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# An approach to interval-valued intuitionistic fuzzy decision making based on induced generalized symmetrical Choquet-Shapley operator

**J. Tang[a] and F. Meng[a,b,*]**

a. *Business School, Central South University, Hunan Changsha 410083, China.*
b. *School of Management and Economics, Nanjing University of Information Science and Technology, Nanjing 210044, China.*

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**Abstract.** Interval-Valued Intuitionistic Fuzzy Variables (IVIFVs) are powerful tools to illustrate the preferred and non-preferred uncertainty degrees of decision-makers. Considering the application of IVIFVs in decision-making, this paper first gives some new operations that can address the shortages of previous ones. Then, an Induced Generalized Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (IG-SIVIFCS) operator is defined, which not only globally considers the importance of the elements, but also reflects their overall interactions. Afterwards, several desirable properties are briefly studied to provide assurance for success in application. In some situations, the weighting information of attributes is incompletely known. Considering this case, the Shapley function-based model for determining the optimal fuzzy measure on the attribute set is constructed. Furthermore, an approach to interval-valued intuitionistic fuzzy decision-making with incomplete weighting information and interactive characteristics is developed to provide a complete theoretical framework. Finally, a practical example is provided to show the concrete practicality and validity of the proposed procedure.



## 1. Introduction

Many researchers have studied the aggregation operators as an important research topic. The Ordered Weighted Averaging (OWA) operator [1] is one of the most important aggregation operators, providing a parameterized family of aggregation operators. Its fundamental aspect is the reordering step; in other words, the input arguments are rearranged in descending order and the weights are merely related to the positions. Since it was first introduced in 1988, several extended forms have been introduced [2-7]. Similar to the OWA operator, Xu and Yager [8] proposed the Ordered Weighted Geometric (OWG) operator to aggregate the arguments. After the seminal work of Xu and Yager [8], many geometric aggregation operators have developed [8-13].

All of the above-mentioned aggregation operators are based on the assumption that elements in a set are independent. However, in some situations, this assumption does not hold and the elements are correlative [14,15]. Thus, we need to find some new ways to deal with this situation where the decision data are correlative. The fuzzy measure proposed by Sugeno [16] is a good tool to address this problem. Using the Choquet integral [17], many intuitionistic fuzzy aggregation operators are developed [18-22]. Although the Choquet integral-based aggregation

*. *Corresponding author. Tel.: +86 18254298903; Fax: +86 0731888879391*
*E-mail address: mengfanyongtjie@163.com. (F. Meng)*

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operator can reflect the correlations among elements, their global interactions cannot be given. Thus, Meng et al. [23] used the Choquet integral and the generalized Shapley function with respect to $\lambda$-fuzzy measure to define two interval-valued intuitionistic fuzzy Choquet Shapley aggregation operators. Meanwhile, Meng et al. [24] introduced an Induced Generalized Interval-Valued Intuitionistic Fuzzy Hybrid Shapley Averaging (IG-IVIFHSA) operator. Besides the above mentioned interval-valued intuitionistic fuzzy aggregation operators, many other aggregation operators are proposed such as the Interval-Valued Intuitionistic Fuzzy Power Weight Heronian Aggregation (IVIFPWHA) operator [25], the Revised Continuous Interval-Valued Intuitionistic Fuzzy OWA (RC-IVIFOWA) operator [26], and the normal interval-valued intuitionistic fuzzy generalized hybrid weighted averaging operator [27]. Furthermore, intuitionistic fuzzy preference relations are discussed in [28-33], and interval-valued intuitionistic uncertain linguistic correlation coefficient is studied in [34]; Meng and Tan [35] researched the intuitionistic hesitant fuzzy linguistic distance measures.

Note that all of the above-mentioned interval-valued intuitionistic fuzzy averaging operators are based on the operational laws in [36]. From the following discussion, we can find that there are some undesirable properties. In particular, these issues may lead to undesirable ranking results. Considering this case, this paper continues to study decision-making with interval-valued intuitionistic fuzzy information and develops a new procedure. To do this, an Induced Generalized Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (IG-SIVIFCS) operator is presented, which is then used to calculate the comprehensive attribute values. To address the situation where the weighting information of attributes is partly known, a model is built for determining the optimal fuzzy measure on the attribute set. The rest can be organized as follows.

In Section 2, some basic concepts are briefly reviewed, including Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs), fuzzy measures, the Choquet integral, two Choquet integral operators, and the generalized Shapley function. Meanwhile, it analyzes the limitations of the previous operations on IVIFSs. In Section 3, the IG-SIVIFCS operator is defined, and several important cases are investigated. Furthermore, some desirable properties are studied. In Section 4, several distance measure-based models for determining the optimal fuzzy measure on the attribute set are established, and an approach to interval-valued intuitionistic fuzzy multi-attribute decision-making with incomplete weighting information is developed that considers the interactions. In Section 5, an illustrative example is provided to show the concrete application of the proposed procedure. The conclusion is made in the end.

## 2. Several basic concepts

### *2.1. Interval-valued intuitionistic fuzzy sets*

In 1989, Atanassov and Gargov [37] first proposed the concept of Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs). To facilitate the discussion, Xu and Chen [38] introduced the concept of Interval-Valued Intuitionistic Fuzzy Values (IVIFVs) and defined several operations.

**Definition 1 [37].** Let $X$ be a non-empty finite set. An IVIFS $A$ in $X$ is expressed as follows:

$$A = \{\langle x, [a(x), b(x)], [c(x), d(x)]\rangle \,|x \in X\},$$

where $[a(x), b(x)] \subseteq [0,1]$ and $[c(x), d(x)] \subseteq [0,1]$ are respectively the interval preferred and non-preferred memberships of element $x \in X$ with $b(x) + d(x) \leq 1$.

An IVIFV $\tilde{\alpha}$ is defined as $\tilde{\alpha} = ([a, b], [c, d])$ [38], where $[a, b] \subseteq [0,1]$ and $[c, d] \subseteq [0,1]$ are, respectively, the interval preferred and non-preferred degrees with $b + d \leq 1$. Let $\Omega$ be the set of all IVIFVs. Considering the order relationship of IVIFVs, Xu and Chen [38] introduced the score and accuracy functions as follows:

$$S(\tilde{\alpha}) = 0.5\,(a - c + b - d)\,,$$

$$H(\tilde{\alpha}) = 0.5\,(a + c + b + d)\,,$$

where $\tilde{\alpha} = ([a, b], [c, d]) \in \Omega$.

Following the score and accuracy functions, the order relationship between any two IVIFVs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ is given as follows [38]:

$$\text{If } S(\tilde{\alpha}_1) < S(\tilde{\alpha}_2), \text{ then } \tilde{\alpha}_1 < \tilde{\alpha}_2,$$

$$\text{If } S(\tilde{\alpha}_1) = S(\tilde{\alpha}_2), \text{ then } \begin{cases} H(\tilde{\alpha}_1) = H(\tilde{\alpha}_2) \Rightarrow \tilde{\alpha}_1 = \tilde{\alpha}_2 \\ H(\tilde{\alpha}_1) < H(\tilde{\alpha}_2) \Rightarrow \tilde{\alpha}_1 < \tilde{\alpha}_2 \end{cases}$$

**Definition 2 [36].** Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFVs in $\Omega$, and then some operations are defined as follows:

1. $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2])$;
2. $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2])$;
3. $r\tilde{\alpha}_1 = ([1 - (1 - a_1)^r, 1 - (1 - b_1)^r], [c_1^r, d_1^r])$ $r > 0$;
4. $\tilde{\alpha}_1^r = ([a_1^r, b_1^r], [1 - (1 - c_1)^r, 1 - (1 - d_1)^r])$ $r > 0$.

As some researchers [39] noted for intuitionistic fuzzy values, there are some undesirable properties of the operational laws listed in Definition 2. For example, the first two operations cannot preserve the order relationship for adding or multiplying some IVIFV.

**Example 1.** Let $\tilde{\alpha} = ([0.3, 0.4], [0.4, 0.6])$, $\tilde{\beta} = ([0.2, 0.3], [0.3, 0.4])$ and $\tilde{\gamma} = ([0.8, 0.9], [0, 0])$. From $E(\tilde{\alpha}) = -0.15$ and $E(\tilde{\beta}) = -0.1$, we have $\tilde{\alpha} \prec \tilde{\beta}$. Because $\tilde{\alpha} \oplus \tilde{\gamma} = ([0.86, 0.94], [0, 0])$ and $\tilde{\beta} \oplus \tilde{\gamma} = ([0.84, 0.93], [0, 0])$, we get $\tilde{\alpha} \oplus \tilde{\gamma} \succ \tilde{\beta} \oplus \tilde{\gamma}$ from $E(\tilde{\alpha} \oplus \tilde{\gamma}) = 0.9$ and $E(\tilde{\beta} \oplus \tilde{\gamma}) = 0.885$. Furthermore, when $\tilde{\alpha} = ([0.4, 0.6], [0.3, 0.4])$, $\tilde{\beta} = ([0.3, 0.4], [0.2, 0.3])$, and $\tilde{\gamma} = ([0.1, 0.1], [0.8, 0.9])$. We have $\tilde{\alpha} \succ \tilde{\beta}$ from $E(\tilde{\alpha}) = 0.15$ and $E(\tilde{\beta}) = 0.1$. However, we obtain $\tilde{\alpha} \otimes \tilde{\gamma} \prec \tilde{\beta} \otimes \tilde{\gamma}$ from $E(\tilde{\alpha} \otimes \tilde{\gamma}) = -0.9$ and $E(\tilde{\beta} \otimes \tilde{\gamma}) = -0.85$, where $\tilde{\alpha} \otimes \tilde{\gamma} = ([0.04, 0.06], [0.86, 0.94])$ and $\tilde{\beta} \otimes \tilde{\gamma} = ([0.03, 0.04], [0.84, 0.93])$.

