Hybrid fuzzy-stochastic approach for multi-product, multi-period, and multi-resource master production scheduling problem: case of a polyethylene pipe and fitting manufacturer

Seyed Hossein Razavi Hajiagha\textsuperscript{a*}, Shide Sadat Hashemi\textsuperscript{b}, Mohammadreza Sadeghi\textsuperscript{c}

\textsuperscript{a} Department of Management, Khatam University, Tehran, Iran
\textsuperscript{b} Department of Management, Saramadan Andishe Avina Co. Tehran, Iran
\textsuperscript{c} Industrial Management Group, Faculty of Management and Accounting, Allameh Tabatabaei University, Tehran, Iran

Abstract

Master production scheduling is an effective phase of production planning which leads to scheduling and magnitude of different products production in a company. This problem requires investigating a wide range of parameters, regarding demand, manufacturing resource usage and costs. Uncertainty is an intrinsic characteristic of these parameters. In this paper, a model is developed for master production scheduling under uncertainty, in which demands, as time-dependent variables, are considered as stochastic variables, while cost and utilization parameters, with cognitive ambiguity, are expressed as fuzzy numbers. A hybrid approach is also proposed to solve the extended model. The application of the proposed method is examined in a practical problem of a polyethylene pipe and fitting Co. in Iran. The result showed a high degree of applicability.

Keywords: Master production scheduling; Stochastic demand; Chance constrained programming; Fuzzy set theory.

1. Introduction

Master production scheduling (MPS) is one of the most important activities in production planning and control \cite{1, 2}. Master production scheduling is a midterm phase in planning which translates the long term aggregate production planning to a plan determining the scheduling and magnitude of different products production. MPS coordinates market demand with internal resources of the company \cite{3}. The main goal of MPS establishment is to increase the productivity of production resources i.e. human resources, costs, production facilities, and to increase some important competitive criteria for company i.e. profit, service level, and etc.

MPS converts the strategic planning defined in a production plan into the tactical operation execution. According to American Production and Inventory Control Society (APICS), MPS is the declaration of what the company expects to be produced in terms of configuration, quantities, and specific dates \cite{4}. It drives the material requirement planning (MRP) and other subsequent activities of a manufacturing company. Figure 1 shows the relation between MPS and the other important activities in production management. Therefore, MPS is a series of managerial decisions that should be made by considering some important issues like forecasted demands,

* Corresponding author, s.hossein.r@gmail.com, Tel:
pending orders, material availability, available capacity, managerial policies, and goals. Figure 2 shows the important inputs, outputs, and considerations in an MPS process.

Using optimization approach is one of the conventional approaches to solve the MPS problems, like many other production management problems. Modeling the problem as an optimization model and then solving the model are the main steps in using optimization approach. To model the MPS problem, many objectives can be involved, which may be conflicted with each other. On the other hand, there can be many constraints that should be involved in modeling and solving the MPS problem. The objectives considered frequently in the previous works are the minimization of production costs, inventory costs and backordering costs, and the main constraints involved in model development are meeting the demands, inventory related constraints, and resource related constraints.

<Please insert Figure 1. The relation between production management activities>

The application of mathematical programming in MPS problems is a well-known and accepted approach. Houghton and Portugal [5] presented an analytic framework for optimum production planning. Vasant [6] proposed a fuzzy linear programming methodology and applied it in a real life industrial production planning problem. Wang and Wu [7] presented a framework to solve multi-period, multi-product, and multi-resource MPS problem. Emani Vieira and Ribas [8] presented a multi objective model and its solution based on simulated annealing. Sawik [9] presented a multi objective production scheduling in make-to-order manufacturing and proposed a lexicographic approach to solve the model. Soares and Vieira [3] presented and developed the application of genetic algorithm to solve the mathematical problem of MPS. Lei [10] reviewed the literature on production scheduling problems. Leu et al. [11] developed a linear programming model for mid-term planning, considering the issues in production and material requirement planning. Kelbel and Hanzalek [12] developed an application of constraint programming in production scheduling with earliness and tardiness penalties. Alfieri et al. [13] proposed an approach based on production process knowledge to extract scheduling information from an aggregate production plan in order to support material procurement. Alfieri et al [14] proposed a two-stage stochastic programming project scheduling approach to support production planning. Ballestin et al. [15] modeled the production planning problem as a project scheduling problem. Moon et al. [16] considered electricity consumption costs in production scheduling and considered two objectives to minimize make span of production and to minimize time-dependent electricity cost. The hierarchical planning decisions are made in a way that the production planning is determined on the integrated models at first, and the scheduling operations are then performed. In this regard, the most complexity comes from the difficulties of synchronization of the production planning and scheduling. Sun et al. [17] have planned a program to reduce the delivery time of cement manufacturing product, which is a key factor in this industry. They designed an MPS model based on BOM, and then proposed an approach to reduce the delivery time.
Sahebjamnia et al. [18] developed a fuzzy stochastic multi-objective linear programming model as a novel fuzzy stochastic programming for a multi-level, capacitated lot-sizing problem in a furniture company. They treated the demand and process-related parameters as fuzzy stochastic parameters. Kim and Lee [19] have proposed an iterative approach to achieve the synchronization in order to coordinate the input and output quantity of the production plan while generating a schedule. For this purpose, they have utilized the input - output quantity and also the production quantity as a coordination factor [19]. In another paper, Menezes et al. [20] have introduced a hierarchical approach to solve the production planning and scheduling problems. In this mathematical model, when scheduling is not feasible, capacity information is forwarded to production planning to modify and show the use of new tasks. This method is proposed and used for transportation of products and stock conditions particularly the situations which involve the flow of products in bulk cargo (iron ore, coal and grains) terminals. Martinez et al. [21] have worked on molded pulp packaging as one sample of multi-stage, multi period, and multi-product manufacturing type. The problem is solved with a mixed integer programming model. For multi-objective optimization of master production scheduling problem, Radhikan et al [22] have used Jaya algorithm as a meta-heuristic problem solving method, which requires only common control parameters, and it does not require any algorithm-specific control parameters. Cho and Jeong [23] have used Genetic algorithm as another meta-heuristic production planning and scheduling method to solve Bi-objective problems. Farrokh et al. [24] proposed novel robust fuzzy stochastic programming approach in loop supply chain network. Gramani et al. [25] proposed an exact method of production planning and compared their findings with industrial practices. Considering the above studied, the main contribution of the current paper can be highlighted as fitting a real statistical distribution over demand data and simultaneously describing the cognitive hesitancy of costs and prices by using fuzzy sets. Then, a hybrid approach of fuzzy-stochastic programming model is proposed to solve the considered problem. Also, the material requirement planning is aggregated in the production planning problem to determine the magnitude and scheduling of material procurement along with developing the production plan.

