A stochastic multi-objective model based on the classical optimal search model for searching for the people who are lost in response stage of earthquake

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ABSTRACT
Although after an earthquake the injured person should be equipped with food, shelter and hygiene activities, before anything must be searched and rescued. But

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disaster management (DM) has focused heavily on emergency logistics and developing an effective strategy for search operations has been largely ignored. In this study, we suggest a stochastic multi-objective optimization model to allocate resource and time for searching the individuals who are trapped in disaster regions. Since in disaster conditions the majority of information is uncertain, our model assumes ambiguity for the locations where the missing people may exist. Fortunately, the suggested model fits nicely into the structure of the classical optimal search model. Hence, we use a stochastic dynamic programming approach to solve this problem. On the other hand, through a computational experiment, we have observed that this model needs heavy computation. Therefore, we reformulate the suggested search model as a multi-criteria decision making (MCDM) problem and employ two efficient MCDM techniques, i.e. TOPSIS and COPRAS to tackle this ranking problem. Consequently, the computational effort is decreased significantly and a promising solution is produced.

*Keywords*: Earthquake response, Multi-objective optimization, Search theory, Dynamic programming, Multi-criteria decision making

1. INTRODUCTION

In recent decades, an increasing rate in the amount of natural catastrophes, people affected, and the economic damages have been reported [1]. For example, during the 1960s and 1970s, more than 3,000,000 people were killed in natural disasters [2] and over 230 billion USD of the world’s wealth was eradicated [3]. Also since the1980s, the rate and the impact of disasters have terribly increased (as shown in Fig.1). We refer to the website of EM-DAT² for valuable statistics about disasters from 1900 to now.

According to World Health Organization³(WHO), a disaster⁴ is a dire trouble in the normal operation of a community, whose effects exceed the capability of community to control the conditions. As pointed out by EM-DAT, between 2000 and 2010 around 8400 disasters happened in the world, which means more than two catastrophes every day. According to WHO, 3.4 billion dwell in regions where at least one natural disaster may terribly shock them. For example, Iran is one of the

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² EM-DAT is a famous database on natural and technological disasters.
⁴ There is difference between terms “disaster” and “catastrophe”. But in this study these terms are employed interchangeably.
most disaster-prone countries and has 10th rank in this regard in the world. This country has experienced 31 out of 40 forms of natural disasters and these catastrophes have destroyed 232 million USD of the wealth of this country in the last decade [4].

In 20th century, earthquakes have killed more than 1,500,000 people around the world [5]. According to [6], two earthquakes occur in the world every minute and as pointed out by United Nations (UN), the millions of people are in danger of earthquakes. For example, over 90% of Iran is built on fault lines. Also Iran is one of the top ten in terms of the rate of earthquakes. On the basis of the number of victims from earthquake, Iran is the number one. Tehran (the capital of Iran) is created above several faults and the likelihood of happening of a very intense earthquake (Mw > 7) is approximately 70% [7].

On the other hand, it is believed that Global Warming will speed up the number of natural calamity shocks [8]. Lay [9] warned that between 2004 and 2014 approximately 1.8 great earthquakes per year happened globally, compared to 0.68 earthquakes per annum from 1900 to 2004. These numbers show a terrifying increase of 265%. Furthermore, Singh [10] pointed out that the world population will increase from the 7 billion in 2011 to 9.30 billion in 2050. Due to this unbridled growth of the population as well as the global urbanization the threat of earthquake will increase [5].

As pointed out by Hou and Shi [11], destruction of an earthquake is not straight determined by its magnitude. For example, from 1980 to 2002, India experienced 14 earthquakes with 32,117 killed while the United States experienced 18 earthquakes with 143 killed [8]. On the basis of Risk’s formula, namely

\[
Risk = Hazard \times (Vulnerability - Resources)
\]

efficient distribution of resources will reduce the likelihood of damage considerably. According to [12], DM (or emergency management) refers a group of actions that is done before, during, and behind a catastrophe with the aim of preventing loss of people, diminishing the disaster shock and coming back earlier to normal conditions. Often, DM is divided into four key steps as shown in Fig. 2. Mitigation is various activities taken to lessen the possibility of it occurring, or decrease its destructive shocks. Preparedness organizes the society to respond when a calamity takes place. Response is the utilization of resources to protect life, assets, the milieu, and the political organization of the society. Recovery is the long term designs to revert to normality [12].

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Obviously, in the early hours after a disaster the response step must concentrate on search and rescue (SAR). SAR activities are utilized to track people after a dreadful catastrophe and when individual is lost [13]. As mentioned, in all SAR activities time is one of the most vital factors. According to [14], numerous human beings trapped under the debris in the earthquake may have a great chance to remain alive if they are saved in golden times, i.e. 72 h after quake. Chang and Nojima [15] pointed out 24 h after quake is the golden rescue phase. According to Chen and Miller-Hooks [16], between the first day and fifth day after the 1976 Tangshan earthquake, the survival rate decreased from 81% to 7.4%. Fiedrich et al. [17] suggested a dynamic frame to approximate such a rate as shown in Fig. 3. This model pointed out that for rescuing the first 72 h are the most serious. Obviously, SAR operations use massive amounts of money and time. As a result, in disaster situation, decision makers must utilize finite resources and limited time efficiently to attain the best relief. However, one significant complexity of the response period is to find the best strategy for assignment of time and resource to SAR operations. In addition, due to the constraints of time and resources any strategy to support search operations contains some aspects of uncertainty (e.g. the number of missing persons or the probable location of them after disaster). Thus, utilizing a practical and efficacious approach to import this stochastic nature into the decision-making procedure is very vital.

Search for individuals lost in disaster fits into the framework of the search theory (ST). ST is the study of how to efficiently use restricted resources when attempting to detect a goal whose position is not exactly known [18]. ST is one of the oldest parts of operations research (OR) [19], but by analyzing the literature, severe scarcity of a search strategy on the basis of OR techniques in DM is revealed. Noteworthy, because the majority of researchers have focused deeply on emergency logistics and more specifically on two main problems i.e., location and transportation [20], developing various effective strategies for search operations has been largely ignored. To reduce this gap, our study focuses on the problem of optimal distribution of time and resources to discover an objective, namely missing people. Fortunately, the search problem for missing people fits nicely into the structure of the classical optimal search model (COSM). Generally, in this model a single motionless goal\(^6\) is in one of given locations. The search consists of a series of discrete investigation until the aim is detected. Look into location \(i\) costs \(C_i\) and \(p_i\) is the probability that the goal is in the location \(i\). The aim is to find out the goal at

\(^6\) In this study, the terms "aim", "target" and "goal" are employed interchangeably.
Among the various measures of efficiency that are employed in ST, the most common factors are expected time to discovery, expected cost to discovery, and probability of discovery. From a mathematical point of view, this problem has been broadly tackled by dynamic programming (DP) method. DP, which was introduced by Bellman [23], is a valuable method to handle multi-step decision processes ([24] and [25]). On the other hand, multi-objectivity is one of the most significant attributes of real-world problems. Generally, decision-making in today’s world needs various compromises among several conflicting goals. In view of the inherent restrictions on resources and time, the problem of search has a multi-objective frame. Consequently, a multi-objective search model in stochastic environment can appropriately satisfy the necessities of this problem in the real world.

The aim of this study is to introduce a stochastic multi-objective model based on COSM for SAR of trapped people immediately after an earthquake. To find a solution to this multi-objective problem (MOP), we employ compromise programming (CP). CP, which is one of the most accepted techniques for coping with the MOP, provides the robust solution, which belongs to Pareto set. According to [26], who reviewed multicriteria optimization in humanitarian help, there are few studies (e.g., [27]) adopting CP in DM field. However, various stochastic problems modeled by DP usually suffer from the “dimensionality” concept, which is a huge increase in the computational efforts as well as required memory when the dimension of the state grows [28]. Thus, it is essential to point out that many MOPs on the basis of DP are close to unfeasible to optimize for a practically sized problem. Fortunately, as pointed out by [29], optimization does not essentially mean obtaining the (global) optimum to a problem, because it can be impracticable owing to the structure of problem. In this paper, because multi-objective stochastic dynamic programming approach to handle a decision making problems is often very difficult, as an alternative solution, an MCDM framework is adapted for the search problem. Noteworthy, in real cases that the exact technique needs a long calculation time MCDM methods act as an approximate approach. The suggested decision framework in this paper is depicted in Fig. 4.

