, little concern about partial coordination. It is well known that one of the main objectives of a coordination contract is to prevent sub-optimization or partial coordination caused by coalitions between some members [6]. This fact probably explains why only very few studies have been dedicated to partial coordination (see, for example, [17,18] for partial coordination between adjacent members in three-echelon supply chains).

As a matter of fact, in practice, a typical supply chain involves multiple echelons [6,7,11,12]. For instance, a typical wood-based panel supply chain is comprised of at least four echelons including multiple suppliers, a manufacturer of Medium Density Fiberboard (MDF) panels, multiple distributers, and multiple retailers. As another real-world example, Ding and Chen [12] stated that some firms, such as Lenovo, HP, and Nokia, procure their components from suppliers and sell finished products through various retailers in a three-echelon supply chain. In another relevant but different context, a laptop supply chain is also considered to be a three-echelon supply chain [7]. Nevertheless, among the few research works on coordination contracts in multi-echelon supply chains, the focus is often placed on supply chains with only three echelons. This can be partly attributed to the complexity of efficiently making an agreement among supply chain members with different and often conflicting goals. In practice, a common type of multi-echelon supply chains is the one with a serial structure, which is already studied in different contexts, from production smoothing to coordination contracts (see, for example [6,9,19-22]). Therefore, this study is conducted to bridge these gaps in part. Specifically, this study not only examines full coordination in a serial N-echelon supply chain and proposes a new profit-sharing contract to coordinate players, but also considers the partial coordination cases.

The rest of the paper is organized as follows: In Section 2, the related literature is reviewed briefly. The previous results of coordination in two- and three-echelon supply chains are revisited and described in Section 3 to lay a foundation of research for more general cases in N-echelon supply chains. Section 4 focuses more narrowly on partial coordination in either upstream or downstream direction in N-echelon supply chains. Full coordination through the spanning profit-sharing contracts and solution concepts are discussed in Section 5. Two numerical examples extracted from the literature are given to illustrate the applicability and effectiveness of the proposed contracts and models in Section 6. Finally, Section 7 presents the conclusions and highlights the main contributions of this study.

2. Literature review

There are three main research streams in the literature pertinent to this study: a newsvendor problem, coordination contracts, and an application of the game theory in supply chains. For a detailed review and discussion on the first research stream, i.e., newsvendor problem, the reader is referred to Silver et al. [23], Khouja [24], Qin et al. [25], Choi [26], and the references therein. In their seminal textbook entitled "Inventory management and production planning and scheduling", Silver et al. [23] presented a partial review of a Single-Period Problem (SPP) in the context of newsvendor-type items. Khouja [24] extended the review of Silver et al. [23] to include the studies that received little or no coverage. Qin et al. [25] presented an extensive review of the newsvendor problem with a focus on customer demand, supplier pricing policies, and buyer risk. Choi [26] recently conducted a research to cover both general and specific topics in the field, including a wide variety of mathematical models, extensions, and some real-world applications of newsvendor problem (cf., the first panel of Table 1).

For a detailed review and discussion on the second research stream, i.e., application of the game theory in supply chains, the reader is referred to Li and Whang [27], Cachon and Netessine [28], Nagarajan and Sošić [29], and the references therein (cf. the second panel of Table 1). A summary is provided in Table 1.

For a detailed review and discussion on the third research stream, i.e., coordination contracts, the reader

Table 1. Selected reviews and surveys on newsvendor problem, application of game theory in supply chains, and coordination contracts.

Research objective	Reference	Context
Newsvendor (NV) problem	Silver et al. [23]	A partial review of a Single-Period Problem (SPP)
	Khouja [24]	A detailed review of SPP, including a taxonomy
	Qin et al. [25]	Review of newsvendor problems
	Choi [26]	Models, extensions, and applications of newsvendor problems
Application of game theory in supply chains	Li and Whang [27]	Application of game theory in OR/MS settings
	Cachon and Netessine [28]	Static and dynamic models
	Nagarajan and Sošić [29]	Review and extensions of cooperative games
Coordination contracts	Tsay, Nahmias, and Agrawal [2]	Review of supply chain contracts including a taxonomy
	Sahin and Robinson [30]	The role of flow coordination and information sharing in supply chains
	Cachon [31]	Investigating different coordination contracts
	Arshinder et al. [16]	Coordination mechanisms including contracts
	Chiu and Choi [32]	Coordination contracts with mean-variance objectives
	Govindan et al. [4]	Review of coordination contracts in forward and reverse supply chains

is referred to Tsay et al. [2], Sahin and Robinson [30], Cachon [31], Arshinder et al. [16], Chiu and Choi [32], Govindan et al. [4], and the references therein. Tsay et al. [2] reviewed the research works on coordination contracts and provided a taxonomy for classifying the related problems. Sahin and Robinson [30] investigated over 100 research works to analyze some coordination contracts in supply chains, especially with a focus on the role of flow coordination and information sharing. Cachon [31] provided an in-depth survey of coordination contracts from different perspectives. Arshinder et al. [16] addressed different coordination mechanisms, including coordination contracts, in supply chains to cope with uncertainty. Chiu and Choi [32] extensively reviewed the literature on some selected coordination contracts with a focus on mean-variance models in supply chains under a single- or multi-period horizon. Govindan et al. [4] presented a comprehensive literature review of 234 research works published between 1961 and 2012 on forward and reverse supply chains and provided a framework for classifying various contracts (see, the third row of Table 1).

As mentioned earlier, in contrast to the increasing number of studies on coordination contracts in two-echelon supply chains, there is rather limited literature on this topic in N-echelon supply chains [6,17]. Table 2 provides a summary of the selected studies related to full and partial coordination in N-echelon supply chains.

To the best of our knowledge, the related works closest to this study are those by Van der Rhee et al. [6] and Seifert et al. [17]. Van der Rhee et al. [6] proposed a new revenue-sharing contract to coordinate all members in a rather convenient manner. Specifically, they found that extending the classic revenue sharing contract from an SC with two echelons to one with N-echelons has major drawbacks, such as the problem of signing simultaneously $N-1$ pairwise contracts. In order to overcome such drawbacks, they proposed a spanning revenue-sharing contract. Seifert et al. [17] studied partial coordination in a three-echelon supply chain consisting of a supplier, a manufacturer, and a retailer.

However, our work is significantly different from the previous studies in the following aspects, as shown in Table 2:

1. Seifert et al. [17] focused on a partial coordination between adjacent members in three-echelon supply chains (i.e., supplier-manufacturer and manufacturer-retailer). In a different context, Zhang and Liu [18] considered two partially

Table 2. Selected studies related to full and partial coordination in N-echelon supply chains.

Reference	Coordination		The number of echelons	Coordination mechanism(s)
	Full coordination	Partial coordination		
Van der Rhee et al. [6]	✓	×	n	Spanning revenue sharing
Feng et al. [7]	✓	×	3	Revenue sharing with reliability (RSR)
Hou et al. [8]	✓	×	3	Revenue sharing (RS)
Van der Rhee et al. [9]	✓	×	4	Spanning Revenue Sharing (SRS)
Giri et al. [11]	✓	×	3	Buy-back with sales rebate and penalty (SRP) Buy-back with two-way SRP
Ding and Chen [12]	✓	×	3	Flexible buy-back
Saha [13]	✓	×	3	Downward direct discount
Seifert et al. [17]	×	✓	3	×
Zhang and Liu [18]	✓	✓	3	Revenue sharing (RS) Shapley value Asymmetric Nash negotiation
Giannoccaro and Pontrandolfo [34]	✓	×	3	Revenue sharing (RS)
This paper	✓	✓	n	Spanning profit sharing (SPS) Myerson value

coordinated cases between adjacent members in a three-echelon green SC and, then, computed the allocations using three methods including the Shapley value. This study considers not only partial coordination, but also full coordination in N-echelon supply chains;

2. It is shown that, due to externality benefit in supply chains as described in later sections for serial supply chains, great care must be exercised in implementing solution concepts, which need the value of coalitions to be taken into account for computing the allocation of profits.

