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Adapted design of experiments for dimension decomposition-based meta-model in structural reliability analysis

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Abstract. Reliability analysis of structures is often problematic for the structures with nonlinear and complex Limit State Functions (LSFs). For these cases, simulation methods often provide accurate failure probability, but with a high number of LSFs in the analysis of the structure. This paper presents an efficient combined meta-model of Monte Carlo Simulation (MCS) and Univariate Dimension Reduction (UDR) to approximate the failure probability of structures with evaluation of few LSFs. For this purpose, the design of the experiment applied in the meta-model was adapted such that the expected failure samples in MCS were approximated with higher accuracy. Several numerical and engineering reliability problems were solved by the proposed approach and the results were verified by MCS. Results showed that the proposed approach highly reduced the required number of structural analyses to provide proper results.

1. Introduction

In structural reliability analysis, the failure probability P_f is defined as [1]:

$$P_f = \int_{G(X)} f_x(X)\, dX, \tag{1}$$

where $f_X(X)$ is the joint probability density function of the vector of basic random variables $X = [x_1, x_2, ..., x_n]^T$, which represents uncertain quantities such as material properties, loads, boundary conditions, and geometry. In Eq. (1), $G(X)$ is the Limit State Function (LSF) in which $G(X) > 0$ represents the safety domain and $G(X) < 0$ represents the failure domain. However, the failure probability of a given problem by means of Eq. (1) is not a straight approach, because the joint probability density function $F_X(X)$ is not always available. In some cases, Eq. (1) cannot be integrated analytically, even if $F_X(X)$ is available, especially for the complex structures with low failure probabilities and implicit LSFs. Therefore, in order to avoid such calculation, various techniques have been proposed, e.g., a) approximation methods (i.e., First Order Reliability Method (FORM) and Second Order Reliability Method (SORM)) and b) simulation methods [2-5]. FORM and SORM are accurate for reliability problems with linear and moderate LSFs, but inaccurate for highly nonlinear LSFs and difficult to solve when the actual implicit LSF cannot be expressed explicitly. Besides, in some cases, FORM and SORM may suffer convergence problems [6]. Hence,

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when an accurate reliability evaluation is required, simulation methods are often employed.

1.1. Monte Carlo simulation

Monte Carlo Simulation (MCS) is considered as the most efficient and accurate simulation method and is commonly used for the evaluation of the probability of failure for structures, either for comparison with other methods or as a standalone reliability analysis tool [7,8]. This method involves sampling the design space based on the mean, variance, and PDF values of random variables. From a mathematical point of view, MCS allows to estimate the expected value of a quantity of interest more specifically. Suppose the goal is to evaluate $\mathbb{E}_f[h(X)]$; an expectation of the function $h : x \to \mathbb{R}$ with respect to the Probability Density Function (PDF) [9] is:

$$\int_X h(X) f_x(X) dx = \mathbb{E}_f[h(X)]. \tag{2}$$

The idea behind MCS is straight forward application of the law of large numbers, which states if $X = [x_1, x_2, ..., x_n]$ is independent from and distributed identically to the PDF $f_x(X)$, then the empirical average $\frac{1}{N}\sum_{i=1}^{N} h(x_i)$ converges to the true value of $\mathbb{E}_f[h(X)]$ when N approaches $+\infty$. Therefore, if the number of samples N is large enough, then $\mathbb{E}_f[h(X)]$ and it can be accurately estimated by the corresponding empirical average:

$$\mathbb{E}_f[h(X)] \approx \frac{1}{N}\sum_{i=1}^{N} h(x_i). \tag{3}$$

The relevance of DMC to the reliability problem (1) follows a simple observation that the failure probability P_f can be written as:

$$P_f = \int_{G(X)} f_x(X)\, dX = \int_X I_f(X) f_x(X) dX$$

$$= E_f[I_f(X)] = \frac{1}{N}\sum_{i=1}^{N} h(x_i), \tag{4}$$

where $X = [x_1, x_2, ..., x_n]$ is independent from and distributed identically to the PDF $f_x(X)$, and I_f is a counting vector with values of zero and unity for samples in the failure and safe regions, respectively. As it is seen, a vast number of simulations have to be performed in order to achieve great accuracy, especially for low values of failure probability. In the efforts to reduce the excessive computation cost of MCS using purely random sampling methodologies, which are considered as the drawback of the method, various variance reduction techniques have been proposed, e.g., importance sampling [10-12], direct sampling [13], line sampling [14,15], Weighted Average Simulation Method (WASM) [16], subset simulation [17], polynomial chaos [18], and stochastic perturbation technique [19]. Unfortunately, most of these techniques are not as generally applicable as MCS. For example, importance sampling requires detailed information about the failure regions for being useful, and it faces difficulties when applied to high-dimension problems [16,20].

