Adaptive dynamic surface control of flexible-joint robot with parametric uncertainties

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ABSTRACT: A new kind of adaptive dynamic surface control (DSC) method is proposed to overcome parametric uncertainties of flexible-joint (FJ) robots. These uncertainties of FJ robots are transformed into linear expressions of inertial parameters which are estimated based on the DSC, and the high-order derivatives in DSC are solved by using first-order filter. The adaptation laws of inertial parameters are designed directly to improve the tracking performance according to the Lyapunov stability analysis. Simulation results for a two-link FJ robot show the better tracking accuracy against model parametric uncertainties. The method used does not need aid of Neural Network (NN), and is simpler and calculation faster than the other adaptive methods.

Keywords: FJ robot; dynamic surface control; inertial parameters; adaptive control; tracking accuracy.

1. Introduction

Flexible-joint (FJ) robots equipped with harmonic drivers, windup shaft and force/torque sensors are used widely in the areas such as aerospace, service robots, and so on [1-3]. Because of the uncertain dynamic parameters such as inertia and flexibility, and the external disturbances, the tracking performance of robots is not ideal generally and even the systems are unstable under the basic control methods.

In the last decades, many researchers have proposed and developed the backstepping control technique because of its advantages of systematic and recursive design methodology in nonlinear control. However its disadvantage is “explosion of complexity” or “explosion of derivative” caused by the repeated differentiation of the virtual control vectors, which results in the heavy computing burden [4, 5]. To overcome the complexity, the DSC is investigated by introducing a first-order filter in the backstepping procedure [6]. Because of the great approximation capability of nonlinear functions, the neural networks (NNs) are widely utilized in the control systems to compensate the uncertainties of parameter, and combined with the DSC and backstepping techniques to design the controller [7-14]. In [14] NN adaptive backstepping controller was proposed, and Radial Basic Function (RBF) was used to approximate the nonlinear unknowns in the backstepping design. Simulation results showed the good tracking performance.

Due to the adaptiveness of fuzzy, it is employed too in the control design [15-19]. In [15] the backstepping control design was suggested, and the adaptive fuzzy DSC was applied to approximate unknown nonlinear control system which was more general for practical applications in the presence of input saturation. The prescribed performance switched adaptive DSC was investigated for a class of switched nonlinear systems and mode-dependent fuzzy logic systems were used to approximate the switching nonlinear functions [16]. The adaptive fuzzy dynamic surface control was investigated for a class of nonlinear systems with fuzzy dead zone, unmodeled dynamics. The Takagi-Sugeno-type fuzzy logic systems were adopted to approximate the unknown functions in system [17]. [20] employed the fuzzy neural networks (FNNs) to approximate the uncertain dynamics and bounding functions when the active DSC was developed to suppress regenerative chatter in micro-milling.
Some disturbance observers (DOB) are employed commonly to design nonlinear controllers because of its distinct physical meaning [21-23]. In [21] a gain scheduled dynamic surface control (GSDSC) based on NN disturbance observer (NNDOB) was proposed for a class of uncertain underactuated mechanical systems, which efficiently solves the mismatches and overcomes the problem of “explosion of complexity”. In [22] a nonlinear disturbance observer was constructed to overcome the unknown environmental disturbances, then controller was designed based on the combination of backstepping and DSC techniques. An observer-based DSC scheme was developed to overcome the problems of model uncertainties, unsteady aerodynamics and actuator saturation in [23].

Sliding mode control (SMC) is one of the efficient control schemes for compensating the external disturbances and parametric uncertainties. In [24, 25] sliding mode control and adaptive control were introduced into the backstepping control technique together with DSC.

Besides, [26] designed an adaptive backstepping controller with dynamic surface method to solve the problem of parameters of permanent magnet synchronous motor and particle swarm optimization algorithm was adopted to adjust and determine the control parameters.

Although with a lot of advantages, the above approximate control schemes exist some problems such as the heavy calculation burden, long training time, approximating error, and so on, which result in the difficulty in choosing the control parameters and guaranteeing system stability.

In [27] the dynamic surface adaptive backstepping control scheme was employed to prevent the uncertainties of nonlinear dynamics of aircraft. By changing the inertial parameters and positions of aircraft center of gravity, the simulation showed the effectiveness of the controller. But the selected parameters were limited and changed slightly.

In this paper, a new adaptive DSC technique for FJ robots is proposed to overcome the problems of parametric uncertainties, explosion of derivative, and calculation inefficiency. First, model uncertainties of FJ robots are expressed as linear equations of its inertial parameters, thus the model uncertainties are transformed to compute the evaluation of inertial parameters. The controller is designed by the backstepping and adaptive DSC techniques. Then the adaptation laws for these inertial parameters are derived according to the Lyapunov stability analysis, which reduce the computing burden efficiently. Finally, simulations of the two-link FJ robot show effectiveness of the adaptive DSC technique.