Defining the parameters is a big challenge in modeling while using the optimization approach. The defined parameters which are used in model development should coincide with the real world, and using the crisp numbers instead of parameters causes the impracticability of the established models. In fact, uncertainty is an intrinsic feature of real world applications. Usually, the uncertainty can occur due to (1) partial or (2) approximate information [26]. Using the fuzzy logic, researchers apply grey numbers and stochastic programming as a solution for this kind of challenge. Each type of uncertainty has its own characteristics and it is appropriate for special cases. While probability is concerned with occurrence of well-defined events, fuzzy sets deal with gradual concepts and describe their boundaries [27]. In production planning framework, the behavior of demand along with time can be assessed with a probability distribution, while the cost parameters ambiguity is often due to lack of knowledge and it does not behave stochastically. Therefore, as it is convenient, the demands are taken into account as stochastic variables, while cost parameters are considered as fuzzy numbers. The aim of this paper is to combine both of them in a singular model.

Demand uncertainty is considered in some previous works on MPS. Tang and Grubbstrom [28] presented an MPS model under demand uncertainty. Fleten and Kristoffersen [29] applied stochastic programming in production planning. Feng et al. [30] studied the MPS problem for single end-product with time-varying demand uncertainty and supply capacity. Liang [31] developed a fuzzy multi objective linear model to solve multi-product and multi-time period
production/distribution problems. Supriyanto and Noche [32] proposed a methodology for MPS problems in which uncertainty is considered under fuzzy information. Körpeoğlu et al. [33] used a multi-stage stochastic programming approach, considering several demand scenarios. Also, Mula et al. [34] reviewed production planning under uncertainty.

In this paper, the main idea is to consider the fuzziness of parameters and objectives and stochastic customer demands in model establishment. Hybrid uncertain methods are extensively applied in different fields [35-38]. After model development, an interactive method based on existing techniques in the optimization literature is developed to solve the proposed model. This paper is organized as follows: Section 2 provides a brief review of used methods in this paper. The problem modeling is discussed in section 3. Section 4 involves the problem solving approach and in section 5, a real world case study of the proposed method is presented. Finally, section 6 contains the conclusion.

2. Preliminaries

2-1- Chance constrained programming

Chance constrained programming (CCP) is introduced by Charnes and Cooper [39] as a conceptual framework to deal with stochastic programming. Their model considers the case when objective function is a deterministic function, while constraints are expressed in stochastic form. Suppose the following problem.

\[
\max f(c, X) \\
\text{Subject to: } \hat{A}X \leq \hat{b} \\
X \geq 0
\]

(1)

Where \( f(c, X) \) is the objective function, \( X \) is the decision vector, \( c \) is the vector of objective coefficients, \( \hat{A} \) is the stochastic matrix of constraint coefficients, and \( \hat{b} \) is the stochastic right hand side vector. The CCP model maximizes the objective function subjected to constraints, which must be satisfied at a prescribed level of probability. A typical CCP model can be expressed as follows:

\[
\max f(c, X) \\
\text{Subject to: } \Pr(\hat{A}X \leq \hat{b}) \geq \alpha, \alpha \in [0,1] \\
X \geq 0
\]

(2)

Here, \( \alpha \) is the prescribed level of probability [39, 40].

2-2- Fuzzy sets

Fuzzy set theory was introduced by Zadeh [41] and has been developed and applied in a wide variety of practical problems. A fuzzy set \( \tilde{A} \) in the universe \( X \) is characterized by its membership function \( \mu_{\tilde{A}} : X \rightarrow [0,1] \), where \( \mu_{\tilde{A}}(x), x \in X \) denotes the membership degree of \( x \) to \( \tilde{A} \).

A fuzzy number is a fuzzy set \( \tilde{a} \) on the real line \( R \) whose membership function \( \mu_{\tilde{a}} \) is a convex, upper semi-continuous function. A trapezoidal fuzzy number is denoted by \( \tilde{a} = (a_1, a_2, a_3, a_4) \) whose membership function is as follows [42]:
A special type of trapezoidal fuzzy number is a triangular fuzzy number, where \( a_2 = a_3 \), and can be shown in the form of \( \tilde{a} = (a_1, a_2, a_3) \). These two types of fuzzy numbers are widely applied in decision making and planning type problems.

The algebraic operations on fuzzy numbers are well-known and reviewed in many references [43, 44]. Therefore, they are not repeated in this paper.

3. Model construction

In this paper, the MPS problem is formulated in the form of a mathematical programming model in order to minimize total production cost, considering different limitations. Emani Vieira and Ribas [8] and Soares and Vieira [3] regarded four considerations in formulating an MPS problem: (1) minimizing the inventory level, (2) minimizing the unfulfilled requirements (demand), (3) minimizing the inventory below the safety stock level, and (4) minimizing the quantity of additional needed resources. In the proposed model, two additional aspects are also considered as (5) minimization of production cost and (6) minimization of resource procurement cost.

3-1- Parameters, variables and notations

The following parameters are used in model construction, where “\(^\wedge\)” denotes the stochastic and “\(\sim\)” denotes the fuzziness of notations. Problem parameters include:

- \( K \): Variety of products
- \( T \): Number of planning periods
- \( R \): Number of productive resources
- \( TH_t \): Time length of each period \( t, t=1,2,\ldots,T \)
- \( TH \): Total planning horizon
- \( POH_k \): On-hand inventory of type \( k \) product at the first period
- \( \hat{d}_k \): Gross requirements for product \( k \) at period \( t \) with expected value of \( \mu_{kt} \) and variance \( \sigma_{kt}^2 \)
- \( c_{rk} \): Available capacity, at resource \( r \) at the first period
- \( \tilde{a}_{rk} \): Capacity used from the resource \( r \) to produce one unit of product \( k \)
- \( \tilde{c}_k^p \): Unit production costs of product \( k \)
- \( \tilde{c}_k^h \): Unit holding costs of product \( k \)
- \( \tilde{c}_k^p \): Penalty costs for each unit of requirement of product \( k \) that is not met
- \( \tilde{u}_{rk} \): Unit costs to obtain one unit of resource \( r \) at period \( t \)
- \( \tilde{c}_{kss} \): Unit costs for each unit of product \( k \) below safety stock at period \( t \)
- \( \tilde{h}_{rt} \): Unit holding costs of resource \( r \) at period \( t \)
- \( SS_{kt} \): Safety inventory level for product \( k \) at period \( t \)