The remainder of this work is arranged as follows: in Section 2, briefly, related literature is remembered. We review the classical optimal search model and multi-objective optimization problem in Section 3 that lead to the suggested method in Section 4. The efficiency of our model is analyzed by a case study in Section 5.
Section 6 provides an alternative framework based on MCDM for search problem. In Section 7, we finish this work with conclusion and future study guidelines.

2. LITERATURE REVIEW

Clearly, during the past 10 years the papers related to logistics of the response period have greatly been expanded. For example, Afshar and Haghani [30] suggested a comprehensive mathematical framework to control the flow of various supplies in the response system. Berkoune et al. [31] considered a compound transportation problem, i.e. multi-vehicle, multi-depot and multi-product for the transportation of humanitarian help. Najafi et al. [32] focused on a stochastic multi-objective, multi-commodity, multi-mode, and multi-period framework to tackle the logistics of injured persons as well as supplies after earthquake. Bozorgi-Amiri et al. [27] developed a multi-objective robust stochastic approach for disaster relief logistics. Abounacer et al. [20] suggested a multi-objective location-transportation framework and proposed an $\varepsilon$-constraint technique to tackle it for disaster response. Bozorgi-Amiri and Asvadi [33] concentrated on selecting optimum sites for relief logistic hubs. Sheu [34] focused on a relief allocation problem in the critical rescue stage. Ma et al. [35] provided a robust transportation framework to minimize the maximum time of rescue for wounded persons.

Altay and Green [12] prepared a significant review of the application of OR approach in DM until 2004. Also Galindo and Batta [36] provided a nice paper as a continuation of Altay and Green’s [12] study. According to these studies, although DM has become a dynamic branch in OR, some gaps can be perceived. For example, only one article related to problem of search can be seen in these papers, i.e. [37]. Jotshi and Batta [37] suggested a heuristic to carry out the search problem for a stationary goal on a network. Before that, Fiedrich et al. [17] introduced a dynamic optimization model with the aim of minimizing the number of victims for the period of SAR. Chen and Miller-Hooks [16] formulated the problem of optimally positioning SAR teams as a multistage stochastic program. The object of this model was to maximize the expected quantity of rescued persons. Berger and Lo [38] proposed a mixed-integer linear framework to optimally handle the multi-agent discrete SAR path problem. Briefly, by analyzing the literature and four comprehensive reviews [12], [36], [39], and [40], we can see the search problems have not been as emphasized as location and transportation problems in DM.

Search for persons who are lost after earthquake falls under the realm of ST. The first developments in ST were produced by Koopman [41] in World War II to
prepare efficacious techniques of finding submarines. Although surprisingly ST and related problems have been vanished for more than twenty years [42], several problems such as searching for a mine land hidden [43], searching for a hostage hidden and searching for an explosive material have emphasized the need for effective search strategies for detecting aims of different forms [40]. As pointed out by [44], because of its stochastic structure and the nonlinearity made by the probability of finding, the problem of obtaining the "best" policy for search is basically very difficult. Several valuable works about ST as well as search problems have been presented in [19], [22], [45], and [46]. A number of parameters in which search scenarios vary are as follows: ([38]; [45]):

1) One-sided (OS) search in which the goal does not react to the act of searcher. In this theme, the most tangible measure of efficiency for the search process is the expected cost or the expected time of the search. First, Black [21], Stone [19] and Washburn [46] concentrated on the OS search framework. Also, OS search can be clarified by various attributes such as:

1-1) discrete search problem (DSP) (e.g., [42]) versus continuous search problem (CSP) (e.g., [47]);
1-2) stationary goal search (e.g., [21]) versus moving goal search
1-3) multiple goals search (e.g., [47]) versus single goal search [22];

2) Two-sided (TS) search in which the objective reacts to the act of searcher (search games).

Also constrained searcher motion is another model which develops for search problem. In this type, some constraints on the motion of searcher are considered. As pointed out by Trummel and Weisinger [48], this category of search problem is NP-hard. A very good survey of search games was provided in [49]. Also Chudnovsky and Chudnovsky [50] presented a review on OS search as well as TS search. Moreover, a survey on pursuit-evasion game in mobile robotics was prepared by [51]. Fig. 5 depicts a general categorization of search parameters.

After presenting the classic model by Black [21], diverse models of DSPs have been extended. For example, Chew [52] analyzed this problem for maximizing the probability of detection the target along with cost restraints. Ross [53] extended the outcome of Chew’s [52] study. He assumes that in the new model a prize $R_i$ is earned if the goal is detected in the location $i$. Smith and Kimeldorf [54] suggested a DSP with an unknown number of objectives. The goal of this work was to minimize the expected cost to discover leastwise one goal. Assaf and Zamir [47] focused on a DSP when there is over one immobile concealed target. Wegener [55] proved that the
general search problem (switching cost problem) with the minimum expected time and switches cost are NP-hard. Kadane [56] proposed a search strategy that maximizes the detection probability of the target considering a restraint on the existing budget.

Various optimization techniques are being employed for solving search problems. Zahl [57] employed Lagrange Multiplier (LM) to tackle a search problem. Kadane [58] developed a branch and bound approach to deal with some limitations in the LM method for a discrete instance. But due to sequential nature of the optimal search model as well as substitute decisions of this optimization frame, Ross [22] utilized DP approach to cope with this model. In contrast to several other techniques such as LM, DP causes no constraint on the non-convex structure of different problems and provides the global solution. In addition, this technique is capable to model sequential decision systems and non-linear structures [42]. However, DP is usually not easy to apply. Further information on DP and applications can be obtained in several well-known books such as [23].

On the other hand, ambiguity has stimulated numerous researchers to address stochastic optimization in disaster response procedures. Mathematically, a stochastic programming (SP) is a framework in which the ambiguities are depicted as random variables with identified probability functions. However, SP techniques are generally avoided because they increase the intricacy of problems. A good survey of SP was provided by Gutjahr and Pichler [59].

The term “stochastic dynamic programming” (SDP) was first utilized by Prof. Richard Bellman. SDP differs from deterministic DP in that the state at the following period is not absolutely specified by the state and decision at the present period. For what the following state will be we use a probability distribution [60]. Cervellera et al. [28] proposed an optimization model for a large-scale water reservoir system using SDP. Li et al. [60] developed an uncertain production planning based on SDP. Ross [22] utilized SDP for solving the search problem. However, Marescot et al. [61] pointed out that despite a growing number of the usage of this technique in different problems, SDP still suffers from a general incomprehension.

Multi-objectivity is a fundamental aspect of engineering optimization and especially DM. Gutjahr and Nolz [26] prepared a comprehensive literature review on the application of this form of optimization to DM. Although real-life problems almost always have multi-objective nature as well as stochastic aspect, these branches, namely stochastic optimization and MOP flourished individually from each other [59]. Hoyos et al. [40] reviewed the literature of DM based on OR methods with stochastic elements. In this review some studies about facility
location, resource allocation, relief distribution and evacuation can be found. But few, if any, have attempted to present a study about search issue in DM. There are only two studies about SAR, i.e. [16] and [37]. Although before them, Richardson and Discenza [62] discussed the utilization of ST in the SAR actions of the U.S. Coast Guard, none of these articles helps directly solve the problem of search generally. Finally, to the best of our knowledge, developing a multi-objective stochastic frame for search in DM literature is completely novel.

3. PRELIMINARIES

In this part, a number of basic models and methods are reviewed.

3.1. The Classical Optimal Search Model [22]

As mentioned by Chew [52], a class of optimization problems, called searching problems, is concerned with a procedure which provides optimal value instead of the value itself. Of this nature, we review classical optimal search model [52] as follows:

An immobile objective is in one of \( n \) locations where \( n \in \mathbb{N} \) is known (\( \mathbb{N} \) is the natural number set). For each location \( i \), \( P_i \) is the probability that the goal is in the location \( i \) where \( \sum_{i=1}^{n} P_i = 1 \) and the cost for every glance in the location \( i \) is \( C_i \). The probability that an objective in the location \( i \) will be discovered on a one glance is \( \alpha_i \) and as a result, the probability of fail to see for location \( i \) is \( 1 - \alpha_i \). The aim of this model is to detect the goal at minimum expected cost. It is the cost-oriented search model. This process finishes when the goal is detected. Clearly, the plan for search is a procedure which determines the search order. As mentioned by Ross [22], this model has a decision procedure whose state is the posterior probability \( P = (P_1, ..., P_n) \), with \( P_i \) indicating the posterior probability, known all that has happened that the target is in region \( i \). Mathematically, \( V(P) \) is minimum expected cost equation and can be defined as follows:

\[
V(P) = \min_i \left[ C_i + (1 - \alpha_i P_i) \times V(T_i(P)) \right]
\]
In the above equation, \( T_i(P) = [(T_i(P))_1, \ldots, (T_i(P))_n] \) is the posterior probabilities given the previous probability \( P \) and given that an inspection of region \( i \) was useless. Hence,

\[
(T_i(P))_j = P( \text{in } j \text{ search of } i \text{ unsuccessful}) = \begin{cases} 
\frac{P_j}{1 - \alpha_i P_i} & \text{if } i \neq j \\
\frac{P_j(1 - \alpha_j)}{1 - \alpha_i P_i} & \text{if } i = j
\end{cases}
\] (2)

For a certain state \( P \), a strategy can be considered as an order of regions with the explanation that the regions are sought in that arrange until the target is discovered.