This paper is organized as follows. In Section 2, the basic dynamic models and properties of FJ robot are introduced. In Section 3, the backstepping control design method is proposed according to the Lyapunov stability theorem. Simulation results and analysis are discussed in Section 4. Finally, some conclusions are given in Section 5.

2. Dynamic model of FJ robots

In general, the dynamic model of n-link FJ robot consists of robot dynamics and actuator dynamics, and they can be expressed as the following forms [28]:

\[ D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q - q_m) = 0 \]  
\[ J\ddot{q}_m + K(q_m - q) = u \]

where \( q, \dot{q}, \ddot{q} \in \mathbb{R}^n \) denote respectively the position, velocity and acceleration vectors of the links, \( D(q) \in \mathbb{R}^{n \times n} \) denotes the inertia matrix, \( C(q,\dot{q}) \in \mathbb{R}^n \) is the Coriolis and centripetal matrix, \( G(q) \in \mathbb{R}^n \) denotes the gravity matrix, \( K \in \mathbb{R}^{n \times n} \) represents a positive definite diagonal constant flexibility matrix, \( q_m, \dot{q}_m \in \mathbb{R}^n \) denote
respectively the position and acceleration vectors for the actuator, $J \in \mathbb{R}^n$ is the actuator inertia matrix, and $u \in \mathbb{R}^b$ is the actual control vector of actuator torque. In the dynamic equations (1) and (2), the joint stiffness terms are assumed to be dominant among all the parameters of the system and the joint damping ones are neglected. As introduced in literature [9], these equations have a few fundamental properties which can be used to make control system design simple. The properties are:

(P1) The link inertia matrix $D(q)$ is positive definite, symmetric and bounded:
$$D_m \leq \|D(q)\| \leq D_M$$

where $D_m$ and $D_M$ denote two different positive constants respectively.

(P2) The Coriolis and centripetal matrix $C(q,\dot{q})$ and gravity matrix $G(q)$ are bounded respectively:
$$\|C(q,\dot{q})\| \leq C_M, \quad \|G(q)\| \leq G_M$$

where $C_M$ and $G_M$ denote positive constants.

(P3) The matrix $\dot{D}(q) - 2C(q,\dot{q})$ is skew-symmetric. For any vector $x$,
$$x^T (\dot{D}(q) - 2C(q,\dot{q})) x = 0$$

(P4) The system parameters can be linear in the equation expressed as coefficients of known functions of $q, \dot{q}$ and $\ddot{q}$ for a rigid joint robot. So the left-hand side of equation (1) can be expressed as
$$D(q)\dddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta_i$$

where $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times r}$ denotes the regression matrix, $\theta_i \in \mathbb{R}^{r\times 1}$ a $r$-dimensional vector of parameters, and $r$ the number of inertia parameters.

Further, the equation (4) can be rewritten according to the literature [29] as:
$$D(q)a_i + C(q,\dot{q})v_i + G(q) = Y(q,\dot{q},v_i,a_i)\theta_i$$

where $a_i$ and $v_i$ are the arbitrary $n$-dimensional vectors.

As a preliminary of the control design, $x_1 = q_m$ and $x_2 = \dot{q}_m$ are defined as the functions of state space variables, and the dynamic system (1) and (2) are described as follows:
$$D(q)\dddot{q} + C(q,\dot{q})\dot{q} + G(q) = Kq = Kx_1$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = J^{-1}u - J^{-1}K(x_1 - q)$$

For developing the controller for the system (6)-(8), some assumptions [8, 22] are proposed.

Assumption 1 All the state space variables can be achieved by some sensors.

Assumption 2 The model parameters of $D(q), C(q,\dot{q}), G(q), K$ in equation (6) are unknown. But there exist the positive constants $K_m, K_M, J_m, J_M$ to make
$$K_m \leq \|K\| \leq K_M, J_m \leq \|J\| \leq J_M.$$
Assumption 3 The desired trajectories of link position $q_d$ are uniformly bounded, differentiable up to second order and satisfies the inequality:

$$\|q_d\|^2 + \|\dot{q}_d\|^2 + \|\ddot{q}_d\|^2 \leq Q$$

where $Q$ is the positive constant.

3. Adaptive DSC system

3.1. Controller Design

The system inertia parameters in FJ robots are classified into four: vectors $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$, which are composed of the elements in the link dynamic parametric matrix, the inverse of joint stiffness matrix, the joint stiffness matrix, and the actuator inertia matrix, respectively.

Thus, the parametric uncertainties in the robot model which are included in $D(q)$, $C(q, \dot{q})$, $G(q)$, $K^{-1}$, $K$ and $J$, can be expressed and solved by using the vectors $\theta_i (i = 1, 2, 3, 4)$.

Here the adaptive DSC algorithm is proposed according to the backstepping control design [4], and the design procedure includes three phases: (1) Design the virtual control vector $\overline{a}_i$ for actuator position $x_i$. (2) Design the virtual control vector $\overline{a}_i$ for actuator velocity $x_i$. (3) Design the actual control vector $u$. Details are shown as follows.