Also, decision variables are:

\[
\mu_3(x) = \begin{cases} (x-a_1)/(a_2-a_1), & \text{if } a_1 \leq x \leq a_2 \\ 1, & \text{if } a_2 \leq x \leq a_3 \\ (x-a_4)/(a_3-a_4), & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}
\]
objective function

Several objectives can be considered in formulating a production planning problem. These objectives mainly implied some costs for manufacturer. For instance, beyond the common production costs associated with direct material, manpower and overhead costs, a manufacturer tolerated some costs due to over or under inventory and etc. In this paper, total production cost is considered as an objective that manufacturer seeks to minimize it. According to the defined parameters and variables, the objective function of MPS problem can be developed as follows:

\[
\text{Minimize } Z = \sum_{k=1}^{K} \left( \sum_{t=1}^{T} \left( c_{kt}^P s_{kt} + c_{kt}^b \text{AIL}_{kt} + \bar{c}_{kt}^b f_{kt} \right) \right) + \sum_{k=1}^{K} \sum_{r=1}^{R} \bar{h}_{rt}^a c_{rt} + \sum_{r=1}^{R} \sum_{t=1}^{T} \bar{h}_{rt}^b r_{rt} + \sum_{k=1}^{K} \sum_{t=1}^{T} \bar{c}_{bsskt} BSS_{kt} \tag{4}
\]

The first term of Eq. (4) represented the production costs of products along with their induced holding cost due to positive amount of inventory level, and the penalty cost due to the inability to satisfy the customers’ demand. While the second term expressed the procurement costs in the whole planning horizon, the third term illustrated the holding cost due to surplus amount of the obtained resources. The last term is the penalty cost which occurred due to inventory level becoming under safety stock in planning horizon.

Model’s constraints

The main constraints that should be satisfied when minimizing the defined objective function are: inventory related constraints, demand related constraints, safety stock related constraints, and resource utilization related constraints.

- Inventory related constraints: for \(k^{th}\) product at period \(t\) Eq.(5) should be satisfied

\[
i_{kt} = POH_k, \quad k=1,2,\ldots,K
\]

And

\[
i_{kt} = f_{kt-1}
\]

Therefore, the average inventory level for each period \(t\) and each product \(k\) will be as follows:

\[
\text{AIL}_{kt} = \frac{i_{kt} + f_{kt}}{2}
\]

- Demand related constraints: for \(k^{th}\) product at period \(t\), Eq.(8) should be satisfied:

\[
x_{kt} + i_{kt} + r_{kt} - f_{kt} \geq d_{kt}, \quad k=1,2,\ldots,K
\]
- The Inventory level of each product remains below the defined safety stock by the satisfaction of Eq. (9):

\[
BSS_{kt} = \max \{0, SS_{kt} - f_{kt}\} \quad k = 1,\ldots,K
\]

- The resource utilization related constraints are defined by Eq. (10):

\[
\sum_{k=1}^{K} \bar{a}_{kt} x_{kt} - s_{rt} = c_{rt} - s_{r(t-1)}, \quad r = 1,\ldots,R
\]

Therefore, the fuzzy-stochastic MPS model can be demonstrated as follows:

Minimize \( \bar{C} = \sum_{k=1}^{K} \sum_{t=1}^{T} \left( c_{kt}^{0} x_{kt} + \bar{c}_{kt}^{1} AIL_{kt} + \bar{c}_{kt}^{2} I_{kt} \right) + \sum_{r=1}^{R} \sum_{t=1}^{T} \bar{a}_{rt} c_{rt} + \sum_{r=1}^{R} \sum_{t=1}^{T} \tilde{h}_{rt} s_{rt} + \sum_{k=1}^{K} \sum_{t=1}^{T} \tilde{c}_{bsskt} BSS_{kt} \)

Subject to

(i) \( i_{k1} = POH_{k} \),

(ii) \( i_{kt} = f_{kt-1} \),

(iii) \( x_{kt} + i_{kt} + r_{kt} - f_{kt} \geq \hat{d}_{kt} \),

(iv) \( BSS_{kt} \leq SS_{kt} - f_{kt} \),

(v) \( \sum_{k=1}^{K} \bar{a}_{kt} x_{kt} + s_{rt} = c_{rt} + s_{r(t-1)} \).

\( x_{kt}, i_{kt}, f_{kt}, r_{kt}, BSS_{kt}, c_{rt}, s_{rt} \geq 0, k = 1,\ldots,K; t = 1,\ldots,T \)

In Eq. (11) that constraints set (iii) are stochastic constraints and the objective function and constraints set (vi) are as fuzzy type.

4. Solving approach

The proposed optimization model for MPS problems, Eq. (11), is a combined stochastic-fuzzy linear programming model. Chance constrained programming [39] is a well-known method to solve stochastic programming problems, while there are some methods to solve fuzzy linear programming problems [45-49]. Since the chance constrained programming method solves the problems in different levels of probability, i.e., \( \alpha, i = 1,\ldots,l \), the applied method to solve hybrid problem will be constructed based on different significance levels. The main advantage of chance constrained programming is its ability to handle and to analyze different statistical distributions. Also, there are not any limitation on randomness of objective function, constraints and right hand side values. In this regard, scholars proposed chance constrained programming as a powerful tool of handling stochastic optimization problems [50-52].