   This study also shows that, in the presence of external benefit, implementing the Shapley value becomes misleading, as wrongly used by some researchers such as Zhang and Liu [18]. In this regard, it is necessary to apply those solution concepts developed for games in Partition Function Form (PFF games) such as the Myerson value [33], which well fits with such situations, as is applied in this study;
3. This study extends the models proposed for partial coordination from a three-echelon chain to serial N-echelon supply chains;
4. This study proposes a new coordination contract, namely a spanning profit-sharing contract, to coordinate serial N-echelon supply chains through only one contract instead of $N-1$ pairwise contracts.

3. Notations, assumptions, and model formulations

3.1. Notations

The technical notations shown in Table 3 are used in the paper.

In addition, superscripts "cc" and "dc" are used to denote the centralized and decentralized chains, respectively. For the sake of clarity, subscripts "sc" and "*" denote the supply chain and the optimal solution, respectively. Other technical notations are introduced when needed.

3.2. Basic assumptions

In order to keep the models mathematically tractable, the following three assumptions are made:

- **Assumption 1.** Without loss of generality, there exists no salvage revenue or holding cost for the left-over inventory, or penalty costs for unsatisfied demand at the end of selling season. This assumption is supported by many independent studies on coordination contracts (see, for example [6,18,35,36]). Interestingly, He and Zhang [35] analytically showed that this assumption did not affect the generality of the discussion and, consequently, simplified the subsequent analysis;
- **Assumption 2.** The most upstream member(s) in a supply chain is the Stackelberg leader, and the most downstream member(s) is the Stackelberg

Table 3. Technical notations.

i, j	Indices representing the position of members in an N-echelon supply chain, $i = 1, 2, \cdots, \|N\|$
w_i	Per-unit wholesale price, decision variables
Q	Retailer's order quantity, a decision variable
c_i	Per-unit marginal cost of member i
c	The total cost of the supply chain, $c = \sum_{i=1}^{\|N\|} c_i$
X	Random variable representing the market demand with support of the form $[L, U]$ for $0 \leq L < U \leq \infty$
$F(x)$	Cumulative probability function of X
$f(x)$	Probability density function of X
$S(Q)$	The expected selling quantity, $S(Q) = E(\min(Q, X))$
π	Random variable representing the profit
Π	Expected profit, $\Pi = E(\pi)$

follower. This assumption is also very common in the coordination contracts literature (see, for example [8, 15,17-19,36]);

- **Assumption 3:** $\boldsymbol{p > c}$, $\boldsymbol{p > w_2 + c_1}$, $\boldsymbol{w_j > w_{j+1} + c_j}$. This common assumption is made to avoid trivial cases and ensure that each member is willing to participate.

Note that these assumptions are not restricted to modeling purposes.

3.3. Model formulations (two benchmark cases)

Before turning to the main subject matter of this study, a brief overview and analysis of research on two-echelon supply chains is presented, followed by a natural extension to three-echelon supply chains (see, for example [17,31,36]). A two-echelon SC consisting of two risk neutral players, namely a manufacturer (she) and a retailer (he), provides a firm theoretical basis for further discussions on full and partial coordination in N-echelon supply chains. The product is assumed to be a single newsvendor-type item characterized by short life-cycle and long lead time (e.g., fashion and fast fashion goods) in a single period facing stochastic demand. Demand X is a random variable with a Probability Distribution Function (PDF), $f(x)$, and a Cumulative Distribution Function (CDF), $F(x)$, with the mean and standard deviation of μ and σ, respectively. In a two-echelon SC, the manufacturer's wholesale price and retailer's order quantity are formally considered as decision variables. All other parameters are assumed to be exogenously given and are common knowledge for all supply chain members.

3.3.1. Centralized case - Benchmark 1

In the centralized case, the entire SC is assumed to be a unified system with a decision-maker having the following profit function:

$$\pi_{sc}^{cc}(Q) = p\min(x, Q) - cQ. \tag{1}$$

Because of the stochastic nature of demand X, the profit of supply chain, $\pi_{sc}^{cc}(Q)$, is also a stochastic variable. Using the expectation operator, it is well known that:

$$S(Q) = E(\min(x, q)) = \int_0^Q (1 - F(x))dx$$

$$= Q - \int_0^Q F(x)dx.$$

Substituting this value in Eq. (1) yields:

$$\Pi_{sc}^{cc}(Q) = E(\pi_{sc}^{cc}(Q)) = pS(Q) - cQ. \tag{2}$$

It is straightforward, from Eq. (2), to show that the SC's expected profit is strictly concave based on the second-order derivative, which is equal to $-pf(Q) < 0$. Therefore, the optimal order quantity of this model is obtained by:

$$Q_*^{cc} = F^{-1}\left(\frac{p - c}{p}\right) = \bar{F}^{-1}\left(\frac{c}{p}\right). \tag{3}$$

This is a standard newsvendor problem with the overage cost (c_o) of $p - c$ and the underage cost (c_u) of c. Substituting Eq. (3) in Eq. (2) yields:

$$\Pi_{sc}^{cc}(Q_*^{cc}) = (p - c)Q_*^{cc} - p\int_0^{Q_*^{cc}} F(x)dx. \tag{4}$$

The results of the centralized case can be used as a benchmark for other cases, as shown in later Sections.

3.3.2. Decentralized case with Wholesale Price (WP) contract - Benchmark 2

In the decentralized case, each player seeks to maximize solely its own expected profit. In order to avoid confusion and provide a consistent approach to extending the results to N-echelon supply chains in the following sections, the buyer's and seller's expected profits are represented by subscripts "f" and "l", respectively, where f stands for the follower and l stands for the leader, as shown below:

$$\Pi_f^{dc}(Q) = pS(Q) - (w_l + c_f)Q, \tag{5}$$

$$\Pi_l^{dc}(Q) = (w_l - c_l)Q. \tag{6}$$

As stated in Assumption 2, several studies considered the upstream member (e.g., a manufacturer) as the Stackelberg leader and, subsequently, the downstream member (e.g., a retailer) as the Stackelberg follower (e.g. [8,15,17-19,36]). Such a setting is briefly called UP-Stackelberg game, in which the entire SC is controlled by a large upstream member (here a large manufacturer).

The sequence of events is as follows:

- First, the manufacturer determines the wholesale price taking the retailer's reaction function into consideration;
- Then, the retailer determines his/her order quantity, Q, and a request for the order quantity, Q, is sent to the manufacturer;
- The order quantity, Q, is shipped in a single shipment from the manufacturer to the retailer;
- Finally, market demand, X, is realized, and the retailer sells the amount $\min(Q, X)$ to market.

When the wholesale price is assumed to be exogenous (see, e.g. [17,34]), given a wholesale price determined by the upstream member (as the Stackelberg leader), the downstream member's optimal order quantity, which maximizes its own expected profit, is given by:

$$Q_*^{dc} = F^{-1}\left(\frac{p - w_l - c_f}{p}\right) = \bar{F}^{-1}\left(\frac{w_l + c_f}{p}\right). \tag{7}$$

It indicates the overage cost of $p - w_l - c_1$ and the underage cost of $w_l + c_1$. Substituting Eq. (7) in Eqs. (5) and (6) yields:

$$\Pi_{sc}^{dc}\left(Q_*^{dc}\right) = pS\left(Q_*^{dc}\right) - cQ_*^{dc} = (p - c)Q_*^{dc} - p\int_0^{Q_*^{dc}} F(x)dx. \tag{8}$$

However, in a more general setting where the wholesale price is assumed to be endogenous (see, e.g. [17,18]), as is the case in this paper, the inverse demand curve is given by:

$$w_l(Q) = p\bar{F}(Q) - c_f. \tag{9}$$

Substituting Eq. (9) into (6) and the first-order condition of Eq. (6) with respect to Q yields:

$$\frac{d\left(\Pi_l^{dc}(Q)\right)}{dQ} = p\bar{F}(Q)\left(1 - \frac{Qf(Q)}{\bar{F}(Q)}\right) - c = 0. \tag{10}$$

The Generalized Failure Rate (GFR)$z(x)$ of a random variable X with distribution $F(x)$ is first defined by Lariviere and Porteus [36] as follows:

$$z(x) = \frac{xf(x)}{\bar{F}(x)}. \tag{11}$$

By definition, when the Generalized Failure Rate (GFR) increases, the respective distribution is called the Increasing Generalized Failure Rate (IGFR) distribution. Argued by Lariviere and Porteus [36], the leader's margin profit is decreasing in Q if $z(Q)$ increases, and then $\Pi_l^{dc}(Q)$ is unimodal. As a result, the optimal order quantity in the decentralized two-echelon SC denoted by Q_*^{dc} satisfies:

$$p\bar{F}(Q) - Qf(Q) - c = 0. \tag{12}$$

Property 1. In a two-echelon supply chain in which the wholesale price is endogenous, if $F(x)$ is assumed to be IGFR, then we have:

(i) $\Pi_l^{dc}(Q)$ is unimodal;

(ii) $Q_*^{dc} < Q_*^{cc}$;

(iii) $\Pi_{sc}^{dc}(Q_*^{dc}) = \Pi_f^{dc}(Q_*^{dc}) + \Pi_l^{dc}(Q_*^{dc}) < \Pi_{sc}^{cc}(Q_*^{cc})$.

