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A queuing theory-based approach to designing cellular manufacturing systems

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Abstract. This paper presents a new cell formation and cell layout problem considering multiple process routings and subcontracting using the principles of queuing theory. It was assumed that each machine operated as an $M/M/1$ queuing system and a queuing network was used to obtain in-process inventories and machine utilization. The problem was formulated as a mixed-integer nonlinear program with the objective of minimizing the total costs, including the production, subcontracting, material handling, machine idleness, and holding costs. Due to the computational complexity of the problem, a heuristic method is suggested to effectively solve the problem. A numerical example is given to clarify the proposed approach, and finally, further instances are solved to verify the performance of the solution method and to accomplish comparisons. The computational results show that the proposed heuristic is both effective and efficient.

1. Introduction

Group Technology (GT) is a manufacturing philosophy that takes advantage of similarities in products design and manufacturing processes. Cellular Manufacturing System (CMS), a successful implementation of the GT, is a hybrid manufacturing system that possesses the advantages of job shops and flow shops. Job shops are usually designed to achieve maximum flexibility and high resource utilization such that a wide variety of products in low volumes could be manufactured. In contrast, flow shops are designed to produce high volumes of products with high production rates and low costs. Job shops and flow shops are not capable of bringing efficiency and flexibility, simultaneously [1]. To deal with such requirements in industries producing high variety, mid-volume product mixes, CMS is an efficient option. In a CMS, the parts requiring similar production processes are grouped together as part families. Each part family is processed by a set of different machines in a manufacturing cell. The main advantages that can be expected from the implementation of CMSs involve reduction in the setup times, in-process inventories, material handling costs, and tool requirements, and improvement in the product quality and production control [2,3].

Cell Formation (CF) process is one of the first and most important steps in designing a CMS. It includes grouping machines into machine cells on the basis of similarities in the manufacturing processes. An ideal CMS configuration involves a set of completely independent machine cells. However, due to the economic and practical reasons, it is usually impossible to process all of the operations of each part family in a single machine cell. Therefore, a common objective in the CF problems is the minimization of costs associated

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with the Exceptional Elements (EEs). An EE is a part that requires to be produced in more than one cell [3].

Although the CF problem is still being studied in the literature, see for example [4-8], in recent years, most researchers have focused on the other important aspects of the CMS design problem, such as scheduling [9-12], production planning [1,13-15], and facility layout [16-19]. The material flow between the cells resulting from EEs is one of the main obstacles to achieving the benefits of CMS [20]. On the other hand, it is estimated that 20-50% of the manufacturing costs come from the material flow [21]. For this reason, in recent years, continuous research efforts have been made towards the facility layout design in CMSs.

Queuing networks consisting of several service stations are more suitable to represent the structure of many systems with a large number of resources. In the manufacturing systems, including CMSs, queuing networks could be employed to analyze and evaluate different specifications of the production system, such as the waiting times, the number of jobs waiting in the system, the utilization level of resources, etc.

Despite the useful characteristics of queuing networks in performance evaluation of manufacturing systems, only a few researchers have applied this approach to the CMS design problems. One of the first attempts in this area was made by Saidi-Mehrabad and Ghezavati [22]. They assumed that the processing time of parts on machines and their inter-arrival times at the cells were exponentially distributed. A queuing theory-based approach was utilized to analyze the manufacturing system, where each machine was considered as an $M/M/1$ queuing system. To deal with EEs, they applied a penalty cost objective function in which the EEs were ignored using outsourcing. Although this assumption made the problem easier to solve, it was not realistic in real-life problems. In the proposed problem, they attempted to minimize the total cost of machine idleness, EEs sub-contracting, and resource underutilization. In another research effort, Ghezavati and Saidi-Mehrabad [23] addressed a similar problem with the objective of maximizing the average machine utilization. Arghish et al. [24] formulated a mathematical model using the concepts of queuing theory for the CF problem. The objective was the minimization of the total idleness cost of machines plus the overall waiting cost of parts in the queue of machines. For the sake of simplicity, they completely ignored the EEs in the problem formulation. As a result, the solution produced by this model was very optimistic, because the impact of EEs was disregarded in the calculation of the waiting time of parts and utilization level of machines. Fardis et al. [25] applied the principles of queuing theory to obtain the waiting time of parts and utilization level of machines in a CF problem. They attempted to minimize the total cost of machine idleness, part holding, EEs subcontracting, and resource underutilization. Fattahi et al. [26] considered a similar problem in which, instead of minimizing the waiting cost of parts, the waiting times were maximized. They argued that this could decrease the number of EEs and finally lead to the formation of optimal cells and part families. However, for such a CF problem, maximizing the waiting times is not an appropriate choice, because the number of EEs could be directed minimized. Ismailnezhad and Fattahi [27] assumed that each machine operated as an $M/G/1$ queuing system, where the inter-arrival time was exponentially distributed and the service time had a general distribution. By considering machine reliability, they presented a CF problem in which the objective was to minimize the number of EEs.

The recent trend in investigating the CF problem using the queuing approach suggests a promising research area that requires further study. Based on the above survey, the following shortcomings in the developed models can be investigated further.

- **Considering single processing route:** In the reviewed papers, it is assumed that each part has a unique processing route. However, in practice, each part can be manufactured through different processing routes. Consideration of multiple processing routes in the CMS design may enhance planning flexibility and throughput rates, and reduce in-process inventory. Furthermore, it could provide the designer with more opportunities to reduce the number of EEs [3,28,29];
- **Neglecting production and outsourcing costs:** Due to the economic reasons and limitation in resource capacities, internal production is not always feasible. Under such circumstances, outsourcing a proportion of demands to external suppliers can be a better alternative. In the reviewed papers, for the sake of simplicity in problem formulation, some researchers considered a subcontracting approach in which the EEs were eliminated by outsourcing some operations. Obviously, this approach is not applicable in a real-life manufacturing system. Thus, addressing a subcontracting approach in which the operational costs and resource capacities are incorporated is necessary;
- **Neglecting facility layout:** As mentioned earlier, facility layout is one of the main aspects of the CMS design problem. According to Tompkins et al. [21], an efficient facility layout can reduce the material handling costs by 10-30%. Nevertheless, this issue is not incorporated in the reviewed papers;
- **Ignoring in-process inventories:** Besides the material flow between the cells, in-process inventory is another obstacle to configuring an efficient CMS.

Reducing in-process inventories can lead to less holding costs and shorter lead times. Although in the reviewed papers, the queuing systems have been used to evaluate the utilization level of machines and waiting time of parts in the queue of machines, the minimization of in-process inventories has not been attempted yet.

With the objective of overcoming all these shortcomings, this paper uses the principles of queuing theory to present a new CF problem in which the cell layout problem is also included. A subcontracting approach is proposed for the situations where the production of parts is not feasible due to either limited machine capacity or high operational cost. Multiple process routings in the production of parts are also taken into consideration. It is assumed that each machine operates as an $M/M/1$ queuing system, and a queuing network is used to obtain in-process inventories and machine utilization. The problem is formulated as a Mixed-Integer Nonlinear Program (MINLP) with the objective of minimizing the total costs, including the production, subcontracting, material handling, machine idleness, and holding costs. Due to the computational complexity of the problem, a heuristic method is developed to solve it. Also, a numerical example is solved for clarification, and finally, further instances selected from the related literature are solved to verify the performance of the solution method and to accomplish comparisons.

2. Open Jackson networks

In this paper, an *open* Jackson network is used to evaluate the in-process inventories and machine utilization. In the following, some explanations are given for the open Jackson network. Then, the description of the problem under study is given.