On the other hand, the last two operations cannot keep the order relationship under multiplication or exponentiation by a scalar.

**Example 2.** Let $\tilde{\alpha} = ([0.3, 0.4], [0.2, 0.3])$, $\tilde{\beta} = ([0.4, 0.5], [0.3, 0.4])$ and $\lambda = 0.7$. Then, $\tilde{\alpha} \prec \tilde{\beta}$ by $E(\tilde{\alpha}) = E(\tilde{\beta}) = 0.1$ and $H(\tilde{\alpha}) = 0.6 < 0.8 = H(\tilde{\beta})$. Meanwhile, we get $\lambda\tilde{\alpha} \succ \lambda\tilde{\beta}$ by $E(\lambda\tilde{\alpha}) = -0.2331 > E(\lambda\tilde{\beta}) = -0.2720$, where $\lambda\tilde{\alpha} = ([0.2209, 0.3006], [0.3241, 0.4305])$ and $\lambda\tilde{\beta} = ([0.3006, 0.3844], [0.4305, 0.5266])$. Furthermore, let $\lambda = 0.3$. When $\tilde{\beta} = ([0.2, 0.5], [0.1, 0.4])$, we derive $\tilde{\alpha} = \tilde{\beta}$ from $E(\tilde{\alpha}) = E(\tilde{\beta}) = 0.1$ and $H(\tilde{\alpha}) = H(\tilde{\beta}) = 0.6$. However, we obtain $\tilde{\alpha}^{\lambda} \succ \tilde{\beta}^{\lambda}$ from $E(\tilde{\alpha}^{\lambda}) = 0.6451 > E(\tilde{\beta}^{\lambda}) = 0.6280$, where $\tilde{\alpha}^{\lambda} = ([0.6968, 0.7597], [0.0647, 0.1015])$ and $\tilde{\beta}^{\lambda} = ([0.6170, 0.8122], [0.0311, 0.1421])$.

To avoid the undesirable properties of the last two operations in Definition 2, some new operations are defined as follows.

**Definition 3.** Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFVs in $\Omega$. Then, some of their operations are defined as follows:

(i) $\lambda_1\tilde{\alpha}_1 \oplus \lambda_2\tilde{\alpha}_2 = ([\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 b_1 + \lambda_2 b_2], [\lambda_1 c_1 + \lambda_2 c_2, \lambda_1 d_1 + \lambda_2 d_2])$, $\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leq 1$;

(ii) $\tilde{\alpha}^{\lambda_1} \otimes \tilde{\beta}^{\lambda_2} = ([a_1^{\lambda_1} a_2^{\lambda_2}, b_1^{\lambda_1} b_2^{\lambda_2}], [c_1^{\lambda_1} c_2^{\lambda_2}, d_1^{\lambda_1} d_2^{\lambda_2}])$, $\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leq 1$.

Next, we consider several properties of operations defined in Definition 3.

**Property 1.** Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ and $\tilde{\alpha}_3 = ([a_3, b_3], [c_3, d_3])$ be any three IVIFVs in $\Omega$. Then, we have:

(i) ***Commutativity:***

$$\lambda_1\tilde{\alpha}_1 \oplus \lambda_2\tilde{\alpha}_2 = \lambda_2\tilde{\alpha}_2 \oplus \lambda_1\tilde{\alpha}_1,$$

$$\tilde{\alpha}_1^{\lambda_1} \otimes \tilde{\alpha}_2^{\lambda_2} = \tilde{\alpha}_2^{\lambda_2} \otimes \tilde{\alpha}_1^{\lambda_1},$$

$$\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leq 1.$$

(ii) ***Associativity:***

$$(\lambda_1\tilde{\alpha}_1 \oplus \lambda_2\tilde{\alpha}_2) \oplus \lambda_3\tilde{\alpha}_3 = \lambda_1\tilde{\alpha}_1 \oplus (\lambda_2\tilde{\alpha}_2 \oplus \lambda_3\tilde{\alpha}_3),$$

$$(\tilde{\alpha}_1^{\lambda_1} \otimes \tilde{\alpha}_2^{\lambda_2}) \otimes \tilde{\alpha}_3^{\lambda_3} = \tilde{\alpha}_1^{\lambda_1} \otimes (\tilde{\alpha}_2^{\lambda_2} \otimes \tilde{\alpha}_3^{\lambda_3}),$$

$$\lambda_1, \lambda_2, \lambda_3 \in [0, 1] \wedge \lambda_1 + \lambda_2 + \lambda_3 \leq 1.$$

(iii) ***Distributivity:***

$$\lambda(\lambda_1\tilde{\alpha}_1 \oplus \lambda_2\tilde{\alpha}_2) = (\lambda\lambda_1)\tilde{\alpha}_1 \oplus (\lambda\lambda_2)\tilde{\alpha}_2,$$

$$(\tilde{\alpha}_1^{\lambda_1} \otimes \tilde{\alpha}_2^{\lambda_2})^{\lambda} = \tilde{\alpha}_1^{\lambda\lambda_1} \otimes \tilde{\alpha}_2^{\lambda\lambda_2},$$

$$\lambda, \lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leq 1.$$

(iv) ***Identity:***

$$\lambda_1\tilde{\alpha}_1 \oplus \lambda_2\tilde{\alpha}_2 = (\lambda_1 + \lambda_2)\tilde{\alpha}_1,$$

$$\tilde{\alpha}_1^{\lambda_1} \otimes \tilde{\alpha}_2^{\lambda_2} = \tilde{\alpha}_1^{\lambda_1 + \lambda_2},$$

$$\begin{cases} a_1 = a_2, b_1 = b_2 \\ c_1 = c_2, c_1 = c_2 \end{cases}$$

$$\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leq 1.$$

***Proof.*** From Definition 3, we can easily derive all of the above conclusions. Considering associativity for example, we have:

$$(\lambda_1\tilde{\alpha}_1 \oplus \lambda_2\tilde{\alpha}_2) \oplus \lambda_3\tilde{\alpha}_3 = ([\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 b_1 + \lambda_2 b_2],$$
$$[\lambda_1 c_1 + \lambda_2 c_2, \lambda_1 d_1 + \lambda_2 d_2])$$
$$\oplus ([\lambda_3 a_3, \lambda_3 b_3], [\lambda_3 c_3, \lambda_3 d_3])$$
$$= ([\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3, \lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3],$$
$$[\lambda_1 c_1 + \lambda_2 c_2 + \lambda_3 c_3, \lambda_1 d_1 + \lambda_2 d_2 + \lambda_3 d_3])$$
$$= ([\lambda_1 a_1 + (\lambda_2 a_2 + \lambda_3 a_3), \lambda_1 b_1 + (\lambda_2 b_2 + \lambda_3 b_3)],$$
$$[\lambda_1 c_1 + (\lambda_2 c_2 + \lambda_3 c_3), \lambda_1 d_1 + (\lambda_2 d_2 + \lambda_3 d_3)])$$
$$= ([\lambda_1 a_1, \lambda_1 b_1], [\lambda_1 c_1, \lambda_1 d_1]) \oplus$$
$$([\lambda_2 a_2 + \lambda_3 a_3, \lambda_2 b_2 + \lambda_3 b_3],$$
$$[\lambda_2 c_2 + \lambda_3 c_3, \lambda_2 d_2 + \lambda_3 d_3])$$
$$= \lambda_1\tilde{\alpha}_1 \oplus (\lambda_2\tilde{\alpha}_2 \oplus \lambda_3\tilde{\alpha}_3),$$

and:

$$\left(\tilde{\alpha}_1^{\lambda_1}\otimes\tilde{\alpha}_2^{\lambda_2}\right)\otimes\tilde{\alpha}_3^{\lambda_3}=\left(\left[a_1^{\lambda_1}a_2^{\lambda_2},b_1^{\lambda_1}b_2^{\lambda_2}\right],\left[c_1^{\lambda_1}c_2^{\lambda_2},d_1^{\lambda_1}d_2^{\lambda_2}\right]\right)$$

$$\otimes\left(\left[a_3^{\lambda_3},b_3^{\lambda_3}\right],\left[c_3^{\lambda_3},d_3^{\lambda_3}\right]\right)$$