Suppose that the decision maker determines the satisfaction level \( \alpha \). The stochastic constraints (iii) can be analyzed based on chance constrained programming: If \( \hat{d}_{kt} \) is a random variable which follows a probability distribution like \( f \), then these constraints can be demonstrated as Eq. (12) in the satisfaction level \( \alpha \):

\[
Pr(\hat{d}_{kt} \leq x_{kt} + i_{kt} - r_{kt} - f_{kt}) \geq \alpha
\]
If \( f_{a}^{-1} \) is the value such that \( Pr(d_{kt} \leq f_{a}^{-1}) = \alpha \), then the Eq. (12) is equivalent to Eq. (13):

\[
x_{kt} + i_{kt} + r_{kt} - f_{kt} \geq f_{a}^{-1}
\]

(13)

By defining Eq. (13), the stochastic constraints (iii) can be transformed to a set of linear constraints. Now, a method using the concept of \( \alpha \)-cuts is developed to deal with fuzziness. If \( \tilde{\mu} \) is a fuzzy set in universe \( U \), characterized by membership function \( \mu_{\tilde{\mu}} \), its \( \alpha \)-cut is defined as

\[
\tilde{\mu}_{\alpha} = \{ x \in U | \mu_{\tilde{\mu}}(x) \geq \alpha \}. 
\]

The \( \alpha \)-cuts can be shown as crisp intervals, which are called \( \alpha \)-level interval:

\[
\tilde{\mu}_{\alpha} = [\mu_{l}, \mu_{u}] = \left[ \min_{x} \{ x \in U | \mu_{\tilde{\mu}}(x) \geq \alpha \}, \max_{x} \{ x \in U | \mu_{\tilde{\mu}}(x) \geq \alpha \} \right]
\]

(14)

For a trapezoidal fuzzy number \( \tilde{\alpha} = (a_{1}, a_{2}, a_{3}, a_{4}) \), its \( \alpha \)-level interval is determined as

\[
\tilde{\alpha}_{\alpha} = (\alpha a_{1}, (1-\alpha)a_{1}, \alpha a_{4}, (1-\alpha)a_{4}).
\]

Applying the concept of \( \alpha \)-level interval and probability level \( \alpha \) (these \( \alpha \) levels totally named satisfaction level), the stochastic-fuzzy problem in Eq. (11) is reduced to an equivalent interval linear programming as represented in Eq. (15).

Minimize \( \tilde{C} = \sum_{k=1}^{K} \sum_{l=1}^{T} \left( c_{k}^{hl} + c_{k}^{hl}_{u} \right) x_{kt} + \left( c_{k}^{hl} + c_{k}^{hl}_{u} \right) \text{ALL}_{kt} + \left( c_{k}^{hl} + c_{k}^{hl}_{u} \right) \text{BSS}_{kt} + \sum_{r=1}^{T} \sum_{l=1}^{T} [t_{rt} + c_{rt}^{u}] BSS_{rt} + \sum_{k=1}^{K} \sum_{l=1}^{T} \left( a_{k}^{l} + a_{k}^{u} \right) x_{kt} + s_{rt} = c_{rt} + s_{r(t-1)} \]

Subject to

(i) \( i_{kt} = \text{POH}_{kt} \),

(ii) \( f_{kt} = f_{kt-1} \),

(iii) \( x_{kt} + i_{kt} + r_{kt} - f_{kt} \geq f_{a}^{-1} \),

(iv) \( \text{BSS}_{kt} \geq \text{SS}_{kt} - f_{kt} \),

(v) \( \sum_{k=1}^{K} \left( a_{k}^{l} + a_{k}^{u} \right) x_{kt} + s_{rt} = c_{rt} + s_{r(t-1)} \)

\( x_{kt}, i_{kt}, f_{kt}, r_{kt}, \text{BSS}_{kt}, c_{rt}, s_{rt} \geq 0, k = 1, 2, \ldots, K; t = 1, 2, \ldots, T \)

Except for (v) constraints in Eq. (15), all other constraints are crisp. Therefore, a method is required to deal with these constraints. Ishibuchi and Tanaka [53] introduced some order relations to compare interval numbers. Considering their definitions, the following equation is substituted for these constraints:

\[
\sum_{k=1}^{K} \left( \frac{a_{k}^{l} + a_{k}^{u}}{2} \right) x_{kt} + s_{rt} = c_{rt} + s_{r(t-1)}
\]

(16)

Replacing Eq. (16) with (v) constraints in Eq. (15), a problem with interval objective function arises. Now, consider the concept of right-center (RC) order relation of Ishibuchi and Tanaka [53]. According to this relation, an interval function \( \left[ C, \bar{C} \right] \) will be minimized when its upper bound \( \bar{C} \) and its center \( \left( \left( C + \bar{C} \right) / 2 \right) \) is minimized. Applying this notion in objective function of the Eq. (15), the objective function upper bound is obtained as:

\[
\bar{C} = \sum_{k=1}^{K} \sum_{l=1}^{T} \left( c_{k}^{hl} x_{kt} + c_{k}^{hl}_{u} \text{ALL}_{kt} + c_{k}^{hl}_{u} r_{kt} \right) + \sum_{k=1}^{K} \sum_{l=1}^{T} \text{BSS}_{kt} + \sum_{k=1}^{K} \sum_{l=1}^{T} \left( a_{k}^{l} + a_{k}^{u} \right) x_{kt} + s_{rt} = c_{rt} + s_{r(t-1)}
\]

(17a)
While its center is
\[
\mathbf{C} + \overline{\mathbf{C}} = \sum_{k=1}^{K} \sum_{t=1}^{T} \left( \frac{c_{k}^{b} + c_{k}^{u}}{2} \right) x_{kt} + \left( \frac{c_{k}^{h}_{1} + c_{k}^{h}_{u}}{2} \right) AIL_{kt} + \left( \frac{c_{k}^{b}_{1} + c_{k}^{h}_{u}}{2} \right) v_{kt} 
\]
\[
+ \sum_{r=1}^{R} \sum_{t=1}^{T} \left( \frac{a_{r}^{l} + a_{r}^{u}}{2} \right) c_{rt} + \sum_{r=1}^{R} \sum_{t=1}^{T} \left( h_{rt}^{l} + h_{rt}^{u} \right) s_{rt} + \sum_{k=1}^{K} \sum_{t=1}^{T} \left( \frac{c_{ks}^{b} \mathbf{BSS}_{kt} + c_{ks}^{h}}{2} \right) \mathbf{BSS}_{kt} 
\]
(17b)

Finally, the hybrid stochastic-fuzzy MSP problem in Eq. (14) and its reduced form interval linear programming problem in Eq. (15) can be restated as a vector minimization problem of the following form:

Minimize \( \left( \overline{\mathbf{C}}, \frac{\mathbf{C} + \overline{\mathbf{C}}}{2} \right) \)

Subject to
\[
\begin{aligned}
\{i_{k1} = POH_{k}, \\
i_{kt} = f_{kt-1}, \\
x_{kt} + h_{kt} + n_{kt} - f_{kt} \geq f_{u}^{-1},
\end{aligned}
\]
FS: \( \mathbf{BSS}_{kt} \geq SS_{kt} - f_{kt}, \)
\[
\sum_{k=1}^{K} \left( \frac{a_{r}^{l} + a_{r}^{u}}{2} \right) x_{kt} + s_{rt} = c_{rt} + s_{rt(\cdot-1)},
\]
\[
x_{kt}, i_{kt}, f_{kt}, n_{kt}, \mathbf{BSS}_{kt}, s_{rt} \geq 0, k = 1,2,\ldots,K; t = 1,2,\ldots,T
\]

