



Effects of viscous dissipation and convective heating on convection flow of a second-grade liquid over a stretching surface: An analytical and numerical study

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Abstract. The effects of viscous dissipation and convective boundary condition on the two-dimensional convective flow of a second-grade liquid over a stretchable surface with suction/injection and heat generation were investigated. The governing partial differential equations were reduced into a dimensionless coupled system of nonlinear ordinary differential equations by appropriate similarity transformation. Then, they were solved analytically by Homotopy Analysis Method (HAM) and numerically by fourth-order Runge-Kutta method with shooting technique. The HAM and numerical results of the local skin friction and local Nusselt number were compared for various emerging parameters. It was found that the momentum boundary layer thickness grew by raising the value of the viscoelastic parameter.

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1. Introduction

The study of boundary layer flow of a viscoelastic liquid over a stretching surface plays a significant role in manufacturing processes including hot rolling, crystal growing, glass blowing, production of synthetic sheets, thermal insulation, thermal control of nuclear reactors, thermal oil recovery, film cooling, design of thrust bearing, and design of radial diffusers. MHD boundary layer flow and energy transfer of a viscoelastic liquid over a stretchy sheet were investigated by Cortell [1]. Several authors investigated the convective flow and energy transfer behavior of viscoelastic liquid with different physical situations, see [2-8]. In nature, heat

transfer is an important process in many technical and industrial applications such as drying, damage of crops because of freezing, food processing, removal of nuclear fuel debris, underground disposal of radioactive waste materials, etc. Because of these applications, several researchers engage with analyzing the heat transfer process in different physical contexts. Some of the important studies that underline such concept can be found in references [9-14].

The problem of free convective flow of a heat generating/absorbing liquid was studied by Chamkha [15]. He found that the thermal boundary layer thickness increased with the rise of the heat generation or absorption parameter. The heat generation effect on boundary layer flow with different fluids has been investigated in some papers [16-21]. Hayat et al. [22] analyzed convective flow of a second-grade liquid over a stretchy wall with convective surface condition. They found that the energy transfer effects were absent when

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Biot number was zero. Few studies, e.g., [23–26], have investigated the boundary layer flow with convective boundary condition.

So far, no attempt has been made to carry out energy transfer analysis on a second-grade liquid over a stretchy surface with viscous dissipation, internal heat generation/absorption, and convective surface condition. Hence, we study this problem both analytically using the homotopy analysis method and numerically using Runge-Kutta method.

2. Mathematical formulation

We consider two-dimensional convective boundary layer flow and energy transfer of a second-grade liquid along a stretchy surface. The surface is stretched with a velocity $u_w(x) = bx$, where b is a real number. It is presumed that the liquid is heat generated or absorbed internally. Also, viscous dissipation is taken into consideration. We choose x -axis along the stretchy surface and y -axis normal to the stretching surface. The governing equations for continuity, momentum, and energy can be written in the following form [2,22]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left(u \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + v \frac{\partial^3 u}{\partial x \partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\alpha_1}{\rho c_p} \left(u \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + v \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right) + \frac{Q}{\rho c_p} (T - T_\infty), \quad (3)$$

where u and v are the components of velocity, x and y are the coordinates, T is temperature, ρ is density of the fluid, α_1 is the second-grade parameter, Q is the coefficient of internal heat absorption (< 0) or generation (> 0) of the fluid, c_p is specific heat, ν is kinematic viscosity, and α_m is thermal diffusivity.