**Lemma 3.1.1**

Suppose that \( V_\delta(P) \) indicate expected cost equation under \( \delta \). Also assume that \((i, j, \delta)\) represent the strategy that first seeks \( i \) then \( j \) and at last \( \delta \). For any strategy \( \delta \), we have: \( V_{(i,j,\delta)}(P) \leq V_{(j,i,\delta)}(P) \Leftrightarrow \frac{\alpha_i P_i}{C_i} \geq \frac{\alpha_j P_j}{C_j} \)

**Proof**: assume \( i \neq j \). Subsequently, we have:

\[
V_{(i,j,\delta)}(P) = C_i + (1 - \alpha_i P_i)[C_j + (1 - \frac{\alpha_j P_j}{1 - \alpha_i P_i})V_\delta(T_j T_i P)] = C_i + C_j + V_\delta(T_j T_i P) - \alpha_i P_i V_\delta(T_j T_i P) + \frac{\alpha_i P_i}{1 - \alpha_i P_j} V_\delta(T_j T_i P) - \alpha_i P_i C_j
\] (3)

\[
V_{(j,i,\delta)}(P) = C_j + (1 - \alpha_j P_j)[C_i + (1 - \frac{\alpha_i P_i}{1 - \alpha_j P_j})V_\delta(T_j T_i P)] = C_i + C_j + V_\delta(T_j T_i P) - \alpha_j P_j V_\delta(T_j T_i P) + \frac{\alpha_j P_j}{1 - \alpha_j P_i} V_\delta(T_j T_i P) - \alpha_j P_j C_i
\] (4)

We know that \( T_j T_i P = T_i T_j P \) and \( \alpha_i P_i C_j \geq \alpha_j P_j C_i \), thus, this completes the proof.

**Proposition 3.1.1**

In the above model, the best strategy is to seek a region with the largest value of \( \frac{\alpha_i P_i}{C_i} \). On the other words, the maximum value of \( \frac{\alpha_i P_i}{C_i} \) minimizes the expected cost to detect.
**Proof:** assume that \( \frac{\alpha_{iP}}{C_i} = \max \frac{\alpha_{iP}}{C_i} \)

Now, think about a strategy such as \( \delta \) that does not seek location 1 at first. This strategy searches location 1 at time \( j \), namely \( \delta = (i_1, ..., i_{j-1}, 1, i_{j+1}, ...) \). By Lemma 3.1.1, clearly a superior policy is acquired by swapping 1 and \( i_{j-1} \), namely \( \delta' = (i_1, ..., 1, i_{j-1}, i_{j+1}, ...) \) is superior than \( \delta \). By iterating this reasoning we can see \( (1, i_1, ..., i_{j-1}, i_{j+1}, ...) \) is superior than \( \delta \). Consequently, for every policy not at first searching 1, we can obtain a better strategy that does start with 1.

### 3.2. Multi-Objective Optimization Problems

In this sub-section, some definitions and famous algorithms for multi-objective optimization problem (known as multi-objective problem in this work), using [63], [64], [65], [66], [20], and [26] have been summarized.

Real problems are often multi-objective and the objectives disagree with each other in these problems. Therefore, nearly all decisions needs trade-offs among various non-commensurable and incompatible objectives. Comparing with a single objective optimization problem (SOP), MOP is more rational in terms of real applications. It can be formulated in a minimization case as follows:

\[
\begin{align*}
\min \quad & f(x) = (f_1(x), f_2(x), ..., f_m(x)) \\
\text{s.t.} \quad & x \in D
\end{align*}
\]

In the above frame, \( m(\geq 2) \) is the number of objectives, \( x = (x_1, x_2, ..., x_n) \) is the vector of variables, and \( D \) is the solution space. A MOP often does not have one optimum, but a group of solutions recognized as the Pareto-optimal set. First of all, some basic definitions in MOP are reviewed. In a minimization form, these notions are clarified as follows [20].

**Definition 1:** Pareto dominance

A vector \( u \in D \) dominates a vector \( v \in D \) in the Pareto logic, iff:

...
\[
\begin{cases}
    f_i(u) \leq f_i(v) & \text{for all } i \in \{1, \ldots, m\} \\
    f_j(u) < f_j(v) & \text{for at least one } j \in \{1, \ldots, m\}
\end{cases}
\]

**Definition 2:** Pareto optimal solution (non-dominated solution or efficient solution)
A solution \( u \in D \) is a Pareto-optimal solution; iff there is no \( v \in D \) that \( v \) dominates \( u \).

**Definition 3:** Pareto optimal set
The Pareto optimal set is specified as \( P = \{ x \in D : x \text{ is a Pareto optimal solution in } D \} \).

**Definition 4:** Pareto front
The Pareto front is specified as \( \mathbb{P_F} = \{ f(x) : x \in P \} \) where \( P \) is the Pareto optimal set.

More formally, a solution is in the Pareto set if no amelioration is achievable in an objective function without deteriorating in another objective function. As mentioned earlier, there are various techniques to cope with MOP. One of the most common techniques for MOP is scalarizing techniques (classical approaches). Scalarizing techniques reduce the MOP into a SOP and optimize this SOP. Obviously, these techniques make very easy the computational effort of MOPs. The weighted-sum (WS) and the \( \varepsilon \)-constrained are among two of the most widespread techniques of this group. Although the \( \varepsilon \)-constrained technique has a number of benefits over the WS method, each of these techniques has various shortcomings. For example, WS technique is not capable to acquire non-dominated solution in a non-convex Pareto front. Also this technique cannot suitably estimate the Pareto-optimum curve ([66]; [67]. On the other hand, \( \varepsilon \)-constrained technique is not suitable for a problem with several objective functions.

The CP is a reputable mathematical programming which works efficiently for MOPs. CP belongs to a group of MOP techniques called “distance-based” approaches, which seeks a solution closest to the reference point (ideal solution) considering a distance index. The outstanding advantage of CP is its very straightforward framework. The CP techniques vary in selection of the distance metric as well as the reference point. The most common option for a reference point is the ideal point that it is the optimum for every objective function separately in a MOP. The distance between a Pareto-optimum (to be searched) and the reference point can be computed by the \( L_p \)-metric which is defined as:
In Model (6), \( m \) shows the number of objective functions, \( w_i \) is the weight of objective \( i \) (the \( L_p \)-metric coefficient), \( z_i \) is non-dominated solution, \( z_i^* \) is the ideal solution and \( p \) indicates the significance of the maximum deviation from the ideal solution. In the other words, as \( p \) increases, the significance of the deviations escalates.

Going back to Model (6), because of the utilization of non-commensurable units in various objectives, normalization is necessary. We normalize model (6) by the reference point \([68]\) as follows:

\[
L_p = \left[ \sum_{i=1}^{m} w_i (z_i^* - z_i)^p \right]^{1/p} \quad 1 \leq p \leq \infty
\]  

(6)

To tackle a MOP by CP, the \( L_p \) given in Model (7) is minimized. On the other words, given \( p \) and \( w_i \), the option with the minimum \( L_p \) will be the best since it is the point nearest to the ideal solution. It should be noted that when \( 1 \leq p < \infty \) and all weights are strictly positive, CP generates Pareto optimal \([64]\). Proofs have been provided in nearly all books about MOP.

4. THE STOCHASTIC MULTI-OBJECTIVE SEARCH PROBLEM

This part explains the main problem of this paper and provides a mathematical form for it.

4.1. Problem Description

Let us begin with notations and explanation of the objectives of suggested framework. Table 1 demonstrates the notations.