1.2. First order reliability method

First Order Reliability Method (FORM) is widely used to approximate the failure probability of structures and has become a basic reliability analysis approach for reliability-based design codes [21,22]. In FORM, structural failure probability is estimated based on the reliability index (β) by linearizing limit state function on the failure surface, i.e., $P_f \approx \Phi(-\beta)$, which corresponds to minimum distance of the origin from the limit state function in the standard normal area [21,23]. Generally, the main goal of FORM is the search for the most probable point (MPP), i.e., $U^*(b = ||U^*||)$ [24,25]. Hasofer Lind proposed a general iterative method for computing reliability index [23], which was extended by Rackwitz and Flessler to include distribution information of random variables [26], called the HL-RF method. This method involves the following steps to estimate the probability of failure [26] based on the HL-RF method:

Step 1. Transform random variables in X-space into U-space by the following relation:

$$u = \frac{x - \mu_x^e}{\sigma_x^e}, \tag{5}$$

where u is the standard normal variable with the mean and standard deviations equal to zero and one, respectively, and μ_x^e and σ_x^e are equivalent mean and standard deviations of the random variable x, respectively; for normal random variable, $\mu_x^e = \mu_x$ and $\sigma_x^e = \sigma_x$. The equivalent mean and standard deviations of non-normal random variables can be determined by the following equations [26-28]:

$$\sigma_x^e = \frac{1}{f_x(x)}\phi\left[\Phi^{-1}\{F_x(x)\}\right], \tag{6}$$

$$\mu_x^e = x - \sigma_x^e \Phi^{-1}[F_x(x)], \tag{7}$$

where $F_x(x)$ is cumulative distribution, $f_x(x)$ is probability distribution, Φ^{-1} is inverse standard normal cumulative distribution, and ϕ is standard normal probability distribution function.

Step 2. Find the reliability index.

The reliability index search is done based on an iterative process that can be reformulated based on design point (U) as:

$$U_{k+1} = \beta_{k+1}\alpha_{k+1}, \tag{8}$$

where:

$$\alpha_{k+1} = -\frac{\nabla^T g(U_k)}{\|\nabla^T g(U_k)\|}, \tag{9}$$

$$\beta_{k+1} = \frac{g(U_k) - \nabla^T g(U_k)U_k}{\|\nabla^T g(U_k)\|}, \tag{10}$$

where $\nabla g(U) = [\partial g/\partial u_1, \partial g/\partial u_2, ..., \partial g/\partial u_n]^T$ is gradient vector of the limit state function at the design point U_k.

Step 3. Calculate the failure probability.

The probability of failure based on FORM can be estimated as $P_f \approx \Phi(-\beta)$ [21,29].

2. Meta-models in structural reliability

When performance evaluation of a structure is computationally expensive, the number of simulation-based function evaluations required for reliability analysis must be carefully controlled. To that end, researchers have explored the use of meta-models, which are simpler approximate models calibrated to sample runs of the original simulation. The approximate model or meta-model can replace the original one, thus reducing the computational burden of evaluating numerous problem [30-35]. Bucher and Bourgund [36] proposed a quadratic polynomial response surface without cross terms. In their model, the response surface represented the LSF along the coordinate axes of the space of standard normal random variables. Nguyen et al. [37] proposed an adaptive RSM based on a double weighted regression technique. For the first iteration, a linear response surface was chosen; for the following iterations, a quadratic response surface with cross terms was considered based on the complementary points. Kang et al. [38] proposed an efficient RSM applying a moving least squares approximation instead of the traditional least squares approximation. Allaix and Carbone [39] discussed the locations of the experimental points used for evaluating parameters of the response surface. Recently, Dimension Reduction Method (DRM) as an efficient approach has been used to reduce the computational costs of the analysis [40-43].