$$=\left(\left[{a_1}^{\lambda_1}{a_2}^{\lambda_2}{a_3}^{\lambda_3},{b_1}^{\lambda_1}{b_2}^{\lambda_2}{b_3}^{\lambda_3}\right],\right.$$

$$\left.\left[{c_1}^{\lambda_1}{c_2}^{\lambda_2}{c_3}^{\lambda_3},{d_1}^{\lambda_1}{d_2}^{\lambda_2}{d_3}^{\lambda_3}\right]\right)$$

$$=\left(\left[{a_1}^{\lambda_1}\left({a_2}^{\lambda_2}{a_3}^{\lambda_3}\right),{b_1}^{\lambda_1}\left({b_2}^{\lambda_2}{b_3}^{\lambda_3}\right)\right],\right.$$

$$\left.\left[{c_1}^{\lambda_1}\left({c_2}^{\lambda_2}{c_3}^{\lambda_3}\right),{d_1}^{\lambda_1}\left({d_2}^{\lambda_2}{d_3}^{\lambda_3}\right)\right]\right)$$

$$=\left(\left[{a_1}^{\lambda_1},{b_1}^{\lambda_1}\right],\left[{c_1}^{\lambda_1},{d_1}^{\lambda_1}\right]\right)$$

$$\otimes\left(\left[{a_2}^{\lambda_2}{a_3}^{\lambda_3},{b_2}^{\lambda_2}{b_3}^{\lambda_3}\right],\left[{c_2}^{\lambda_2}{c_3}^{\lambda_3},{d_2}^{\lambda_2}{d_3}^{\lambda_3}\right]\right)$$

$$=\tilde{\alpha}_1^{\lambda_1}\otimes\left(\tilde{\alpha}_2^{\lambda_2}\otimes\tilde{\alpha}_3^{\lambda_3}\right).$$

**Remark 1.** Without special explanation, this paper uses the operational laws in Definition 3.

### *2.2. Fuzzy measure and the Choquet integral*

Fuzzy measure [16] is useful for measuring the interactions between elements [18-24,40,41] defined as follows.

**Definition 4 [16].** A fuzzy measure, $\mu$, on the finite set $X = \{x_1, x_2, ..., x_n\}$ is a set function $\mu : P(X) \to [0, 1]$ satisfying:

(i) $\mu(\phi) = 0$, $\mu(X) = 1$,

(ii) If $A, B \in P(X)$ and $A \subseteq B$, then $\mu(A) \leq \mu(B)$,

where $P(X)$ is the power set of $X$.

When $X$ is the attribute set of a decision-making problem, the fuzzy measure $\mu$ does not only give the importance of each attribute, but also define the importance of their combinations.

With regard to fuzzy measures, fuzzy integrals are important to calculate the comprehensive aggregation values that use fuzzy measures [16,42-44], among which the Choquet integral [17] is the most important one. In 1996, Grabisch [15] put forward the following concept of the Choquet integral on discrete sets:

**Definition 5 [15].** Let $f$ be a positive real-valued function on $X = \{x_1, x_2, ..., x_n\}$ and $\mu$ be a fuzzy measure on $N$. The discrete Choquet integral of $f$ with respect to $\mu$ is defined as follows:

$$C_\mu(f(x_{(1)}), f(x_{(2)}), ..., f(x_{(n)}))$$

$$= \sum_{i=1}^{n} f(x_{(i)})(\mu(A_{(i)}) - \mu(A_{(i+1)})),$$

where (.) indicates a permutation on $N = \{1, 2, ..., n\}$, such that $f(x_{(1)}) \leq f(x_{(2)}) \leq ... \leq f(x_{(n)})$ and $A_{(i)} = \{i, ..., n\}$ with $A_{(n+1)} = \phi$.

### *2.3. Several interactive interval-valued intuitionistic fuzzy operators*

Let $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i])$, $i = 1, 2, ..., n$, be a collection of IVIFVs in $\Omega$. Xu [41] defined the Interval-Valued Intuitionistic Fuzzy Correlated Averaging (IVIFCA) operator as follows:

$$\text{IVIFCA}(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \overset{n}{\underset{i=1}{\oplus}} (\mu(A_{(i)}) - \mu(A_{(i+1)}))\tilde{\alpha}_{(i)}, \tag{1}$$

where (.) indicates a permutation on $X$ such that $\tilde{\alpha}_{(1)} \leq \tilde{\alpha}_{(2)} \leq .... \leq \tilde{\alpha}_{(n)}$ and $A_{(i)} = \{i, ..., n\}$ with $A_{n+1} = \phi$. $\mu$ is the fuzzy measure on the index set $N = \{1, 2, ..., n\}$.

Xu [41] and Tan [40] presented the following Interval-Valued Intuitionistic Fuzzy Geometric Aggregation (IVIFGA) operator, where:

$$\text{IVIFGA}(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \overset{n}{\underset{i=1}{\otimes}} \tilde{\alpha}_{(i)}^{\mu(A_{(i)}) - \mu(A_{(i+1)})}. \tag{2}$$

Herein, (.) indicates a permutation on $N$ such that $\tilde{\alpha}_{(1)} \leq \tilde{\alpha}_{(2)} \leq .... \leq \tilde{\alpha}_{(n)}$ and $A_{(i)} = \{i, ..., n\}$ with $A_{n+1} = \phi$. $\mu$ is the fuzzy measure on the index set $N = \{1, 2, ..., n\}$.

Although IVIFCA and IVIFGA operators consider the interactions between elements in a set, they cannot globally reflect their dependent or correlative characteristics.

Overall, to measure the interactions between coalitions rather than elements, Marichal [45] introduced the generalized Shapley function as follows:

$$\Phi_S(\mu, N) = \sum_{T \subseteq N \setminus S} \frac{(n - s - t)!t!}{(n - s + 1)!}(\mu(S \cup T) - \mu(T)),$$

$$\forall S \subseteq N, \tag{3}$$

where $\mu$ is a fuzzy measure on $N = \{1, 2, ..., n\}$, and $s$, $t$, and $n$ are the cardinalities of $S$, $T$, and $N$, respectively.

When $s = 1$, Eq. (3) degenerates to the Shapley function [46]:

$$\varphi_i(\mu, N) = \sum_{S \subseteq N \setminus i} \frac{(n - s - 1)!s!}{n!}(\mu(S \cup i) - \mu(S)),$$

$$\forall i \in N. \tag{4}$$

It is not difficult to find that Eq. (3) is an expectation value of the overall interactions between coalition $S$ and every coalition in $N \setminus S$, and Eq. (4) is an expectation value of the overall interactions between element $i$ and every coalition in $N \setminus i$ [47].

## 3. An induced generalized symmetrical interval-valued intuitionistic fuzzy Choquet-Shapley operator

Using the operational laws in Definition 3, the generalized Shapley function [45], and the Choquet integral [15], this section defines the Induced Generalized Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (IG-SIVIFCS) operator as follows.

**Definition 6.** Let $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i])$, $i = 1, 2, ..., n$, be a collection of IVIFVs in $\Omega$, and $\Phi$ be the associated generalized Shapley function with respect to the fuzzy measure $\mu$ on $N = \{1, 2, ..., n\}$. The IG-SIVIFCS operator of dimension $n$ is a mapping IG-SIVIFCS: $\Omega^n \rightarrow \Omega$ is defined to aggregate the set of the second arguments of two tuples $< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >$:

$$\mathrm{IG-SIVIFCS}_{\Phi}(< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >) = \left( \overset{n}{\underset{i=1}{\oplus}} \left( \Phi_{A_{(i)}}(\mu,N) - \Phi_{A_{(i+1)}}(\mu,N) \right) \tilde{\alpha}_{(i)}^{\gamma} \right)^{1/\gamma}, \tag{5}$$

where $\gamma \in \mathrm{R}_+$, (.) indicates a permutation on $N$ such that $u_{(j)} \leq u_{(j+1)}$, $u_{(j)}$ is the $j$th least value of $u_i$, and $A_{(i)} = \{i, ..., n\}$ with $A_{(n+1)} = \phi$.