Suppose that a motionless target is located in one of the locations. It is essential to point out that the number of locations is known and \( n \in \mathbb{N} \). The goal’s precise location is unknown, but to the searcher the probability that the goal is in that
location is known. Also we have \( \sum_{i=1}^{n} P_i = 1 \). The solution (search) space \( \chi \) is discrete as well as search cost and search time of location \( i \) are \( C_i \) and \( t_i \), respectively. The search problem is to obtain in what order should locations be checked to minimize the expected cost as well as expected time of the search?

Avoiding unimportant cases we suppose that no \( C_i \), \( t_i \), or \( P_i \) are zero. Let us point out that although Ross [53] relaxed the condition \( C_i > 0 \) and analyzed this problem, in our suggested model any \( C_i = 0 \) (or \( t_i = 0 \)) is meaningless.

4.2. Solution Procedure

Designing an applicable search procedure is a subset of resource distribution problem. After earthquake, resources for search are almost always inadequate. Thus, there is a need for experts to decide how to apportion the resources among the various strategies in order to minimize the impact of the earthquake. Furthermore, the search plan in the response stage contains another goal. After the majority of earthquakes, life of missing persons depend on minutes and the chance to discover survivors after golden time is tremendously low. Therefore, decision maker aims to minimize the time of unserved missing persons.

With the above postulation, we concentrate on two objectives for the suggested model: to minimize expected search cost and expected search time.

On the basis of the aforementioned goals, we present the following stochastic multi-objective model for search problem.

Two objectives are considered for the suggested search problem as clarified below:

(1) Expected search cost, which is the expected cost of search in the DM.

The first objective is prepared to minimize the expected search cost. It is the mathematical expectation of search cost and takes the following model:

\[
V(P) = \min_{i} \left[ C_i + (1-\alpha_i)P_i \times V(T_i(P)) \right]
\]  

(8)  

The objective function \( V(P) \) takes into account and minimizes the total cost of search operation including the transportation cost, expert cost, etc.
(2) Expected search time, which is the expected time of search in the DM.

Now, the second objective is prepared to minimize the expected search time. It is the mathematical expectation of search time and takes the following model:

\[
W(P) = [t_i + (1 - \alpha_i)P_i \times W(T_i(P))] \tag{9}
\]

The objective function \( w(P) \) takes into account and minimizes the total time of search operation including transportation time, deployment time, etc.

As a result, the stochastic multi-objective search problem will be formulated as follows:

\[
\begin{aligned}
\text{Minimize} & \\
& \left\{ \\
& \quad V(P) = [C_i + (1 - \alpha_i)P_i \times V(T_i(P))] \\
& \quad \text{expected search cost function} \\
& \quad W(P) = [t_i + (1 - \alpha_i)P_i \times W(T_i(P))] \\
& \quad \text{expected search time function} \\
\right. \\
\end{aligned} 
\tag{10}
\]

Two objectives are simultaneously minimized in the absence of certain restriction. Therefore, a multi-objective technique is indispensable when handling our problem. After building our stochastic multi-objective framework by Model (10), CP is employed to tackle the suggested MOP for the search problem. For doing so, first, we solve suggested MOP as two single objective models separately. After that, we reformulate our MOP as a SOP by a weighted aggregation. This SOP minimizes the sum of the difference between an objective and its optimum when each objective function is normalized. With recalling model (7) for \( p = 1 \), the \( L_p \)-metrics function will be obtained as follows:

\[
Z = \min \left[ w \frac{V^*(P) - V(P)}{V^*(P)} + (1 - w) \frac{W^*(P) - W(P)}{W^*(P)} \right] 
\tag{11}
\]

According to Proposition 3.1.1, the policy which seeks a location with the largest value of \( \frac{\alpha_i P_i}{C_i} \) minimizes the expected search cost in Model (8). Also Sadi-Nezhad et al. [43] by using similar approach shown that the plan which searches a location with
the largest value of \( \frac{a_i}{P_{t_i}} \) can minimize the expected search time in Model (9). Now, we employ Model (11) with identical relative weights, i.e., 0.5 to solve the proposed multi-objective search problem. Let us point out that when there are no preferences for the objectives, obtaining a group of Pareto optimal solutions of a MOP is challenging. In this problem decision maker is not willing to give a preference of an objective to another objective. For this reason, to highlight the significance of simultaneously considering the cost and time of search, we set \( w \) equal to 0.5 in Model (11). Reader is referred to [63] for a classification of MOP techniques on the basis of the role of the decision maker in a solution procedure. To obtain a desirable solution such that the expert can compromise between cost and time on the basis of the results, Model (11) can be solved while changing the \( L_p \)-metrics coefficient several times. The solution will be one of the search scenarios\(^7\) which minimized the above \( L_p \)-metrics function. As mentioned, \( \chi \) is the set of possible solutions and solution space is any search sequence of the locations. Let us recall that the solution in the suggested problem is a procedure which provides optimal value instead of the value itself. Hence, we will have \( k = n! \) possible scenario. Clearly, the suggested model determines the best scenario for search process among the possible scenarios while trying to minimize search cost and search time. Thus, in the problem setting, a scenario-based approach is used to contain all discrete scenarios. Noteworthy, scenario-based SP optimizes the expected value of the objectives with no directly employing the expert’s priorities [69]. In the scenario-based SP a group of discrete policies and their related probabilities will be considered, but in this problem we take into account all possible scenarios. Obviously, due to the scenario-based nature of the search problem, CP is better than other multi-objective methods in terms of calculation time. Now, in this step, we obtain all possible search scenarios as solution space. In our problem, search scenario is any search sequence of the locations. In these sequences, each location will appear only once. For example, [location1, location3, location4, location2] or [location4, location1, location2, location3] are some feasible search scenarios for four given locations. Clearly, Model (11) is nearly impossible to handle directly. Hence, all possible scenarios are separately placed in Model (11). Finally, the scenario with the minimum \( Z \) will be the best. It is obvious that Model (11) is feasible as well as bounded. It is very straightforward to prove.

\(^7\) There is difference between terms “scenario” and “policy”. But in this study, these terms are employed interchangeably.
In the next section, we prepare a numerical example to elucidate the working of the proposed model.

5. NUMERICAL EXAMPLE

In this section to explain how the suggested model works we analyze a realistic application.

5.1. Data Base Formation

This case is adapted from a real event after an earthquake\(^8\). At first, we asked an experienced expert in the field of DM to score these locations by using the experiences of the 1990 Manjil, the 2003 Bam and the 2012 Ahar and Varzaghan earthquakes in Iran. This expert had more than 30 years of experience for various topics of DM in Iran and fortunately was familiar with the basic concepts of OR. Finally, the senior manager who verified those responses had above 20 years of experience in the relief activities.

5.2. Case Study

After an earthquake, a search committee in the relief distribution center (RDC) is organized to search and rescue a missing family. According to the some evidences, they probably have been trapped during the earthquake in one of the following locations: a cottage far from city (location 1), a forest near the city (location 2), and a mountain near the city (location 3).

The probability that they are in each of the above locations are as follows (Table 2):

As explained, the family is in one of these locations. Thus, we have \( \sum_{i=1}^{3} P_i = 1 \). Due to the need for experts as well as special equipments the mountain search is more expensive compared to other locations. Considering this fact, the search cost of each location is estimated shown Table 3.

Helicopter cannot land in the near of cottage. Thus, access to the cottage is possible only using a road. The distance between the cottage and RDC is more than 175 km. Therefore, due to the impossibility of using helicopter, road damage, and

---

\(^8\) Because of some social and political concerns the earthquake's name remains confidential.
remoteness, more time will be spent to search of cottage compared to other locations. The search time of each location is estimated as shown Table 4.

As mentioned by Chew [52], this class of optimization problems is concerned with a procedure which obtains optimal value instead of the value itself. Therefore, the key problem is how to select a policy to minimum simultaneously the expected cost and expected time of searching the family.