**Phase 1**

Choose a virtual control vector $\overline{a}_i$ for $x_i$ as

$$\overline{a}_i = q + \hat{K}^{-1}u_i$$

(9)

where $(\cdot)$ is the estimated value of $(\cdot)$, the auxiliary control vector $u_i$ is defined as

$$u_i = \hat{D}(q)a_i + \hat{C}(q, \dot{q})v_i + \hat{G}(q) - \Lambda r_i$$

(10)

where $v_i = \dot{q}_d - Aq$, $a_i = \dot{v}_i$, $r_i = \dot{q} - v_i = \dot{q} + A\dot{q}$, $q = q - q_d$, $A_i \in \mathbb{R}^{n \times n}$ and $A_i \in \mathbb{R}^{n \times n}$ are diagonal positive definite matrix [16], $q_d$ is the desired trajectory of link position.

The equation (10) can be rewritten by using (P4) as

$$u_i = Y(q, \dot{q}, v_i, a_i) \hat{\theta}_i - A_r r_i$$

(11)

To avoid the problem of “explosion of derivative”, the estimated value of derivative of $\overline{a}_i$ is derived by using the first order filter in the DSC. That is,

$$\tau, \dot{\alpha}_i + \alpha_i = \overline{a}_i, \quad \alpha_i (0) = \overline{a}_i (0)$$

(12)

where $\tau_i$ denotes a diagonal positive define matrix, and $\alpha_i$ the filtering control vector.

Define

$$z_i = x_i - \alpha_i$$

(13)

Substituting equation (13) into equation (6), it can be obtained as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + Kq = K\alpha_i + Kz_i$$

(14)
From (9), (12) and (14), yield
\[
D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + Kq + K\tilde{K}^{-1}u_i + Kz_i - K\tau_i\dot{\alpha_i} = Kq + u_i + K\left(\tilde{K}^{-1} - K^{-1}\right)u_i + Kz_i - K\tau_i\dot{\alpha_i} \tag{15}
\]
To get the estimated value of \(\hat{K}^{-1}\), the definition is given by
\[
\hat{K}^{-1}u_i = \text{diag}(u_i)\hat{\theta}_z \tag{16}
\]
where \(\text{diag}(\bullet)\) denotes the diagonal matrix composed of all the elements of \(\bullet\).

Then, substituting (16) and (P4) into (15) to yield
\[
D(q)\ddot{q} + C(q, \dot{q})\dot{q} + \Lambda_z r_i = D(q)\tilde{q} + C(q, \dot{q})v_i + \tilde{G}(q) + K\text{diag}(u_i)\hat{\theta}_i + Kz_i - K\tau_i\dot{\alpha_i}
\]
\[
= Y(q, \dot{q}, v, a_i)\hat{\theta}_i + K\text{diag}(u_i)\hat{\theta}_i + Kz_i - K\tau_i\dot{\alpha_i} \tag{17}
\]
where \((\bullet) = (\bullet) - (\bullet)\).

From (9) and (16), the equation can be described as
\[
\bar{\alpha}_i = q + \text{diag}(u_i)\hat{\theta}_i \tag{18}
\]

**Phase 2**

Differentiating (13) to yield
\[
\dot{z}_i = \dot{x}_i - \dot{\alpha}_i = x_2 - \dot{\alpha}_i \tag{19}
\]
Design the second virtual control vector \(\bar{\alpha}_z\) as
\[
\bar{\alpha}_z = \alpha_i - \Lambda_z z_i \tag{20}
\]
where \(\Lambda \in \mathbb{R}^{n\times n}\) denotes diagonal positive definite matrix.

And \(\bar{\alpha}_z\) is obtained using the first order filter again as
\[
\alpha_z + \alpha_z = \bar{\alpha}_z, \quad \alpha_z(0) = \bar{\alpha}_z(0) \tag{21}
\]
where \(\alpha_z\) is diagonal positive definite vector, and \(\bar{\alpha}_z\) the filtering control vector.

Define
\[
z_2 = x_2 - \alpha_z \tag{22}
\]
Substituting (20), (21) and (22) into (19) to yield
\[
\dot{z}_i = z_2 - \tau_2\dot{\alpha}_z - \Lambda_z z_i \tag{23}
\]

**Phase 3**

Differentiating (22) and substituting it into (8) to yield
\[
\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_z = J^{-1}u - J^{-1}K(x_1 - q) - \dot{\alpha}_z \tag{24}
\]
Multiply both sides of equation (24) by \(J\),
\[ J \hat{z}_2 = u - K(x_i - q) - J \hat{\alpha}_2 \]  

(25)

Thus, the actual control vector is designed as

\[ u = \hat{K}(x_i - q) + \hat{J} \hat{\alpha}_2 - \Lambda_z z_2 \]  

(26)

where \( \Lambda_z \in \mathbb{R}^{n \times n} \) denotes a diagonal positive definite matrix.