Proof. The proof is straightforward and, thus, omitted here.

Property 1 asserts that $\Delta = \Pi_{sc}^{cc}(Q_*^{cc}) - \Pi_{sc}^{dc}(Q_*^{dc}) > 0$, which is consistent with the findings of Van der Rhee et al. [6]. The value of $\Delta > 0$ is called the "surplus" in the game theory literature [37]. From the practical point of view, the question is how to share this surplus among involved members [12,18]. This study as shown in Section 5 provides new answers to this crucial question.

Similar to interactions between members in a two-echelon SC, it is important to investigate the interactions among a supplier, a manufacturer, and a retailer in a three-echelon SC in which a supplier sells some raw material to a manufacturer who in turn produces some newspaper-type products in order to sell to a retailer who resells them to the market (see, Figure 1). In the three-echelon SC, the supplier's wholesale price of raw materials, the manufacturer's wholesale price of the finished product, and the retailer's order quantity are all considered as decision variables.

The sequence of events is as follows:

- First, the supplier determines the wholesale price of raw materials taking the manufacturer's reaction function into consideration;

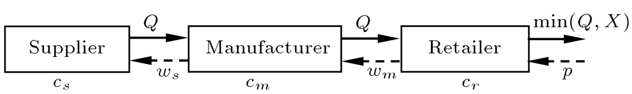

Figure 1. Three-echelon supply chain.

- Then, the manufacturer determines the wholesale price of the finished products taking the retailer's reaction function into consideration;
- Subsequently, the retailer determines his/her order quantity, Q, and a request for Q is sent to the manufacturer;
- A request for the same order quantity, Q, is sent from the manufacturer to the supplier;
- The order quantity, Q, is shipped in a single shipment from the supplier all the way to the retailer;
- Finally, market demand, X, is realized and the retailer sells the amount, $\min(Q,X)$, to market.

In the following, subscript "s" denotes the supplier, "m" denotes the manufacturer, and "r" denotes the retailer. Additionally, superscripts "(SMR)" and "(S,M,R)" denote the centralized and decentralized cases, respectively. The expected profit for each member is as follows:

$$\Pi_r^{(S,M,R)}(Q) = pS(Q) - (w_m + c_r)Q, \tag{13}$$

$$\Pi_m^{(S,M,R)}(Q) = (w_m - c_m - w_s)Q, \tag{14}$$

$$\Pi_s^{(S,M,R)}(Q) = (w_s - c_s)Q. \tag{15}$$

The total expected profit in the decentralized three-echelon supply chain is given by:

$$\Pi_{sc}^{(S,M,R)}(Q) = \Pi_r^{(S,M,R)}(Q) + \Pi_m^{(S,M,R)}(Q) + \Pi_s^{(S,M,R)}(Q) = pS(Q) - cQ. \tag{16}$$

It is worth noting that the total expected profit and the optimal order quantity in the fully coordinated (i.e., centralized) three-echelon supply chain are the same as that in the two-echelon supply chain, as shown by Eqs. (2) and (3), respectively. Certainly, this will be true in a serial N-echelon SC regardless of the number of echelons. Now, let us consider the decentralized case, in which both the supplier and manufacturer offer a Wholesale Price (WP) contract to their respective downstream member. When the wholesale price is assumed to be endogenous, using the first-order condition of Eq. (13) with respect to Q, the inverse demand curve for the retailer is as follows:

$$w_m(Q) = p\bar{F}(Q) - c_r. \tag{17}$$

Now, the manufacturer's expected profit anticipating the response function of the retailer is given by:

$$\Pi_m^{(S,M,R)}(Q) = pQ\bar{F}(Q) - (w_s + c_r + c_m)Q. \tag{18}$$

Subsequently, the first-order condition of Eq. (18) with respect to Q yields:

$$\frac{d\left(\Pi_m^{(S,M,R)}(Q)\right)}{dQ} = p\bar{F}(Q) - pQf(Q) - (w_s + c_r + c_m) = 0. \tag{19}$$

Then, the inverse demand curve for the manufacturer is given by:

$$w_s(Q) = p\bar{F}(Q) - pQf(Q) - (c_r + c_m). \tag{20}$$

The supplier's profit anticipating the response function of the manufacturer is given by:

$$\Pi_s^{(S,M,R)}(Q) = pQ\bar{F}(Q) - pQ^2 f(Q) - cQ. \tag{21}$$

The first-order condition of Eq. (21) with respect to Q yields:

$$\frac{d\left(\Pi_s^{(S,M,R)}(Q)\right)}{dQ} = p\bar{F}(Q) - p\left[3Qf(Q) + Q^2 f'(Q)\right] - c = 0. \tag{22}$$

As a result, the supplier's order quantity satisfies:

$$p\bar{F}(Q) - p\left[3Qf(Q) + Q^2 f'(Q)\right] = c. \tag{23}$$

Eq. (23) can be rewritten by:

$$p\bar{F}(Q)\left[1 - \left(3z(Q) + \frac{Q^2 f'(Q)}{\bar{F}(Q)}\right)\right] = c.$$

Lemma 1. Let us assume that $F(x)$ is IGFR and $\left(3z(Q) + \frac{Q^2 f'(Q)}{\bar{F}(Q)}\right)$ is increasing in Q. Then:

(i) $\Pi_s^{dc}(Q)$ is concave in $[0, \bar{Q}_s]$ and decreasing in $[\bar{Q}_s, \infty]$, where $\bar{Q}_s$ is the greatest $Q \geq 0$, for which the following inequality holds:

$$\left(3z(Q) + \frac{Q^2 f'(Q)}{\bar{F}(Q)}\right) \leq 1.$$

(ii) Any solution Q_s^* of Eq. (23) is unique and must lie in the interval $[0, \bar{Q}_s]$. The optimal order quantity for the supplier is Q_s^* according to Eq. (23).

Proof. See Seifert et al. [17] for the proof of their Theorem 3.

Theorem 1. Suppose that $F(x)$ is IGFR, and the term $\left(3z(Q) + \frac{Q^2 f'(Q)}{\bar{F}(Q)}\right)$ increases in Q. Then:

(i) The optimal order quantity in a decentralized three-echelon SC (i.e., the (S,M,R) case) is less than that in a decentralized two-echelon SC;

(ii) The expected profit of a decentralized three-echelon SC is less than that of a decentralized two-echelon SC.

Proof

(i) The proof follows from comparing Eqs. (12) and (23) by considering the assumptions specified above;

(ii) The proof follows from the proof of part (i) together with the fact that the expected profit of the decentralized chain increases in Q.

The proof is complete. □

Theorem 1 implies that the efficiency of a decentralized three-echelon SC is less than that of a decentralized two-echelon SC. It is not hard to prove that the results of Theorem 1 can be generalized.

Remark 1. The efficiency of a decentralized N-echelon SC is strictly decreasing in N.