The velocity and thermal boundary conditions can be expressed as follows:

$$u = u_w(x) = ax, \quad v = v_w, \quad -k \frac{\partial T}{\partial y} = h_c(T_f - T),$$

$$\text{at } y = 0,$$

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty,$$

$$\text{as } y \rightarrow \infty,$$

where a is a positive constant, k is thermal conductivity of the fluid, v_w is suction velocity (> 0) or injection

velocity (< 0), h_c is heat transfer coefficient, and T_f is temperature of the hot fluid. We define:

$$\eta = \sqrt{\frac{a}{\nu}} y, \quad u = axf'(\eta),$$

$$v = -\sqrt{av}f(\eta), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}. \quad (4)$$

Substituting Eq. (4) into Eqs. (2) and (3), we obtain:

$$f''' + ff'' - f'^2 + K(2ff''' - f''^2 - ff^{iv}) = 0, \quad (5)$$

$$\theta'' + \text{Pr}f\theta' + \text{PrEc}f''^2 + \text{PrEc}K(f'f''^2 - ff''f''') + \text{Pr}Hg\theta = 0, \quad (6)$$

with boundary conditions:

$$f(0) = f_w, \quad f'(0) = \frac{b}{a} = c, \quad \theta'(0) = -\text{Bi}[1 - \theta(0)],$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0,$$

where $K = \frac{\alpha_1 a}{\mu}$ is viscoelastic parameter, $\text{Pr} = \frac{\nu}{\alpha_m}$ is the Prandtl number, $\text{Ec} = \frac{u_w^2}{c_p(T_f - T_\infty)}$ is the Eckert number, $Hg = \frac{Q}{a\rho c_p}$ is heat generation (> 0) and absorption (< 0) parameter, $f_w = -\frac{v_w}{\sqrt{av}}$ is suction (> 0) and injection (< 0) parameter, $c = \frac{b}{a}$ is stretching ratio, $\text{Bi} = \frac{h_c}{k} \left(\frac{\nu}{a} \right)^{\frac{1}{2}}$ is Biot number, and $\text{Re} = \frac{u_w x}{\nu}$ is local Reynolds number.

The local skin friction coefficient (Cf) and the local Nusselt number (Nu) are defined as:

$$Cf = \frac{2\tau_w}{\rho u_w^2}, \quad \text{Nu} = \frac{xq_w}{k(T_f - T_\infty)},$$

where τ_w is wall shear stress and q_w is the heat flux, which are given by:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} + \frac{\alpha_1}{\rho} \left(u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 v}{\partial y^2} - 2 \frac{\partial v}{\partial y} \frac{\partial u}{\partial y} \right)_{y=0},$$

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}.$$

Accordingly, the dimensionless form of local skin friction coefficient and the local Nusselt number are:

$$\frac{1}{2} Cf \text{Re}^{\frac{1}{2}} = (1 + 3K) f''(0),$$

$$\text{Nu}/\text{Re}^{\frac{1}{2}} = -\theta'(0).$$

3. Solutions

3.1. HAM solution

The solution to the governing system of ordinary

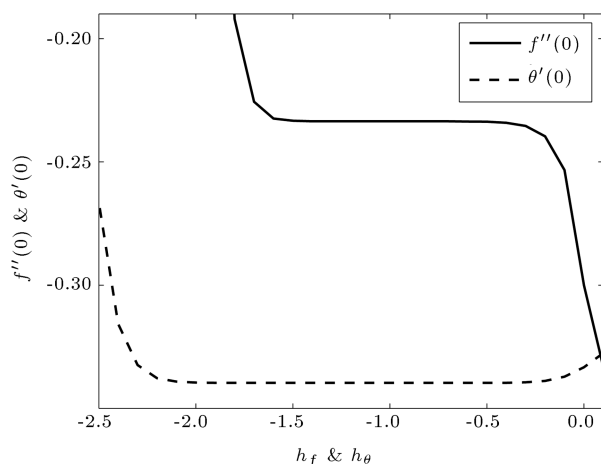


Figure 1. h curves of $f''(0)$ and $\theta'(0)$ with $K = 0.2$, $c = 0.3$, $f_w = 0.5$, $Ec = 0.3$, $Hg = -0.5$, and $Bi = 0.5$.

Table 1. Convergence of homotopy analysis solution for different orders of approximation when $K = 0.2$, $c = 0.3$, $f_w = 0.5$, $Ec = 0.3$, $Hg = -0.5$, and $Bi = 0.5$.