Considering to deal with \( \hat{K}, \hat{J} \) simply, the similar expressions as (16) are given

\[ \hat{K}(x_i - q) = \text{diag}(x_i - q) \hat{\theta}_i \]

(27)

\[ \hat{J} \hat{\alpha}_2 = \text{diag}(\hat{\alpha}_2) \hat{\theta}_i \]

From equations (27) and (26), it follows that

\[ u = \text{diag}(x_i - q) \hat{\theta}_i + \text{diag}(\hat{\alpha}_2) \hat{\theta}_i - \Lambda_z z_2 \]  

(28)

Substituting (28) and (27) into (25), it follows that

\[ J \hat{z}_2 = (\hat{K} - K)(x_i - q) + (\hat{J} - J) \hat{\alpha}_2 - \Lambda_z z_2 = \text{diag}(x_i - q) \hat{\theta}_i + \text{diag}(\hat{\alpha}_2) \hat{\theta}_i - \Lambda_z z_2 \]  

(29)

3.2. Stability Analysis

Define the error-surface vectors as

\[ s_i = \alpha_i - \bar{\alpha}_i, \quad i = 1, 2 \]

(30)

Using the equations (12) and (21), the equation (30) can be rewritten as

\[ s_i = -r_i \alpha_i, \quad i = 1, 2 \]

(31)

After taking the first-order differential of equation (30) with respect to time and referring to the equations (9), (20) and (31), the equations can be obtained as

\[ \dot{s}_1 = \dot{\alpha}_i - \bar{\alpha}_i = -r_i s_1 + \delta_i \]

(32)

\[ \dot{s}_2 = \dot{\alpha}_2 - \bar{\alpha}_2 = -r_2 s_2 + \delta_2 \]

(33)

where \( \delta_i = d\left(q_i + \text{diag}(u_i) \hat{\theta}_i\right)/dr \) and \( \delta_i = \Lambda_i \dot{z}_i - \bar{\alpha}_i \) are continuous bounded vectors.

Choose the global Lyapunov function

\[ V = V_1 + V_2 + V_3 \]

(34)

where

\[ V_1 = \frac{1}{2} r_1^T D(q) r_1 + \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T J z_2 \]

(35)

\[ V_2 = \frac{1}{2} s_1^T s_1 + \frac{1}{2} s_2^T s_2 \]

(36)

\[ V_3 = \frac{1}{2} \dot{\theta}_1^T \Gamma_1^{-1} \dot{\theta}_1 + \frac{1}{2} \dot{\theta}_2^T K \Gamma_2 \dot{\theta}_2 + \frac{1}{2} \dot{\theta}_3^T \Gamma_3^{-1} \dot{\theta}_3 + \frac{1}{2} \dot{\theta}_4^T \Gamma_4^{-1} \dot{\theta}_4 \]

(37)

and \( \Gamma_i \) is a diagonal positive definite matrix, \( i = 1, 2, 3, 4 \).

**Theorem**
Considering the FJ robot described as equations (1) and (2) with parametric uncertainties and the initial condition of the Lyapunov function $V(0) \leq p$ ($p$ denotes the arbitrary positive constant), the adaptive laws can be determined using the control law (26) as

$$
\begin{align*}
\dot{\theta}_i &= \hat{\theta}_i - \Gamma_i^T r_i - \eta_i \Gamma_i \hat{\theta}_i \\
\dot{\theta}_2 &= \hat{\theta}_2 - \Gamma_2 \text{diag}(u_i) r_i - \eta_2 \Gamma_2 \hat{\theta}_2 \\
\dot{\theta}_3 &= \hat{\theta}_3 - \Gamma_3 \text{diag}(x_i - q) z_2 - \eta_3 \Gamma_3 \hat{\theta}_3 \\
\dot{\theta}_4 &= \hat{\theta}_4 - \Gamma_4 \text{diag}(\hat{\phi}(z_2)) z_2 - \eta_4 \Gamma_4 \hat{\theta}_4
\end{align*}
$$

(38)

Where $\eta_i$ are the scalar coefficients, $\eta_i > 0$, and $i = 1,2,3,4$.

If $A_i, \Gamma_i, \eta_i (i = 1,2,3,4)$ and $\tau_i (i = 1,2)$ are selected properly, it is easy to guarantee the uniformly semi-globally boundedness of all closed-loop signals, and the tracking errors will converge to a compact set whose size can be adjusted to be arbitrarily small.