**Remark 2.** When $\gamma \rightarrow 0^+$ the IG-SIVIFCS operator degenerates to the Induced Geometric Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (IG-SIVIFCS) operator:

$$\mathrm{IG-SIVIFCS}_{\Phi}(< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >) = \overset{n}{\underset{i=1}{\otimes}} \tilde{\alpha}_{(i)}^{\Phi_{A_{(i)}}(\mu,N) - \Phi_{A_{(i+1)}}(\mu,N)}.$$

Furthermore, if $u_i = \tilde{\alpha}_i$, $i = 1, 2, ..., n$, then IG-SIVIFCS operator degenerates to the Geometric Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (G-SIVIFCS) operator:

$$\mathrm{G-SIVIFCS}_{\Phi}(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \overset{n}{\underset{i=1}{\otimes}} \tilde{\alpha}_{(i)}^{\Phi_{A_{(i)}}(\mu,N) - \Phi_{A_{(i+1)}}(\mu,N)}.$$

**Remark 3.** When $\gamma = 1$, the IG-SIVIFCS operator reduces to the Induced Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (I-SIVIFCS) operator:

$$\mathrm{I-SIVIFCS}_{\Phi}(< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >) = \overset{n}{\underset{i=1}{\oplus}} \left( \Phi_{A_{(i)}}(\mu,N) - \Phi_{A_{(i+1)}}(\mu,N) \right) \tilde{\alpha}_{(i)}.$$

Furthermore, if $u_i = \tilde{\alpha}_i$, $i = 1, 2, ..., n$, then IG-SIVIFCS operator degenerates to the Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (SIVIFCS) operator:

$$\mathrm{SIVIFCS}_{\Phi}(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \overset{n}{\underset{i=1}{\oplus}} \left( \Phi_{A_{(i)}}(\mu,N) - \Phi_{A_{(i+1)}}(\mu,N) \right) \tilde{\alpha}_{(i)}.$$

**Remark 4.** When $\gamma = 2$, the IG-SIVIFCS operator degenerates to the Induced Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley Quadratic (I-SIVIFCSQ) operator:

$$\mathrm{I-SIVIFCSQ}_{\Phi}(< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >) = \left( \overset{n}{\underset{i=1}{\oplus}} \left( \Phi_{A_{(i)}}(\mu,N) - \Phi_{A_{(i+1)}}(\mu,N) \right) \tilde{\alpha}_{(i)}^{2} \right)^{1/2}.$$

**Remark 5.** When fuzzy measure $\mu$ is additive, the IG-SIVIFCS operator reduces to the induced generalized symmetrical interval-valued intuitionistic fuzzy OWA (IG-SIVIFOWA) operator:

$$\mathrm{IG-SIVIFOWA}_{w}(< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >) = \left( \overset{n}{\underset{i=1}{\oplus}} w_i \tilde{\alpha}_{(i)}^{\gamma} \right)^{1/\gamma},$$

where $w = (w_1, w_2, ..., w_n)^T$ is the associated weighting vector with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

**Theorem 1.** Let $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i])$, $i = 1, 2, ..., n$, be a collection of IVIFVs in $\Omega$, and $\Phi$ be the associated generalized Shapley function with respect to the fuzzy measure $\mu$ on $N = \{1, 2, ..., n\}$. Then, their aggregated value using the IG-SIVIFCS operator is an IVIFV, denoted by:

$$\begin{aligned} &\mathrm{IG-SIVIFCS}_{\Phi}(< u_1, \tilde{\alpha}_1 >, < u_2, \tilde{\alpha}_2 >, ..., < u_n, \tilde{\alpha}_n >) \\ &= \left( \left[ \left( \sum\nolimits_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) a_i^{\gamma} \right)^{1/\gamma}, \right.\right. \\ &\left. \left( \sum\nolimits_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) b_i^{\gamma} \right)^{1/\gamma} \right], \\ &\left[ \left( \sum\nolimits_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) c_i^{\gamma} \right)^{1/\gamma}, \right. \\ &\left.\left. \left( \sum\nolimits_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) d_i^{\gamma} \right) \right]^{1/\gamma} \right). \end{aligned} \tag{6}$$

The notations are shown in Definition 6.

***Proof.*** From Definition 3, we have:

$$\text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\alpha}_1>, <u_2, \tilde{\alpha}_2>, ..., <u_n, \tilde{\alpha}_n>)$$

$$= \left( \overset{n}{\underset{i=1}{\oplus}} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) \tilde{\alpha}^{\gamma}_{(i)} \right)^{1/\gamma}$$

$$= \left( \overset{n}{\underset{i=1}{\oplus}} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) ([a^{\gamma}_{(i)}, b^{\gamma}_{(i)}], [c^{\gamma}_{(i)}, d^{\gamma}_{(i)}]) \right)^{1/\gamma}$$

$$= \left( \overset{n}{\underset{i=1}{\oplus}} \left[ \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) a^{\gamma}_{(i)}, \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) b^{\gamma}_{(i)} \right], \left[ \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) c^{\gamma}_{(i)}, \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) d^{\gamma}_{(i)} \right] \right)^{1/\gamma}$$

$$= \left( \left[ \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) a^{\gamma}_{(i)}, \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) b^{\gamma}_{(i)} \right], \left[ \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) c^{\gamma}_{(i)}, \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) d^{\gamma}_{(i)} \right] \right)^{1/\gamma}$$

$$= \left( \left[ \left( \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) a^{\gamma}_{(i)} \right)^{1/\gamma}, \left( \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) b^{\gamma}_{(i)} \right)^{1/\gamma} \right], \left[ \left( \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) c^{\gamma}_{(i)} \right)^{1/\gamma}, \left( \sum_{i=1}^{n} \left( \Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \right) d^{\gamma}_{(i)} \right)^{1/\gamma} \right] \right).$$

We can easily prove that the IG-SIVIFCS operator is commutative, monotonic, bounded, and idempotent, which are presented as follows.

**Theorem 2.** Let $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i])$ and $\tilde{\beta}_i = ([e_i, f_i], [g_i, h_i])$, $i = 1, 2, ..., n$, be two collections of IVIFVs in $\Omega$, and $\Phi$ be the associated generalized Shapley function with respect to the fuzzy measure $\mu$ on $N = \{1, 2, ..., n\}$.

1. **Commutativity:** Let $\tilde{\alpha}'_i = ([a'_i, b'_i], [c'_i, d'_i])$, $i = 1, 2, ..., n$, be a permutation of $\tilde{\alpha}_i$, and then:

$$\text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\alpha}_1>, ..., <u_n, \tilde{\alpha}_n>) = \text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\alpha}'_1>, ..., <u_n, \tilde{\alpha}'_n>).$$

2. **Idempotency:** If all $\tilde{\alpha}_i$, $i = 1, 2, ..., n$, are equal, i.e. $\tilde{\alpha}_i = \tilde{\alpha} = ([a, b], [c, d])$ for all $i$, then:

$$\text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\alpha}_1>, <u_2, \tilde{\alpha}_2>, ..., <u_n, \tilde{\alpha}_n>) = \tilde{\alpha}.$$

3. **Comonotonicity:** If $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ are comonotonic, namely, $\tilde{\alpha}_{(1)} \leq \tilde{\alpha}_{(2)} \leq ... \leq \tilde{\alpha}_{(n)}$ if $\tilde{\beta}_{(1)} \leq \tilde{\beta}_{(2)} \leq ... \leq \tilde{\beta}_{(n)}$ for all $i$, where (.) is a permutation on $N$ such that $u_{(j)}$ is the $j$th least value of $u_i$, $i = 1, 2, ..., n$, then:

$$\text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\alpha}_1>, ..., <u_n, \tilde{\alpha}_n>) \leq \text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\beta}_1>, ..., <u_n, \tilde{\beta}_n>).$$

4. **Boundary:** Let:

$$\tilde{\alpha}^- = \left( \left[ \min_{i \in N} a_i, \min_{i \in N} b_i \right], \left[ \max_{i \in N} c_i, \max_{i \in N} d_i \right] \right),$$

and:

$$\tilde{\alpha}^+ = \left( \left[ \max_{i \in N} a_i, \max_{i \in N} b_i \right], \left[ \min_{i \in N} c_i, \min_{i \in N} d_i \right] \right),$$

and then:

$$\tilde{\alpha}^- \leq \text{IG} - \text{SIVIFCS}_{\Phi}(<u_1, \tilde{\alpha}_1>, <u_2, \tilde{\alpha}_2>, ..., <u_n, \tilde{\alpha}_n>) \leq \tilde{\alpha}^+.$$

***Proof.*** Following Property 1 and Theorem 1, we can easily derive all of the above conclusions.

## 4. A new approach to interval-valued intuitionistic fuzzy multi-attribute decision-making

### *4.1. Models for obtaining the optimal fuzzy measure*

When the weight of each attribute is given, we can use some aggregation operator to get the optimal alternative(s). However, because of time pressure and the expert's limited expertise about the problem domain, the weighting information of attributes may be partly known.

Considering a decision-making problem, let $A = \{a_1, a_2, ..., a_m\}$ be the set of alternatives, and $C = \{c_1, c_2, ..., c_n\}$ be the set of attributes. Assume that the judgment of the alternative $a_i$, $i = 1, 2, ..., m$, for attribute $c_j$, $j = 1, 2, ..., n$, is given as IVIFV $\tilde{\alpha}_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$. Then, we obtain the Interval-Valued Intuitionistic Fuzzy Decision Matrix (IVIFDM) $A = (\tilde{\alpha}_{ij})_{m\times n}$. Similar to the above analysis, let $\tilde{\alpha}^+ = \{\tilde{\alpha}_1^+, \tilde{\alpha}_2^+, ..., \tilde{\alpha}_n^+\}$ and $\tilde{\alpha}^- = \{\tilde{\alpha}_1^-, \tilde{\alpha}_2^-, ..., \tilde{\alpha}_n^-\}$, where:

$$\tilde{\alpha}_j^+ = \left(\left[\max_{1\le i\le m} a_{ij}, \max_{1\le i\le m} b_{ij}\right], \left[\min_{1\le i\le m} c_{ij}, \min_{1\le i\le m} d_{ij}\right]\right),$$

and:

$$\tilde{\alpha}_j^- = \left(\left[\min_{1\le i\le m} a_{ij}, \min_{1\le i\le m} b_{ij}\right], \left[\max_{1\le i\le m} c_{ij}, \max_{1\le i\le m} d_{ij}\right]\right),$$

for all $j = 1, 2, ..., n$.