In order to use the capabilities of MCS and simultaneously reduce the computational efforts, this paper presents a framework that efficiently employs the DRM to evaluate the reliability of structures. The proposed framework is presented after a brief review of DRM.

3. Dimension reduction method

DRM is a newly developed technique to calculate statistical moments of the output performance function [40-43]. There are several DRMs depending on the level of dimension reduction: (1) Univariate Dimension Reduction Method (UDRM), which is an additive decomposition of N-dimensional performance function into one-dimensional functions; (2) Bivariate Dimension Reduction (BDR), which is an additive decomposition of N-dimensional performance function into at most two-dimensional functions; (3) Multivariate Dimension Reduction (MDR), which is an additive decomposition of N-dimensional performance function into at most S-dimensional functions, where $S \leq N$.

According to UDRM, any N-dimensional performance function $h(X)$ can be additively decomposed into one-dimensional functions as [44]:

$$h(X) \cong \hat{h}(X) \equiv \sum_{i=1}^{N} h(\mu_1, \ldots, \mu_{i-1}, x_i, \mu_{i+1}, \ldots, \mu_N) - (N-1)\, h(\mu_1, \ldots, \mu_N), \tag{11}$$

where μ_i is the mean value of a random variable X_i and N is the number of design variables.

4. Proposed framework

This study employs the efficiency of the UDR-based meta-modeling in conjunction with the accuracy of MCS to provide a suitable framework for structural reliability analysis. The idea is to concentrate the experiments of the UDR-based meta-model on the region with high failure probability to correctly approximate the performance function value for the samples that are expected to be in the failure set. The following steps could be conducted to provide the desirable results.

4.1. Axial Design Of Experiments (DOE) based on the desired reliability index

UDR-based meta-model requires axial DOE to approximate the LSF. Determination of the location of experience samples in the proposed approach is based on the perception created by conducting the MCS sampling and excluding the safe area part. In this approach, as shown in Figure 1, which is presented in standard normal space (U), the space is divided into two separate regions D_1 and D_2 and it is assumed that D_1 is selected such that no failure occurs in this region. Here, D_1 is chosen as the region inside a sphere with radius β [45].

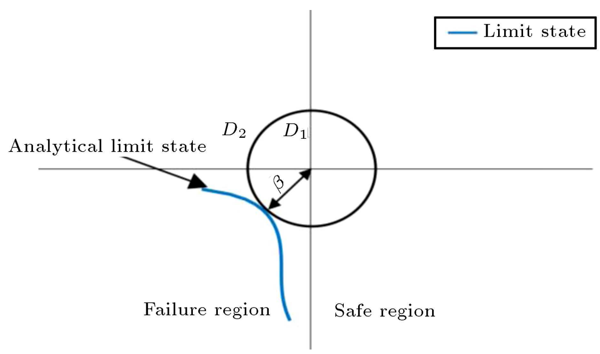

Figure 1. Excluding the safe area part.

To reduce the computational cost, the location of DOEs could be considered such that the corpus of experience samples in each axis is condensed within the boundaries of D_1 and D_2 regions and regions with the highest possibilities of failure. These samples should be used with the aim of interpolating DRM for the separated term in each dimension. By employing an anticipated reliability index and mapping the proposed experience samples in the physical space (original design space), the location of experiences for each variable is as follows:

$$X_{DOE} = \mu \pm \frac{\sqrt{2}}{2}.\beta_{\text{target}}.\zeta.\sigma, \tag{12}$$

where μ and σ are the mean value and standard deviation of random variable, respectively, and ζ is the location coefficient.

4.2. Generation of random sample based on crude MCS

After conducting the proposed step in the design of the experiment, in the next step, crude MCS should be performed to generate random samples for approximating the failure probability. Figure 2 schematically shows the proposed DOEs and the generated samples for a two-dimensional problem. According to the proposed approach, the approximated performance function corresponding to each sample is achievable by employing the UDR-based meta-model.