A goal programming based approach is now developed to solve this problem. The problem seeks to minimize total costs of production; therefore, its goal value will be equal to zero. The soft constraints of the problem are formulated as follows:

\[
\begin{aligned}
\overline{\mathbf{C}} + d_{l}^{+} - d_{l}^{-} &= 0 \\
\overline{\mathbf{C}} + \overline{\mathbf{C}} + d_{l}^{+} - d_{l}^{-} &= 0
\end{aligned}
\]
(19)

Finally, the goal programming formulation of the problems is extended as:

Minimize \( d_{l}^{+} + d_{l}^{-} \)
\[
\overline{\mathbf{C}} + d_{l}^{+} - d_{l}^{-} = 0
\]
\[
\overline{\mathbf{C}} + \overline{\mathbf{C}} + d_{l}^{+} - d_{l}^{-} = 0
\]
(20)

Where, \( x \) represents the solution vector. Solving the above problem will determine the optimal production plan.

Note that both for stochastic and fuzzy constraints, smaller value for \( \alpha \) conclude with larger feasible region, and the objective function can be improved. Therefore, the decision maker faces two conflicting objectives: (1) to improve the objective function value and (2) to improve the satisfaction level of constraints. It is notable that since there are different fuzzy parameters in the model, there isn’t any dependency among the \( \alpha \) values of these values. However, it seems reasonable that while solving the model in one time, all the \( \alpha \) values determined equally. The
considered $\alpha$ value can be predetermined by decision maker based on his/her cognition about uncertainty. However, Jimenez et al. [47] proposed an interactive procedure in a similar situation. Following Kaufmann and Gil Aluja [54], Jimenez et al. [47] applied an eleven-point scale for sufficient distinction between satisfaction levels:

- 0 Unacceptable solutions
- 0.1 Practically unacceptable solution
- 0.2 Almost unacceptable solution
- 0.3 Very unacceptable solution
- 0.4 Quite unacceptable solution
- 0.5 Neither acceptable nor unacceptable solution
- 0.6 Quite acceptable solution
- 0.7 Very acceptable solution
- 0.8 Almost acceptable solution
- 0.9 Practically acceptable solution
- 1 Completely acceptable solution

Firstly, the ordinal linear programming model (20) is solved for each $\alpha_k = \alpha_0 + 0.1k, k = 0.1, ..., (1 - \alpha_0)/0.1$. By solving the model (20) for different values of $\alpha$, a set of optimal fuzzy values $\bar{C}_\alpha$, will be obtained. To compare these solutions and to choose an acceptable solution, the Yager [55] method is used. Yager [55] index is defined as:

$$K_G(z^*) = \frac{\int_{-\infty}^{+\infty} \mu_G(x) \cdot \mu_a(z) dx}{\int_{-\infty}^{+\infty} \mu_a(z) dx}$$

(21)

Where the denominator is the area under $\mu_a(z)$, and, in the numerator, the possibility of occurrence $\mu_a(z)$ of each crisp value $z$ is weighted by its satisfaction degree $\mu_G(x)$ of the goal $G$. This is an extension of the widely accepted center of gravity defuzzification method, using the goal function $\mu_G(x)$ as a weighting value. $K_G(z^*)$ illustrates the compatibility of a decision in satisfaction level $\alpha$, with decision maker’s aspiration defined by $\mu_G(x)$. To balance the impact of satisfaction level, the fuzzy decision in the assumed satisfaction level is determined by the following membership degree in decision space:

$$\mu_D(x^*_{\alpha}) = \alpha * K_G(z^*_{\alpha})$$

(22)

Where $*$ is an operator like minimum, product, etc. The final decision is chosen with the highest membership degree in decision space. i.e. $x^*$ is chosen when:

$$\mu_D(x^*) = \max_{\alpha} \mu_D(x^*_{\alpha})$$

(23)

An algorithmic scheme of the above problem can be stated as follows:

1. Prepare the information needed to formulate the problem, Eq. (11), include fuzzy cost parameters, demand statistical distributions, resources usage of products, and maximum available amount of resources.
2. Choose a satisfaction level $\alpha_k = \alpha_0 + 0.1k, k = 0.1, ..., (1 - \alpha_0)/0.1$. 


3. In the specified satisfaction level \( \alpha_k \), transform the stochastic constraints, using Eq. (13) to a set of equivalent linear constraints.

4. Constitute the problem in Eq. (15) by using fuzzy numbers \( \alpha \)-cuts.

5. Constitute and solve the problem in Eq. (16) at different \( \alpha \)-levels and find the most compromising solution, using Eqs. (21) – (23)

Figure 3 illustrate the algorithm used to solve the proposed model in a flowchart.

**5. Practical example; A real world case study**

In this section, an application of the proposed method is presented. The Semnan polyethylene pipe and fitting Co. (SPP and F) was founded in 1994, following the major demand of the polyethylene pipes and irrigation tools for the implementation of under pressure irrigation, water supply and gas supply. This company produces many types of polyethylene products, using different types of materials; therefore, it requires a production scheduling program for its products. The SPP and F products include 16 items as follow:

1) Bubbler net (BN)
2) Dripper sided pot (DSP)
3) Easy block coupling (EBC)
4) Variable plastic nozzles (VPN)
5) Bubbler (B)
6) End closure (EC)
7) Offtake (O)
8) Adaptor (A)
9) Equal tee (E)
10) Drum dripper (DD)
11) Adjustable dripper (AD)
12) Spray jets (SJ)
13) Puncher (P)
14) Bubbler stake (BS)
15) In line dripper (ILD)
16) Male adopter (MA)

The manufacturing process begins with the granular raw materials entered into molding machine. After being melted and shaped, the product will come out of machine. The process continues with packaging, and then the products are stocked. Three types of main raw materials which are used to produce these products include the following:

1) PP (Polypropylene): price per Kg is approximated as (1.5, 1.7, 2.05) $ with a holding cost of (0.18, 0.204, 0.246) $ per Kg;
2) Poly acetal (or Poly oxy methylene): price per Kg is approximated as (0.65, 1, 1.4) $ with a holding cost of (0.13, 0.2, 0.28) $ per Kg;
3) ABS (Acrylonitrile-butadiene-styrene): price per Kg is approximated as (3.5, 4, 5) $ with a holding cost of (0.595, 0.68, 0.85) $ per Kg. Also, the company has the capacity for buying and holding 300 tons of material per month.