Order of approximation	$-f''(0)$	$-\theta'(0)$
1	0.24600	0.33541
5	0.23397	0.33958
10	0.23349	0.33958
15	0.23347	0.33958
20	0.23347	0.33958
25	0.23347	0.33958
30	0.23347	0.33958
35	0.23347	0.33958

differential equations (5) and (6) is obtained by using homotopy analysis method. In HAM, first, we choose the initial approximations as $f_0(\eta) = f_w + c(1 - e^{-\eta})$ and $\theta_0(\eta) = \frac{Bi e^{-\eta}}{1 + Bi}$. The auxiliary linear operators are $L_f = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}$ and $L_\theta = \frac{d^2 \theta}{d\eta^2} - \theta$. After substituting the m th order deformation equations, the resulting system is solved using HAM with the 15th order of approximations. All the symbolic computations are carried out by MATLAB.

The solutions strongly depend on the auxiliary parameters h_f and h_θ . In order to see the rates for admissible values of h_f and h_θ , the h -curves of the functions $f''(0)$ and $\theta'(0)$ are plotted in Figure 1. The figure indicates that the ranges of admissible values of h_f and h_θ are $-1.4 \leq h_f \leq -0.6$ and $-1.8 \leq h_\theta \leq -0.5$, respectively. If we choose the values of auxiliary parameter (h) from this range, we acquire more precise results. Further, the series converges in the whole region of η when $h_f = h_\theta = -1.0$. Table 1 presents the order of approximations which are necessary for a convergent solution. It is clear that the

15th order of approximation is sufficient for the velocity and temperature profiles.

3.2. Numerical solution

Eqs. (5) and (6) are highly nonlinear ordinary differential equations. It is critical to solve these equations. The resulting system of equations with the boundary conditions (7) is numerically solved using Runge-Kutta fourth-order method with initial guessing $f''(0)$ and $\theta'(0)$. This process is repeated until we get the required degree of accuracy. The analytical and numerical results are compared in Tables 2 and 3. They assure our analytical and numerical results.

4. Results and discussion

In this section, we present the graphical results for velocity profile ($f'(\eta)$), temperature profile ($\theta(\eta)$), local skin friction ($\frac{1}{2}C_f Re^{\frac{1}{2}}$), and local Nusselt number ($Nu/Re^{\frac{1}{2}}$) with different values of viscoelastic parameter, Eckert number, suction/injection parameter, heat absorption/generation parameter, stretching ratio, and Biot number and fixed value of Prandtl number ($Pr = 1.0$).

Table 2 shows the local skin friction and Nusselt number for various values of K , c , and f_w . It is observable that the local skin friction diminishes on raising the values of K , c , and f_w . It is also clear that the local Nusselt number increases on raising the values of K and f_w . The heat transfer rate is enhanced for slow stretching $c \leq 0.3$; after this, energy transfer rate diminishes. Table 3 represents the local Nusselt number for various values of Hg and Bi . It is seen that the local Nusselt number rises by raising the Biot number. The heat transfer rate diminishes in the heat absorption case, because the fluid absorbs the heat and reduces the energy transfer rate along the boundary. The high heat transfer rate in internal heat generation case is visualized for $Hg \geq 0.5$.

Figure 2(a) and (b) displays the variations of suction ($f_w > 0$) and injection ($f_w < 0$) parameters in velocity profile. It is seen from these graphs that the fluid velocity decreases on increasing the suction parameter. However, the velocity increases with increase in the injection parameter. Therefore, we can control the boundary layer flow using suction. The suction stabilizes the boundary layer growth and the injection destabilizes the boundary layer growth. Figure 3(a) and (b) shows the effects of heat generation ($Hg > 0$) and heat absorption ($Hg < 0$) parameters on temperature profile. It is found that the temperature increases with increase in the heat generation parameter. However, temperature is a decreasing function of heat absorption parameter. The outcome of internal heat generation is the rise in the rate of energy

Table 2. The local skin friction coefficient and the local Nusselt number for different values of K , c , and f_w when $Ec = 0.3$, $Hg = -0.5$, and $Bi = 0.5$.