An open Jackson network consists of M nodes, each with one or several servers. In our problem, a node in the network stands for a machine. Jobs arrive from outside following a Poisson process with the rate of $\lambda \geq 0$. Each job is independently routed to node k with probability $p_{0k} \geq 0$, and $\sum_{k=1}^{M} p_{0k} = 1$. Upon service completion at node k, a job may go to another node k' with probability $p_{kk'}$ or leave the network with probability $p_{k0} = 1 - \sum_{k'=1}^{M} p_{kk'}$. The arrival rate for node k, denoted by λ_k, is calculated by adding the arrival rate from outside and arrival rates from all the other nodes; see Figure 1. Thus, the overall arrival rate at node k can be written as:

$$\lambda_k = \lambda p_{0k} + \sum_{k'=1}^{M} \lambda_{k'} p_{k'k}, \qquad \forall\ k = 1, \cdots, M. \tag{1}$$

Let $N_k(t)$ denote the number of jobs in node k at time t, and $\mathbf{N} = (N_1, \cdots, N_M)$. The possible values are denoted by $\mathbf{n} = (n_1, \cdots, n_m)$. The equilibrium distribution of $\mathbf{N}$, $\pi(\mathbf{n}) = \Pr(\mathbf{N} = \mathbf{n})$, is determined by:

$$\pi(\mathbf{n}) = \prod_{k=1}^{M} \Pr(N_k = n_k) = \prod_{k=1}^{M} \pi_k(n_k)$$

$$= \prod_{k=1}^{M} (1 - \rho_k) \rho_k^{n_k}, \tag{2}$$

where $\rho_k = \lambda_k / \mu_k$ is defined as the utilization level in node k [30].

As it can be seen, Eq. (2) is the joint probability mass function of M independent geometric random variables. This implies that the network behaves as if it were composed of M dependent $M/M/1$ queuing systems. Therefore, the average number of jobs at node k, denoted by L_k, is computed by:

$$L_k = \frac{\rho_k}{1 - \rho_k}, \qquad k = 1, \cdots, M. \tag{3}$$

In the next section, this useful result is utilized to obtain the in-process inventory of parts and utilization level of machines in a CMS design problem.

3. Proposed problem

This research combines and extends the ideas presented by Mahootchi et al. [31] and Ghezavati and Saidi-Mehrabad [23] to address a new CF and cell layout problem. It is assumed that P parts, each having an uncertain demand with Poisson distribution, are produced by M machines [23]. Each part is allowed to be produced through R_i processing routes that are

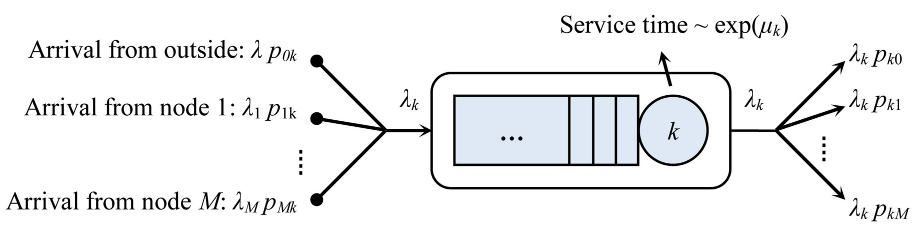


Figure 1. Node k in an open Jackson network.

known in advance. In each route, several operations are done according to a pre-specified sequence of machines. Besides the production, an outsourcing option is available to cope with part demand. In other words, not only each part can be produced, but also it can be outsourced to an external supplier [31]. Machines are grouped into a maximum of $C^{\max}$ cells and no more than NM machines are allowed in each cell. Similar to Ghezavati and Saidi-Mehrabad [23], we assume that the service times are exponentially distributed. Thus, an *open* Jackson network composed of M nodes can be used to analyze the entire system in the steady state. Each node in this network is equivalent to a specific machine which serves as an $M/M/1$ queuing system with a known service rate. The objective function is to minimize the sum of production costs, outsourcing costs, material handling costs, machine idleness costs, and inventory holding costs. Hereafter, the following notations are used throughout the paper.

Indices:

- i — Index of parts ($i = 1, \cdots, P$, where P is the number of parts);
- j — Index of processing routes ($j = 1, \cdots, R_i$, where R_i is the number of processing routes of part i);
- k, k' — Index of machines ($k, k' = 1, \cdots, M$, where M is the number of machines);
- l, l' — Index of cells ($l, l' = 1, \cdots, C^{\max}$, where $C^{\max}$ is the maximum permissible number of cells).

Parameters:

- d_i — Demand rate of part i;
- μ_k — Service rate of machine k;
- a_{ijk} — Coefficient for adjusting the arrival rate of part i on machine k in route j;
- c_{ij}^P — Unit production cost of part i through route j;
- c_i^O — Unit outsourcing cost of part i;
- c_i^H — Unit holding cost of part i;
- c_k^I — Unit idleness cost of machine k;
- $E_{ll'}$ — Distance between cells l and l';
- $c_{ikk'}^A$ — Unit intra-cell material flow cost for transporting part i between machines k and k';
- $c_{ikk'}^E$ — Unit inter-cell material flow cost for transporting part i between machines k and k' ($c_{ikk'}^E \geq c_{ikk'}^A$);
- $f_{ijkk'}$ — Number of times that part i in route j is transported between machines k and k';
- NM — Maximum allowable number of machines in a cell;
- TC — Total costs.

Decision variables:

- z_{kl} — Equals 1 if machine k is assigned to cell l; 0 otherwise;
- λ_{ij} — Arrival rate of part i in route j;
- o_i — Volume of part i which is outsourced to the external supplier;
- ρ_k — Average utilization level of machine k (auxiliary variable);
- L_i — Average number of parts i in the system (auxiliary variable).

In order to illustrate how ρ_k and L_i are calculated, a small example is given. This example consists of four parts and eight machines. Figure 2 depicts the processing routes of parts. In this figure, each colored arrow is associated with a part. The dashed arrows mean that another route is also available for the

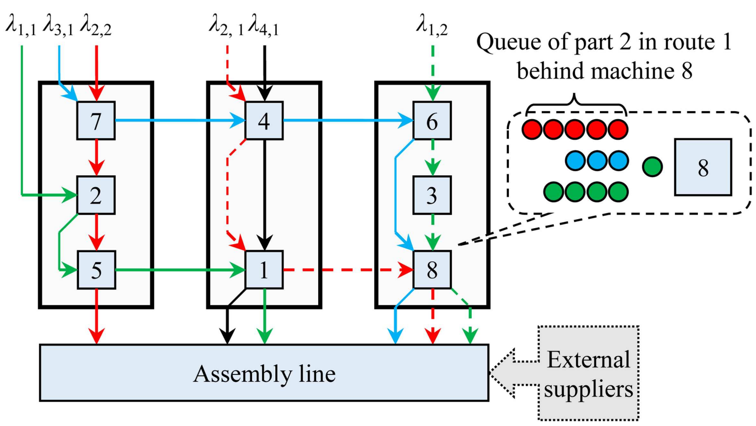


Figure 2. Illustration of the proposed problem.

Table 1. Effective arrival on machines for the example given in Figure 2.