**Proof**

Differentiating $V_1$ with respect to time and simplifying it based on (P3), (17), (23), (29) and (31), thus

$$
\dot{V}_1 = r_i^T Y \hat{\theta}_i + r_i^T \text{diag}(u_i) \hat{\theta}_2 - r_i^T A_r r_i + r_i^T K z_i + r_i^T K s_i - z_i^T A z_i - z_i^T A z_i + z_i^T z_i + z_i^T s_i
$$

(39)

Differentiating $V_2$ and referring to (32), (33)

$$
\dot{V}_2 = -s_i^T r_i^T - s_i^T r_i^T z_i - r_i^T s_i + s_i^T \delta_i + s_i^T \delta_i
$$

(40)

Same to differentiate $V_3$ as above,

$$
\dot{V}_3 = \hat{\theta}_i^T \Gamma_i^{-1} \hat{\theta}_i + \hat{\theta}_2^T \Gamma_2^{-1} \hat{\theta}_2 + \hat{\theta}_3^T \Gamma_3^{-1} \hat{\theta}_3 + \hat{\theta}_4^T \Gamma_4^{-1} \hat{\theta}_4
$$

(41)

Therefore,

$$
\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 = -r_i^T A_r r_i - z_i^T A z_i - z_i^T z_i - s_i^T r_i^T - s_i^T r_i^T s_i + r_i^T K z_i + r_i^T K s_i + z_i^T z_i
$$

(42)

By Young inequality, for arbitrary vector $a \in \mathbb{R}^{n+1}, b \in \mathbb{R}^{n+1}$ and diagonal positive definite matrix $H \in \mathbb{R}^{n \times n}$, the inequality exists as

$$
a^T H b \leq \frac{1}{4} a^T H a + b^T H b
$$

(43)

Moreover, the inertia parameters set $\theta_i$, the estimated set $\hat{\theta}_i$, and estimated errors set $\bar{\theta}_i$ satisfy the following inequality:

$$
2 \hat{\theta}_i^T \hat{\theta}_i \geq \bar{\theta}_i^T \bar{\theta}_i - \hat{\theta}_i^T \hat{\theta}_i
$$

(44)

Using (42), (43) and (44), the derivative of Lyapunov function can be rewritten as
\[ V \leq -r_i^T A_i r_i - z_i^T A_i z_i - s_i^T \tau_i^{-1} s_i - s_i^T r_i s_i + \frac{1}{4} r_i^T K r_i + z_i^T K z_i + \frac{1}{2} J r_i^T K r_i + s_i^T K s_i + z_i^T z_i + \]
\[ - \frac{1}{2} \eta_i \bar{\theta}_i^T K \bar{\theta}_i. \]

According to (P1) and Assumption 2, the equation (45) is derived as follows:
\[ \dot{V} \leq -r_i^T D(q) \left( A_i - \frac{1}{2} K_M \right) r_i - z_i^T \left( A_i - K_M I - 2I \right) z_i - s_i^T J \left( A_i - \frac{1}{2} I \right) z_i - s_i^T \left( \tau_i^{-1} - K_M I - I \right) s_i \]
\[ - \frac{s_i^T \left( \tau_i^{-1} - \frac{5}{4} I \right) s_i}{4} - \frac{1}{2} \sum_{i=1}^4 \eta_i \bar{\theta}_i^T \Gamma^{-1} \Gamma \bar{\theta}_i - \frac{1}{2} \eta_i \bar{\theta}_i^T K \Gamma^{-1} \Gamma \bar{\theta}_i + \frac{1}{2} \sum_{i=1}^4 \eta_i \bar{\theta}_i^T \theta_i + \frac{1}{4} \delta_i^T \delta_i \]
\[ + \frac{1}{2} \eta_i \bar{\theta}_i^T K \bar{\theta}_i. \]

Define
\[ A_i = \frac{1}{2} K_M I + A^*_i, \]
\[ A_i = 2I + K_M I + A^*_i, \]
\[ A_i = \frac{1}{4} I + A^*_i, \]
\[ \tau_i^{-1} = K_M I + I + \tau_i^{-1}, \]
\[ \tau_i^{-1} = \frac{5}{4} I + \tau_i^{-1}, \]

where \( A^*_i > 0 \) and \( \tau_i > 0 \).

Therefore,
\[ V \leq -r_i^T D(q) A_i r_i - z_i^T A_i z_i - s_i^T \tau_i s_i - s_i^T r_i s_i - \frac{1}{2} \sum_{i=1}^4 \eta_i \bar{\theta}_i^T \Gamma^{-1} \lambda_m \bar{\theta}_i \]
\[ - \frac{1}{2} \eta_i \bar{\theta}_i^T K \Gamma^{-1} \lambda_m \bar{\theta}_i + \frac{1}{2} \sum_{i=1}^4 \eta_i \bar{\theta}_i^T \theta_i + \frac{1}{2} \sum_{i=1}^4 \eta_i \bar{\theta}_i^T \theta_i + \frac{1}{2} \eta_i \bar{\theta}_i^T K \theta_i, \]

where \( \lambda_m \) is the minimum eigenvalue of \( \Gamma \).