5.3. Solution

As mentioned, it is assumed that the family is in one of 3 locations \( \sum_{i=1}^{3} P_i = 1 \). On the other hand, each policy for searching the family is a search order of locations. In this case, we have 3 locations. Therefore, there are \( 6 (k=3!) \) possible scenarios to search. Suppose that [location 1, location 2, location 3] indicates that at the start location 1 is searched; if the family is not detected, then location 2 is searched and if the family is not detected in location 2, then location 3 is searched. These three locations will be referred as 1, 2 and 3, abbreviated as [1, 2, 3]. As mentioned, there are 6 possible policies to search as follows:

\[
\{(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)\} \in \chi
\]

For simplicity, we suppose that the probability that the family in the location \( i \) will be discovered on a single glance is one. Thus, \( \forall i, \alpha_i = 1 \), namely according to this scenario firstly location \( i \) is searched, if the family is not detected, this location will be omitted from more consideration. Now, Model (11) is used to obtain the solution. But Model (11) is nearly impossible to solve directly. Therefore, all six possible scenarios are separately placed in Model (11) and the scenario with the minimum for \( Z \) will be the best. Model (11) has been coded by Python 3.3 for each strategy. Python has a very lucid syntax and provides the majority of the functionality. Also this interpreter is free. A comparison between Python and MATLAB (another high-level language) prepared by Fangohr [70]. Noteworthy, our suggested model is carried out on a laptop with Intel Core i5, CPU 2.5 GHz and 4GB RAM. Now, we calculate the value of Model (11) for each policy as follows:
At first, by using proposition 3.1.1 we obtain \( \max_{i} \frac{\alpha_i P_i}{C_i} \) as well as \( \max_{i} \frac{\alpha_i P_i}{t_i} \) for \( i = 1, 2, 3 \), due to \( \forall i, \alpha_i = 1 \) therefore, we have:

\[
\begin{align*}
\frac{P_1}{C_1} &= \frac{0.38}{23} = 0.0165, & \frac{P_2}{C_2} &= \frac{0.28}{25.5} = 0.0109, & \frac{P_3}{C_3} &= \frac{0.34}{30} = 0.0113 \\
\frac{P_1}{t_1} &= \frac{0.38}{19} = 0.02, & \frac{P_2}{t_2} &= \frac{0.28}{17.5} = 0.016, & \frac{P_3}{t_3} &= \frac{0.34}{16} = 0.0212,
\end{align*}
\]

therefore, we consider all possible scenarios as follows:

Policy 1: Model (11) for scenario \([1,2,3]\):

\[
Z_{[1,2,3]} = \frac{C_1 + (1-\alpha_1 P_1)(C_2 + (1-\frac{\alpha_2 P_2}{1-\alpha_1 P_1})C_3) - 0.0165}{0.0165} + \frac{t_1 + (1-\alpha_1 P_1)(t_2 + (1-\frac{\alpha_2 P_2}{1-\alpha_1 P_1})t_3) - 0.021}{0.021}
\]

\[
= 4450.28 \Rightarrow Z_{[1,2,3]} = 4450.28
\]

Policy 2: Model (11) for scenario \([1,3,2]\):

\[
Z_{[1,3,2]} = \frac{C_1 + (1-\alpha_1 P_1)(C_3 + (1-\frac{\alpha_3 P_3}{1-\alpha_1 P_1})C_2) - 0.0165}{0.0165} + \frac{t_1 + (1-\alpha_1 P_1)(t_3 + (1-\frac{\alpha_3 P_3}{1-\alpha_1 P_1})t_2) - 0.021}{0.021}
\]

\[
= 4372.05 \Rightarrow Z_{[1,3,2]} = 4372.05
\]

Policy 3: Model (11) for scenario \([2,1,3]\):

\[
Z_{[2,1,3]} = \frac{C_2 + (1-\alpha_2 P_2)(C_1 + (1-\frac{\alpha_1 P_1}{1-\alpha_2 P_2})C_3) - 0.0165}{0.0165} + \frac{t_2 + (1-\alpha_2 P_2)(t_1 + (1-\frac{\alpha_1 P_1}{1-\alpha_2 P_2})t_3) - 0.021}{0.021}
\]

\[
= 4702.99 \Rightarrow Z_{[2,1,3]} = 4702.99
\]

Policy 4: Model (11) for scenario \([2,3,1]\):
\[ Z_{[2,3,1]} = \frac{C_2 + (1 - \alpha_2 P_2)[C_3 + (1 - \frac{\alpha_3 P_3}{1 - \alpha_2 P_2}) \times C_1] - 0.0165}{0.0165} + \frac{t_2 + (1 - \alpha_2 P_2)[t_3 + (1 - \frac{\alpha_3 P_3}{1 - \alpha_2 P_2}) \times t_1] - 0.021}{0.021} \]

\[ = 4903.68 \Rightarrow Z_{[2,3,1]} = 4903.68 \]

**Policy 5:** Model (11) for scenario [3,2,1]:

\[ Z_{[3,2,1]} = \frac{C_3 + (1 - \alpha_3 P_3)[C_2 + (1 - \frac{\alpha_2 P_2}{1 - \alpha_3 P_3}) \times C_1] - 0.0165}{0.0165} + \frac{t_3 + (1 - \alpha_3 P_3)[t_2 + (1 - \frac{\alpha_2 P_2}{1 - \alpha_3 P_3}) \times t_1] - 0.021}{0.021} \]

\[ = 4825.44 \Rightarrow Z_{[3,2,1]} = 4825.44 \]

**Policy 6:** Model (11) for scenario [3,1,2]:

\[ Z_{[3,1,2]} = \frac{C_3 + (1 - \alpha_3 P_3)[C_2 + (1 - \frac{\alpha_2 P_2}{1 - \alpha_3 P_3}) \times C_1] - t_3 + (1 - \alpha_3 P_3)[t_2 + (1 - \frac{\alpha_2 P_2}{1 - \alpha_3 P_3}) \times t_1] - 0.021}{0.021} \]

\[ = 4572.73 \Rightarrow Z_{[3,1,2]} = 4572.73 \]

The value of all strategies is graphically shown in Fig. 6. Consequently, according to the order [1, 3, 2], the locations should be checked to minimize the expected cost and expected time of the search.

Unfortunately, the above procedure is practically not useful. Obviously, this combination of SDP and MOP is not simple to carry out for medium or even small values of \( n \). On the other words, this problem is too hard to solve in the ordinary sense. Noteworthy, for optimization problem diverse origins of difficulty including the huge number of solutions and the intricacy of objective functions can be considered. Clearly, we faced with all these factors in the suggested model. After the sensitivity analysis, we will try to diminish this difficulty.

### 5.4. Sensitivity Analysis
To facilitate investigation of the performance of the suggested model, a sensitivity analysis is carried out. Sensitivity analysis is the study of how change of a parameter can influence the solution generated by the model when the others are constant. Now, we conduct the sensitivity analysis with variations in the cost and time of the model. On the other words, when time (or cost) is changed and the others are constant, varying the result of the model demonstrates its sensitivity relating to that parameter. According to Fig.7, increasing the value of cost is insensitive after near 43% of its range, namely 32.9. Please see Fig.7 for $C_1 = 25, 28, 30, 32, 32.9$, respectively for as example.

Also according to Fig.8, increasing the value of time is insensitive after near 74% of its range, namely 33.1. Please see Fig.8 for $t_1 = 20, 25, 30, 33, 33.1$, respectively. Clearly, the sensitivity analysis has been investigated just to this very small case of the search problem and the greater cases for larger quantities of $n$ require too much calculation time. As explained by many researchers, stochastic multi-objective structure for most problems is often hard to implement. Also it is essential to mention that several MOPs on the basis of DP are close to unfeasible to optimize for practically sized problem. Noteworthy, as pointed out by [10], DP approach has been restricted to the problems with 2 or 3 state variables. Although for small cases we try to employ exact method based on SDP, the computational efforts rise exponentially with the number of locations. On the other hand, in emergency situations an operational problem should be solved within a limited time. But exact algorithms such as SDP are time-consuming and very difficult to use in such problems. As stated before, optimization does not fundamentally emphasize just obtaining the global optimum to a problem. For example, Su et al. [71] mentioned that the optimum of the manifold emergency resources problem based on integer programming is very difficult and lengthy to obtain. Thus, they concentrated on another method to solve this problem to find a suboptimal solution however, in a rational time. Consequently, in the next section, we will suggest an approximate approach on the basis of MCDM for the suggested search problem.