Let:

$$d_{ij} = \frac{d_{ij}^+(\tilde{\alpha}_{ij}, \tilde{\alpha}_j^+)}{d_{ij}^+(\tilde{\alpha}_{ij}, \tilde{\alpha}_j^+) + d_{ij}^-(\tilde{\alpha}_{ij}, \tilde{\alpha}_j^-)},$$

where:

$$d_{ij}^+(\tilde{\alpha}_{ij}, \tilde{\alpha}_j^+) = \frac{\left[\begin{array}{c} |a_{ij} - \max\limits_{1\le i\le m} a_{ij}| + |b_{ij} - \max\limits_{1\le i\le m} b_{ij}| \\ +|c_{ij} - \min\limits_{1\le i\le m} c_{ij}| + |d_{ij} - \min\limits_{1\le i\le m} d_{ij}| \end{array}\right]}{4},$$

and:

$$d_{ij}^-(\tilde{\alpha}_{ij}, \tilde{\alpha}_j^-) = \frac{\left[\begin{array}{c} |a_{ij} - \min\limits_{1\le i\le m} a_{ij}| + |b_{ij} - \min\limits_{1\le i\le m} b_{ij}| \\ +|c_{ij} - \max\limits_{1\le i\le m} c_{ij}| + |d_{ij} - \max\limits_{1\le i\le m} d_{ij}| \end{array}\right]}{4}.$$

Because the optimal fuzzy measure makes the optimal comprehensive value of each alternative as "the bigger, the better", when the weighting information of the attributes is partly known, the following model is built to obtain the optimal fuzzy measure $\mu$ with respect to alternative $a_i$:

$$\min \sum_{j=1}^{n} d_{ij}\varphi_{c_j}(\mu, C),$$

$$\text{s.t.} \begin{cases} \mu(C) = 1 \\ \mu(S) \le \mu(T) \quad \forall S, T \subseteq C \quad \text{s.t. } S \subseteq T \\ \mu(c_j) \in U_j, \quad \mu(c_j) \ge 0 \end{cases} \tag{7}$$

where $i = 1, 2, ..., m$, $\varphi_{c_j}(\mu, C)$ is the Shapley value of attribute $c_j$ given as Eq. (4), $\mu$ is the fuzzy measure on attribute set $C$, and $U_j$ is the known weighting information of attribute $c_j$.

Because all alternatives are non-inferior, build the following model is built even further for determining the optimal fuzzy measure $\mu$ on attribute set $C$:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\varphi_{c_j}(\mu, C),$$

$$\text{s.t.} \begin{cases} \mu(C) = 1 \\ \mu(S) \le \mu(T) \quad \forall S, T \subseteq C \text{ s.t.} S \subseteq T \\ \mu(c_j) \in U_j, \quad \mu(c_j) \ge 0, j = 1, 2, ..., n \end{cases} \tag{8}$$

where the notations are shown in Model (7).

In Models (7) and (8), we use the Shapley values of attributes as their weights, which globally consider the importance of each attribute. When there is no interaction among them, Model (8) degenerates to model for the optimal additive weighting vector, where:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} w_{c_j},$$

$$\text{s.t.} \begin{cases} \sum\limits_{j=1}^{n} w_{c_j} = 1 \\ w_{c_j} \in U_j, \quad w_{c_j} \ge 0, \ j = 1, 2, ..., n. \end{cases} \tag{9}$$

### 4.2. A decision-making procedure

Based on the IG-SIVIFCS operator and models for calculating the optimal fuzzy measure on the attribute set, an approach is developed for interval-valued intuitionistic fuzzy multi-attribute decision-making with incomplete weighting information and interactive characteristics.

## 5. A numerical example

Assume that a manufacturing company seeks to select the best global supplier according to the core competencies of suppliers [18]. Now, suppose that there are four suppliers $A = \{a_1, a_2, a_3, a_4\}$ whose core competencies are evaluated using the following four attributes $C = \{c_1, c_2, c_3, c_4\}$:

(i) The level of technology innovation, $c_1$;

(ii) The control ability of flow, $c_2$;

(iii) The ability of management, $c_3$;

(iv) The level of service, $c_4$.

There is an expert team that is invited to evaluate the core competence of these four candidates following the above four attributes, where the IVIFDM is given as shown in Box I.

$$\tilde{A}=\begin{pmatrix}([0.2,0.4],[0,0.2]) & ([0.4,0.6],[0.3,0.5]) & ([0.4,0.6],[0.2,0.4]) & ([0.6,0.8],[0.1.0.2])\\ ([0.2,0.4],[0.3,0.5]) & ([0.3,0.5],[0.2,0.4]) & ([0.5,0.6],[0.2,0.4]) & ([0.7,0.9],[0,0.1])\\ ([0.4,0.6],[0.3,0.5]) & ([0.4,0.6],[0.3,0.4]) & ([0.3,0.6],[0,0.2]) & ([0.5,0.7],[0.1,0.3])\\ ([0.1,0.3],[0.4,0.6]) & ([0.3,0.5],[0.1,0.3]) & ([0.5,0.8],[0.1,0.2]) & ([0.6,0.8],[0.1,0.2])\end{pmatrix}.$$

Box I

The weighting range of each attribute is offered as follows:

$$U_1=[0.3,0.5],\quad U_2=[0.15,0.35],$$

$$U_3=[0.27,0.47],\quad U_4=[0.1,0.3].$$

In the following, Algorithm 1 can be utilized to derive the most desirable supplier(s).

**Step 1:** Because all attributes are benefits, there is no need to modify IVIFDM , $\tilde{A}$ namely $\tilde{R}=\tilde{A}$.

**Step 2:** From $\tilde{A}$ we get:

$$\tilde{\alpha}_1^+=([0.4,0.6],[0,0.2]),$$

$$\tilde{\alpha}_2^+=([0.4,0.6],[0.1,0.3]),$$

$$\tilde{\alpha}_3^+=([0.5,0.8],[0,0.2]),$$

$$\tilde{\alpha}_4^+=([0.7,0.9],[0,0.1]),$$

and:

$$\tilde{\alpha}_1^-=([0.1,0.3],[0.4,0.6]),$$

$$\tilde{\alpha}_2^-=([0.3,0.5],[0.3,0.5]),$$

$$\tilde{\alpha}_3^-=([0.3,0.6],[0.2,0.4]),$$

$$\tilde{\alpha}_4^-=([0.5,0.7],[0.1,0.3]).$$

According to $\tilde{\alpha}^+$ and $\tilde{\alpha}^-$ the distance matrices are derived as follows:

$$D^+=\begin{pmatrix}0.2 & 0.2 & 0.35 & 0.2\\ 0.5 & 0.2 & 0.3 & 0\\ 0.3 & 0.15 & 0.2 & 0.35\\ 0.7 & 0.1 & 0.05 & 0.2\end{pmatrix},$$

$$D^-=\begin{pmatrix}0.5 & 0.1 & 0.05 & 0.15\\ 0.2 & 0.1 & 0.1 & 0.35\\ 0.4 & 0.15 & 0.2 & 0\\ 0 & 0.2 & 0.35 & 0.15\end{pmatrix}.$$

---

**Step 1:** If all attributes $c_j$, $j=1,2,...,n$, are benefits (i.e., the larger, the greater preference), then do not normalize the attribute values. Otherwise, the IVIFDM $A=(\tilde{\alpha}_{ij})_{m\times n}$ will be normalized into:

$$R=\begin{pmatrix}\tilde{r}_{11} & \tilde{r}_{12} & \dots & \tilde{r}_{1n}\\ \tilde{r}_{21} & \tilde{r}_{22} & \dots & \tilde{r}_{2n}\\ \dots & \dots & \dots & \dots\\ \tilde{r}_{m1} & \tilde{r}_{m2} & \dots & \tilde{r}_{mn}\end{pmatrix},$$

where $\tilde{r}_{ij}=\begin{cases}\tilde{\alpha}_{ij} & \text{for benefit attribute } c_j\\ (\tilde{\alpha}_{ij})^C & \text{cost attribute} c_j\end{cases}$, $i=1,2,...,m$; $j=1,2,...,n$, and $(\tilde{\alpha}_{ij})^C=([c_{ij},d_{ij}],[a_{ij},b_{ij}])$ is the complement of $\tilde{\alpha}_{ij}$ and $r_{ij}=([e_{ij},f_{ij}],[g_{ij},h_{ij}])$, $i=1,2,...,m$; $j=1,2,...,n$ [8].