Select the real number \( \lambda \) to satisfy the inequality as
\[ 0 < \lambda \leq \min \left( \frac{\| A_k \|}{D_m} \| A^*_k \| \| \bar{r}_i \| \| \bar{r}_i \|, \frac{\lambda_{m1}}{2}, \frac{\lambda_{m2}}{2}, \frac{\lambda_{m3}}{2}, \frac{\lambda_{m4}}{2} \right) \]

(48)

Because \( \delta_i, \delta_2, \theta_1, \theta_2, \theta_3 \) and \( \theta_1 \) are bounded, the inequality can be written as
\[ \frac{1}{4} \sum_{i=1}^4 \delta_i^T \delta_i + \frac{1}{2} \sum_{i=1}^4 \eta_i \bar{\theta}_i^T \theta_i + \frac{1}{2} \eta_i \bar{\theta}_i^T K \theta_i \leq \Phi \]

(49)

where \( \Phi \) is the constant.

The equation (47) can be written according to (48), (49) as
\[ V \leq -2\lambda V + \Phi \]

(50)

Consider the following compact set,
\[ \Omega := \left\{ (q, \bar{q}, \bar{q}) : \| q \|^2 + \| \bar{q} \|^2 \leq \chi \right\} \]
\[ \Omega_2 := \{ V \leq p \} \]

where \( p \) is an arbitrary positive constant.

Select \( \lambda \geq \Phi / 2p \). If \( V = p \), \( \dot{V} \leq 0 \). So \( V \leq p \) presents an invariant set, i.e. if \( V(0) \leq p \), then \( V(t) \leq p \) for all \( t > 0 \).

Solving the inequality (50) to yield

\[ V \leq \frac{\Phi}{2\lambda} + \left( V(0) - \frac{\Phi}{2\lambda} \right) e^{-2\lambda} \]  

(51)

Obviously the all closed-loop signals are uniformly semi-globally bounded, and

\[ \lim_{t \to \infty} V(t) \leq \frac{\Phi}{2\lambda} \]  

(52)

Therefore by adjusting the parameters \( \Theta(i=1,2,3,4) \), \( \tau(i=1,2) \), \( \Gamma(i=1,2,3,4) \) and \( \eta(i=1,2,3,4) \) to make \( \lambda \) bigger, the tracking error will be smaller.

4. Simulation results and analysis

The two-link FJ robot shown in Fig.1 with parametric uncertainties is used to test the feasibility of the proposed control method. The dynamic equations are derived based on Kane method, and numerical calculation is finished in Simulink of MATLAB.

The robot dynamic parameters are given as follows:

\[ m_1 = 6.07, m_2 = 5.76 \]

\[ p_{c1} = (0.169,0,0.0026)^T, p_{c2} = (0.162,0,0)^T \]

\[ p_{x1} = (0,0,0.055)^T, p_{x2} = (0.3,0,0.06)^T \]

Here, \( m_i(i=1,2) \) is the \( i \)-th link mass, \( p_{c(i=1,2)} \) is the mass center of \( i \)-th link relative to the \( i \)-th link coordinate. \( p_{x(i=1,2)} \) is the relative position between \( i \)-th joint and \( i-1 \)-th joint.

And

\[
I_{c1} = \begin{bmatrix}
0.0074 & 0 & -0.002 \\
0 & 0.0718 & 0 \\
-0.002 & 0 & 0.0742
\end{bmatrix},
I_{c2} = \begin{bmatrix}
0.0065 & 0 & 0 \\
0 & 0.0654 & 0 \\
0 & 0 & 0.0685
\end{bmatrix}
\]

\[ J = \text{diag}(0.0197, 0.0197) \]

\[ g = (0, -9.8, 0)^T \]

\[ K_1 = K_2 = 100 \]

Now, the minimum inertial parameters can be solved as

\[ \Theta_1 = (2.84, 0, 0.766, 0.9331, 0, 0.2197)^T \]

\[ \Theta_2 = (0.01, 0.01)^T, \Theta_3 = (100, 100)^T \]

\[ \Theta_4 = (0.0197, 0.0197)^T \]
The ideal trajectories are given as \( q_{sd} = \sin(2\pi t) \) and \( q_{sd2} = \sin(2\pi t) \), the initial positions are specified as \( q_1(0) = q_2(0) = 0 \), \( q_{m1}(0) = q_{m2}(0) = 0 \), and the controller parameters are chosen by equation (46) as
\[
\eta_1 = \eta_2 = \eta_3 = \eta_4 = 0.0005, \quad r_1 = r_2 = \text{diag}(0.001, 0.001), \quad \Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = \text{diag}(0.00005, 0.00005), \quad A_1 = \text{diag}(60, 60), \quad A_2 = \text{diag}(60, 60), \quad A_3 = \text{diag}(150, 150), \quad A_4 = \text{diag}(5, 5)
\]

The initial estimated conditions of the inertia parameters are set as \( \hat{\theta}_1(0) = (0, 0, 0, 0, 0)^T \), \( \hat{\theta}_2(0) = (0, 0)^T \), \( \hat{\theta}_3(0) = (0, 0)^T \), and \( \hat{\theta}_4(0) = (0, 0)^T \).