6. ALTERNATIVE MODEL BASED ON MCDM FRAMEWORK FOR SEARCH PROBLEM

Although SDP prepares a potent tool for stochastic optimization, it is not easy to apply to large scale search problem. Also we have observed during a computational case that stochastic multi-objective frame for search problem usually needs huge
computational effort. In this part, as an alternative solution an MCDM framework is adapted for search problem. The model discussed in this part is the assessment of locations for optimal search. To obtain this, a ranking of locations needs to be found. On the other words, the goal of this problem is to order choices (locations) in the presence of various attributes. Therefore, an MCDM approach that has the aptitude to give a complete ordering of options is essential. Generally, MCDM methods refer to obtain the best choice from the possible options by considering diverse conflicting decision factors [72]. It should be noted that usually the complexities of MOP and SDP is obvious as well as software for solving these methods is relatively scarce. Fortunately, nearly all MCDM methods are relatively straightforward and software for solving them is available. Now, we reformulate the search problem as an MCDM framework, namely we reduce the MOP into a MCDM problem. Consequently, the computational effort is decreased considerably and a good solution is produced.

6.1. Multi-Criteria Decision Making (MCDM)

Let us begin with explanation of the typical MCDM model. An MCDM framework can be depicted as follows:

\[
D = \begin{bmatrix}
A_1 & A_2 & \ldots & A_m \\
x_{11} & x_{12} & \ldots & x_{1n} \\
x_{21} & x_{22} & \ldots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \ldots & x_{mn}
\end{bmatrix}
\]

\[
W = [w_1, w_2, \ldots, w_n]
\]

(12)

Where \( D \) is the decision matrix, \( A_1, A_2, \ldots, A_m \) are feasible options among which expert has to select, \( C_1, C_2, \ldots, C_n \) are criteria with which choices are measured, \( x_{ij} \) is the rating of \( A_i \) regarding to \( C_j \) and \( w_j \) is the weight of \( C_j \).

First of all, \( D \) should be normalized thereby it becomes dimensionless to allow for comparisons. There are various normalization techniques [73]. Also obtaining
the suitable weight for all attributes is one of the key steps in the majority of MCDM methods. These techniques can be divided into two main categories [74]: (1) Subjective methods obtain weights only considering the preference experts. (2) Objective methods obtain weights by solving mathematical approaches without reflection of the expert’s preferences.

Obviously, the expert’s opinion should be considered in the majority of real problems. Therefore subjective techniques can generally be more desirable. However, when providing a set of reliable subjective weights is not easy, the application of objective weights is beneficial. Among the objective techniques Shannon Entropy is one the most popular. In this work, the weight of each attribute is identified using Shannon Entropy. Also in MCDM approaches, the criteria will be categorized into two groups. Criteria that should be maximized are benefit factors and criteria that should be minimized are cost factors.

As mentioned by [75] more than 70 MCDM methods have been presented. Although the goal of all these techniques is to help making a good decision, as pointed out by [76], various MCDM approaches may result conflicting outcomes when exerted to the same problem. Vincke [77] mentioned that the choosing of an MCDM technique should be performed skillfully based on the nature of problem, measurement scales, dependency among criteria, the amount of alternatives, form of ambiguity, and expectation of the experts. Løken [78] pointed out that often we cannot derive that one method is better than the others for a general problem. On the other words, no one MCDM method is selected as the most appropriate for all problems. But a number of MCDM methods better suit to a given problem than others. Therefore, the comparisons of MCDM techniques have been presented in several studies. However, the majority of researchers avoid selecting single MCDM technique over another, because such assertion would require a firm theoretical base or evaluation on a numerous real cases [79].

As mentioned, up to now, several MCDM techniques have been introduced. But the selection of a proper MCDM technique is an intricate MCDM problem [75]. Transparency is one of the most essential factors that should be addressed for choosing an MCDM technique [80]. If an expert does not realize what is taking place within the MCDM procedure, the outcome may be that the expert does not trust in the suggestion from that approach [78]. Simplicity and computation time are two other important criteria for selecting an MCDM technique either. As pointed out by [80], it is recommended not to employ a very intricate MCDM approach with lack of transparency, because it makes very hard for a user to recognize any mistake made through the computation procedure.
Chatterjee et al. [80] compared COPRAS, TOPSIS, VIKOR and AHP in a material selection problem on the basis of simplicity, transparency, calculation time and so on. According to that study, COPRAS is straightforward to employ and very good in transparency. Also COPRAS has a low calculation time. On the other hand, although AHP is one of the most broadly used MCDM techniques, it is a very controversial method. For example, AHP method suffers from a numerous pairwise evaluation [78]. Also if the pairwise assessment is recognized to be inconsistent, expert should execute this task again. On the other words, AHP mathematically has an intricate procedure [80].

Antucheviciene et al. [81] evaluated the results of VIKOR, TOPSIS and COPRAS in building redevelopment problem and deduced that COPRAS and TOPSIS are superior to VIKOR. Noteworthy, VIKOR is one of the popular MCDM methods. But Huang et al. [82] warned that VIKOR may generate an incorrect ranking in some cases. Also according to [83], TOPSIS method can has higher distinguishing ability due to vector normalization. Peng [84] appraised TOPSIS, VIKOR, ELECTRE, PROMETHEE, GRA, and WSM in the earthquake vulnerability problem. According that work, TOPSIS was chosen as the most trustable technique. Sun and Li [75] assessed 24 MCDM techniques such as TOPSIS, ELECTRE, AHP, SAW, etc. in the aircraft selection problem. Based on this study, TOPSIS was chosen as the most proper technique. On the basis of an extensive literature review, Mousavi-Nasab and Sotoudeh-Anvari [85] revealed that TOPSIS and COPRAS are the best MCDM methods for general material selection problem. However, in some problems, there is not a solid reason to select a specific MCDM method. For example, Athawale and Chakraborty [86] compared 10 MCDM techniques such as VIKOR, AHP, TOPSIS, GRA, ELECTRE, PROMETHEE, WPM, etc. in robot selection problem. According to this comparison, all aforementioned techniques provided very similar orderings of the alternatives. Mulliner et al. [83] analyzed some MCDM methods such as revised AHP, TOPSIS, COPRAS, WSM, etc. in a sustainable housing affordability problem. Based on this study they deduced that none of these techniques are perfect. Therefore, Mulliner et al. [83] suggested that more than one technique should be employed to a problem to give a more powerful decision. However, when usage of several MCDM techniques is not feasible, they recommend the employment of COPRAS. Mela et al. [79] presented a comparison among VIKOR, WPM, TOPSIS, SAW, and PROMETHEE for a building design problem. But the authors inferred that the best MCDM techniques will hardly be acquired. Finally, reader can see [72] and [87] for two comparative studies of MCDM methods.
It is a well-known approach to employ more than one MCDM methods to handle a given prioritizing problems \[88\]. Thus, we select two MCDM approaches, namely COPRAS and TOPSIS to prioritize the search locations for several reasons. First of all, the ordering concurred by two MCDM techniques is more reliable than a result produced by one MCDM technique. Furthermore, COPRAS and TOPSIS allow for benefit as well as cost factors to be combined with one analysis without complexity. But for example, cost factors in AHP and WSM should be converted into benefit factor before normalization \[83\]. In addition, COPRAS and TOPSIS can provide a perfect ranking of options. But for example, ELECTRE and PROMETHEE cannot often give a full ordering of the choices. Also these two methods require large expert interaction in solving procedures. Hence, ELECTRE and PROMETHEE are unsuitable for our problem. In contrast, TOPSIS and COPRAS will be performed well in spite of numerous choices and criteria \([72\) and \([83\)\]. Let us point out that the only aim of this subsection is not to declare which MCDM technique is the best. Rather, our key aim is to highlight that the use of two (or more) MCDM techniques can generate more reliable results.

In brief, some main reasons why we choose TOPSIS and COPRAS for this problem are as follows:

1- The notion behind of TOPSIS and COPRAS is logical and easy to realization and use. In contrast, Løken [78] mentioned that for many decision makers, ELECTRE is complicated to understand as well as utilization. Velasquez and Hester [89] pointed out that one of the important shortcomings of ELECTRE is that its procedure and results can be difficult to elucidate for a layman. Also AHP and PROMETHEE are computationally intricate and need user’s intervention. In these methods, subjective judgments have large influence on the results.

2- Calculation procedures of TOPSIS and COPRAS can be simply programmed.

3- TOPSIS and COPRAS can be employed efficiently when the amount of alternatives or selection factors is large. In contrast, AHP, PROMETHEE and ELECTRE are time-consuming in this situation and their performance reduces rapidly when the quantity of options or attributes is large \([72\) and \([85\)\]).

4- TOPSIS and COPRAS are easily applied for positive and negative decision factors with one procedure. In contrast, in AHP and SAW, negative criteria must be transformed into positive criteria before normalization. However, Millet and Schoner [90] revealed that this transformation may produce a computational difficulty and extract incompatible outcomes in AHP.