**Step 2:** Adopt Model (8) to calculate the optimal fuzzy measure on attribute set $C$.

**Step 3:** Calculate the Shapley value of each attribute by using Eq. (4).

**Step 4:** For each $i=1,2,...,m$, reorder $\{\tilde{r}_{ij}\}_{j=1,2,...,n}$ such that $\varphi_{(1)}(\mu,C)\le\varphi_{(2)}(\mu,C)\le...\le\varphi_{(n)}(\mu,C)$. According to $\varphi_j(\mu,C)$, $j=1,2,...,n$, the generalized Shapley values are calculated.

**Step 5:** Let $u_j=\varphi_j(\mu,C)$, $j=1,2,...,n$ and use the IG-SIVIFCS operator:

$$\tilde{\alpha}_i=\text{IG}-\text{SIVIFCS}_\Phi(<u_1,\tilde{r}_{i1}>,<u_2,\tilde{r}_{i2}>,...,<u_n,\tilde{r}_{in}>)=\left(\bigoplus_{j=1}^{n}\left(\Phi_{A_{(j)}}(\mu,C)\quad-\Phi_{A_{(j+1)}}(\mu,C)\right)(\tilde{r}_{i(j)})^\gamma\right)^{1/\gamma},$$

to calculate the synthetical IVIFV $\tilde{\alpha}_i=([u_i,v_i],[x_i,y_i])$ of the alternative $a_i$, $i=1,2,...,m$, where $A_{(j)}=\{c_j,...,c_n\}$ and $A_{(n+1)}=\{\phi\}$.

**Step 6:** According to the IVIFVs $\tilde{\alpha}_i$, $i=1,2,...,m$, calculate score function $S(\tilde{\alpha}_i)$ and accuracy degree $H(\tilde{\alpha}_i)$ to rank the alternatives $a_i$, $i=1,2,...,m$, and, then, select the best one.

**Step 7:** End.

---

**Algorithm 1.** Ranking alternatives from IVIFDMs based on the IG-SIVIFCS operator.

From $D^+$ and $D^-$, the following relative distance matrix is obtained:

$$D=\begin{pmatrix} 0.286 & 0.667 & 0.875 & 0.571 \\ 0.714 & 0.667 & 0.75 & 0 \\ 0.429 & 0.5 & 0.5 & 1 \\ 1 & 0.333 & 0.125 & 0.571 \end{pmatrix}.$$

Following Model (8), the following linear programming is derived:

$$\begin{aligned}\min\ & 0.061\,(\mu(c_1)-\mu(c_2,c_3,c_4)) \\ & -0.027\,(\mu(c_2)-\mu(c_1,c_3,c_4)) \\ & +0.001\,(\mu(c_3)-\mu(c_1,c_2,c_4)) \\ & -0.035\,(\mu(c_4)-\mu(c_1,c_2,c_3)) \\ & +0.017\,(\mu(c_1,c_2)-\mu(c_3,c_4)) \\ & +0.031\,(\mu(c_1,c_3)-\mu(c_2,c_4)) \\ & +0.013\,(\mu(c_1,c_4)-\mu(c_2,c_3))+2.25,\end{aligned}$$

$$\text{s.t.}\begin{cases}\mu(C)=1\\ \mu(S)\leq\mu(T)\ \ \forall S,T\subseteq C=\{c_1,c_2,c_3,c_4\}\\ \mu(c_1)\in[0.3,0.5], \quad \mu(c_2)\in[0.15,0.35]\\ \mu(c_3)\in[0.27,0.47], \quad \mu(c_4)\in[0.1,0.3]\end{cases}$$

By solving the above linear programming using LINGO, we obtain:

$$\begin{aligned}\mu(c_1)&=\mu(c_2)=\mu(c_4)=\mu(c_1,c_2)=\mu(c_1,c_3)\\ &=\mu(c_1,c_4)=\mu(c_2,c_3)=\mu(c_3,c_4)\\ &=\mu(c_1,c_2,c_3)=\mu(c_1,c_3,c_4)=0.3,\end{aligned}$$

$$\mu(c_3)=0.27,$$

$$\mu(c_2,c_4)=\mu(c_1,c_2,c_4)=\mu(c_2,c_3,c_4)=\mu(C)=1.$$

**Step 3:** From the fuzzy measure $\mu$ obtained in Step 2, the following Shapley values are obtained:

$$\varphi_{c_1}(\mu,C)=0.078, \quad \varphi_{c_2}(\mu,C)=0.43,$$

$$\varphi_{c_3}(\mu,C)=0.063, \quad \varphi_{c_4}(\mu,C)=0.43.$$

**Step 4:** Because $\varphi_{c_2}(\mu,C)=\varphi_{c_4}(\mu,C)$, we rearrange them according to the index in ascending order, where $\varphi_{c_3}(\mu,C)<\varphi_{c_1}(\mu,C)<\varphi_{c_2}(\mu,C)=\varphi_{c_4}(\mu,C)$.

From $\{\varphi_{c_j}(\mu,C)\}_{j=1,2,3,4}$, we obtain:

$$\Phi_C(\mu,C)=1, \quad \Phi_{C\backslash c_3}(\mu,C)=0.865,$$

$$\Phi_{\{c_2,c_4\}}(\mu,C)=0.805, \quad \Phi_{c_4}(\mu,C)=0.43.$$

**Step 5:** Let $\gamma=2$. The synthetical IVIFVs $\tilde{\alpha}_i$, $i=1,2,3,4$, are obtained by using the I-SIVIFCSQ operator. For example, let $i=1$, we have:

$$\begin{aligned}\tilde{\alpha}_1=&\mathrm{I-SIVIFCSQ}_\Phi(<u_1,\tilde{r}_{11}>,\\ &<u_2,\tilde{r}_{12}>,<u_3,\tilde{r}_{13}>,<u_4,\tilde{r}_{14}>)\\ =&\Bigg(\bigg[\big(0.135\cdot(a_{13}^2)+0.06\cdot(a_{11}^2)\\ &+0.375\cdot(a_{12}^2)+0.43\cdot(a_{14}^2)\big)^{1/2},\\ &\big(0.135\cdot(b_{13}^2)+0.06\cdot(b_{11}^2)\\ &+0.375\cdot(b_{12}^2)+0.43\cdot(b_{14}^2)\big)^{1/2}\bigg],\\ &\bigg[\big(0.135\cdot(c_{13}^2)+0.06\cdot(c_{11}^2)\\ &+0.375\cdot(c_{12}^2)+0.43\cdot(c_{14}^2)\big)^{1/2},\\ &\big(0.135\cdot(d_{13}^2)+0.06\cdot(d_{11}^2)\\ &+0.375\cdot(d_{12}^2)+0.43\cdot(d_{14}^2)\big)^{1/2}\bigg]\Bigg)\\ =&([0.4887,0.6844],[0.2084,0.3674]).\end{aligned}$$

Similar to the calculation of $\tilde{\alpha}_1$ the following synthetical IVIFVs are derived:

$$\tilde{\alpha}_2=([0.5297,0.7073],[0.1606,0.3176]),$$

$$\tilde{\alpha}_3=([0.4350,0.6449],[0.2084,0.3451]),$$

$$\tilde{\alpha}_4=([0.4721,0.6788],[0.1378,0.2792]).$$

**Step 6:** According to $\tilde{\alpha}_i$, $i=1,2,3,4$, the following scores are derived:

$$S(\tilde{\alpha}_1)=0.299, \quad S(\tilde{\alpha}_2)=0.379,$$

$$S(\tilde{\alpha}_3)=0.263, \quad S(\tilde{\alpha}_4)=0.367,$$

where $S(\tilde{\alpha}_2)>S(\tilde{\alpha}_4)>S(\tilde{\alpha}_1)>S(\tilde{\alpha}_3)$. Thus, the ranking order is $a_2\succ a_4\succ a_1\succ a_3$, and supplier $a_2$ is the best choice.