The tracking trajectories of link 1 and link 2 are placed in Fig. 2(a) and (b) together with their ideal trajectories. The simulation results are shown in Fig. 2(c) and (d). These figures show that the proposed control method results in the very small link tracking errors (\( e_1 \) and \( e_2 \) are both about 5% of the ideal trajectories) and ensures the global boundedness of all closed-loop signals with uncertainties.

1) Uncertainties of the initial values

An initial values of \( \hat{\theta}_i \) are not restrained theoretically, that is, they can be selected arbitrary. For verifying its effect, keep the other parameters unchanged and set initial values of \( \hat{\theta}_i \) to be 0%, 50% and 80% respectively of the nominal value \( \theta_i \) as
\[
(1) \hat{\theta}_1 = (0, 0, 0, 0, 0)^T, \hat{\theta}_2 = (0, 0)^T, \hat{\theta}_3 = (0, 0)^T, \hat{\theta}_4 = (0, 0)^T.
\]
\[
(2) \hat{\theta}_1 = (1.42, 0.38, 0.47, 0.11)^T, \hat{\theta}_2 = (0.005, 0.005)^T, \hat{\theta}_3 = (50, 50)^T, \hat{\theta}_4 = (0.01, 0.01)^T.
\]
\[
(3) \hat{\theta}_1 = (2.27, 0.061, 0.75, 0.18)^T, \hat{\theta}_2 = (0.008, 0.008)^T, \hat{\theta}_3 = (80, 80)^T, \hat{\theta}_4 = (0.016, 0.016)^T.
\]

The tracking errors in the simulation are shown in Fig. 3(a) and (b). They reveal that the tracking errors of link 1 and 2 respectively decrease as the estimated values are closer to the nominal value of \( \hat{\theta}_i \), and their simulation computing times are equal almost. Further, the tracking errors can be lower if the estimated values of inertia parameters are closer to the nominal values.

2) Uncertainties of parameters of the control laws

For verifying the effectiveness of the proposed adaptive laws, we change these parameters continuously as
\[
\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = \text{diag}(0.00005, 0.00005) \ast \sin(t), \quad \eta_1 = \eta_2 = \eta_3 = \eta_4 = 0.0005 \ast \sin(t)
\]

The tracking errors are shown in Fig. 4(a) and (b), where the \( e_1 \) and \( e_2 \) represent the tracking errors of link 1 and 2 respectively with the original adaptive laws parameters, and the \( e_{11} \) and \( e_{22} \) represent the errors with the changed parameters.

It is evident that the tracking errors of link 1 and 2 are keeping small even though the adaptive laws parameters are changed.

3) Uncertainties of external disturbances

Add the external disturbances on the joints of link 1 and 2 with the functions \( 50 \sin(2\pi t) \) and \( 35 \sin(4\pi t) \) respectively.

The tracking errors are shown in Fig. 5(a) and (b), where the \( e_1 \) and \( e_2 \) represent the tracking errors of link 1 and 2 respectively without the external disturbances, and the \( e_{11} \) and \( e_{22} \) represent the errors with the external disturbances.
It is clear that the external disturbances cause the tracking errors of the links slightly, and tracking errors become weak quickly in 0.1sec. In addition, the errors of links are less than 15% of desired amplitude at the beginning phase whether the inertia parameters, control laws parameters or external disturbance are changed. In Literature [14], we investigated an adaptive backstepping control method based on NN, the maximum tracking errors of links are more than 20% of input amplitude and time-consuming is at least twice as much as what this paper suggests.

5. Conclusions

In this paper, the robot model uncertainties are estimated by using the inertia parameters of FJ robots. The adaptive DSC for FJ robot is proposed based on backstepping control method, in which the virtual control vectors are derived by using the first order filter in order to avoid the “explosion of derivative”. The control vectors and adaptive laws in the DSC are expressed as the functions of inertia parameters in FJ robots, and the joint tracking errors could be very small even though the initial estimated conditions of the inertia parameters are set to be far away from the real ones, the parameters of the control laws are changed, or the external disturbances are added in the system.

The simulations of the two-link FJ robot show that the proposed control system gets (1) Less tracking error, better tracking performance and robustness against model uncertainties, (2) Simpler calculation to avoid high order derivative and iterative regression matrix, (3) Shorter computing time to respond, and (4) Easier selection of control parameters. Moreover, the tracking performance in a short time can be better if the inertia parameters can be selected precisely.

Conflict of interest

There are no conflicts of interest.

Contributors

Chenggang Li had proposed the adaptive dynamic surface control applied on the flexible-joint robot, and derived the adaptation laws. Wen Cui did the simulation and calculation. Dengdeng Yan studied the control method based on Neural network and discussed the uncertainties of FJ robots. Yan Wang built the dynamics of robot. Chunming Wang verified the dynamics and simulation in the paper about the adaptive control.