	Node	To: Outside	1	2	3	4	5	6	7	8	Effective arrival
From	Outside	0	0	$\lambda_{1,1}$	0	$\lambda_{2,1}+\lambda_{4,1}$	0	$\lambda_{1,2}$	$\lambda_{2,2}+\lambda_{3,1}$	0	—
	1	$\lambda_{1,1}+\lambda_{4,1}$	0	0	0	0	0	0	0	$\lambda_{2,1}$	$\lambda_{1,1}+\lambda_{2,1}+\lambda_{4,1}$
	2	0	0	0	0	0	$\lambda_{1,1}+\lambda_{2,2}$	0	0	0	$\lambda_{1,1}+\lambda_{2,2}$
	3	0	0	0	0	0	0	0	0	$\lambda_{1,2}$	$\lambda_{1,2}$
	4	0	$\lambda_{2,1}+\lambda_{4,1}$	0	0	0	0	$\lambda_{3,1}$	0	0	$\lambda_{2,1}+\lambda_{3,1}+\lambda_{4,1}$
	5	$\lambda_{2,2}$	$\lambda_{1,1}$	0	0	0	0	0	0	0	$\lambda_{1,1}+\lambda_{2,2}$
	6	0	0	0	$\lambda_{1,2}$	0	0	0	0	$\lambda_{3,1}$	$\lambda_{1,2}+\lambda_{3,1}$
	7	0	0	$\lambda_{2,2}$	0	$\lambda_{3,1}$	0	0	0	0	$\lambda_{2,2}+\lambda_{3,1}$
	8	$\lambda_{1,2}+\lambda_{2,1}+\lambda_{3,1}$	0	0	0	0	0	0	0	0	$\lambda_{1,2}+\lambda_{2,1}+\lambda_{3,1}$

production of the corresponding part. For example, according to Figure 2, the sequences of machines in the first and second routes of part 1 are $2 \rightarrow 5 \rightarrow 1$ and $6 \rightarrow 3 \rightarrow 8$, respectively. Table 1 shows the effective arrival rate on each machine calculated by Eq. (1). For example, the effective arrival rate on machine 8 equals the sum of arrival rates of parts 1, 2, and 3, i.e., $\lambda_{1,2}+\lambda_{2,1}+\lambda_{3,1}$. The average utilization level of machine k is obtained by dividing the effective arrival on machine k to its service rate. For example, suppose that $\lambda_{1,2}=10$, $\lambda_{2,1}=5$, $\lambda_{3,1}=10$ and $\mu_8=30$; therefore, the utilization level of machine 8 is $\rho_8=(\lambda_{1,2}+\lambda_{2,1}+\lambda_{3,1})/\mu_8=(10+5+10)/30=0.833$. Also, the average number of parts in the queue of each machine can be computed by Eq. (3). For instance, the average number of parts in the queue of machine 8 is $\rho_8/(1-\rho_8)=0.833/(1-0.833)=5$ parts, where 40% $(\lambda_{1,2}/(\lambda_{1,2}+\lambda_{2,1}+\lambda_{3,1})\times 100=40)$ of this number, which equals 2 parts, are associated with part 1; the remainder 60% correspond to parts 2 and 3. Finally, the average in-process inventory of part i equals the sum of the average numbers of part i in the queue of all M machines. For example, the average in-process inventory of part 1 in the system equals the sum of the average number of part 1 in the queue of machines 2, 5, and 1, plus that in the queue of machines 6, 3, and 8.

According to the explanations given above, the mathematical model of the proposed problem, hereafter called *original model*, is as follows:

$$\begin{aligned}\min\quad TC=&\sum_{i=1}^{P}\sum_{j=1}^{R_i}c_{ij}^{P}\lambda_{ij}+\sum_{i=1}^{P}c_i^{O}o_i+\sum_{i=1}^{P}c_i^{H}L_i\\&+\sum_{k=1}^{M}c_k^{I}(1-\rho_k)+\sum_{i=1}^{P}\sum_{j=1}^{R_i}\sum_{k=1}^{M-1}\sum_{k'=k+1}^{M}\\&f_{ijkk'}\left(c_{ikk'}^{A}\sum_{l=1}^{C^{\max}}\lambda_{ij}z_{kl}z_{k'l}\right.\\&+c_{ikk'}^{E}\sum_{l=1}^{C^{\max}-1}\sum_{l'=l+1}^{C^{\max}}E_{ll'}\lambda_{ij}(z_{kl}z_{k'l'}\\&\left.+z_{kl'}z_{k'l})\vphantom{\sum_{l=1}^{C^{\max}}}\right),\end{aligned}\tag{4}$$

subject to:

$$\sum_{l=1}^{C^{\max}}z_{kl}=1,\qquad\forall\ k,\tag{5}$$

$$\sum_{k=1}^{M}z_{kl}\le NM,\qquad\forall\ l,\tag{6}$$

$$o_i+\sum_{j=1}^{R_i}\lambda_{ij}=d_i,\qquad\forall\ i,\tag{7}$$

$$\rho_k=\frac{\sum_{i=1}^{P}\sum_{j=1}^{R_i}a_{ijk}\lambda_{ij}}{\mu_k},\qquad\forall\ k,\tag{8}$$

$$L_i=\sum_{k=1}^{M}\left(\frac{\sum_{j=1}^{R_i}a_{ijk}\lambda_{ij}}{\mu_k-\sum_{i'=1}^{P}\sum_{j'=1}^{R_{i'}}a_{i'j'k}\lambda_{i'j'}}\right),\qquad\forall\ i,\tag{9}$$

$$0\le\rho_k<1,\qquad\forall\ k,\tag{10}$$

$$\lambda_{ij},o_i,L_i\ge 0,\qquad\forall\ i,j,\tag{11}$$

$$z_{kl}\in\{0,1\},\qquad\forall\ k,l.\tag{12}$$

In the model above, objective function (4) minimizes the sum of the production cost, outsourcing cost, holding cost, machine idleness cost, and material handling cost. The set of Constraints (5) ensures that each machine is assigned to one cell. The set of Constraints (6) constraints ensures that no more than NM machines are assigned to each cell. The set of Constraints (7) represents that the demand for parts is satisfied by production and subcontracting. The set of Constraints (8) calculates the utilization level of machines. The set of Constraints (9) calculates the average number of each part type in the system. The set of Constraints (10) ensures that the utilization level of machines is a value between 0 and 1. Finally, the sets of Constraints (11) and (12) indicate the type of decision variables.

4. Solution method

The model presented in Section 3 is an MINLP, due to the existence of some nonlinear terms in objective function (4) and Constraints (9). The term $\lambda_{ij} z_{kl} z_{k'l'}$ in objective function (4) can be linearized using additional auxiliary variables and constraints. However, the nonlinear term in Constraints (9) cannot be linearized using the exact linearization methods. On the other side, it is well known that both the CF and layout problems belong to the class of NP-hard problems [32]. Thus, a heuristic method is developed to effectively solve the problem. Also, a mathematical model is presented to obtain a lower bound on the objective value of the problem.

4.1. A heuristic method

To develop the heuristic method, the model proposed in Section 3 is decomposed into two sub-models, Models I and II, which are easier to optimally solve than the original model is.

In Model I, binary variable z_{kl} is assumed to be fixed, denoted by $\bar{z}_{kl}$. Now, by substituting o_i, ρ_k, and L_i with their equivalent terms, see Eqs. (7)-(9), in objective function (4) and Constraint (10), the problem can be rewritten as follows:

Model I:

$$\min \quad TC_1 = \sum_{k=1}^{M} c_k^I + \sum_{i=1}^{P} c_i^O d_i + \sum_{i=1}^{P}\sum_{j=1}^{R_i}\left(F_{ij} + c_{ij}^P - c_i^O + \sum_{k=1}^{M}\left(\frac{a_{ijk} c_i^H}{\mu_k - \sum_{i'=1}^{P}\sum_{j'=1}^{R_{i'}} a_{i'j'k}\lambda_{i'j'}} - \frac{a_{ijk} c_k^I}{\mu_k} \right)\right)\lambda_{ij}, \tag{13}$$

subject to:

$$0 \leq \sum_{j=1}^{R_i} \lambda_{ij} \leq d_i, \qquad \forall\, i, \tag{14}$$

$$0 \leq \frac{\sum_{i=1}^{P}\sum_{j=1}^{R_i} a_{ijk}\lambda_{ij}}{\mu_k} < 1, \qquad \forall\, k, \tag{15}$$

where F_{ij} is calculated by Eq. (16):

$$F_{ij} = \sum_{k=1}^{M-1}\sum_{k'=k+1}^{M} f_{ijkk'}\left(c_{ikk'}^A \sum_{l=1}^{C^{\max}} \bar{z}_{kl}\bar{z}_{k'l} + c_{ikk'}^E \sum_{l=1}^{C^{\max}-1}\sum_{l'=l+1}^{C^{\max}} E_{ll'}\left(\bar{z}_{kl}\bar{z}_{k'l'} + \bar{z}_{kl'}\bar{z}_{k'l}\right)\right), \quad \forall\, i, j. \tag{16}$$

In Eq. (16), parameter $\bar{z}_{kl}$ is a solution derived by solving either Model II or approximation model; the models will be presented later. Although Model I is a nonlinear program (NLP), a high-performance NLP solver like CONOPT can optimally solve it in a small amount of time.