5- TOPSIS and COPRAS offer a full ranking of options. In contrast, ELECTRE and PROMETHEE are sometimes unable to determine the best choice. For example,
Özcan et al. [72] pointed out that since ELECTRE does not give a complete ranking in some situations, it may suggest plural solution as the best choice. A plural solution is two or more options that find the same ranking.

6- The TOPSIS and COPRAS results are not influenced by any additional parameter. In contrast, for example, VIKOR result relies on parameter $\nu$ severely [79]. Also in PROMETHEE, an unsuitable parameters tuning can generate incoherent outcomes.

It is important to mention that literature review shows that SAW and AHP can be considered as the most frequently used MCDM techniques [89]. However, some points about these two MCDM methods should be pointed out. Since AHP was introduced there has been a widespread argument about the theoretical truth of this technique. These arguments have focused on four sectors: the axiomatic foundation, the true meanings of criteria weights, the measurement scale, and the rank reversal phenomenon [91]. Also although SAW is the simplest and probably most generally used MCDM technique, Velasquez and Hester [89] warned that in some cases, outcome provided by SAW may not be reasonable. Besides, Mulliner et al. [83] pointed out WPM is a straightforward method, although the disadvantage of WPM is that benefit-type and cost-type criteria should not be utilized simultaneously.

### 6.2. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS, one of the most famous MCDM techniques, was at first suggested by Hwang and Yoon [92]. The preferred choice is closest to the ideal solution and farthest from the anti-ideal solution. TOPSIS is a compensatory approach and allows tradeoffs among decision attributes. Noteworthy, TOPSIS has been effectively employed in tremendous real applications ([84]; [85]; [93]; [94]).

The steps in TOPSIS are explained as below:

**Step 1:** Normalize the decision matrix ($D$) using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}}} \quad (i = 1,2,\ldots,m; j = 1,2,\ldots,n)$$

**Step 2:** Provide weight to this matrix by:

$$v_{ij} = w_j \times r_{ij} \quad i = 1,2,\ldots,m; j = 1,2,\ldots,n.$$
Where $w_j$ is the weight of attribute $j$ and $\sum_{j=1}^{n} w_j = 1$.

**Step 3:** Specify the ideal solution and anti-ideal solution using Model (13).

$$A^+ = \{v^+_1, v^+_2, \ldots, v^+_n\} = \{(\max_{j} v_{ij} | i \in I'), (\min_{j} v_{ij} | i \in I'')\}$$

$$i = 1,2, \ldots, m; j = 1,2, \ldots, n.$$  

$$A^- = \{v^-_1, v^-_2, \ldots, v^-_n\} = \{(\min_{j} v_{ij} | i \in I'), (\max_{j} v_{ij} | i \in I'')\}$$

$$i = 1,2, \ldots, m; j = 1,2, \ldots, n.$$  

(15)

Where $I'$ is related to benefit factors, and $I''$ is related to cost factors.

**Step 4:** Obtain the distance of each option from $A^+$ and $A^-$ using the following equations (the n-dimensional Euclidean distance):

$$D^+_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^+_j)^2} \quad i = 1,2, \ldots, m$$  

(16)

$$D^-_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^-_j)^2} \quad i = 1,2, \ldots, m$$

**Step 5:** The comparative closeness to the ideal solution can be obtained by:

$$CC^*_i = \frac{D^-_i}{D^-_i + D^+_i} \quad i = 1,2, \ldots, m$$  

(17)

**Step 6:** Rank the choices according to $CC^*_i$. The bigger $CC^*_i$, the better choice $A_i$.

### 6.3. Complex Proportional Assessment (COPRAS)

COPRAS method, which was introduced by Zavadskas et al. [95], considers straight (direct) and relative (proportional) dependences of the priority (significance) and utility degree of the options considering the mutually incompatible attributes. This technique chooses the best option considering the ideal solution and the ideal-worst solution [80]. Similar to TOSIS, COPRAS is a compensatory technique. Also owing to
its distinguishing features, COPRAS has been effectively employed in the various fields ([80]; [81]; [83]; [85]).
The procedure of COPRAS is explained as below [80]:

*Step 1:* Similar to most MCDM methods, the first stage is normalization of $D$ by:

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$$

(18)

*Step 2:* Provide weight to this matrix by:

$$y_{ij} = w_j \times r_{ij} \quad (i = 1, 2, ..., m; j = 1, 2, ..., n)$$

In the above-mentioned equation, $r_{ij}$ is the normalized value of choice $i$ on attribute $j$ and $w_j$ is the weight of factor $j$. Using this alteration, the total of the dimensionless weighted values of every attribute equals the weight of that attribute:

$$\sum_{i=1}^{m} y_{ij} = w_j$$

(19)

*Step 3:* Obtain sums of weighted normalized values (WNV) by:

$$S_{+i} = \sum_{j=1}^{n} y_{+ij}$$

$$S_{-i} = \sum_{j=1}^{n} y_{-ij}$$

(20)

Where $y_{+ij}$ and $y_{-ij}$ are the WNV for the positive (beneficial) criteria as well as negative (non-beneficial) criteria, correspondingly. The greater $S_{+i}$, the superior is the option and the lower $S_{-i}$, the better that choice. Note that $S_{+i}$ and $S_{-i}$ indicate the level of goals reached by every choice.
**Step 4:** Obtain the priority of the choices based on defining the positive options $S_{+i}$ and negative options $S_{-i}$ traits. The comparative significance $Q_i$ of each $A_i$ is calculated by:

$$Q_i = S_{+i} + \frac{S_{-i} \sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{S_{-i}}{S_{-i}}}$$  \quad (i = 1, 2,..., m)

In the above-mentioned equation, $S_{-\text{min}}$ indicates the minimum of $S_{-i}$. The greater $Q_i$, the higher the significance of the option. Thus, the choice with the highest relative priority ($Q_{\text{max}}$) is the best option.

**Step 5:** Determine the level of utility ($U_i$) for choice $i$.

The $U_i$ (absolute prioritizing) is calculated by:

$$U_i = \frac{Q_i}{Q_{\text{max}}} \times 100$$

To specify the adaptability of these two MCDM techniques, namely TOPSIS and COPRAS, to deal with this problem their comparative accomplishment are compared by the Spearman’s rank correlation test.

### 6.4. Spearman’s Rank Correlation Coefficient (SRCC)

SRCC is employed to obtain the measure of relationship (agreement) between results (ranks) generated by various MCDM techniques. If $R_i$ and $R_j$ indicate the results obtained by two various MCDM techniques for alternative $i$ and $m$ is the number of options, then SRCC will be calculated as follows:

$$r_s = 1 - 6 \sum_{i=1}^{m} \frac{(R_i - R_j)^2}{m(m^2 - 1)}$$

(23)
The bigger the $r_s$, the better the association is between the two techniques. Please note that $r_s = 1$ shows complete agreement, $r_s = -1$ shows complete disagreement and $r_s = 0$ indicates no relationship between the results. Reader can refer to [81] and references therein to study further about SRCC.

### 6.4. MCDM-based Method for Search Problem

In this part, we introduce a multiple criteria evaluation framework for search problem.

#### 6.4.1. Problem description and solution

Suppose that a motionless target is located or hidden in location $i$ and this goal does not react to the searcher’s activity. Please note that the number of locations is known. The probability that the goal is in the location $i$ is $P_i$, and it is clear that $\sum_{i=1}^{n} P_i = 1$. A search cost and search time of location $i$ are $C_i$ and $t_i$, respectively. Again, in this problem a wounded man trapped under debris is the “target” and the positions where a missing person has been probably there before a disaster are as “locations”. Now, the aim of this problem is the prioritization of the search of locations. In the other words, the model prioritizes given locations based on three weighted attributes, i.e., search cost, search time and probability of finding the goal. It should be noted that various criteria are significant while studying the search problem as a MCDM problem. From the [14], [17], [26], [32], and [43] the criteria for the prioritization of locations can be identified. More formally, we select four of them which correspond to previous two objectives as follows:

1. Search cost
2. Search time
3. Probability that the goal is in the location $i$
4. Overlook probability

Let us reconsider the case that was presented in sub-section 5.2. We consider three attributes, i.e., search cost, search time and probability of detecting the goal. In this case, without loss of generality, we suppose the probability of overlook is zero,
i.e. \( \forall i, \alpha_i = 1 \). Among these three attributes, probability of finding the goal is beneficial criterion. On the other hand, search cost and search time are non-beneficial criteria. Three locations were considered as the candidate choices. From these three choices, the highest priority should be chosen. Table 5 demonstrates the ratings of the locations regarding to selected attributes on the basis of Tables 2, 3, and 4.