4.3. LSF approximation by the UDRM-based meta-model and reliability evaluation

Eq. (11) is employed at this step to approximate the LSF. The following is the resulting function for a limit state function with two random variables X_1 and X_2:

$$h(X) \cong \hat{h}(X) \equiv h(x_1, \mu_2) + h(\mu_1, x_2) - h(\mu_1, \mu_2), \tag{13}$$

in which for the samples produced by MCS, through employing the experiments and a proper interpolation

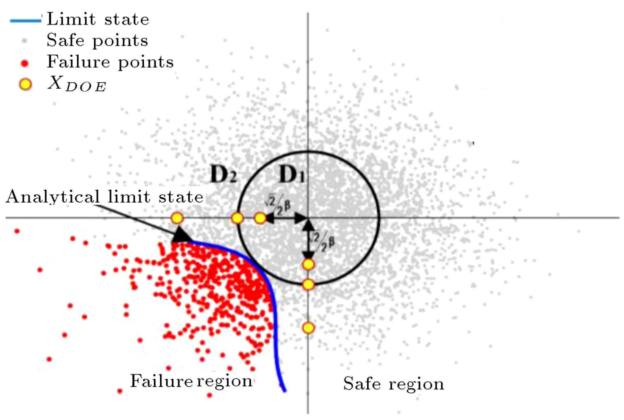

Figure 2. DOE and the generated samples based on MCS.

Table 1. The employed interpolation techniques.

Sign	Interpolation method
#1	Spline
#2	PCHIP
#3	Kriging
#4	Linear
#5	Cubic
#6	V5cubic

technique, the values of $h(x_1, \mu_2)$ and $h(\mu_1, x_2)$ for each dimension are achievable thorough interpolation. Then, the value of performance function for each sample could be approximated using Eq. (13). Then, the failure probability could be approximate by Eq. (4).

In this study, the effectiveness of various interpolation techniques is also investigated to suggest a proper technique for use in the proposed framework. It consists in several one-dimensional interpolations implemented by MATLAB toolbox, which are presented in Table 1. The kriging method (method #3) as a newly developed approximation method is also used in the approach and compared with common interpolation techniques.

5. Kriging method

Kriging meta-model is an interpolation technique based on statistical theory, which consists in a parametric linear regression model and a non-parametric stochastic process. It needs a design of experiments to define the stochastic parameters and then, predictions of the response can be completed on any unknown point. An initial DOE $X = [x_1, x_2, ..., x_{N_0}]$ is given with $x_i \in R^n (i = 1, 2, ..., N_0)$ as the ith experiment and $G = [G(x_1), G(x_2), ..., G(x_{N_0}),]$ with $G(x_i) \in R$ as the corresponding response to X [46]. The approximate relationships between any experiment X and the response $G(x)$ can be denoted as:

$$\widehat{G}(x) = F(\beta, x) + z(x) = f^T(x)\beta + z(x), \tag{14}$$

where $\beta^T = [\beta_1, ..., \beta_p]$ is a regression coefficient vector. Built by response surface method similar to the polynomial, $f^T(x) = [f_1(x), f_2(x), ..., f_p(x)]^T$ makes a global simulation in the design space. In the ordinary kriging, $F(\beta, x)$ is a scalar always taken as $F(\beta, x) = \beta$. Hence, the estimated $\widehat{G}(x)$ can be simplified as:

$$\widehat{G}(x) = F(\beta, x) + z(x) = \beta + z(x). \tag{15}$$

Here, $z(x)$ is a stationary Gaussian process [46]. The statistic characteristics can be denoted as:

$$E(z(x)) = 0, \tag{16}$$

$$Var(z(x)) = \sigma_z^2, \tag{17}$$

$$Cov\left[Z(x_i), Z(x_j)\right] = \sigma_z^2 R(x_i, x_j), \tag{18}$$

where σ_z^2 is the process variance; x_i, and x_j are discretional points from the whole samples X; and $R(x_i, x_j)$ is the correlation function about x_i and x_j with a correlation parameter vector θ [46].

6. Numerical and engineering examples

Five numerical and engineering problems are investigated in this section. For each example, the results obtained by using the proposed approach are compared with those by FORM and the accurate solution provided by using the MCS.

6.1. Example 1

This example is presented to investigate the effect of the different interpolation and prediction methods on the function approximation in the proposed approach. The performance function is presented as $f(x_1, x_2) = x_1^3 + x_2^3 - 18$ and the distribution of random variables are $x_1 = N(10, 5)$, and $x_2 = N(9.9, 5)$ [47].