In Model II, positive variable λ_{ij} is assumed to be fixed, denoted by $\bar{\lambda}_{ij}$. Now, in order to linearize the nonlinear term $z_{kl}z_{k'l'} + z_{kl'}z_{k'l}$ in objective function (4), auxiliary variable $\theta_{kk'll'}$ is introduced and sets of Constraints (18)-(20) are added to the model. Therefore, the resultant model is as follows:

Model II:

$$\min \quad TC_2 = \overline{TC} + \sum_{i=1}^{P}\sum_{j=1}^{R_i}\sum_{k=1}^{M-1}\sum_{k'=k+1}^{M} f_{ijkk'}\bar{\lambda}_{ij}\left(c_{ikk'}^A \sum_{l=1}^{C^{\max}} \theta_{kk'll} + c_{ikk'}^E \sum_{l=1}^{C^{\max}-1}\sum_{l'=l+1}^{C^{\max}} E_{ll'}\theta_{kk'll'}\right), \tag{17}$$

subject to Eqs. (5), (6), and (12):

$$\theta_{kk'll'} - z_{kl} - z_{k'l'} + 1 \geq 0, \qquad \forall\, k' > k, \quad l' \geq l, \tag{18}$$

$$\theta_{kk'll'} - z_{kl'} - z_{k'l} + 1 \geq 0, \qquad \forall\, k' > k, \quad l' \geq l, \tag{19}$$

$$\theta_{kk'll'} \geq 0, \qquad \forall\, k' > k, \quad l' \geq l, \tag{20}$$

where $\bar{\lambda}_{ij}$ is a solution derived by solving Model I, and $\overline{TC}$ is calculated by Eq. (21):

$$\overline{TC}=\sum_{k=1}^{M}c_k^I+\sum_{i=1}^{P}c_i^O d_i+\sum_{i=1}^{P}\sum_{j=1}^{R_i}\left(c_{ij}^P-c_i^O+\sum_{k=1}^{M}\left(\frac{a_{ijk}c_i^H}{\mu_k-\sum\limits_{i'=1}^{P}\sum\limits_{j'=1}^{R_{i'}}a_{i'j'k}\bar{\lambda}_{i'j'}}-\frac{a_{ijk}c_k^I}{\mu_k}\right)\right)\bar{\lambda}_{ij}. \qquad (21)$$

Model II is a Mixed-Integer Program (MIP); thus, it can be solved by a high-performance MIP solver such as GUROBI.

To find a good initial value of $\bar{z}_{kl}$ at the beginning of the solution procedure, an approximation solution of the original problem is obtained. As it was mentioned earlier, the nonlinear term in Constraint (9) cannot be linearized using the exact linearization methods. However, it can be approximated using some additional binary variables. Let L_{ik} denote the average number of parts i in the queue of machine k. Therefore, Eq. (9) can be rewritten as $L_i=\sum_{k=1}^{M}L_{ik}$, where:

$$L_{ik}=\sum_{j=1}^{R_i}a_{ijk}\lambda_{ij}/(\mu_k-\sum_{i'=1}^{P}\sum_{j'=1}^{R_{i'}}a_{i'j'k}\lambda_{i'j'}).$$

It is reasonable to assume that L_{ik} is bounded on the interval $[0,d_i]$. Thus, L_{ik} could be approximated using a new set of binary variables, $y_{ik\alpha}$, as shown in Eq. (22) (see also Figure 3):

$$L_{ik}=\frac{\sum\limits_{j=1}^{R_i}a_{ijk}\lambda_{ij}}{\mu_k-\sum\limits_{i'=1}^{P}\sum\limits_{j'=1}^{R_{i'}}a_{i'j'k}\lambda_{i'j'}}=\frac{d_i}{2^N-1}\sum_{\alpha=1}^{N}2^{\alpha-1}y_{ik\alpha}-\varepsilon_{ik}, \qquad \forall\ i,k, \qquad (22)$$

where ε_{ik} is the approximation error ($0\le\varepsilon_{ik}\le d_i/(2^N-1)$) and N controls the accuracy of approximation, that is, the larger N is, the better the approximation tends to be.

Now, by omitting positive variables ε_{ik} from Eq. (22), the following set of inequalities can be derived:

$$\sum_{\alpha=1}^{N}2^{\alpha-1}\mu_k y_{ik\alpha}-\sum_{i'=1}^{P}\sum_{j'=1}^{R_{i'}}\sum_{\alpha=1}^{N}2^{\alpha-1}a_{i'j'k}\lambda_{i'j'}y_{ik\alpha}-\frac{2^N-1}{d_i}\sum_{j=1}^{R_i}a_{ijk}\lambda_{ij}\ge 0, \qquad \forall\ i,k. \qquad (23)$$

As it can be seen, Inequality (23) contains the multiplication of positive variable $\lambda_{i'j'}$ by binary variable $y_{ik\alpha}$, which can be linearized using additional auxiliary variables and constraints [33]. As a result, by substituting the non-linear term $\sum_{j=1}^{R_i}a_{ijk}\lambda_{ij}/(\mu_k-\sum_{i'=1}^{P}\sum_{j'=1}^{R_{i'}}a_{i'j'k}\lambda_{i'j'})$ with $\frac{d_i}{2^N-1}\sum_{\alpha=1}^{N}2^{\alpha-1}y_{ik\alpha}$, the approximation problem can be formulated as the following MIP:

Approximation model:

$$\min \quad TC_A=\sum_{k=1}^{M}c_k^I+\sum_{i=1}^{P}c_i^O d_i+\sum_{i=1}^{P}\sum_{j=1}^{R_i}\left(c_{ij}^P-c_i^O-\sum_{k=1}^{M}\frac{a_{ijk}c_k^I}{\mu_k}\right)\lambda_{ij}+\sum_{i=1}^{P}\sum_{k=1}^{M}\sum_{\alpha=1}^{N}\frac{2^{\alpha-1}d_i c_i^H y_{ik\alpha}}{2^N-1}+\sum_{i=1}^{P}\sum_{j=1}^{R_i}\sum_{k=1}^{M-1}\sum_{k'=k+1}^{M}f_{ijkk'}\left(c_{ikk'}^A\sum_{l=1}^{C^{\max}}\varphi_{ijkk'll}+c_{ikk'}^E\sum_{l=1}^{C^{\max}-1}\sum_{l'=l+1}^{C^{\max}}\varphi_{ijkk'll'}\right), \qquad (24)$$

subject to Eqs. (5), (6), (12), (14), and (15):

$$\varphi_{ijkk'll'}-E_{ll'}\lambda_{ij}+E_{ll'}d_i(2-z_{kl}-z_{k'l'})\ge 0, \qquad \forall\ i,j,k'>k, \quad l'>l, \qquad (25)$$

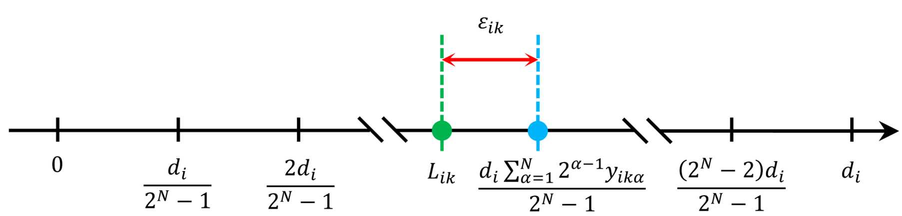


Figure 3. Illustration of approximating L_{ik} via binary variables $y_{ik\alpha}$ within the interval $[0,d_i]$.