4. Partial coordination

4.1. Partial coordination in a three-echelon SC

As mentioned before, rather than coordinating all SC members using some coordinating contracts (e.g., revenue-sharing and buy-back contracts), a few researchers (e.g. [17,18]) studied partially coordinated cases between only two (adjacent) members in a three-echelon SC. The superscript "(S, MR)" denotes the downstream coordinated case and "(SM, R)" denotes the upstream coordinated case. First, consider the downstream coordinated case in which the manufacturer and retailer form a small coalition and are coordinated with a total expected profit, $\Pi_{mr}(Q)$. To be consistent with the notations used earlier, the expected profits in the downstream case denoted by the setting (S, MR) can be expressed by:

$$\Pi_{mr}^{(S,MR)}(Q) = pS(q) - (w_s + c_m + c_r)Q, \tag{24}$$

$$\Pi_{s}^{(S,MR)}(Q) = (w_s - c_s)Q. \tag{25}$$

Setting (S, MR) is modeled using a Stackelberg game, in which the supplier acts as the Stackelberg leader and the manufacturer-retailer coalition acts as the Stackelberg follower. It is worth noting that $\Pi_{mr}(Q)$ and $\Pi_s(Q)$ can be written as $\Pi_f(Q)$ and $\Pi_l(Q)$, respectively, in Eqs. (5) and (6), with $c_f = c_m + c_r$, $c_l = c_s$, and $w_l = w_s$. As a matter of fact, by using this simplified technique first proposed by Seifert et al. [17], a partially coordinated three-echelon SC can be converted to a two-echelon SC and can be solved more efficiently.

Then, let us consider the upstream coordinated case, in which the supplier and manufacturer form a small coalition and are coordinated with a total profit, $\Pi_{sm}(Q)$, as follows:

$$\Pi_{sm}^{(SM,R)}(Q) = (w_m - c_m - c_r)Q, \tag{26}$$

$$\Pi_{r}^{(SM,R)}(Q) = pS(Q) - (w_m + c_r)Q. \tag{27}$$

Setting (SM, R) is then modeled using a Stackelberg game, in which the supplier-manufacturer coalition acts as the Stackelberg leader and the retailer acts as the Stackelberg follower. It is also worth noting that $\Pi_r(Q)$ and $\Pi_{sm}(Q)$ can be written as $\Pi_f(Q)$ and $\Pi_l(Q)$, respectively, in Eqs. (5) and (6), with $c_f = c_r$, $c_l = c_m + c_s$, $w_l = w_m$.

Property 2. Suppose a three-echelon SC with two upstream and downstream coordinated cases. Then:

(i) The optimal order quantity and the expected profit of the scheme (S, MR) are less (more) than those of the centralized (decentralized) case, that is:

$$Q_*^{(S,M,R)} < Q_*^{(S,MR)} < Q_*^{(SMR)},$$

and:

$$\Pi_{sc}^{(S,M,R)}(Q) < \Pi_{sc}^{(S,MR)}(Q) < \Pi_{sc}^{(SMR)}(Q).$$

(ii) The optimal order quantity and the expected profit of the scheme (SM, R) are less (more) than those of the centralized (decentralized) case, that is:

$$Q_*^{(S,M,R)} < Q_*^{(SM,R)} < Q_*^{(SMR)},$$

and:

$$\Pi_{sc}^{(S,M,R)}(Q) < \Pi_{sc}^{(SM,R)}(Q) < \Pi_{sc}^{(SMR)}(Q).$$

Proof. The proof is straightforward and, thus, omitted here. □

Theorem 2. Suppose that $F(x)$ is IGFR in a three-echelon SC with two upstream and downstream coordinated cases. Then:

(i) $\Pi_s^{(S,MR)}(Q) > \Pi_s^{(S,M,R)}(Q)$,

(ii) $\Pi_r^{(SM,R)}(Q) > \Pi_r^{(S,M,R)}(Q)$.

Proof

(i) The proof follows from the result of Lemma 1 (i) and Property 2 (i);

(ii) In a similar manner, the proof of this part follows from the result of Lemma 1 (ii) and Property 2 (ii). □

Theorem 2 implies the occurrence of a paradoxical phenomenon in partially coordinated cases. This phenomenon that is called "external benefit" in economics (see, for example [38-40]) has been less investigated in the SCM literature. In other words, when two members form a small coalition in a three-echelon SC to weaken the position of the third member, the one left out of the coalition, the result is surprisingly quite different in a sense that the third member also benefits from the coalition of two other members. This Theorem also implies that small coalitions are potentially vulnerable and, thus, unstable. Therefore, the members of the small coalition have a strong incentive to disrupt the small coalition. The practical importance of this theorem is that the structure of a serial SC is collusion-proof, which means that if the grand coalition is formed, there is then no reason for the SC members to leave the grand coalition. To deal with these coalition-related issues is, however, beyond the scope of this study and is left for future works.

Remark 2. Externality benefit in a serial N-echelon supply chain reflects the indirect benefit that the coalition of $N-1$ adjacent players imposes on the player who is left out of the coalition.

4.2. Partial coordination in an N-echelon SC

4.2.1. Problem description

After extending the model from two-echelon to three-echelon SC, now the next step is to generalize the results, in a natural way, to model a general N-echelon SC with finite members. Without loss of generality, Member 1 and member n, respectively, denote the most downstream and the most upstream member. Member 1 is a retailer facing stochastic demand in a market, and member n is a supplier providing raw materials to a manufacturer. On intermediary links of the chain, there exists a wide variety of different members, such as various manufacturers, processing units (e.g., assembling, sorting, inspecting, packaging, and labeling), and many distributors. The member j ($j = 1, 2, \cdots, n$) is denoted by j, where subscript j denotes the position of an associated member in a serial N-echelon SC, such that $2 \leq n < \infty$. Following the previous notations, the profit functions of SC members can be written as follows:

$$\Pi_1(Q) = pS(Q) - (w_2 + c_1)Q, \tag{28}$$

$$\Pi_j(Q) = (w_j - c_j - w_{j+1})Q,$$

$$\text{for} \quad j = 2, 3, \cdots, n, \tag{29}$$

where c_j and w_j denote the marginal cost and selling price of member j, respectively.

Note that since the expected profit of member n is $\Pi_n(Q) = (w_n - c_n)Q$, it is assumed like [6] that $w_{n+1} = 0$.

4.2.2. Partial coordination

In the upstream scheme, all SC members except for the retailer join together to form the upstream coalition. The expected profit function of the upstream coalition is as follows:

$$\Pi_{\text{upstream}}(Q) = \left(w_2 - \sum_{j=2}^{n} c_j \right) Q. \tag{30}$$

Comparing Eqs. (30), (5) and (6), one can clearly see that $c_f = c_1 = c_r$, $c_l = \sum_{j=2}^{n} c_j$, and $w_l = w_2$. Therefore, the procedure for obtaining the optimal order quantity of the upstream coordinated case is straightforward.

In a downstream scheme, all SC members except for the supplier join together to form the downstream coalition. The expected profit function of the downstream coalition is as follows:

$$\Pi_{\text{downstream}}(Q) = pS(Q) - \left(w_n + \sum_{j=1}^{n-1} c_j \right) Q. \tag{31}$$

Comparing Eqs. (31), (5) and (6), one can clearly see that $c_f = \sum_{j=1}^{n-1} c_j$, $c_l = c_n$, and $w_l = w_n$. Therefore, the procedure for obtaining the optimal order quantity of the downstream coordinated case is also straightforward.