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References


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Figure and table captions
Fig. 1 3D model of two-link FJ robot

Fig. 2 the trajectories and tracking errors of link 1 and link 2 with the designed parameters. (a) Ideal and tracking trajectory of link 1. (b) Ideal and tracking trajectory of link 1. (c) Errors of link 1. (d) Errors of link 2.

Fig. 3 the tracking errors of link 1 and link 2 with the different initial value of $\hat{\theta}(i = 1 \sim 4)$. (a) Errors of link 1. (b) Errors of link 2.

Fig. 4 the tracking errors of link 1 with different parameters of the control laws. (a) Errors of link 1 with the original and changed parameters. (b) Errors of link 2 with the original and changed parameters

Fig. 5 the tracking errors of link 1 with different external disturbances. (a) Errors of link 1 without and with external disturbance. (b) Errors of link 2 without and with external disturbance.

All figures and tables together with their numbers

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(a) (b) (c) (d)

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disturbance. (b) Errors of link 2 without and with external disturbance.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q \cdot \dot{q} \cdot \ddot{q} )</td>
<td>Position, velocity and acceleration vectors of the links</td>
</tr>
<tr>
<td>( D(q) )</td>
<td>Inertia matrix</td>
</tr>
<tr>
<td>( C(q, \dot{q}) )</td>
<td>Coriolis and centripetal matrix</td>
</tr>
<tr>
<td>( G(q) )</td>
<td>Gravity matrix</td>
</tr>
<tr>
<td>( K )</td>
<td>Positive definite diagonal constant flexibility matrix</td>
</tr>
<tr>
<td>( q_m \cdot \ddot{q}_m )</td>
<td>Position and acceleration vectors for the actuator</td>
</tr>
<tr>
<td>( J )</td>
<td>Actuator inertia matrix</td>
</tr>
<tr>
<td>( u )</td>
<td>Actual control vector</td>
</tr>
<tr>
<td>( D_m \cdot D_M \cdot C_M \cdot G_M )</td>
<td>Positive constant</td>
</tr>
<tr>
<td>( Y(q, \dot{q}, \ddot{q}) )</td>
<td>Regression matrix</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>( r )-dimensional vector of parameters</td>
</tr>
<tr>
<td>( r )</td>
<td>Number of inertia parameters</td>
</tr>
<tr>
<td>( a_1 \cdot v_i )</td>
<td>Arbitrary ( n )-dimensional vectors</td>
</tr>
<tr>
<td>( x_1 \cdot x_2 )</td>
<td>Functions of state space variables</td>
</tr>
<tr>
<td>( K_m, K_m, J_m, J_m )</td>
<td>Positive constants</td>
</tr>
<tr>
<td>( q_d )</td>
<td>Desired trajectory of link position</td>
</tr>
<tr>
<td>( Q )</td>
<td>Positive constant</td>
</tr>
<tr>
<td>( \bar{a}, \bar{a}_2 )</td>
<td>Virtual control vectors</td>
</tr>
<tr>
<td>( \alpha, \alpha_2 )</td>
<td>Filtering virtual control vectors</td>
</tr>
<tr>
<td>( u_r )</td>
<td>Auxiliary control vector</td>
</tr>
<tr>
<td>( A, A_1 \cdot \tau, A_2 \cdot \tau_2, H )</td>
<td>Diagonal positive definite matrices</td>
</tr>
<tr>
<td>( \delta, \delta_2 )</td>
<td>Continuous bounded vectors</td>
</tr>
<tr>
<td>( \Gamma_i )</td>
<td>Diagonal positive definite matrix, ( i = 1, 2, 3, 4 )</td>
</tr>
<tr>
<td>( \eta_i )</td>
<td>Scalar coefficients, ( \eta_i &gt; 0, i = 1, 2, 3, 4 )</td>
</tr>
<tr>
<td>( a, b )</td>
<td>Arbitrary vectors</td>
</tr>
<tr>
<td>( \theta, \tilde{\theta}, \hat{\theta} )</td>
<td>Inertia parameters set, estimated values set and estimated errors set</td>
</tr>
<tr>
<td>( \lambda_m )</td>
<td>Minimum eigenvalue of ( \Gamma_i )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Real number</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Constant</td>
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<tr>
<td>( p )</td>
<td>Arbitrary positive constant</td>
</tr>
<tr>
<td>( m_i ) (( i = 1, 2 ))</td>
<td>( i )-th link mass</td>
</tr>
<tr>
<td>( \mathbf{p}_i (i = 1, 2) )</td>
<td>Mass centers of ( i )-th link relative to the ( i )-th link coordinate</td>
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<td>---</td>
</tr>
<tr>
<td>( \mathbf{p}_{i-1,i} (i = 1, 2) )</td>
<td>Relative position between ( i )-th joint and ( i-1 )-th joint</td>
</tr>
<tr>
<td>( q_{1d} \cdot q_{2d} )</td>
<td>Ideal trajectories</td>
</tr>
<tr>
<td>( e_1, e_2 )</td>
<td>Link tracking errors</td>
</tr>
</tbody>
</table>