K	c	f_w	Cf			Nu		
			HAM	Numerical	Error (%)	HAM	Numerical	Error (%)
0.0	0.3	0.5	-0.255625	-0.254190	0.56	0.339481	0.338971	0.15
0.1			-0.316714	-0.315082	0.52	0.339546	0.339013	0.16
0.2			-0.373553	-0.372914	0.17	0.339581	0.339158	0.12
0.3			-0.426924	-0.426092	0.19	0.339595	0.339242	0.10
0.2	0.0	0.5	0.000000	0.000000	0.00	0.333333	0.333298	0.01
	0.3		-0.373553	-0.372914	0.17	0.339581	0.339158	0.12
	0.5		-0.713504	-0.713011	0.07	0.337943	0.338024	0.02
	1.0		-1.737367	-1.731987	0.31	0.310850	0.310762	0.03
	1.5		-2.922988	-2.922194	0.03	0.248224	0.248067	0.06
	2.0		-4.216972	-4.216525	0.01	0.150971	0.151000	0.02
0.2	0.3	-2.0	4.019258	4.021158	0.05	0.133571	0.133382	0.14
		-1.0	-0.115475	-0.115019	0.39	0.223018	0.222957	0.03
		-0.5	-0.167777	-0.166943	0.50	0.263387	0.263951	0.21
		0.0	-0.255377	-0.256191	0.32	0.303996	0.303845	0.05
		0.5	-0.373553	-0.372914	0.17	0.339581	0.339158	0.12
		1.0	-0.490922	-0.490879	0.01	0.368094	0.367924	0.05
		1.5	-0.587097	-0.586917	0.03	0.389865	0.388455	0.36
		2.0	-0.662446	-0.661953	0.07	0.411623	0.410958	0.16

Table 3. The local Nusselt number for different values of Hg and Bi when $K = 0.2$, $c = 0.3$, $f_w = 0.5$, and $Ec = 0.3$.

Hg	Bi	Nu		
		HAM	Numerical	Error (%)
-1.0	0.5	0.362380	0.361992	0.11
-0.7		0.350318	0.351846	0.44
-0.5		0.339581	0.338957	0.18
-0.3		0.325061	0.325001	0.02
0.0		0.286097	0.285952	0.05
0.3		0.101896	0.101657	0.23
0.5		0.836841	0.835927	0.11
0.7		6.589394	6.590428	0.02
-0.5	0.0	0.000000	0.000000	0.00
	0.1	0.090796	0.091582	0.87
	0.5	0.339581	0.338953	0.18
	1.0	0.516477	0.516090	0.07
	1.5	0.625004	0.624991	0.00
	2.0	0.698378	0.697753	0.09
	2.5	0.751299	0.749852	0.19
	3.0	0.791272	0.790581	0.09
	4.0	0.847646	0.846928	0.08
	5.0	0.885499	0.884398	0.12
	10.0	0.972341	0.971573	0.08

transport to the liquid, thereby raising the temperature of the liquid. However, heat absorption is to diminish the rate of energy transport to the liquid, thereby decreasing the temperature of the liquid. The effect of

Eckert number on temperature profile is displayed in Figure 4. It is determined that the thermal boundary layer thickness increases on increasing the Eckert number. That is, the enthalpy inside the boundary layer increases by raising the values of Ec and this produces thicker thermal boundary layer.

Figure 5 shows the various values of Biot number on temperature profile. The fluid temperature is zero for $Bi = 0$, that is, the heat transfer coefficient is zero here. It is observed that the fluid temperature and thermal boundary layer thickness increase on increasing the Biot number (heat transfer coefficient). Figure 6(a)-(c) show the variation of local skin friction for different values of viscoelastic parameter, stretching ratio parameter, and suction/injection parameter. It is observed that the local skin friction decreases on raising the values c , f_w , and K . The local skin friction slowly decreases when raising the values of f_w up to $f_w = -0.5$, and it decreases rapidly on increasing f_w .