5. Full coordination in an N-echelon SC

An N-echelon SC as depicted in Figure 2 comprises n members with Member 1 (as the Stackelberg follower)

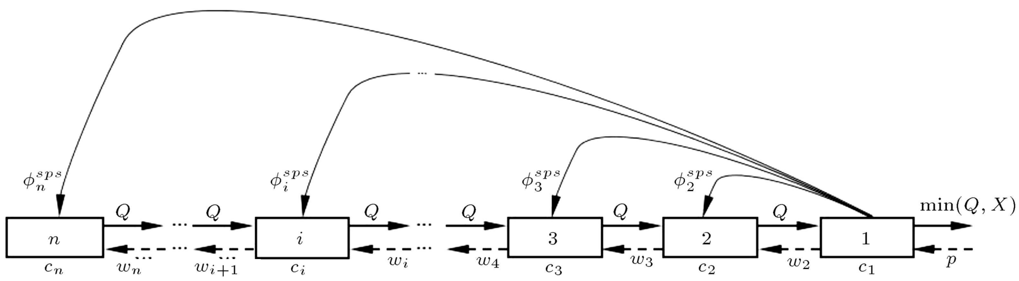

Figure 2. Spanning profit sharing contract in an N-echelon supply chain.

in the most downstream position and member n (as the Stackelberg leader) at the most upstream position. Each member j ($j = 2, 3, \cdots, n-1$) acts as the follower of its immediate upstream member and the leader of its immediate downstream member. The sequence of events is as follows:

- First, member n (i.e., supplier) determines its wholesale price w_n, taking member $n-1$'s reaction function into consideration;
- Then, member j ($j = n-1, n-2, \cdots 2$) determines its wholesale price w_j, taking member $j-1$'s reaction function into consideration;
- Subsequently, Member 1 (i.e., retailer) decides an order quantity, Q, and a request for the order quantity is sent to Member 2;
- A request for the same order quantity, Q, is sent from Member 2 to Member 3 and, in the same way, is passed on from one to another until it reaches member n;
- The order quantity, Q, is shipped in a single shipment from member n all the way to Member 1;
- Finally, market demand, X, is realized, and Member 1 sells the amount $\min(Q, X)$ to the market.

In this section, the full coordination of the supply chain is examined by proposing a new coordination mechanism, namely the Spanning Profit Sharing (SPS) contract that is characterized by a set of parameters $(w_2^{sps}, \cdots, w_n^{sps}; \phi_2^{sps}, \cdots, \phi_n^{sps})$, where w_j is the wholesale prices of member j ($j = 2, \cdots, n$), and ϕ_j is the percentage of the profit of Member 1 shared with member j, such that $0 \leq \phi := \sum_{j=2}^{n} \phi_j \leq 1$ (see Figure 2). The expected profit of each member under the proposed SPS contract is as follows:

$$\Pi_1^{sps}(Q) = (1-\phi)\left(pS(Q) - (w_2^{sps} + c_1)\,Q\right), \qquad (32)$$

$$\Pi_j^{sps}(Q) = \left(w_j^{sps} - c_j - w_{j+1}^{sps}\right) Q + \phi_j^{sps}(pS(Q) - (w_2^{sps}+c_1)Q), \quad \text{for} \quad j=2,\cdots,n. \qquad (33)$$

Before accepting a new coordination contract, each member needs to make sure that a lower threshold or a target level of profit, which is sometimes called Reservation Accepted Profit (RAP), meets the contract terms [41]. Some researchers have considered the expected profit of each member in the decentralized case as a reasonable value for the RAP (see e.g. [41,42]), and some other researchers have considered an exogenous value as RAP (see e.g. [43]). In fact, since none of these two approaches affects the results of the problem, without loss of generality, one of them can be chosen depending on the application. In this study, the former is chosen, which is $\text{RAP}_j = \Pi_j^{dc}(Q_*^{dc})$, for $j = 1, 2, \cdots, n$.

Remark 3. When $\phi_j^{sps} = 0$, for $j = 2, 3, \cdots, n$, the SPS contract reduces to a wholesale price contract. In other words, the wholesale price contract studied in Section 3.3.2 is a special case of the SPS contract with $\phi_j^{sps} = 0$, for $j = 2, 3, \cdots, n$.

Theorem 3. The SPS contract can coordinate the supply chain with the following parameters:

$$w_2^{sps} = c - c_1 = \sum_{j=2}^{n} c_j, \qquad (34)$$

$$w_{j+1}^{sps} \leq w_j^{sps} - c_j + \left(\phi_j^{sps}\Pi_{sc}^{cc}(Q_*^{cc}) - \Pi_j^{dc}(Q_*^{dc})\right)/Q^{sps}; \quad \text{for} \quad j=2,3,\cdots,n. \qquad (35)$$

Proof. We first prove that, under Conditions (34) and (35), the SPS contract is able to remain coordinated with the SC; then, we show that these conditions are certainly satisfied. The retailer determines the optimal order quantity to maximize his expected profit as given by Eq. (32). The optimal order quantity under the SPS contract is given by:

$$Q_*^{sps} = F^{-1}\left(\frac{p - w_2^{sps} - c_1}{p}\right). \qquad (36)$$

Under Condition (34), $Q_*^{sps} = Q_*^{cc}$. Therefore, the total expected profit of the supply chain under the SPS contract becomes:

$$\Pi_{sc}^{sps}(Q_*^{sps}) = \sum_{j=1}^{n} \Pi_j^{sps}(Q_*^{sps}) = \Pi_{sc}^{cc}(Q_*^{cc}),$$

which satisfies the necessary condition (full coordination) for coordination. In order to prove the sufficient condition (win-win situation), it is enough to show that implementing the SPS contract leads to $\Pi_i^{sps}(Q) \geq \Pi_j^{dc}(Q_*^{dc})$, for $i = 1, \cdots, n$. The expected profit of Member 1 using Eq. (32) under Condition (34) can be written as:

$$\Pi_1^{sps}(Q_*^{sps}) = (1-\phi)\Pi_{sc}^{cc}(Q_*^{cc}).$$

Subsequently, the expected profit of member j ($j = 2, \cdots, n$), using Eq. (33), can be written as in:

$$\Pi_j^{sps}(Q_*^{sps}) = \left(w_j^{sps} - c_j - w_{j+1}^{sps}\right) Q_*^{sps} + \phi_j^{sps}\Pi_{sc}^{cc}(Q_*^{cc}).$$

In order to show that the condition $\Pi_j^{sps}(Q_*^{sps}) \geq \Pi_j^{dc}(Q_*^{dc})$, or equivalently, the condition $w_{j+1}^{sps} \leq w_j^{sps} - c_j + (\phi_j^{sps}\Pi_{sc}^{cc}(Q_*^{cc}) - \Pi_j^{dc}(Q_*^{dc}))/Q^{sps}$ holds, it is sufficient to show that the set of parameters

$(w_2^{sps}, \cdots, w_n^{sps}; \phi_2^{sps}, \cdots, \phi_n^{sps})$ is non-empty. One feasible set of parameters can be defined as:

$$w_j^{sps} = c - \sum_{i=1}^{j-1} c_i = \sum_{i=j}^{n} c_i,$$

and:

$$\phi_j^{sps} = \left(\Pi_j^{dc}(Q_*^{dc}) + \frac{\Delta}{n}\right) / \Pi_{sc}^{cc}(Q_*^{cc}),$$

for $j = 2, 3, \cdots, n$, respectively. This set of parameters always guarantees that:

$$\Pi_j^{sps}(Q_*^{sps}) = \Pi_j^{dc}(Q_*^{dc}) + \frac{\Delta}{n} \geq \Pi_j^{dc}(Q_*^{dc}),$$

for $j = 2, 3, \cdots, n$. Substituting:

$$\phi = \sum_{j=2}^{n} \phi_j = \sum_{j=2}^{n} \left[\left(\Pi_j^{dc}(Q_*^{dc}) + \frac{\Delta}{n}\right) / \Pi_{sc}^{cc}(Q_*^{cc})\right],$$

into:

$$\Pi_1^{sps}(Q_*^{sps}) = (1 - \phi)\Pi_{sc}^{cc}(Q_*^{cc}),$$

and some straightforward algebra yields:

$$\Pi_1^{sps}(Q_*^{sps}) = \Pi_1^{dc}(Q_*^{dc}) + \frac{\Delta}{n},$$

which completes the proof. □

In terms of sharing the surplus, Theorem 3 implies that there is a wide variety of the set of parameters under the SPS contract that can arbitrarily be chosen depending on the negotiation power of players, as discussed by Cachon [31], Giannoccaro and Pontrandolfo [34] and Van der Rhee et al. [6] for other coordination contracts. In this regard, Van der Rhee et al. [6] proposed two special cases for tuning the set of parameters in the spanning revenue sharing contract:

1. An equal absolute increase in profit;
2. An equal relative increase in profit.

The same approach can be adopted for the SPS contract proposed in this study. Therefore, two above-mentioned special cases for tuning the set parameters are examined here (see Theorems 4 and 5). Additionally, another relevant, yet different, approach to sharing the surplus will be obtained by using the Myerson value, which is a fundamental solution concept from game theory in the presence of the externality benefit (see Section 5.1).