$$\varphi_{ijkk'll'} - E_{ll'}\lambda_{ij} + E_{ll'}d_i(2 - z_{kl'} - z_{k'l}) \geq 0,$$

$$\forall\, i, j, k' > k, \quad l' > l, \tag{26}$$

$$\varphi_{ijkk'll} - \lambda_{ij} + d_i(2 - z_{kl} - z_{k'l}) \geq 0,$$

$$\forall\, i, j, k' > k, l, \tag{27}$$

$$\sum_{\alpha=1}^{N} 2^{\alpha-1}\mu_k y_{ik\alpha} - \sum_{i'=1}^{P}\sum_{j'=1}^{R_{i'}}\sum_{\alpha=1}^{N} 2^{\alpha-1}\beta_{ii'j'k\alpha}$$

$$-\frac{2^N - 1}{d_i}\sum_{i=1}^{P}\sum_{j=1}^{R_i} a_{ijk}\lambda_{ij} \geq 0, \qquad \forall\, i, k, \tag{28}$$

$$\beta_{ii'j'k\alpha} - a_{i'j'k}(\lambda_{i'j'} - d_{i'}(1 - y_{ik\alpha})) \geq 0,$$

$$\forall\, i, i', j', k, \alpha, \tag{29}$$

$$\beta_{ii'j'k\alpha} - a_{i'j'k}(\lambda_{i'j'} + d_{i'}(1 - y_{ik\alpha})) \leq 0,$$

$$\forall\, i, i', j', k, \alpha, \tag{30}$$

$$\beta_{ii'j'k\alpha} - a_{i'j'k}d_{i'}y_{ik\alpha} \leq 0, \qquad \forall\, i, i', j', k, \alpha, \tag{31}$$

$$\varphi_{ijkk'l'l}, \beta_{ii'j'k\alpha} \geq 0, \tag{32}$$

$$\forall\, i, i', j', \quad k' > k, l' \geq l, \alpha,$$

$$y_{ik\alpha} \in \{0, 1\}, \qquad \forall\, i, k, \alpha. \tag{33}$$

In the model above, $\varphi_{ijkk'l'l}$ and $\beta_{ii'j'k\alpha}$ are new auxiliary variables used for the linearization purpose.

It should be noted that at optimality, we have $TC_A^*(n) < TC^*$ and $\lim_{n\to\infty} TC_A^*(n) = TC^*$, where $TC_A^*(n)$ is the optimal objective value of approximation model assuming $N = n$ and TC^* is optimal objective value of the problem. Although it is still difficult to optimally solve approximation model for a relatively large N (e.g., $N = 30$), for a small N (e.g., $N = 5$), a high-performance MIP solver can be employed to solve it in a specified amount of time so as to find a good starting solution to the heuristic method.

Given the above explanations, the steps of the heuristic method are summarized as follows:

Step 1. Solve approximation model by an MIP solver and obtain z_{kl}^*. Let $TC^* = \infty$ and $\bar{z}_{kl} = z_{kl}^*$. Go to Step 2;

Step 2. Solve Model I by an NLP solver and obtain λ_{ij}^* and TC_1^*. If $TC_1^* < TC^*$, let $\bar{\lambda}_{ij} = \lambda_{ij}^*$, $TC^* = TC_1^*$, and go to step 3; otherwise, go to Step 4;

Step 3. Solve Model II by an MIP solver and obtain z_{kl}^* and TC_2^*. If $TC_2^* < TC^*$, let $\bar{z}_{kl} = z_{kl}^*$, $TC^* = TC_2^*$, and go to step 2; otherwise, go to Step 4;

Step 4. Report $\bar{\lambda}_{ij}$, $\bar{z}_{kl}$, and TC^*. Stop.

Also, a conceptual framework of the methodology proposed for solving the problem is shown in Figure 4.

It should be noted that the proposed heuristic method does not necessarily lead to the optimal so-

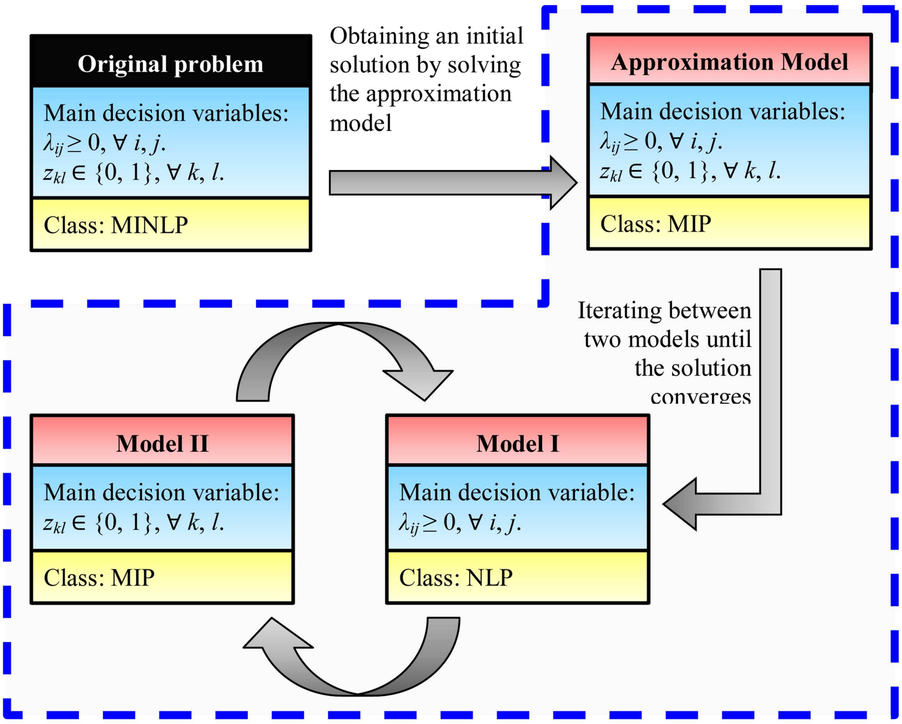


Figure 4. Conceptual framework of the proposed heuristic.

lution. This comes from two reasons. The first one is that the initial solution derived from approximation model might be non-optimal due to the approximation error or the computational complexity of the problem. The other reason is that when a non-optimal solution is improved by solving Models I and II, the final result might be still non-optimal, because in these models, the decision concerning decision variables λ_{ij} and z_{kl} is made separately.

4.2. A lower bound

Finding a tight lower bound on the objective value of a minimization problem can provide a useful criterion to assess a solution in hand. In a real-life CMS design problem, it is reasonable to assume that $c^E_{ikk'} \geq c^A_{ikk'}$. Now, to obtain a lower bound on the objective value of the problem, we let $c^E_{ikk'} = c^A_{ikk'}$. By this assumption, binary variable z_{kl}, as well as sets of Constraints (5) and (6), in the original model can be dropped from the model. Thus, the problem reduces to the following NLP.

Model LB:

$$\begin{aligned} \min \quad TC_L = & \sum_{k=1}^{M} c^I_k + \sum_{i=1}^{P} c^O_i d_i \\ & + \sum_{i=1}^{P}\sum_{j=1}^{R_i}\Bigg(c^P_{ij} - c^O_i \\ & + \sum_{k=1}^{M}\Bigg(\frac{a_{ijk}c^H_i}{\mu_k - \sum\limits_{i'=1}^{P}\sum\limits_{j'=1}^{R_{i'}} a_{i'j'k}\lambda_{i'j'}} - \frac{a_{ijk}c^I_k}{\mu_k} \\ & + \sum_{k'=k+1}^{M} c^A_{ikk'} f_{ijkk'}\Bigg)\Bigg)\lambda_{ij}, \end{aligned} \tag{34}$$

subject to Eqs. (14) and (15).