In the above example, when $\gamma = 1$, the I-SIVIFCS operator is applied to calculate the synthetical IVIFVs $\tilde{\alpha}_i$, $i = 1, 2, 3, 4$, where:

$$\tilde{\alpha}_1 = \text{I} - \text{SIVIFCS}_\Phi$$
$$(<u_1, \tilde{r}_{11}>, <u_2, \tilde{r}_{12}>, <u_3, \tilde{r}_{13}>, <u_4, \tilde{r}_{14}>)$$
$$= ([0.135 \cdot a_{13} + 0.06 \cdot a_{11} + 0.375 \cdot a_{12} + 0.43 \cdot a_{14}, 0.135 \cdot b_{13} + 0.06 \cdot b_{11} + 0.375 \cdot b_{12} + 0.43 \cdot b_{14}], [0.135 \cdot c_{13} + 0.06 \cdot c_{11} + 0.375 \cdot c_{12} + 0.43 \cdot c_{14}, 0.135 \cdot d_{13} + 0.06 \cdot d_{11} + 0.375 \cdot d_{12} + 0.43 \cdot d_{14}]) = ([0.4740, 0.6740], [0.1825, 0.3395]).$$

Similarly, the following synthetical IVIFVs are derived:

$$\tilde{\alpha}_2 = ([0.4930, 0.6795], [0.1200, 0.2770]),$$
$$\tilde{\alpha}_3 = ([0.4295, 0.6430], [0.1735, 0.3360]),$$
$$\tilde{\alpha}_4 = ([0.4440, 0.6575], [0.1180, 0.2615]).$$

From $\tilde{\alpha}_i$, $i = 1, 2, 3, 4$, the following scores are obtained:

$$S(\tilde{\alpha}_1) = 0.313, \quad S(\tilde{\alpha}_2) = 0.388,$$
$$S(\tilde{\alpha}_3) = 0.282, \quad S(\tilde{\alpha}_4) = 0.361,$$

namely, $S(\tilde{\alpha}_2) > S(\tilde{\alpha}_4) > S(\tilde{\alpha}_1) > S(\tilde{\alpha}_3)$ and $a_2 \succ a_4 \succ a_1 \succ a_3$. The ranking results are the same as those obtained from the I-SIVIFCSQ operator, and supplier $a_2$ is the best choice.

When $\gamma \to 0^+$, the IG-SIVIFCS operator is used to calculate the synthetical IVIFVs $\tilde{\alpha}_i$, $i = 1, 2, 3, 4$, where:

$$\tilde{\alpha}_1 = \text{IG} - \text{SIVIFCS}_\Phi$$
$$(<u_1, \tilde{r}_{11}>, <u_2, \tilde{r}_{12}>, <u_3, \tilde{r}_{13}>, <u_4, \tilde{r}_{14}>)$$
$$= ([a_{13}^{0.135} \cdot a_{11}^{0.06} \cdot a_{12}^{0.375} \cdot a_{14}^{0.43}, b_{13}^{0.135} \cdot b_{11}^{0.06} \cdot b_{12}^{0.375} \cdot b_{14}^{0.43}], [c_{13}^{0.135} \cdot c_{11}^{0.06} \cdot c_{12}^{0.375} \cdot c_{14}^{0.43}, d_{13}^{0.135} \cdot d_{11}^{0.06} \cdot d_{12}^{0.375} \cdot d_{14}^{0.43}])$$
$$= ([0.4568, 0.6627], [0, 0.3097]).$$

Similarly, the following synthetical IVIFVs are obtained:

$$\tilde{\alpha}_2 = ([0.4516, 0.6510], [0, 0.2234]),$$
$$\tilde{\alpha}_3 = ([0.4235, 0.6411], [0, 0.3262]),$$
$$\tilde{\alpha}_4 = ([0.4054, 0.6324], [0.1087, 0.2487]).$$

According to $\tilde{\alpha}_i$, $i = 1, 2, 3, 4$, we have the following scores:

$$S(\tilde{\alpha}_1) = 0.405, \quad S(\tilde{\alpha}_2) = 0.440,$$
$$S(\tilde{\alpha}_3) = 0.369, \quad S(\tilde{\alpha}_4) = 0.340,$$

by which $S(\tilde{\alpha}_2) > S(\tilde{\alpha}_1) > S(\tilde{\alpha}_3) > S(\tilde{\alpha}_4)$ and $a_2 \succ a_1 \succ a_3 \succ a_4$ are derived. The ranking results are different from those derived from the I-SIVIFCSQ and I-SIVIFCSQ operators. However, the best choice is still supplier $a_2$.

When the Interval-Valued Intuitionistic Fuzzy Correlated Averaging (IVIFCA) operator [41] is adopted to calculate the synthetical IVIFVs, we get:

$$\tilde{\alpha}_1 = \text{IVIFCA}(\tilde{r}_{11}, \tilde{r}_{12}, \tilde{r}_{13}, \tilde{r}_{14})$$
$$= \Big( \Big[ 1 - (1 - a_{13})^{0.7} \cdot (1 - a_{11})^{0} \cdot (1 - a_{12})^{0} \cdot (1 - a_{14})^{0.3}, 1 - (1 - b_{13})^{0.7} \cdot (1 - b_{11})^{0} \cdot (1 - b_{12})^{0} \cdot (1 - b_{14})^{0.3} \Big], [c_{13}^{0.7} \cdot c_{11}^{0} \cdot c_{12}^{0} \cdot c_{14}^{0.3}, d_{13}^{0.7} \cdot d_{11}^{0} \cdot d_{12}^{0} \cdot d_{14}^{0.3}] \Big)$$
$$= ([0.47, 0.68], [0, 0.38]).$$

Similar to the calculation of $\tilde{\alpha}_1$ the following synthetical IVIFVs are obtained:

$$\tilde{\alpha}_2 = ([0.46, 0.69], [0, 0.26]),$$
$$\tilde{\alpha}_3 = ([0.43, 0.63], [0, 0.37]),$$
$$\tilde{\alpha}_4 = ([0.41, 0.62], [0.1, 0.27]).$$

From $\tilde{\alpha}_i$, $i = 1, 2, 3, 4$, the scores are:

$$S(\tilde{\alpha}_1) = 0.385, \quad S(\tilde{\alpha}_2) = 0.445,$$
$$S(\tilde{\alpha}_3) = 0.345, \quad S(\tilde{\alpha}_i) = 0.330.$$

Thus, $S(\tilde{\alpha}_2) > S(\tilde{\alpha}_1) > S(\tilde{\alpha}_3) > S(\tilde{\alpha}_4)$ and $a_2 \succ a_1 \succ a_3 \succ a_4$, which are the same as those derived from the IG-SIVIFCS with $a_2$, are the best choice.

If the Interval-Valued Intuitionistic Fuzzy Geometric Aggregation (IVIFGA) operator [40,41] is applied to calculate the synthetical IVIFVs $\tilde{\alpha}_i$, $i = 1, 2, 3, 4$, we derive:

$$\begin{aligned}\tilde{\alpha}_1 =&\text{IVIFGA}(\tilde{r}_{11},,\tilde{r}_{12},\tilde{r}_{13},\tilde{r}_{14})\\ =&\Big(\left[a_{13}^{0.7}\cdot a_{11}^{0}\cdot a_{12}^{0}\cdot a_{14}^{0.3}, b_{13}^{0.7}\cdot b_{11}^{0}\cdot b_{12}^{0}\cdot b_{14}^{0.3}\right],\\ &\Big[1-(1-c_{13})^{0.7}\cdot(1-c_{11})^{0}\cdot(1-c_{12})^{0}\cdot(1-c_{14})^{0.3},\\ &1-(1-d_{13})^{0.7}\cdot(1-d_{11})^{0}\cdot(1-d_{12})^{0}\cdot(1-d_{14})^{0.3}\Big]\Big)\\ =&([0.45,0.65],[0.25,0.42]).\end{aligned}$$

Similarly, the following synthetical IVIFVs are derived:

$$\tilde{\alpha}_2 = ([0.39,0.6],[0.14,0.32]),$$

$$\tilde{\alpha}_3 = ([0.43,0.63],[0.25,0.37]),$$

$$\tilde{\alpha}_4 = ([0.37,0.58],[0.1,0.27]).$$

With respect to $\tilde{\alpha}_i$, $i = 1,2,3,4$, the scores are:

$$S(\tilde{\alpha}_1) = 0.215, \quad S(\tilde{\alpha}_2) = 0.265,$$

$$S(\tilde{\alpha}_3) = 0.220, \quad S(\tilde{\alpha}_4) = 0.290,$$

by which we obtain $S(\tilde{\alpha}_4) > S(\tilde{\alpha}_2) > S(\tilde{\alpha}_3) > S(\tilde{\alpha}_1)$. Thus, $a_4 \succ a_2 \succ a_3 \succ a_1$, by which the best supplier is $a_4$. The ranking results as well as the best choice are different from that derived from the four above-listed aggregation operators, where the best choice is supplier $a_2$.

When we assume there is no interaction between the weights of the attributes, the additive weighting vector is $w = (0.3,0.15,0.27,0.28)$. When the Interval-Valued Intuitionistic Fuzzy Weighted Geometric (IIFWG) operator [36] is used to calculate the collective values of alternatives that adopt the operations in Definition 2, we obtain:

$$\begin{aligned}\tilde{\alpha}_1 =& \text{IIFWG}(\tilde{r}_{11},\tilde{r}_{12},\tilde{r}_{13},\tilde{r}_{14})\\ =& \Big(\left[a_{11}^{0.3}\cdot a_{12}^{0.15}\cdot a_{13}^{0.27}\cdot a_{14}^{0.28}, b_{11}^{0.3}\cdot b_{12}^{0.15}\cdot b_{13}^{0.27}\cdot b_{14}^{0.28}\right],\\ &\Big[1-(1-c_{11})^{0.3}\cdot(1-c_{12})^{0.15}\cdot(1-c_{13})^{0.27}\\ &\cdot(1-c_{14})^{0.28}, 1-(1-d_{11})^{0.3}\cdot(1-d_{12})^{0.15}\\ &\cdot(1-d_{13})^{0.27}\cdot(1-d_{14})^{0.28}\Big]\Big)\\ =& ([0.3321,0.5472],[0.1335,0.3102]).\end{aligned}$$

In a similar way, we have:

$$\tilde{\alpha}_2 = ([0.3427,0.5403],[0.1819,0.3636]),$$

$$\tilde{\alpha}_3 = ([0.3595,0.5953],[0.1730,0.3590]),$$

$$\tilde{\alpha}_4 = ([0.2666,0.5183],[0.2031,0.3631]).$$

Thus, the scores include $S(\tilde{\alpha}_1) = 0.218$, $S(\tilde{\alpha}_2) = 0.169$, $S(\tilde{\alpha}_3) = 0.211$, and $S(\tilde{\alpha}_4) = 0.109$; the ranking order is $a_1 \succ a_3 \succ a_2 \succ a_4$, namely, the best supplier is $a_1$.