Theorem 4. Consider an SPS contract in a serial N-echelon supply chain with the set of parameters $(w_2^{sps}, \cdots, w_n^{sps}; \phi_2^{sps}, \cdots, \phi_n^{sps})$. In addition, suppose that the following conditions hold for $j = 2, \cdots, n$:

$$w_j^{sps} = c - \sum_{i=1}^{j-1} c_i = \sum_{i=j}^{n} c_i, \quad \text{for} \quad j = 2, \cdots, n, \tag{37}$$

(i) If for $j = 2, \cdots, n$:

$$\phi_j^{sps} = \frac{\Pi_j^{dc}(Q_*^{dc}) + \frac{\Delta}{n}}{\Pi_{sc}^{cc}(Q_*^{cc})}, \tag{38}$$

and $\phi = \sum_{j=2}^{n} \phi_j$, then, under the SPS contract, each member earns an equal absolute increase in profit (i.e., $\Pi_i^{sps}(Q) = \Pi_i^{dc}(Q) + \frac{\Delta}{n}$, for $i = 1, \cdots, n$);

(ii) If for $j = 2, \cdots, n$:

$$\phi_j^{sps} = \frac{\left(1 + \frac{\Delta}{\Pi_{sc}^{dc}(Q_*^{dc})}\right) \Pi_j^{dc}(Q_*^{dc})}{\Pi_{sc}^{cc}(Q_*^{cc})}, \tag{39}$$

and $\phi = \sum_{j=2}^{n} \phi_j$, then, under the SPS contract, each member earns an equal relative increase in profit (i.e., $\Pi_i^{sps}(Q) = \left(1 + \frac{\Delta}{\Pi_{sc}^{dc}(Q_*^{dc})}\right) \Pi_i^{dc}(Q_*^{dc})$, for $i = 1, \cdots, n$).

Proof. The proofs are omitted for brevity and are available on request from the authors. □

Theorem 5. Consider a spanning profit sharing contract in a serial N-echelon supply chain with the set of parameters $(w_2^{sps}, \cdots, w_n^{sps}; \phi_2^{sps}, \cdots, \phi_n^{sps})$. In addition, suppose that the following conditions hold for $j = 2, \cdots, n$:

$$w_2^{sps} = c - c_1, \qquad \text{and} \qquad w_j^{sps} = 0,$$

$$\text{for} \qquad j = 3, \cdots, n. \tag{40}$$

(i) If for $j = 2, \cdots, n$:

$$\phi_j^{sps} = \frac{\Pi_j^{dc}(Q_*^{dc}) + \frac{\Delta}{n} - (w_j^{sps} - c_j)Q_*^{sps}}{\Pi_{sc}^{cc}(Q_*^{cc})}, \tag{41}$$

and $\phi = \sum_{j=2}^{n} \phi_j$, then, under the SPS contract, each member earns an equal absolute increase in profit (i.e., $\Pi_i^{sps}(Q_*^{sps}) = \Pi_i^{dc}(Q_*^{dc}) + \frac{\Delta}{n}$, for $i = 1, \cdots, n$);

(ii) If for $i = 2, \cdots, n$:

$$\phi_i^{sps} = \frac{\left(1 + \frac{\Delta}{\Pi_{sc}^{dc}(Q_*^{dc})}\right) \Pi_i^{dc}(Q_*^{dc}) - (w_j^{sps} - c_j)Q_*^{sps}}{\Pi_{sc}^{cc}(Q_*^{cc})},$$

$$\text{for} \quad j = 2, \cdots, n, \tag{42}$$

and $\phi = \sum_{j=2}^{n} \phi_j$, then, under the SPS contract, each member earns an equal relative increase in profit (i.e., $\Pi_i^{sps}(Q_*^{sps}) = \left(1 + \frac{\Delta}{\Pi_{sc}^{dc}(Q_*^{dc})}\right) \Pi_i^{dc}(Q_*^{dc})$, for $i = 1, \cdots, n$).

Proof. The proofs are omitted for brevity and are available on request from the authors. □

Note that the results obtained under the SPS contract are entirely consistent with those obtained under the SRS contract proposed by Van der Rhee et al. [6]. Additionally, the SPS contract is rather simpler and more convenient than the SRS contract.

5.1. Myerson value

As already mentioned, due to externality benefit in small coalitions, implementing the Shapley value in such situations is misleading [37]. In this section, the Myerson value, as one of the most appealing solution concepts in PFF games, is used to share the profit of a centralized N-echelon SC among members. Myerson [33] axiomatically proved a modified Shapley value for PFF games given by:

$$\psi_i^{\text{Myerson}}(v)=\sum_{(S,T)\in ECL}(-1)^{|T|-1}(|T|-1)!\left(\frac{1}{|N|}-\sum_{\substack{\tilde{S}\in T\\ \ni\tilde{S}\neq S\\ n\notin\tilde{S}}}\frac{1}{(|T|-1)\left(|N|-\left|\tilde{S}\right|\right)}\right)v(S,T),\tag{43}$$

where ECL denotes the set of embedded coalitions, that is, $ECL=\{(S,T)|S\in T\in PT\}$, in which PT is referred to as the set of partitions of N; $v(S,T)$ denotes the value of coalition S in partition T. The interested reader can find the details in Myerson [33].

In addition to the Myerson value, there exist a number of solution concepts developed for PFF games; however, discussion and comparison of these are obviously beyond the scope of this study (see, for example [37,44] for more details).

Lemma 2. In a serial three-echelon SC, the Myerson value for each player is given by Eqs. (44)-(46) as follows:

$$\begin{aligned}\psi_1^{\text{Myerson}}(v)=&\frac{1}{3}v_{\{1,2,3\},\{\{1,2,3\}\}}+\frac{1}{6}v_{\{1,2\},\{\{1,2\},\{3\}\}}\\&-\frac{1}{3}v_{\{3\},\{\{1,2\},\{3\}\}}+\frac{2}{3}v_{\{1\},\{\{1\},\{2,3\}\}}\\&-\frac{1}{3}v_{\{2,3\},\{\{1\},\{2,3\}\}}+\frac{1}{6}v_{\{2\},\{\{1\},\{2\},\{3\}\}}\\&+\frac{1}{6}v_{\{3\},\{\{1\},\{2\},\{3\}\}}-\frac{1}{3}v_{\{1\},\{\{1\},\{2\},\{3\}\}},\end{aligned}\tag{44}$$

$$\begin{aligned}\psi_2^{\text{Myerson}}(v)=&\frac{1}{3}v_{\{1,2,3\},\{\{1,2,3\}\}}+\frac{1}{6}v_{\{2,3\},\{\{1\},\{2,3\}\}}\\&-\frac{1}{3}v_{\{1\},\{\{1\},\{2,3\}\}}+\frac{1}{6}v_{\{1,2\},\{\{1,2\},\{3\}\}}\\&-\frac{1}{3}v_{\{3\},\{\{1,2\},\{3\}\}}+\frac{1}{6}v_{\{1\},\{\{1\},\{2\},\{3\}\}}\\&+\frac{1}{6}v_{\{3\},\{\{1\},\{2\},\{3\}\}}-\frac{1}{3}v_{\{2\},\{\{1\},\{2\},\{3\}\}},\end{aligned}\tag{45}$$

$$\begin{aligned}\psi_3^{\text{Myerson}}(v)=&\frac{1}{3}v_{\{1,2,3\},\{\{1,2,3\}\}}+\frac{1}{6}v_{\{2,3\},\{\{1\},\{2,3\}\}}\\&-\frac{1}{3}v_{\{1\},\{\{1\},\{2,3\}\}}+\frac{2}{3}v_{\{3\},\{\{3\},\{1,2\}\}}\\&-\frac{1}{3}v_{\{1,2\},\{\{3\},\{1,2\}\}}+\frac{1}{6}v_{\{2\},\{\{1\},\{2\},\{3\}\}}\\&+\frac{1}{6}v_{\{1\},\{\{1\},\{2\},\{3\}\}}-\frac{1}{3}v_{\{3\},\{\{1\},\{2\},\{3\}\}}.\end{aligned}\tag{46}$$

Proof. The proof is directly obtained using Eq. (43) for $|N|=3$ and considering the fact that $v_{\{1,3\},\{\{1,3\},\{2\}\}}=v_{\{2\},\{\{1,3\},\{2\}\}}=0$, because there is no coalition between Players 1 and 3.