Obviously, the optimal objective value of Model LB is a lower bound on the objective value of the original model, that is, $TC^*_L \leq TC^*$.

5. A numerical example

To provide a better understanding of the proposed approach, a numerical example adopted from [34] is solved. This problem consists of 14 machines, 20 parts, and 45 processing routes. Table 2 gives the dataset of the numerical example including the routing and operation sequences of parts, as well as the other necessary parameters added to the original dataset. The service rate of all machines (μ_k) is assumed to be 400. The maximum number of machines in each cell (NM) and the maximum number of cells ($C^{\max}$) are limited to 5 and 3, respectively. The cell layout is assumed to be a linear single-row layout. As a result, the distance between each pair of cells l and l' ($l' > l$) can be calculated by subtracting l' from l (i.e., $E_{ll'} = l' - l$, for $l' > l$). The material handling costs are assumed $c^A_{ikk'} = 0.3$ and $c^E_{ikk'} = 0.45$ for $i, k' > k$.

Figure 5 illustrates the solution obtained by the heuristic method. Based on this solution, it can be observed that the demand for parts 4, 9, and 19 is totally fulfilled by subcontracting; the demand for parts 3, 5, 6, 8, 10-13, 15-18, and 20 is totally fulfilled by production; and the demand for parts 1, 2, 7, and 14 is satisfied by a combination of both. Also, it can be seen that each of the parts 8 and 20 is produced through two distinct routes. The average utilization level of each machine is given in percentage in Figure 5. Machine 11 is the busiest one with $\rho_{11} = 0.967$, whereas machines 1 and 9 jointly have the lowest utilization level. On the other hand, part 13 has the highest level of in-process inventory with $L_{13} = 23.5$.

6. Computational results

To verify the performance of the heuristic method and to demonstrate the advantage of the proposed approach, 12 instances collected from the literature are solved and the results are presented. As our problem is different from those in the literature, some parameters may not be available in the original datasets. Thus, these parameters are generated according to Table 3 and added to the original dataset. Table 4 gives the sizes of instances as well as their sources. In this table, the numbers declared in column '$\sum_i R_i$' indicate the total number of processing routes in each instance. Also, the values given in columns 'A', 'B', and 'C' are the parameters required in Table 3 for dataset generation. Problem 1 is the smallest instance with 7 machines and 10 parts, and problem 12 is the largest one with 30 machines and 40 parts. Also, problem 7 has the greatest number of total processing routes, with 402 processing routes. All the mathematical models, including the original model, Model I, Model II, approximation model, and Model LB were coded in GAMS 24.5 IDE and implemented on a PC with Intel® Core™ i7-4790K@4.00 GHz processor, 16 GB of memory, and Windows 10 operating system. In the GAMS software, BARON 15.9/LINDOGLOBAL 9.0, CONOPT 3, and GUROBI 6.0 were selected as the default MINLP, NLP, and MIP solvers, respectively; also, the *thread* option was set to 0 in order to use all cores of the processor, 4 cores with 8 threads, if possible.

6.1. Performance evaluation

The solution of the heuristic method for the selected instances is compared to the best solution derived

Table 2. Dataset of the numerical example.

Part	Route	d_i	c_i^H	c_i^O	c_{ij}^P	Operation sequence (machine (a_{ijk}))
1	1	117	0.54	10.86	7.34	6(0.9)→5(0.5)→3(0.7)→12(0.7)→8(0.5)→11(0.9)
	2				8.69	6(0.1)→14(0.3)→3(0.7)→12(0.2) →8(0.6)→11(0.9)
2	1	184	0.44	8.88	7.10	10(0.3)→11(0.4)→6(0.9)→5(0.5)→7 (0.1)
	2				6.60	10(0.7)→11(0.6)→6(0.9)→14(0.4) →7(0.4)
3	1	155	0.54	10.89	7.58	10(0.2)→2(0.8)→4(0.9)→1(0.6)→ 5(0.1)→11(0.5)
	2				7.45	10(0.1)→13(0.5)→4(0.2)→1(0.3) →5(0.8)→11(0.2)
	3				7.70	10(0.3)→2(0.8)→4(0.2)→1(0.1) →14(0.8)→11(0.6)
	4				8.71	10(0.2)→13(0.6)→4(0.9)→1(0.2) →14(0.6)→11(0.6)
4	1	130	0.38	7.67	6.13	4(0.8)→1(0.7)→10(0.6) →3(0.8)→6(0.1)
5	1	129	0.37	7.50	5.00	12(0.9)→2(0.8)→6(0.1)
	2				6.00	12(0.8)→13(0.3)→6(0.2)
6	1	122	0.44	8.73	6.16	8(0.4)→5(0.1)→2(0.8)→6(0.8)
	2				6.98	8(0.1)→5(0.2)→13(0.1)→6(0.5)
	3				6.53	8(0.9)→14(0.4)→2(0.3)→6(0.4)
	4				5.26	8(0.2)→14(0.9)→13(0.6)→6(0.3)
7	1	135	0.27	5.35	4.28	12(0.9)→8(0.8)
8	1	186	0.28	5.63	4.32	9(0.4)→2(0.3)→4(0.6)
	2				4.50	9(0.5)→13(0.7)→4(0.9)
9	1	107	0.46	9.17	7.34	2(0.1)→7(0.3)→3(0.3)→11(0.7)→ 12(0.4)
	2				6.87	13(0.7)→7(0.5)→3(0.6)→11(0.8)→12(0.5)
10	1	150	0.42	8.40	6.72	1(0.1)→7(0.9)→4(0.4)→2(0.7)→ 9(0.7)
	2				6.70	1(0.4)→7(0.2)→4(0.3)→13(0.5)→9(0.2)
11	1	200	0.54	10.83	7.26	12(0.4)→3(0.1)→2(0.9) →11(0.2)→8(0.7)→5(0.2)
	2				7.30	12(0.8)→3(0.9)→13(0.1)→11(0.2)→8(0.2)→ 5(0.3)
	3				8.18	12(0.4)→3(0.1)→2(0.9)→11(0.5) →8(0.8)→14(0.3)
	4				8.66	12(0.1)→3(0.2)→13(0.8)→11(0.4) →8(0.9)→14(0.3)
12	1	158	0.40	7.91	5.46	11(0.4)→10(0.8)→5(0.5)→8(0.4)
	2				6.33	11(0.9)→10(0.9)→14(0.5)→8(0.3)
13	1	199	0.41	8.19	6.55	10(0.8)→7(0.9)→11(0.4)→5(0.1)
	2				5.61	10(0.9)→7(0.9)→11(0.1)→14(0.3)
14	1	176	0.33	6.53	5.22	3(0.4)→4(0.9)→10(0.1)→7(0.7)

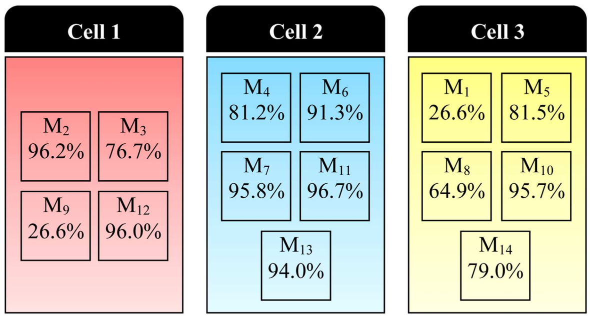
by solving the original model by both BARON and LINDOGLOBAL solvers. As some problems might not be optimally solvable in a reasonable computational time, the time limit for solving the original model and approximation model was set to 7200 and 1000 seconds, respectively. Table 5 reports the computational results. In this table, the values declared in column 'Diff.' indicate the relative differences in percentage between the objective value gained from the heuristic method and that obtained from the original model, where Diff. = $100 \times (TC_S - TC_H)/TC_S$.

The computational results suggest that in all the

Table 2. Dataset of the numerical example (continued).