Now, the weight of attributes should be provided. In this work, the criteria weights are obtained by using Shannon Entropy. Shannon Entropy value of attributes is calculated as follows:

We have \( E_j = -\frac{1}{\ln m} \sum_{j=1}^{n} p_{ij} \ln p_{ij} \) where \( p_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}} \) and the weights will be calculated as \( w_j = \frac{1-E_j}{\sum_{j=1}^{n} 1-E_j} \). Thus, to prioritize the locations by TOPSIS and COPRAS, weight of attributes are specified using Shannon Entropy as shown in Table 6.

Finally, we run COPRAS as well as TOPSIS and obtain the ranking of alternatives separately. According to Table 7, COPRAS gives the ranking of locations as 1-3-2. Also TOPSIS gives the ranking of locations as 1-3-2. Hence, according to the order [1,3,2], the locations should be checked.

Fig. 9 demonstrates graphically the ranking of locations. Clearly, the value of SRCC between TOPSIS and COPRAS is 1. Therefore, an important observation related to the use of various MCDM techniques is that the outcome of TOPSIS is in excellent agreement with the result of COPRAS in this problem. Noteworthy, it is consistent with Mousavi-Nasab and Sotoudeh-Anvari’s [85] study.

Finally, despite the same results, in a comparison between stochastic multi-objective model and MCDM-based model for suggested search problem the calculation effort and computation time is intuitively smaller for MCDM-based model.

7. CONCLUSION

Although SAR is one of the most vital operations in the earthquake DM, various models for SAR are not as accentuated as models for transportation or location in this field. In this paper, we suggested a stochastic multi-objective model to optimize the search operation in the response stage of DM based on COSM. Due to sequential nature of the optimal search problem and stochastic organization of this model we
employed SDP technique to tackle this problem similar to COSM. This proposed framework minimized expected cost and expected time of search procedure successfully. However, the outcome obtained in the suggested model illustrated that computing time in real cases will be large. On the other words, the stochastic multi-objective nature of the problem makes it very intricate to find the optimal solution even for small-sized cases using SDP. Therefore, we reformulated the search problem as an MCDM problem. This study has shown that not only MCDM methods are workable for search problem but also the use of MCDM methods seems inevitable for coping with large cases. Owing to outstanding features, TOPSIS and COPRAS were employed to rank the locations. The observation indicated an excellent agreement between TOPSIS and COPRAS in search problem.

Let us now note a number of directions of future research. The development of the proposed stochastic multi-objective framework for search problem in the presence of various constraints is remarkable. Also heuristic methods can be proposed to handle large problems. Furthermore, it can be very interesting to introduce a MCDM framework when MCDM techniques produce conflicting results.

REFERENCES


Biographies

**Alireza Sotoudeh-Anvari** is PhD student in the Department of Industrial Engineering at Islamic Azad University, Science and Research Branch, Tehran. He holds an MSc degree in Industrial Management Institute of Iran. His research interests are in the areas of multicriteria decision making and fuzzy logic. He has published several papers in international journals including *Materials and Design, Journal of Intelligent & Fuzzy Systems, Journal of Cleaner Production*, etc.

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**Soheil Sadi-Nezhad** received his PhD in Industrial Engineering at the IAU, Sciences & Research Branch in 1999. He has been working as a Post-Doctoral Fellow in Waterloo University. He used to be a faculty member of Sciences & Research Branch, Islamic Azad University, and Industrial Management Institute of Iran. Most of his recent studies have focused on decision-making and multi-objects optimization under uncertainty, imprecision, and partial truth, especially as it involves human perceptions or risk.
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Adapted from United Nations Office for Disaster Risk Reduction
(http://www.unisdr.org/we/inform/publications/47804)
Fig. 2 The disaster management cycle and some classic activities
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Instituting expert panel from RDC for obtaining an optimal search policy for the persons who are lost

Identifying potential locations, cost search, time search, and the probability of finding the person

Optimization (Ranking) approach

MCDM model
- Ranking locations by TOPSIS and COPRAS
- Analysis of the results using SRCC

Stochastic multi-objective model
- Minimizing expected search cost and expected search time
- The optimum is an order of locations.

Discussion on results

Fig. 4 The proposed decision framework for search problem
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### Tables

**Table 1. Notations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>Number of locations</td>
</tr>
<tr>
<td>( \mathbb{N} )</td>
<td>The natural number set</td>
</tr>
<tr>
<td>( i )</td>
<td>Index of locations ( i = 1, 2, \ldots, n )</td>
</tr>
<tr>
<td>( C_i )</td>
<td>Search cost of locations ( i )</td>
</tr>
<tr>
<td>( l_i )</td>
<td>Search time of locations ( i )</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Probability that a goal in the location ( i ) will be discovered on a single glance</td>
</tr>
<tr>
<td>( P )</td>
<td>Posterior probability vector that the goal is in location ( i )</td>
</tr>
<tr>
<td>( V(P) )</td>
<td>Expected cost function</td>
</tr>
<tr>
<td>( W(P) )</td>
<td>Expected time function</td>
</tr>
<tr>
<td>( T_i(P) )</td>
<td>Posterior probability vector considering prior probability vector and failure to discover the target in location ( i )</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Solution space</td>
</tr>
<tr>
<td>( k )</td>
<td>The number of possible scenario</td>
</tr>
<tr>
<td>( Z )</td>
<td>The value of ( L_p )-metrics function</td>
</tr>
<tr>
<td>( w )</td>
<td>( L_p )-metrics coefficient</td>
</tr>
</tbody>
</table>

**Table 2. The probability of finding the goal**

<table>
<thead>
<tr>
<th>Locations</th>
<th>Probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location 1 (cottage)</td>
<td>38</td>
</tr>
<tr>
<td>Location 2 (forest)</td>
<td>28</td>
</tr>
<tr>
<td>Location 3 (mountain)</td>
<td>34</td>
</tr>
</tbody>
</table>
### Table 3. The search cost of each location

<table>
<thead>
<tr>
<th>Locations</th>
<th>Search cost (thousand USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location 1 (cottage)</td>
<td>23</td>
</tr>
<tr>
<td>Location 2 (forest)</td>
<td>25.5</td>
</tr>
<tr>
<td>Location 3 (mountain)</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 4. The search time of each location

<table>
<thead>
<tr>
<th>Locations</th>
<th>Search time (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location 1 (cottage)</td>
<td>19</td>
</tr>
<tr>
<td>Location 2 (forest)</td>
<td>17.5</td>
</tr>
<tr>
<td>Location 3 (mountain)</td>
<td>16</td>
</tr>
</tbody>
</table>

### Table 5. The decision matrix

<table>
<thead>
<tr>
<th>Locations</th>
<th>Search cost (thousand USD)</th>
<th>Search time (hour)</th>
<th>Probability of finding the goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location 1</td>
<td>23</td>
<td>19 h</td>
<td>38%</td>
</tr>
<tr>
<td>Location 2</td>
<td>25.5</td>
<td>17.5 h</td>
<td>28%</td>
</tr>
<tr>
<td>Location 3</td>
<td>30</td>
<td>16 h</td>
<td>34%</td>
</tr>
</tbody>
</table>

### Table 6. Weights of criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Search cost</th>
<th>Search time</th>
<th>Probability that the goal is in the location</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.373</td>
<td>0.152</td>
<td>0.473</td>
<td></td>
</tr>
<tr>
<td>Location</td>
<td>Rank</td>
<td>Similarity ratio</td>
<td>Location</td>
<td>Rank</td>
</tr>
<tr>
<td>-------------------</td>
<td>------</td>
<td>------------------</td>
<td>-------------------</td>
<td>------</td>
</tr>
<tr>
<td>Location 1 (cottage)</td>
<td>1</td>
<td>0.868</td>
<td>Location 1 (cottage)</td>
<td>1</td>
</tr>
<tr>
<td>Location 2 (forest)</td>
<td>3</td>
<td>0.308</td>
<td>Location 2 (forest)</td>
<td>3</td>
</tr>
<tr>
<td>Location 3 (mountain)</td>
<td>2</td>
<td>0.436</td>
<td>Location 3 (mountain)</td>
<td>2</td>
</tr>
</tbody>
</table>