6. Numerical examples

In order to demonstrate the validity and efficiency of the proposed models and contracts, in this section, two examples taken from the literature are presented. The first example (Section 6.1) deals with the application of partial coordination in computing the allocations among players, whereas the second one (Section 6.2) concerns the full coordination of an N-echelon SC under the proposed SPS contract.

6.1. Example 1

The dataset of this example is taken from Zhang and Liu [18]. The results are shown in Table 4, in which first column represents four different settings:

1. (S,M,R) or decentralized case;
2. (S,MR) or downstream coordinated case;
3. (SM,R) or upstream coordinated case;
4. (SMR) or centralized case.

The second column represents the total expected profit for each case. The next three columns represent the expected profits of the supplier, the manufacturer, and the retailer, respectively. Finally, the last column gives the well-known efficiency ratio that is defined as:

$$\text{Efficiency}=\Pi_y/\Pi^{cc},$$

for:

$$y\in\{(S,M,R),(S,MR),(SM,R),(SMR)\}.$$

Table 4. Results of the models in decentralized, centralized, and sub-coordination cases.

Cases	Π_{sc}	Π_s	Π_m	Π_r	Efficiency
(S, M, R) or decentralized case	$\frac{7}{64}\kappa$[a]	$\frac{1}{16}\kappa$	$\frac{1}{32}\kappa$	$\frac{1}{64}\kappa$	6.25
(S, MR) or downstream coordinated case	$\frac{3}{16}\kappa$	$\frac{1}{8}\kappa$	$\pi_{mr} =$	$\frac{1}{16}\kappa$	25
(SM, R) or upstream coordinated case	$\frac{3}{16}\kappa$	$\pi_{sm} =$	$\frac{1}{8}\kappa$	$\frac{1}{16}\kappa$	25
(SMR) or centralized case	$\frac{1}{4}\kappa$	—	—	—	100

[a] κ is a constant.

Table 5. Comparison of allocations computed by the Shapley value and Myerson value.

Allocations	Zhao and Liu's results (Shapley value)	Our results (Myerson value)	Difference (%)
Player 1	$\varphi_1 = 0.385\Pi_{SC} = 0.096\kappa$[a]	$\psi_1 = 0.531\Pi_{SC} = 0.133\kappa$	+37.84
Player 2	$\varphi_2 = 0.448\Pi_{SC} = 0.112\kappa$	$\psi_2 = 0.219\Pi_{SC} = 0.055k$	-51.11
Player 3	$\varphi_3 = 0.167\Pi_{SC} = 0.042\kappa$	$\psi_3 = 0.250\Pi_{SC} = 0.062\kappa$	+49.70
Total Profit	$\Pi_{SC} = \varphi_1 + \varphi_2 + \varphi_3 = 0.250k$	$\Pi_{SC} = \psi_1 + \psi_2 + \psi_3 = 0.250k$	0

[a] κ is a constant.

As can be seen in Table 4, the decentralized or (S, M, R) case has the least efficiency ratio, while the centralized or (SMR) case has the most one. The significant difference between these two values shows the high importance of full coordination across the supply chain. Two partially coordinated cases have efficiency ratios that fall within the range of these two extreme values. Note that the efficiency ratios of partially coordinated cases are more than that of the decentralized case; however, both are substantially less than that of the centralized case. This further emphasizes the significance of full coordination among players. Two important observations are worth mentioning:

1. Surprisingly, in both (S, MR) and (SM, R) cases, the non-participating member benefits from the small coalition formed between the other two players, verifying the occurrence of externality benefit;
2. The rate of improvement in profit of the non-participating member is much more than that of coalition members for both partially coordinated cases.

For instance, consider the (S, MR) case in which the coalition "MR" is formed by the manufacturer and the retailer in the hope of achieving higher profits and weakening the position of the supplier, which is outside the coalition. The total profit of the coalition "MR" becomes $\frac{1}{16}\kappa$, which amounts to an about 33.3% increase compared to that in the (S, M, R) case. The profit of the supplier becomes $\frac{1}{8}\kappa$, which amounts to an about 100% increase compared to that in the (S, M, R) case. Therefore, the third player (i.e., supplier) is reluctant to join a new coalition with the manufacturer. The same situation will be reached in the (SM, R) case.

In summary, there are a number of fundamental challenges faced by SC members when small coalitions are taken into account out of the "grand" coalition. As a side result, the partially coordinated cases are unstable in serial N-echelon supply chains. These observations are consistent with the studies of Zhang and Liu [18] and Seifert et al. [17] with different interpretations of the meaning. As mentioned earlier, in order to overcome the large gap between the centralized and the decentralized cases, there are two main approaches in the literature in general:

1. Cooperative games and using solution concepts to divide the profits after cooperation;
2. Coordination contracts and using contract parameters to divide the profits after coordination.

Zhang and Liu [18] adopted the former and used some solution concepts, including the Shapley value, to divide the profits among the players. The results are summarized in the second column of Table 5.

Although widely used, the Shapley value has significant limitations when dealing with PFF games [37,44]. However, a common alternative in such cases is to apply the modified solution concepts appropriate for PFF games, as is the case in this example. In this study, a modified Shapley value is used (also known as the Myerson value), as developed by Myerson [33]. The results, using Eqs. (44)-(46), are summarized in the third column of Table 5. As expected, the results obtained by these two well-known solution concepts

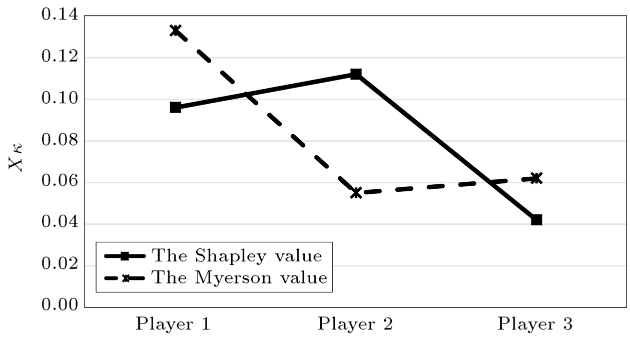

Figure 3. Allocations based on the Shapley and Myerson values.

(i.e., the Shapley value and the Myerson value) are naturally different, because they come from different backgrounds. It can be seen when using the Shapley value that the manufacturer and supplier have the least and the most profits, respectively. On the other hand, when using the Myerson value, the supplier and retailer have the least and the most profits, respectively. In fact, when the classic Shapley value is used in a three-echelon SC in the presence of externality benefit, the manufacturer gains much more benefit at the expense of two other players (see Figure 3). That is the reason why there has been a growing interest in the cooperative game literature to develop new and reliable solution concepts that are appropriate for PFF games (see, for example [44] for more details).

6.2. Example 2

The data set of this example is taken from Van der Rhee et al. [6] for a serial three-echelon supply chain in which the demand is assumed to be uniformly distributed with the support $[0, B]$. The optimal order quantity and the expected profit in both the centralized and decentralized cases are presented in Table 6.

Note that the optimal order quantity in the centralized and decentralized cases is identical with the values obtained by Eqs. (3) and (23), respectively. Similarly, the expected profits in the centralized and decentralized cases, respectively, are identical with the values obtained by Eqs. (4) and (16), demonstrating a large gap between these two expected profits ($\Delta = \Pi_{sc}^{cc} - \Pi_{sc}^{dc} = \frac{7}{16}\Pi_{sc}^{cc}$). The spanning profit sharing contract can be implemented to bridge this large gap, as shown in Table 7.