Part	Route	d_i	c_i^H	c_i^O	c_{ij}^P	Operation sequence (machine (a_{ijk}))
15	1	113	0.36	7.18	5.01	5(0.4)→2(0.8)→4(0.6)
	2				4.32	5(0.5)→13(0.7)→4(0.1)
	3				5.75	14(0.8)→2(0.9)→4(0.5)
	4				4.53	14(0.3)→13(0.3)→4(0.7)
16	1	164	0.45	8.99	6.57	6(0.5)→7(0.3)→11(0.4)→3(0.1)→ 2(0.9)
	2				7.19	6(0.8)→7(0.4)→11(0.5)→3(0.9)→13(0.8)
17	1	116	0.40	8.06	6.45	2(0.8)→3(0.4)→11(0.4)→6(0.4)
	2				6.26	13(0.1)→3(0.8)→11(0.9)→6(0.6)
18	1	125	0.31	6.16	4.93	4(0.1)→8(0.8)→5(0.9)
	2				4.83	4(0.8)→8(0.3)→14(0.9)
19	1	167	0.41	8.29	6.24	3(0.9)→2(0.2)→10(0.6)→9(0.3)→ 12(0.9)
	2				6.63	3(0.4)→13(0.7)→10(0.9)→9(0.1) →12(0.7)
20	1	144	0.29	5.85	4.09	6(0.7)→7(0.7)→2(0.9)
	2				4.68	6(0.2)→7(0.2)→13(0.5)

Part	Route	Operation sequence	λ_{ij}	L_i	o_i
1	1	$M_6 \rightarrow M_5 \rightarrow M_3 \rightarrow M_{12} \rightarrow M_8 \rightarrow M_{11}$	12.8	2.0	104.2
2	2	$M_{10} \rightarrow M_{11} \rightarrow M_6 \rightarrow M_{14} \rightarrow M_7$	85.3	12.0	98.7
3	2	$M_{10} \rightarrow M_{13} \rightarrow M_4 \rightarrow M_1 \rightarrow M_5 \rightarrow M_{11}$	155.0	8.8	–
4	–	–	–	–	130.0
5	1	$M_{12} \rightarrow M_2 \rightarrow M_6$	129.0	14.5	–
6	4	$M_8 \rightarrow M_{14} \rightarrow M_{13} \rightarrow M_6$	122.0	5.6	–
7	1	$M_{12} \rightarrow M_8$	110.0	6.9	25.0
8	1	$M_9 \rightarrow M_2 \rightarrow M_4$	164.8	5.7	–
8	2	$M_9 \rightarrow M_{13} \rightarrow M_4$	21.2		–
9		–	–	–	107.0
10	2	$M_1 \rightarrow M_7 \rightarrow M_4 \rightarrow M_{13} \rightarrow M_9$	150.0	5.9	–
11	2	$M_{12} \rightarrow M_3 \rightarrow M_{13} \rightarrow M_{11} \rightarrow M_8 \rightarrow M_5$	200.0	17.0	–
12	1	$M_{11} \rightarrow M_{10} \rightarrow M_5 \rightarrow M_8$	158.0	13.7	–
13	2	$M_{10} \rightarrow M_7 \rightarrow M_{11} \rightarrow M_{14}$	199.0	23.5	–
14	1	$M_3 \rightarrow M_4 \rightarrow M_{10} \rightarrow M_7$	21.6	1.4	154.4
15	2	$M_5 \rightarrow M_{13} \rightarrow M_4$	113.0	4.2	–
16	1	$M_6 \rightarrow M_7 \rightarrow M_{11} \rightarrow M_3 \rightarrow M_2$	164.0	20.3	–
17	2	$M_{13} \rightarrow M_3 \rightarrow M_{11} \rightarrow M_6$	116.0	11.4	–
18	2	$M_4 \rightarrow M_8 \rightarrow M_{14}$	125.0	2.9	–
19		–	–	–	167.0
20	1	$M_6 \rightarrow M_7 \rightarrow M_2$	94.1	13.4	–
20	2	$M_6 \rightarrow M_7 \rightarrow M_{13}$	49.9		–



Production cost=\$12447.446
Outsourcing cost=\$6510.325
Machine idleness cost=\$297.598
Inventory holding cost=\$69.698
Total material handling cost=\$2603.321
Total costs=21928.389\$

Figure 5. Detailed solution of the numerical example.

problems, the objective value obtained by the heuristic method is better than or at least equal to that obtained for the original model using MINLP solver; see column 'Diff.' in Table 5. Also, it can be seen that the CPU time of the heuristic method is reasonable even for a large-scale instance like problem 11 or problem 12. On the other side, it can be observed that Model LB can provide good lower bounds in a short amount of computational time. For problems 2, 3, and 4, which are small-scale problems, Model LB does not gain a better lower bound than the solvers do. Nevertheless, as the problem size increases, Model LB gives better lower bounds than the solvers so. To examine the null hypothesis $H_0 : LB - LB_S \leq 0$ against alternative hypothesis $H_1 : LB - LB_S > 0$, the paired t-test was carried out. The p-value associated with this test is 0.0067. Therefore, it is concluded that the null hypothesis is rejected at the 0.01 significance level, that is, for the proposed set of the problems, the average LB is larger than the average LB_S. The relative optimality gap derived by each method is also plotted in Figure 6. For instance, in problem 12, the relative

Table 3. Parameter generation for incomplete datasets in the literature*.

Parameter	Value
d_i	100, for i
c_{ij}^p	No. of operations+uniform (1,3), for i, j
c_i^o	$A \times \max_j \{c_{ij}^P\}$, for i
$c_{ikk'}^A$	$B \times \text{uniform}(0.5, 1)$, for i, k, k'
μ_k	C, for k
c_i^H	$0.5 \times c_i^o$, for i
c_k^I	100, for k
$c_{ikk'}^E$	$1.5 \times c_{ikk'}^A$, for i, k, k'
$E_{ll'}$	$l' - l$, for $l' > l$
α_{ijk}	Uniform(0.1,0.9), for i, j, k

*: Constants A, B, and C are chosen according to Table 4.

optimality gap based on lower bound of the solver is $(32337.813 - 25428.937)/32337.813 = 0.214$, while this value reduces to $(32337.813 - 31854.792)/32337.813 = 0.015$ when using Model LB.

6.2. Comparison results

Traditionally, the CF problems are investigated under an assumption in which for each part type, only one route is allowed to be selected from multiple routes. In other words, the possibility of simultaneous production through multiple routes is not considered. In order to perform a fair comparison between these two approaches, Model I is modified so as to make it capable of obtaining the total costs based on the CF and routing results reported in the literature. Table 6 contains the summary of results. In this table, the values declared in columns 'TC_P' and 'TC_O' show the total costs based on the proposed approach and approaches in the literature, respectively. Also, the improvement percentages achieved in the total costs are given in column 'Imp.', where Imp. $= 100 \times (TC_O - TC_P)/TC_O$. The comparison demonstrates that the proposed approach gives a better solution in terms of the total costs than the conventional approaches in the literature do; see column 'Imp.' in Table 6. For instance, based on the solution in the literature, the total costs calculated for problem 11 are \$22749.4. However, when the parts are allowed to be simultaneously produced in multiple routes, this cost reduces to \$20420.9 by 10.2%. To examine the null hypothesis $H_0 : TC_O - TC_P \leq 0$ against the alternative hypothesis $H : TC_O - TC_P > 0$, the paired t-test was carried out. The p-value associated with this test is 0.0006. Therefore, it is concluded that the null hypothesis is rejected at the

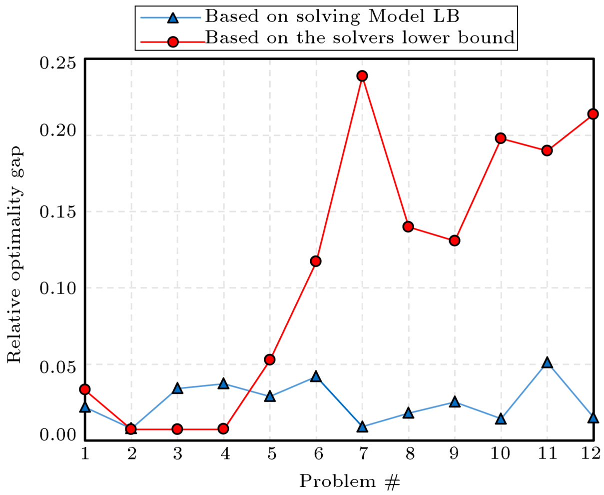


Figure 6. Relative optimality gaps obtained for the selected problems.