Figure 4 presents a scheme of SPP and F production shop.
<Please insert Figure 4. The production shop in SPP and F>

The aim of the problem is to determine the production plan for a period of 3 months with 22, 21, and 22 working days, each of which has two 8-hour working shifts. Table 1 shows the information about the operation process of each product. It is notable that the operation time, per product per operation, is approximated with a fuzzy number, in which the time variations are considered with a triangular fuzzy number. On the other hand, the material usage of each granular in each product is approximated by considering the approximated material waste and trash, which is characterized with a triangular fuzzy number. The required costs information regarding each product is given in table 2.

<Please insert Table 1. Production and operation's information>

<Please insert Table 2. Costs information>

The marketing department analyzed the historical data on products demand and it fitted appropriate probability distribution on demands per product. Table 3 shows the probability distribution of products for a three-month period. Initially, a total of sale information for a period of 120 months is analyzed and a probability distribution is fitted for each product demand. Then the distribution parameters are evaluated for each month by using a Maximum Likelihood Estimation (MLE) method in similar months in previous years.

<Please insert Table 3. Products demand information for a three-month period>

The model (20) for SPP and F model includes 198 variables and 104 constraints. SPP and F Managers needed to solve the production planning in three satisfaction levels of 0.7, 0.8, and 1.0. Also, the values of \( f_a^{-1} \) for probability distributions of table 3 at different satisfaction levels are determined using MINITAB 16 package. The obtained fuzzy objective functions in these three levels are equal to (108902859, 134883301, 155675144), (127388473, 156915809, 180137255), and (159099140, 194737221, 222036669).

To determine a preferable solution, an aspiration membership function \( \mu_G \) is constructed as follows:

\[
\mu_G(x) = \begin{cases} 
\frac{222036669 - x}{222036669} & 0 \leq x \leq 222036669 \\
0 & \text{otherwise}
\end{cases}
\]
Applying Eq. (21), compatibility index of each solution with DMs aspiration $\mu_G$ is computed as $K_G(x^*_0.7)=0.4003$, $K_G(x^*_0.8)=0.6036$, and $K_G(x^*_0.8)=0.1355$. The membership of each decision in decision space is specified by Eq. (22) as $\mu_D(x^*_0.7)=0.28021$, $\mu_D(x^*_0.7)=0.48288$, and $\mu_D(x^*_0.7)=0.12195$. Finally, the optimal decision is selected with the highest degree in 0.8 satisfaction level.

In satisfaction level 0.8, the optimal master production plan is shown in table 4.

Please insert Table 4. Optimal production plan at satisfaction level 0.8

Note that as a result of the proposed model, the values of $c_{ij}$ in table 4, which show the net requirements of different resources in each period, can directly enter the process of material requirement planning.

As this problem is solved to create a suitable situation for production planning, the results of this planning are compared with the same period in the previous year, and illustrates the existing gaps between production amounts and the occurred sale, and also the purchased materials and consumed amounts. Consequently, the real data indicates that PP and ABS have been purchased 1.5 and 0.7 tons more than the required materials respectively, which led to additional holding costs. On the other hand, some products such as EBC, EC, DD, and SJ were produced less than the demands, and on the contrary, DSP, VPN, D, BS, and ILD were produced more than the required amounts, in which both diversions caused remarkable costs due to the backorder holding costs. However, with an 80% confidence, the proposed method does not impose any costs due to not met demands, and holding costs are maintained at their lowest level.

Regarding these results, the production plan is determined as an exact guide for production managers. Also, a purchase manager can schedule his/her buying process according to these results.

6. Conclusion

Master production scheduling is a roadmap in the hands of production managers to schedule their operations and to get the required materials and resources. Various parameters and variables are considered in MPS like products demand, cost parameters, material requirement parameters, capacity limitations, and etc. These parameters usually face uncertainty and they are not determined exactly. A model is proposed to deal with the situation, according to which products demands behave stochastically, while cost and resource utilization parameters are determined as fuzzy numbers. These assumptions are logical in practical situations. While demand follows a time dependent behavior which can be captured soundly with probability distributions, the cost and capacity constraints deal with recognition-based uncertainty due to lack of knowledge and they can be handled with fuzziness. The problem is modeled and its solution approach is proposed based on an interactive procedure, in which the formulated MPS problem is solved in different satisfaction levels, and finally the preferable solution is chosen based on the decision maker’s aspiration. The results of the proposed method determine the scheduling and the magnitude of the manufacturer’s different products production in each period. Also, one of the advantages of the model is the possibility to determine the material net requirement in each period, which can be used directly in material requirement planning. Application of the proposed model is shown in a real world case study composed of production scheduling of 16 products.
with stochastic demand in a time horizon of 3 months. The best results are obtained at a satisfaction level of 0.8. The proposed method has a good conformity with real situations, in which products demand have a stochastic characteristic and cost parameters are not determined exactly. Another advantage of the proposed model is its feasibility to accept new restrictions such as warehouse space, outsourcing, and etc. This model can be applied in manufacturing companies that produce multiple products; the companies whose critical issue is planning and scheduling these products and getting the required resources. Future researches can be focused on formulating multi-objective production planning models considering other objectives like production progress.

REFERENCES


**Figure 1.** The relation between production management activities

**Figure 2.** Inputs and outputs of MPS

**Figure 3.** The production shop in SPP and F

**Figure 4.** The production shop in SPP and F

**Table 1.** Production and operation's information

**Table 2.** Costs information

**Table 3.** Products demand information for a three-month period

**Table 4.** Optimal production plan at satisfaction level 0.8
**Figure 1.** The relation between production management activities

**Figure 2.** Inputs and outputs of MPS
Figure 3. The production shop in SPP and Fα is predetermined?

Find α-cuts of fuzzy parameters

Formulate the interval programming equivalent

Formulate the crisp bi-objective equivalent of interval mode

Formulate the equivalent goal programming model

Solve the model

Calculate the Yager index of the solution

Based on Eq. (15)

Based on Eq. (18)

Based on Eq. (20)

Based on Eq. (21)

No

Production and material requirement plan is on hand

Solution is satisfying?