Figure 7(a)-(d) shows the variation of the local Nusselt number for various combinations of suction/injection, heat generation/absorption, Eckert number, and Biot number. It is observed from Figure 7(a) that the surface temperature gradient increases on increasing the Biot number. The increment in Nusselt number with Bi is higher in the presence of suction than in the presence of injection. The heat transfer rate is almost constant at low Biot number ($Bi = 0$) on increasing f_w . However, heat transfer rate increases rapidly on increasing f_w for higher values of

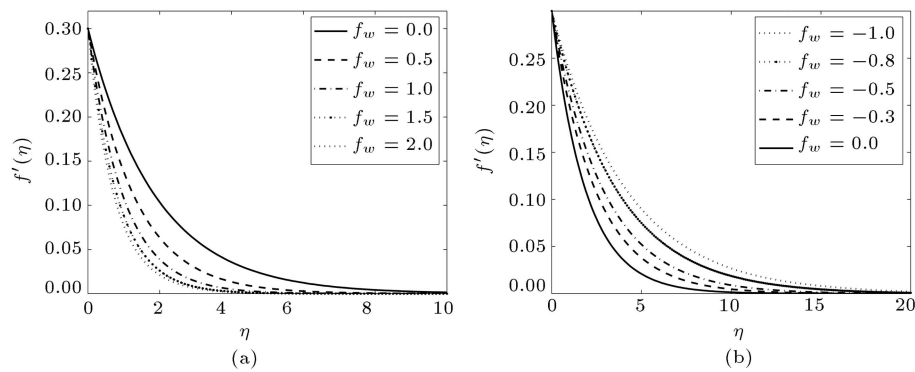


Figure 2. Velocity profile for different values of (a) suction parameter ($f_w > 0$) and (b) injection parameter ($f_w < 0$); $c = 0.3$, $Ec = 0.3$, $Hg = -0.5$, and $Bi = 0.5$.

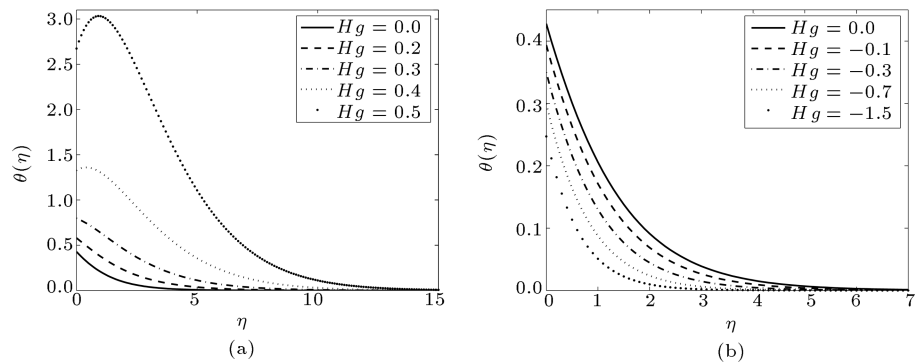


Figure 3. Temperature profile for different values of (a) heat generation parameter ($Hg > 0$) and (b) heat absorption parameter ($Hg < 0$); $K = 0.2$, $c = 0.3$, $f_w = 0.5$, $Ec = 0.3$, and $Bi = 0.5$.

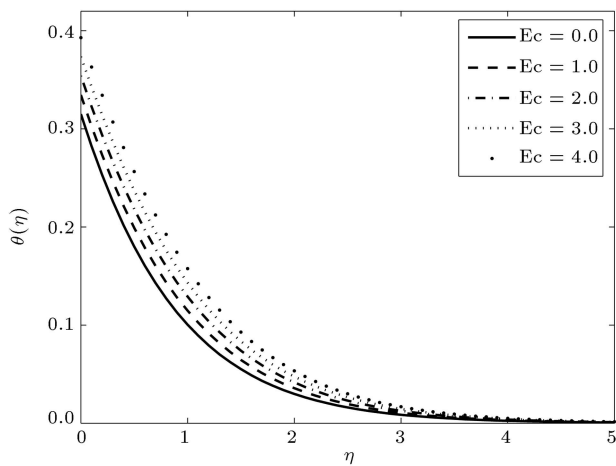


Figure 4. Temperature profile for different values of Eckert number (Ec) with $K = 0.2$, $c = 0.3$, $f_w = 0.5$, $Hg = -0.5$, and $Bi = 0.5$.