The example is solved by three approaches and the results are presented in Table 2. The six employed interpolation methods are also shown in this table based on their ranks to provide an accurate solution. According to Table 2 and as shown in Figure 3, among various interpolation methods used in the proposed approach, method #1 presents a suitable approximation in such a manner that by 13 times function evaluation, the provided results are in good agreement with MCS with 10^4 function evaluations. Result shows that the FORM requires few LSF evaluations to provide solution, but the obtained solution is highly different from the accurate result provided by the MCS and the proposed approach.

6.2. Example 2

A nonlinear limit state with two independent standard normal variables is considered [48].

$$g(X) = -0.16(X_1 - 1)^3 - X_2 + 4 - 0.04\cos(X_1.X_2). \tag{19}$$

In this example, the accuracy of the method for a nonlinear limit state function is investigated. The results are presented in Table 3. According to Table 3 and as shown in Figure 4, the proposed method provides acceptable results when four techniques are used to interpolate the results in Step 2. Results presented in the table show that the proposed method has provided accurate solution to the problem, although the number of its function evaluations is even less than that required by FORM. It should be noted that due to the nonlinearity of the LSF, the reliability index determined by FORM is higher than the correct reliability index.

6.3. Example 3

An implicit reliability problem with highly nonlinear performance function is investigated in this example. Figure 5 shows the problem of a four-story building excited by a single-period sinusoidal impulse of ground acceleration. The building contains isolated equipment on the second floor. The motion of the lowest floor is resisted by a nonlinear hysteresis force in base isolation bearings of the building and an additional stiffness force, if its displacement exceeds d_c. Each floor has a mass of m_f and between floors, the stiffness and damping coefficients are k_f and c_f, respectively. The statistical parameters of the basic random variables are listed in Table 4. All variables are assumed to be lognormal and independent. The limit state function is defined by [48]:

Table 2. Reliability results for Example 1.

Method		P_f	Reliability index	No. of function evaluations
FORM		0.0832	1.384	13
MCS		0.0057	2.530	10^4
UDRM (Interpolation method)	#1	0.0057	2.530	13
	#5	0.0043	2.628	13
	#2	0.0042	2.636	13
	#3	0.0026	2.794	13
	#6	0.0024	2.820	13
	#4	0.0020	2.878	13

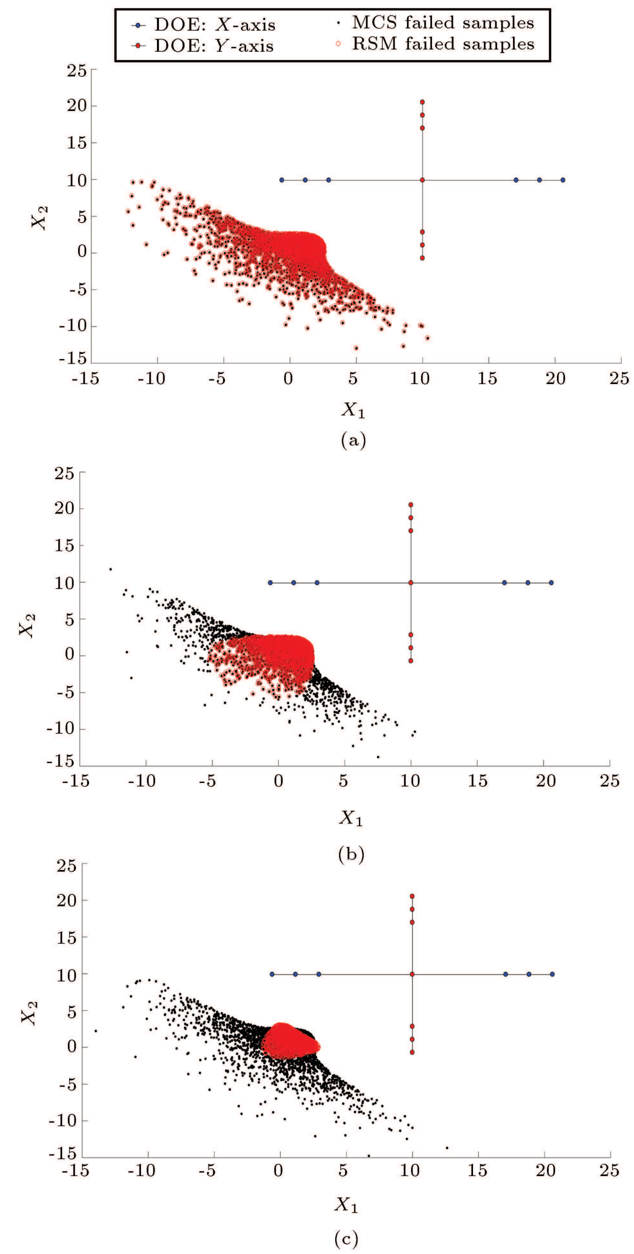

Figure 3. Failure region in the MCS and the proposed approach by using: (a) Spline, (b) PCHIP, and (c) kriging interpolation for Example 1.