Yes

k=k+1

No

Determine inverse cumulative demand distribution

Set α₀=0

For k=0, 1, ..., (1-α₀)/0.1

α_k = α₀ + 0.1k

Start

Estimating fuzzy cost parameters

Fitting demand distribution over historical information

Model formulation

α is predetermined?

Yes

No

Based on Eq. (11)
**Figure 4.** The production shop in SPP and F

**Table 1.** Production and operation’s information

<table>
<thead>
<tr>
<th>Products</th>
<th>Molding Machine (time per unit, $S$)</th>
<th>Packaging (time per unit, $S$)</th>
<th>Material per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>time of per unit ($S$)</td>
<td>PP (gr)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ABS (gr)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Polyacetal (gr)</td>
</tr>
<tr>
<td>BN</td>
<td>2.5</td>
<td>[0.6, 0.75, 1]</td>
<td>10.72</td>
</tr>
<tr>
<td>DSP</td>
<td>8.8</td>
<td>[0.6, 0.72, 0.8]</td>
<td>112</td>
</tr>
<tr>
<td>EBC</td>
<td>25</td>
<td>[0.6, 0.7, 0.8]</td>
<td>16.3</td>
</tr>
<tr>
<td>VPN</td>
<td>22.5</td>
<td>[0.4, 0.6, 0.75]</td>
<td>28</td>
</tr>
<tr>
<td>B</td>
<td>27</td>
<td>[0.7, 0.8, 0.9]</td>
<td>25.46</td>
</tr>
<tr>
<td>EC</td>
<td>2.75</td>
<td>[0.65, 0.75, 0.88]</td>
<td>29.28</td>
</tr>
<tr>
<td>O</td>
<td>4.38</td>
<td>[0.8, 0.9, 1]</td>
<td>19.2</td>
</tr>
<tr>
<td>A</td>
<td>13.34</td>
<td>[0.65, 0.75, 0.85]</td>
<td>21.12</td>
</tr>
<tr>
<td>E</td>
<td>9.88</td>
<td>[0.55, 0.65, 0.75]</td>
<td>22.08</td>
</tr>
<tr>
<td>DD</td>
<td>10</td>
<td>[0.4, 0.6, 0.75]</td>
<td>23.58</td>
</tr>
<tr>
<td>AD</td>
<td>19.25</td>
<td>[0.55, 0.65, 0.75]</td>
<td>28</td>
</tr>
<tr>
<td>SJ</td>
<td>2.083</td>
<td>[0.45, 0.6, 0.7]</td>
<td>9.6</td>
</tr>
<tr>
<td>P</td>
<td>15</td>
<td>[0.7, 0.85, 0.95]</td>
<td>11.44</td>
</tr>
<tr>
<td>BS</td>
<td>21.88</td>
<td>[0.5, 0.6, 0.75]</td>
<td>30.6</td>
</tr>
<tr>
<td>ILD</td>
<td>10.5</td>
<td>[0.7, 0.75, 0.9]</td>
<td>102.16</td>
</tr>
<tr>
<td>MA</td>
<td>4.63</td>
<td>[0.5, 0.65, 0.8]</td>
<td>25.84</td>
</tr>
</tbody>
</table>
Table 2. Costs information

<table>
<thead>
<tr>
<th>Products</th>
<th>$\tilde{c}_p^p$</th>
<th>$\tilde{c}_c^h$</th>
<th>$\tilde{c}_c^h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
<td>[16, 19, 32]</td>
<td>[2.4, 2.85, 4.8]</td>
<td>[17.76, 21.09, 35.52]</td>
</tr>
<tr>
<td>DSP</td>
<td>[15, 18, 23]</td>
<td>[1.5, 1.8, 2.3]</td>
<td>[16.65, 19.98, 25.53]</td>
</tr>
<tr>
<td>EBC</td>
<td>[17, 21, 25]</td>
<td>[3.4, 4.2, 4.6]</td>
<td>[18.02, 22.26, 26.5]</td>
</tr>
<tr>
<td>VPN</td>
<td>[90, 100, 150]</td>
<td>[22.5, 25, 37.5]</td>
<td>[103.5, 115, 172.5]</td>
</tr>
<tr>
<td>B</td>
<td>[120, 139, 152]</td>
<td>[12, 13.9, 15.2]</td>
<td>[130.8, 151.51, 165.68]</td>
</tr>
<tr>
<td>EC</td>
<td>[50, 79, 92]</td>
<td>[10, 15.8, 18.4]</td>
<td>[55.5, 87.69, 102.12]</td>
</tr>
<tr>
<td>O</td>
<td>[65, 80, 102]</td>
<td>[9.75, 12, 18.4]</td>
<td>[72.8, 89.6, 114.24]</td>
</tr>
<tr>
<td>A</td>
<td>[150, 173, 186]</td>
<td>[37.5, 43.25, 46.5]</td>
<td>[166.5, 192.03, 206.46]</td>
</tr>
<tr>
<td>E</td>
<td>[45, 63, 92]</td>
<td>[9, 12.6, 18.4]</td>
<td>[48.6, 68.04, 99.36]</td>
</tr>
<tr>
<td>DD</td>
<td>[80, 109, 118]</td>
<td>[12, 16.35, 17.7]</td>
<td>[90.4, 123.17, 133.34]</td>
</tr>
<tr>
<td>AD</td>
<td>[100, 110, 120]</td>
<td>[10, 11, 12]</td>
<td>[106, 116.6, 127.2]</td>
</tr>
<tr>
<td>SJ</td>
<td>[78, 90, 100]</td>
<td>[19.5, 22.5, 25]</td>
<td>[84.24, 97.2, 108]</td>
</tr>
<tr>
<td>P</td>
<td>[90, 109, 116]</td>
<td>[18, 21.8, 23.2]</td>
<td>[94.5, 114.45, 121.8]</td>
</tr>
<tr>
<td>BS</td>
<td>[68, 79, 90]</td>
<td>[6.8, 7.9, 9]</td>
<td>[76.16, 88.48, 100.8]</td>
</tr>
<tr>
<td>ILD</td>
<td>[20, 35, 42]</td>
<td>[5, 8.75, 10.5]</td>
<td>[21.4, 37.45, 44.94]</td>
</tr>
<tr>
<td>MA</td>
<td>[32, 49, 56]</td>
<td>[4.8, 7.35, 8.4]</td>
<td>[36.16, 55.37, 63.28]</td>
</tr>
</tbody>
</table>