Table 4. Characteristics of the instances selected from the literature*.

Problem #	Size $M \times P$	$\sum_i R_i$	$C^{\max}$	NM	Solution source	A	B	C
1	7×10	23	4	3	[35]	1.25	1	—
2	8×13	26	3	3	[36]	1.25	1	40
3	10×10	24	3	4	[17]	1.25	0.75	250
4	12×20	26	3	5	[17]	1.5	0.75	400
5	14×20	45	3	5	[17]	1.25	0.5	400
6	15×15	27	3	6	[37]	1.5	0.5	70
7	17×16	402	2	10	[38]	1.25	1	300
8	17×30	61	4	5	[17]	1.25	0.5	500
9	18×30	59	3	7	[17]	1.25	0.75	300
10	20×20	51	5	5	[16]	1.25	0.75	300
11	26×28	71	6	7	[39]	1.25	0.5	300
12	30×40	89	6	7	[36]	1.25	0.75	400

*: Constants A, B, and C are used in Table 3 to generate additional data.

Table 5. Summary of computational results.

Problem #	Model LB		Heuristic method		Approximation model				Original model[a]			Diff.[h] (%)
	LB[b]	CPU time (s)	TC_H[c]	CPU time (s)	N[d]	TC_A[e]	Opt. gap	CPU time (s)	TC_S[f]	CPU time (s)	LB_S[g]	
1	1651.136	0.002	1688.101	0.220	5	1688.415	1.070	1000	1688.101	7200	1632.218	0.000
2	8470.147	0.015	8537.246	0.182	5	8538.067	0.000	814	8537.246	7200	8476.187	0.000
3	7390.532	0.031	7651.321	0.185	5	7652.506	0.000	696	7663.994	7200	7596.617	0.165
4	17700.702	0.094	18385.992	0.405	5	18391.185	0.365	1000	18385.992	7200	18252.510	0.000
5	21960.713	0.094	22613.390	0.306	5	22622.804	8.324	1000	22613.390	7200	21424.609	0.000
6	31199.559	0.007	32565.505	0.623	5	32581.804	13.941	1000	32565.505	7200	28752.625	0.000
7	3802.007	0.515	3836.540	0.854	5	3903.211	25.125	1000	3836.540	7200	2921.443	0.000
8	23768.901	0.063	24203.459	3.313	4	24391.435	15.082	1000	24203.459	7200	20820.573	0.000
9	21767.314	0.109	22331.198	0.894	5	22337.825	13.483	1000	22436.274	7200	19413.923	0.468
10	13495.159	0.054	13690.4225	11.793	6	13713.457	16.151	1000	13761.017	7200	10982.973	0.513
11	19412.713	0.094	20459.704	604.105	6	20539.437	18.658	1000	20599.072	7200	16578.213	0.677
12	31854.792	0.059	32337.813	588.744	6	33041.323	0.000	1000	33535.972	7200	25428.937	3.573

[a]: The best solution derived from BARON and LINDOGLOBAL solvers;
[b]LB: Lower bound obtained by solving Model LB;
[c]TC_H: Objective value of the solution obtained by the heuristic method;
[d]N: Accuracy level; [e]TC_A: Objective value of the solution obtained by Approximation Model;
[f]TC_S: Objective value of the solution obtained by the original model;
[g]LB_S: Lower bound obtained by the solver for the original model;
[h]Diff. $= 100 \times (TC_S - TC_H)/TC_S$.

Table 6. Summary of comparison results.

Problem #	Proposed approach						Other approaches						Imp.[h] (%)
	TC_P[a]	TP[b]	TS[c]	TI[d]	TH[e]	TM[f]	TC_O[g]	TP	TS	$T1I$	TH	TM	
1	1688.1	1223.4	73.9	54.0	33.2	303.6	1902.8	897.0	578.4	226.2	17.2	184.0	11.3
2	8537.3	5065.6	1900.5	95.2	32.7	1443.3	8694.7	4543.8	2651.6	174.0	31.6	1293.9	1.8
3	7651.3	4535.2	1299.6	386.3	26.3	1404.0	8073.8	5670.2	714.9	350.4	16.9	1321.4	5.2
4	18386.0	11812.6	2333.1	227.8	75.1	3937.5	19212.3	9229.9	6582.0	420.9	40.5	2939.0	4.3
5	15270.5	10303.4	1698.1	367.1	45.7	2856.3	16093.5	10213.7	2561.8	442.4	40.2	2835.4	5.1
6	32565.5	18627.9	7649.5	120.3	202.7	5965.1	34059.2	18295.1	9548.3	235.6	146.5	5833.8	4.4
7	3836.5	1851.6	845.6	255.2	90.7	793.5	4360.7	1776.3	1222.3	443.5	60.9	857.7	12.0
8	24203.5	17145.1	2897.8	465.9	68.8	3625.9	24914.8	14645.2	6815.4	587.7	51.7	2814.8	2.9
9	22331.2	10015.8	8460.1	672.9	36.5	3145.9	23080.4	9207.3	10309.6	907.4	27.0	2629.1	3.3
10	13690.4	10136.6	0.0	800.0	20.5	2733.3	14004.67	10095.69	459.17	459.17	27.91	2613.10	2.24
11	20420.9	14796.9	948.1	1066.6	52.4	3556.9	22749.4	12991.7	5835.3	1200.0	18.1	2704.3	10.2
12	32339.3	20465.1	4663.8	1382.0	63.0	5765.4	34161.8	20624.4	6728.8	1304.3	64.1	5440.2	5.3

[a]TC_P: Total costs in the proposed approach;
[b]TP: Total production costs;
[c]TS: Total subcontracting costs;
[d]TI: Total idleness costs;
[e]TH: Total holding costs;
[f]TM: Total material handling costs;
[g]TC_O: Total costs calculated for the solutions reported in the literature;
[h]Imp. $= 100 \times (TC_O - TC_P)/TC_O$.

0.001 significance level, that is, for the proposed set of the problems, the average improvement in the total costs is positive.

7. Conclusions

In this paper, a new approach was presented to designing a cellular manufacturing system based on the principles of queuing theory with the consideration of subcontracting. For the sake of effective utilization of resources, parts were allowed to be simultaneously produced through multiple routes. It was assumed that each machine operated as an $M/M/1$ queuing system, and a Jackson network was utilized to obtain the in-process inventory of parts in the system and utilization level of machines. The objective was to find the cell formation, cell layout, and production volume of parts such that the sum of production, outsourcing, material handling, machine idleness, and holding costs would be minimized. The computational complexity of the problem motivated us to develop a heuristic-based solution method. A mathematical model, called Model LB, was proposed to obtain a lower bound on the objective value of the original problem. To verify the performance of the heuristic method, and to accomplish a comparison against existing approaches in the literature, several instances were selected from the related literature and solved. The computational results indicated that the solution gained by the proposed heuristic method was better than or equal to that derived by solving the original problem using BARON and LINDOGLOBAL solvers. The results also indicated that Model LB gave better lower bounds than the solvers did, especially for large-sized instances. Finally, the comparison demonstrated that the proposed approach was able to generate better solutions in terms of the total costs than the existing approaches in the literature were.

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