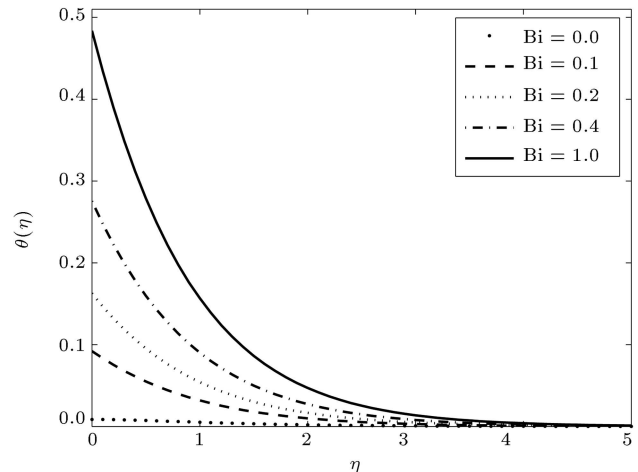


Figure 5. Temperature profile for different values of Biot number (Bi) with $K = 0.2$, $c = 0.3$, $f_w = 0.5$, $Ec = 0.3$, and $Hg = -0.5$.

Bi . It is seen from Figure 7(b) that the local Nusselt number decreases on raising the values of Hg . This is because surface temperature gradient diminishes on increasing Hg . The heat transfer rate vanishes when $Hg = 0.371$. It is found that the critical value of Hg is 0.371. At this point ($Hg_{crit} = 0.371$), the

local Nusselt number vanishes. Therefore, the internal heat generation rate is equal to surface temperature at $Hg_{crit} = 0.371$. Therefore, there is no energy transport between the surface and liquid medium. This is an important result for thermal engineering applications. Before $Hg_{crit} = 0.371$, the decrease

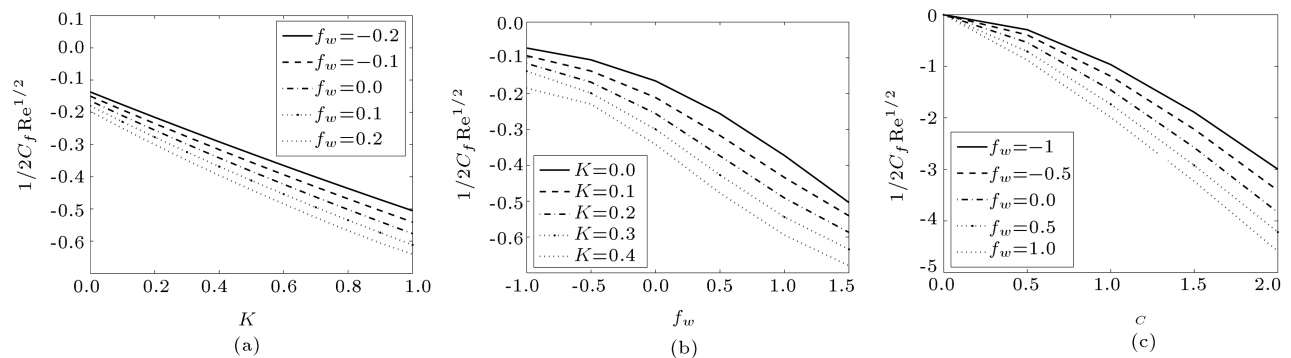


Figure 6. Variation of the local skin friction for different values of K , c , and f_w .