Table 3. Products demand information for a three-month period

<table>
<thead>
<tr>
<th>Products</th>
<th>Probability distribution</th>
<th>Parameters</th>
<th>Month 1</th>
<th>Month 2</th>
<th>Month 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
<td>Weibull</td>
<td>Shape 1.6125</td>
<td>1.8</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scale 46045</td>
<td>44320.5</td>
<td>47342</td>
<td></td>
</tr>
<tr>
<td>DSP</td>
<td>Normal</td>
<td>Mean 19217</td>
<td>18500</td>
<td>20562</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Variance 166916</td>
<td>124568</td>
<td>103791</td>
<td></td>
</tr>
<tr>
<td>EBC</td>
<td>Weibull</td>
<td>Shape 1.7238</td>
<td>1.982</td>
<td>2.045</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scale 6032</td>
<td>7000.32</td>
<td>8931</td>
<td></td>
</tr>
<tr>
<td>VPN</td>
<td>Poisson</td>
<td>Mean 2560</td>
<td>2800</td>
<td>2011</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Variance 30966</td>
<td>28690.7</td>
<td>32019</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Normal</td>
<td>Mean 191068</td>
<td>145678.2</td>
<td>58960</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Variance 419279</td>
<td>47542</td>
<td>72940</td>
<td></td>
</tr>
<tr>
<td>EC</td>
<td>Logistic</td>
<td>Location 4772</td>
<td>3864</td>
<td>2901</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scale 51929</td>
<td>47542</td>
<td>72940</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Normal</td>
<td>Mean 53790</td>
<td>68700</td>
<td>80173</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Exponential</td>
<td>Mean 51929</td>
<td>47542</td>
<td>72940</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Normal</td>
<td>Mean 53790</td>
<td>68700</td>
<td>80173</td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td>Exponential</td>
<td>Mean 20125</td>
<td>29340</td>
<td>17811</td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>Uniform</td>
<td>Lower bound 10000</td>
<td>8500</td>
<td>13000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper bound 15000</td>
<td>12000</td>
<td>17000</td>
<td></td>
</tr>
<tr>
<td>SJ</td>
<td>Log logistic</td>
<td>Location 9.33</td>
<td>10.465</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scale 0.01989</td>
<td>0.3</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>Products</td>
<td>Probability distribution</td>
<td>Parameters</td>
<td>Month</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------</td>
<td>------------</td>
<td>-------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Lognormal</td>
<td>Location</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BS</td>
<td>Logistic</td>
<td>Location</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ILD</td>
<td>Normal</td>
<td>Location</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA</td>
<td>Constant</td>
<td>Location</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Optimal production plan at satisfaction level 0.8

<table>
<thead>
<tr>
<th>Products</th>
<th>Months</th>
<th>Material</th>
<th>Months (Ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>BN</td>
<td>61853</td>
<td>57733</td>
<td>65018</td>
</tr>
<tr>
<td>DSP</td>
<td>19561</td>
<td>18797</td>
<td>20834</td>
</tr>
<tr>
<td>EBC</td>
<td>7950</td>
<td>8901</td>
<td>11278</td>
</tr>
<tr>
<td>VPN</td>
<td>2603</td>
<td>2845</td>
<td>2049</td>
</tr>
<tr>
<td>B</td>
<td>31084</td>
<td>29012</td>
<td>32224</td>
</tr>
<tr>
<td>EC</td>
<td>58545</td>
<td>52899</td>
<td>76962</td>
</tr>
<tr>
<td>O</td>
<td>44890</td>
<td>51424</td>
<td>66028</td>
</tr>
<tr>
<td>A</td>
<td>94262</td>
<td>74678</td>
<td>45554</td>
</tr>
<tr>
<td>E</td>
<td>21706</td>
<td>18798</td>
<td>32694</td>
</tr>
<tr>
<td>DD</td>
<td>32390</td>
<td>47221</td>
<td>28666</td>
</tr>
<tr>
<td>AD</td>
<td>14000</td>
<td>11300</td>
<td>16200</td>
</tr>
<tr>
<td>SJ</td>
<td>14935</td>
<td>53151</td>
<td>111729</td>
</tr>
<tr>
<td>P</td>
<td>6088</td>
<td>16951</td>
<td>43257</td>
</tr>
<tr>
<td>BS</td>
<td>89351</td>
<td>102693</td>
<td>84843</td>
</tr>
<tr>
<td>ILD</td>
<td>32005</td>
<td>37854</td>
<td>36153</td>
</tr>
<tr>
<td>MA</td>
<td>24000</td>
<td>20000</td>
<td>19800</td>
</tr>
</tbody>
</table>

- The results are rounded upward.

Seyed Hossein Razavi Hajiagha, PhD in industrial management at Allameh Tabatabaei University, assistant professor in Khatam University, editorial board and reviewer of several international journals and chairman of several international conferences. He is interested in MADM, operational research, and DEA.

Shide Sadat Hashemi, MA. In industrial management at Allameh Tabatabaei University, executive manager in Saramadan Andishe AVINA Co, and the senior member of consultant
team in different managerial projects. She is reviewer of international journals. She is interested in BPMN, strategy, MCDM and FS.

Mohammadreza Sadeghi, PhD in industrial management at Allameh Tabatabaei University, manager of managerial research team in Sa-Avina CO., has written several international papers and managed many managerial projects in MCDM, DEA, and PRODUCTION PLANNING.

Seyed Hossein Razavi Hajijaghah\textsuperscript{a,}\textsuperscript{†},
Shide Sadat Hashemi\textsuperscript{b},
Mohammadreza Sadeghi\textsuperscript{c}

\textsuperscript{a} Department of Management, Khatam University, Tehran, Iran,
No.30, Hakim Azam St., North Shiraz St., Mollasadra Ave., Tehran, Iran
Email: s.hossein.r@gmail.com

\textsuperscript{b} Department of Management, Saramadan Andishe Avina Co. Tehran, Iran
Unit 401, No. 15, Modares Science and technology park, Gordafarid Ave. Tehran Iran.
Email: shide_hashemi@yahoo.com
Tel: +982166919151-401

\textsuperscript{c} Department of Management, Saramadan Andishe Avina Co., Unit 401, No. 15, Modares Science and technology park, Gordafarid Ave. Tehran Iran.
Email: m.r.sadeghi@st.atu.ac.ir

\textsuperscript{†} Corresponding author, s.hossein.r@gmail.com