The parameters for achieving an equal absolute increase in profit can be obtained in two ways:

1. Based on Theorem 4 (i), the parameters are set to be $(w_2^{sps}, w_3^{sps}; \phi_2^{sps}, \phi_3^{sps}) = (c_2 + c_3, c_3; \frac{5}{16}, \frac{7}{16})$, clearly implying that $\Phi_1^{sps} = \frac{1}{4}$. Subsequently, the expected profit of each member, Π_j^{sps}, is calculated according to Eqs. (32) and (33). The values of w_j^{sps}, ϕ_j^{sps}, and π_j^{sps} are presented in the second to fourth columns of Table 7. As can be seen in the fifth column of this table, the absolute increase of the profit of each member is equal, which is consistent with the results of Theorem 4 (i);
2. Based on Theorem 5 (i), the parameters are set to be $(w_2^{sps}, w_3^{sps}; \phi_2^{sps}, \phi_3^{sps}) = (c_2 + c_3, 0; \frac{5}{16} - \frac{c_3}{p-c}, \frac{7}{16} + \frac{c_3}{p-c})$, clearly implying that $\phi_1^{sps} = \frac{1}{4}$. Subsequently, the expected profit of each member Π_j^{sps} is calculated according to Eqs. (32) and (33). The values of π_j^{sps} are essentially identical with those presented in the fifth column of Table 7, clearly leading to equal absolute increases in the profit of each member, which is entirely consistent with the results of Theorem 5 (i).

The parameters for achieving an equal relative increase in profit can be obtained in two ways:

1. Based on Theorem 4 (ii), the parameters are set to be $(w_2^{sps}, w_3^{sps}; \phi_2^{sps}, \phi_3^{sps}) = (c_2 + c_3, c_3; \frac{2}{7}, \frac{4}{7})$, clearly implying that $\phi_1^{sps} = \frac{1}{7}$. Subsequently, the

Table 6. Results of the models in the decentralized and centralized cases.

Settings	Q	Π_1	Π_2	Π_3	Total	Efficiency
(1, 2, 3) or decentralized case	$B\left(\frac{p-c}{4p}\right)$	$\frac{1}{16}\Pi_{cc}$	$\frac{1}{8}\Pi_{cc}$	$\frac{1}{4}\Pi_{cc}$	$\Pi_{sc}^{dc} = \frac{7}{16}\Pi_{sc}^{cc}$	6.25
(123) or centralized case	$B\left(\frac{p-c}{p}\right)$	—	—	—	$\Pi_{sc}^{cc} = B\frac{(p-c)^2}{p}$	100

Table 7. Expected profits of three-echelon SC members under the SPS contract.

SC	Equal absolute increase in profit				Equal relative increase in profit			
members	ϕ_j^{sps}	w_j^{sps}	Π_j^{sps}	$\Pi_j^{sps} - \Pi_j^{dc}$	ϕ_j^{sps}	w_2^{sps}	Π_j^{sps}	$(\Pi_j^{sps} - \Pi_j^{dc})/\Pi_j^{dc}$
Player 1	$\frac{1}{4}$	N/A[a]	$\frac{B(p-c)^2}{8p}$	$\frac{3}{16}\Pi_{sc}^{cc}$	$\frac{1}{7}$	N/A	$\frac{B(p-c)^2}{14p}$	$\frac{9}{7} = (1 + \Delta/\Pi_{sc}^{dc})$
Player 2	$\frac{5}{16}$	$c_2 + c_3$	$\frac{5B(p-c)^2}{32p}$	$\frac{3}{16}\Pi_{sc}^{cc}$	$\frac{2}{7}$	$c_2 + c_3$	$\frac{B(p-c)^2}{7p}$	$\frac{9}{7} = (1 + \Delta/\Pi_{sc}^{dc})$
Player 3	$\frac{7}{16}$	c_3	$\frac{7B(p-c)^2}{32p}$	$\frac{3}{16}\Pi_{sc}^{cc}$	$\frac{4}{7}$	c_3	$\frac{2B(p-c)^2}{7p}$	$\frac{9}{7} = (1 + \Delta/\Pi_{sc}^{dc})$
Total	1	—	$\frac{B(p-c)^2}{2p}$	$\frac{9}{16}\Pi_{sc}^{cc}$	1	—	$\frac{B(p-c)^2}{2p}$	—

[a] "N/A" means Not Applicable.

expected profit of each member, Π_j^{sps}, is calculated according to Eqs. (32) and (33). The values of w_j^{sps}, ϕ_j^{sps}, and Π_j^{sps} are presented in the sixth to eighth columns of Table 7. As can be seen in the last column of this table, the relative increases in profit of each member are equal, which is consistent with the results of Theorem 4 (ii);

2. Based on Theorem 5 (ii), the parameters are set to be $(w_2^{sps}, w_3^{sps}; \phi_2^{sps}, \phi_3^{sps}) = (c_2 + c_3, 0; \frac{2}{7} - \frac{c_3}{p-c}, \frac{4}{7} + \frac{c_3}{p-c})$, clearly implying that $\phi_1^{sps} = \frac{1}{7}$.

Subsequently, the expected profit of each member, Π_j^{sps}, is calculated according to Eqs. (32) and (33). The values of Π_j^{sps} are essentially identical with those presented in a column next to the column of Table 7 and clearly lead to equal relative increases in the profit of each member, which is entirely consistent with the results of Theorem 5 (ii).

7. Conclusion

This study is one of few efforts to characterize ordering and pricing decisions for both full and partial coordination cases in N-echelon supply chains. For partial coordination purposes, two cases were examined, namely upstream and downstream coordination cases. Then, the optimal order quantity and wholesale prices of the N-echelon supply chain were obtained, using the results of models in a two-echelon supply chain. In addition, the findings of this study showed that, due to externality benefit in partially coordinated cases, neither the upstream coordinated case nor the downstream coordinated one was a stable coalition, thus demonstrating the necessity of full coordination among all members. Moreover, great care should be exercised when dealing with partially coordinated cases and, more specifically, when applying solution concepts. In this regard, the Myerson value as one of the most appealing solution concepts developed for games in the partition function form is applied to take the externality benefit into consideration. In addition, the centralized and decentralized cases were examined to serve as two common benchmarks. The large gap between the profits of the centralized and decentralized cases indicates the necessity of coordination to improve the supply chain performance under demand uncertainties. Furthermore, for full coordination purposes, a new profit-sharing contract was proposed and proved to be very effective in coordinating N-echelon supply chains in a rather simple and convenient way, resulting in win-win situations for all members, too. The proposed Spanning Profit Sharing (SPS) contract is flexible enough in a sense that appropriate tuning of contract parameters leads to desired allocations by arbitrarily dividing the total expected profit of the supply chain into various ratios among supply chain members. Moreover, numerical analysis results verified the effectiveness and validity of the proposed contracts and models for improving the supply chain performance under demand uncertainties.

The main contributions of this study can be summarized as follows:

1. Considering not only full coordination, but also partial coordination in serial N-echelon supply chains;
2. Examining the effect of externality benefit on the expected profits of both participating and non-participating members in small coalitions (i.e., partially coordinated cases), and implementing the appropriate solution concepts for dividing the profit of the entire supply chain among members in the presence of such situations;
3. Extending the proposed fully and partially coordinated models from the three-echelon to serial N-echelon supply chains;
4. Proposing a Spanning Profit Sharing (SPS) contract to coordinate all members in serial supply chains through only one contract instead of $N-1$ pairwise coordination contracts.

The findings of this study can be a good starting point for future research works on developing other spanning coordination contracts in N-echelon supply chains. In addition, investigating partial coordination between non-adjacent members was beyond the scope of this study and, therefore, left for future studies. Considering the risk attitude of supply chain members can also be an interesting option to pursue. Further extensions can include predicting possible coalitions in supply chains, particularly when the grand coalition is hard to form due to some practical limitations such as information sharing between supply chain members.

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