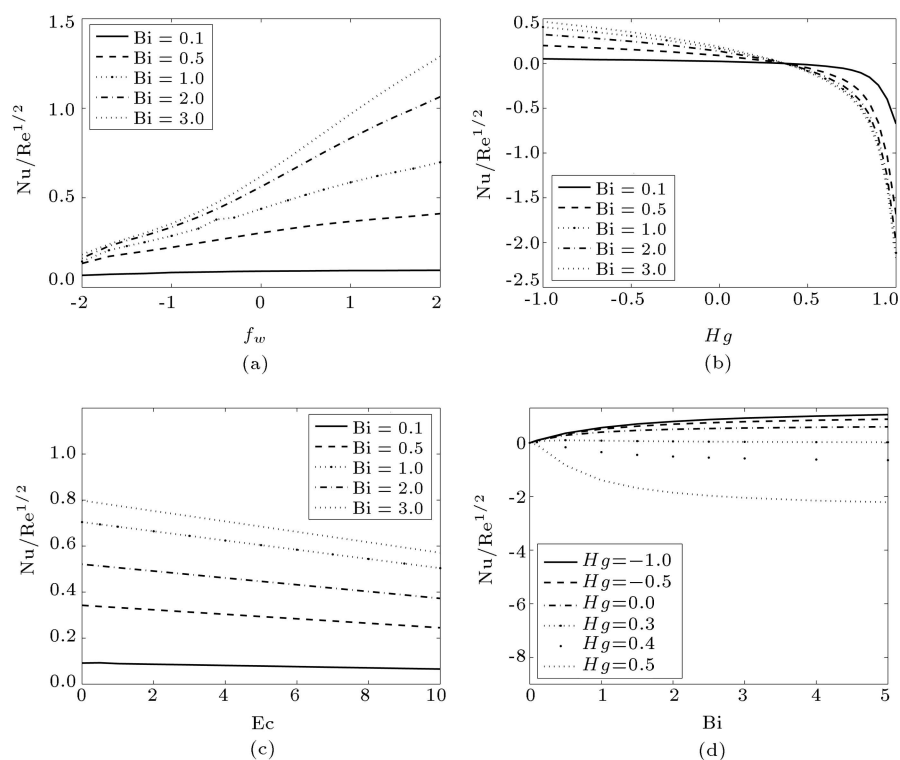


Figure 7. Variation of the local Nusselt for different values of f_w , Ec , Hg , and Bi with $K = 0.2$ and $c = 0.3$.

in heat transfer rate is significantly small and after that, it is very high. The Nusselt number decreases exponentially when Hg approaches unity. The heat transfer rate is reduced slowly on increasing Hg up to $Hg = 0.5$. However, after that, heat transfer rate decreases exponentially. It is found from Figure 7(c) that the higher values of Eckert number suppress the heat transfer rate. The rate of decrement in Nusselt number with Ec increases for higher values of Biot number. It is concluded from Figure 7(d) that the local Nusselt number decreases on increasing Hg . In heat absorption case, the local Nusselt number is enhanced with increasing the values of Biot number. The local Nusselt number rapidly decreases with increasing the low values of Biot number for the heat generation case.

5. Conclusions

The present study elaborated the effects of viscous dissipation and convective heating on the boundary layer flow and heat transfer of a second-grade liquid. The governing boundary layer systems were transformed into a set of ordinary differential equations with the help of similarity variables. Then, these equations were solved analytically by using homotopy analysis method and numerically by using the Runge-Kutta 4th order method. The following results were concluded from the study:

- The momentum boundary layer thickness was controlled by the suction and it increased by increasing the injection;

- The thermal boundary layer thickness was enhanced in the presence of heat generation parameter and it decreased when increasing the heat absorption parameter;
- The thermal boundary layer thickness increased with the help of the Eckert and Biot numbers due to the heat dissipation;
- The heat transfer rate increased with increase in the Biot number, and suction/injection parameters, and decreased with increase in the Eckert number and heat generation/absorption parameter.

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