Furthermore, when Liu's method based on the IVIFPWHA operator [25] is adopted, the comprehensive IVIFVs are derived as follows:

$$\tilde{\alpha}_1 = ([0.5537,0.3601],[0.0000,0.3055]),$$

$$\tilde{\alpha}_2 = ([0.4931,0.3126],[0.0000,0.2830]),$$

$$\tilde{\alpha}_3 = ([0.5638,0.3439],[0.0000,0.3076]),$$

$$\tilde{\alpha}_4 = ([0.5443,0.3156],[0.1303,0.2692]),$$

where $p = q = 2$ is used in the IVIFPWHA operator that is the same as that in [25].

Following the comprehensive IVIFVs, we have $S(\tilde{\alpha}_1) = 0.304$, $S(\tilde{\alpha}_2) = 0.261$, $S(\tilde{\alpha}_3) = 0.300$, $S(\tilde{\alpha}_4) = 0.230$, and $a_1 \succ a_3 \succ a_2 \succ a_4$. Thus, supplier $a_1$ is the best choice. Note that the IVIFPWHA operator adopts different weights of attributes for different alternatives.

Now, Lin and Zhang's method [26] should be considered. Because Lin and Zhang [26] did not consider the situation where the weighting information is not exactly known, the weight vector on the attribute set is defined the same as that used by the IIFWG operator [36], namely, $w = (0.3,0.15,0.27,0.28)$. When Lin and Zhang's method based on the RC-IVIFOWA operator [26] is applied, the comprehensive intuitionistic fuzzy values are obtained as follows:

$$\tilde{\alpha}_1 = (0.4856,0.1520), \quad \tilde{\alpha}_2 = (0.5392,0.1639),$$

$$\tilde{\alpha}_3 = (0.4805,0.1829), \quad \tilde{\alpha}_4 = (0.4925,0.2008),$$

where $\lambda = 1/3$ that is used in the RC-IVIFOWA operator, which are the same as in [26].

According to the comprehensive intuitionistic fuzzy values, the scores of suppliers are $S(\tilde{\alpha}_1) = 0.167$, $S(\tilde{\alpha}_2) = 0.188$, $S(\tilde{\alpha}_3) = 0.149$, and $S(\tilde{\alpha}_4) = 0.146$, and the ranking order is $a_2 \succ a_1 \succ a_3 \succ a_4$. Thus, the best supplier is $a_2$ that is the same as that obtained from our aggregation operators. However, the ranking values are different. Note that the RC-IVIFOWA operator causes information loss by only considering one point in the interval preferred and non-preferred judgments.

To show the ranking values and the ranking orders based on different aggregation operators clearly, please see Table 1.

The numerical results show that the different optimal alternatives may be yielded by using different aggregation operators. Thus, the decision-maker can properly select a desirable alternative according to his/her interest and the actual needs. Note that the previous interval-valued intuitionistic fuzzy geometric aggregation operators adopt the operational laws listed

**Table 1.** Ranking values and ranking orders based on different aggregation operators.

| Methods | Ranking values | | | | Ranking orders |
|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | |
| Liu's method based on the IVIFPWHA operator [25] | 0.304 | 0.261 | 0.300 | 0.230 | $a_1 \succ a_3 \succ a_2 \succ a_4$ |
| Lin and Zhang's method based on the RC-IVIFOWA operator [26] | 0.167 | 0.188 | 0149 | 0.146 | $a_2 \succ a_1 \succ a_3 \succ a_4$ |
| Xu's method based on the IIFWG operator [36] | 0.218 | 0.169 | 0.211 | 0.109 | $a_1 \succ a_3 \succ a_2 \succ a_4$ |
| Tan and Xu's methods based on the IVIFGA operator [40,41] | 0.215 | 0.265 | 0.220 | 0.290 | $a_4 \succ a_2 \succ a_3 \succ a_1$ |
| Xu's method based on the IVIFCA operator [41] | 0.385 | 0.445 | 0.345 | 0.330 | $a_2 \succ a_1 \succ a_3 \succ a_4$ |
| Our method based on the I-SIVIFCSQ operator | 0.299 | 0.379 | 0.263 | 0.367 | $a_2 \succ a_4 \succ a_1 \succ a_3$ |
| Our method based on the I-SIVIFCS operator | 0.313 | 0.388 | 0.282 | 0.361 | $a_2 \succ a_4 \succ a_1 \succ a_3$ |
| Our method based on the IG-SIVIFCS operator | 0.405 | 0.440 | 0.369 | 0.340 | $a_2 \succ a_1 \succ a_3 \succ a_4$ |

**Table 2.** Comparison of several related methods.

| Methods | Do undesirable properties exist? | Are the weights the same for different alternatives? | Are the interactive characteristics of the weighting information considered? | Are the overall interactions reflected? | Is decision-making with incomplete weighting information studied? | Does information loss exist? |
|---|---|---|---|---|---|---|
| Method [25] | Yes | No | No | No | Yes | No |
| Method [26] | Yes | Yes | No | No | No | Yes |
| Method [36] | Yes | Yes | No | No | No | No |
| Method [40] | Yes | Yes | Yes | No | No | No |
| Method [41] | Yes | Yes | Yes | No | No | No |
| Our method | No | Yes | Yes | Yes | Yes | No |

in Definition 2, meaning that they have the limitations as pointed out in Subsection 2.2. Thus, the decision-makers are advised to apply the new operators to calculate the comprehensive attribute values.

To show the differences between the listed methods in Table 1, they are compared even further with respect to their principles as shown in Table 2.

## 6. Conclusion

An interval-valued intuitionistic fuzzy aggregation operator was defined, called the Induced Generalized Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (IG-SIVIFCS) operator. It is worth pointing out that this operator overcomes the limitations of some existing aggregation operators by using new operations. To ensure its successful application reasonably, some desirable properties were researched. When the weighting information of attributes is partly known, models for determining the optimal weighting vector on the attribute set are established. Based on the above discussion, an approach to interval-valued intuitionistic fuzzy multi-attribute decision-making with incomplete weighting information and correlative characteristics was developed, expanding the rational application of decision-making with interval-valued intuitionistic fuzzy information. The main contributions of this paper include:

(i) Two new operational laws are defined that avoid the undesirable properties existing in previous ones;

(ii) Following the new operations, the IG-SIVIFCS operator is defined, which globally considers the importance of the elements and, overall, reflects their interactions;

(iii) When the weighting information is not exactly known, models are built for determining the fuzzy measure on the attribute set;

(iv) A new method to interval-valued intuitionistic fuzzy decision making is proposed that avoids the constraining issues of previous ones.

However, this study only researched one induced generalized symmetrical interval-valued intuitionistic fuzzy operator, and it is essentially useful to study other interval-valued intuitionistic fuzzy operators by using the Choquet integral and the generalized Shapley function, such as Pythagorean fuzzy power aggregation operator [48,49], cosine similarity measure [50], Pythagorean fuzzy Hamacher aggregation operator [51], and Bonferroni mean operator [52]. Furthermore, the application of the new method in some other fields shall continue.

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## Biographies

**Jie Tang** is doing her PhD degree in School of Business, Central South University. She received her master degree in School of Management, Qingdao Technological University, China in 2008. Currently, she has contributed 15 journal articles to professional journals including Computers & Industrial Engineering, Knowledge-Based Systems, Journal of the Operational Research Society, International Journal of Intelligent Systems, Neural Computing and Applications, International Journal of Computational Intelligence Systems, and Journal of Intelligent & Fuzzy Systems. Her research interests include uncertain multi-attribute decision making, cluster analysis, and pattern recognition.

**Fanyong Meng** is a Professor in School of Business, Central South University. He received his master degree in School of Mathematics and Information Science, Guangxi University, China in 2008, and earned his PhD degree in School of Management and Economics, Beijing Institute of Technology, China in 2011. He finished his postdoctoral work in School of Business, Central South University, China, in 2016. Currently, he has contributed over 100 journal articles to professional journals such as Omega, Fuzzy Sets and Systems, IEEE Transactions on Systems, and so on. His research interests include fuzzy mathematics, decision-making, and games theory.