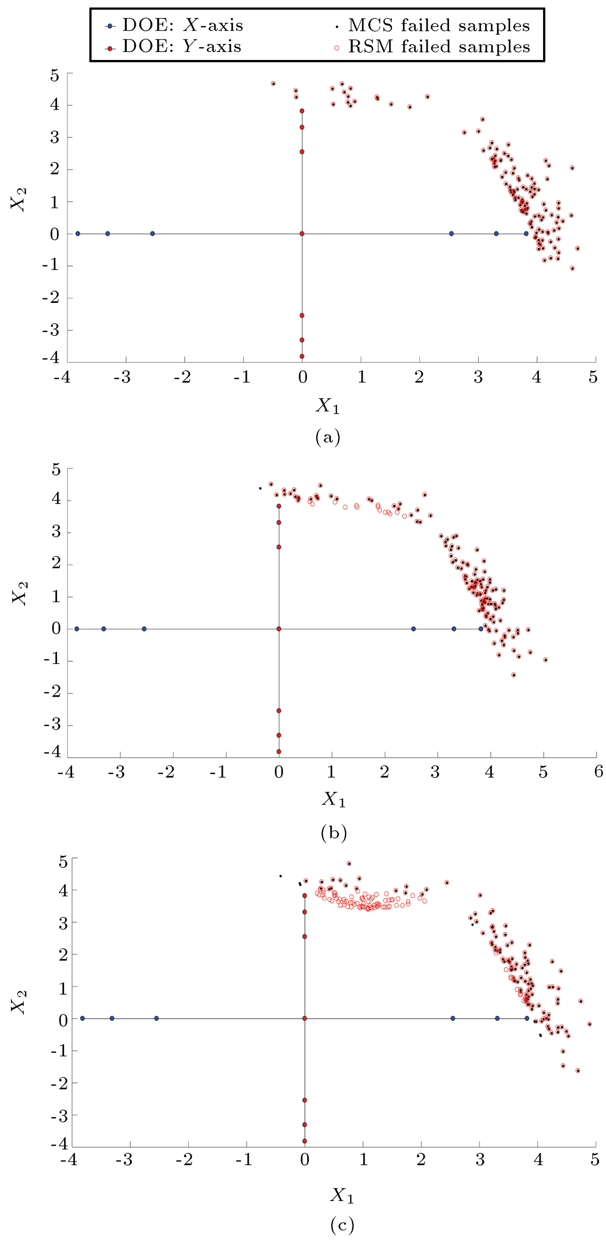

Figure 4. Failure region in the MCS and the proposed approach by using: (a) Spline, (b) PCHIP, and (c) kriging interpolation for Example 2.

$$g(X) = 12.5(0.04 - \max|r_{f_i}(t) - r_{f_{i-1}}(t)|)_{i=2,3,4} + (0.5 - \max|\ddot{z}_g(t) - \ddot{r}_{m_2}(t)|) + 2(0.25 - \max|r_{f_2}(t) - r_{m_1}(t)|), \quad (20)$$

where r_{fi} refers to the displacement of the ith floor and $r_{fi}(t) - r_{fi-1}(t)$ is the inter-story displacement of two consecutive floors. The accelerations $\ddot{z}_g$ and $\ddot{r}_{m2}$ are of the ground and the smaller mass block, respectively. The displacement r_{m1} is of the larger mass block and represents the displacement of the equipment isolation system. The limit state function in Eq. (20) is the sum of three expressions of failure modes. The first term describes damage to the structural system due to excessive deformation. The second term represents damage to equipment, which is caused by excessive acceleration.

The last term represents the damage to the isolation system. They are multiplied by weighing factors, which emphasize the three failure modes equally. As Eq. (20) states, it is desirable that:

1. None of the inter-story displacements exceed 0.04 m;

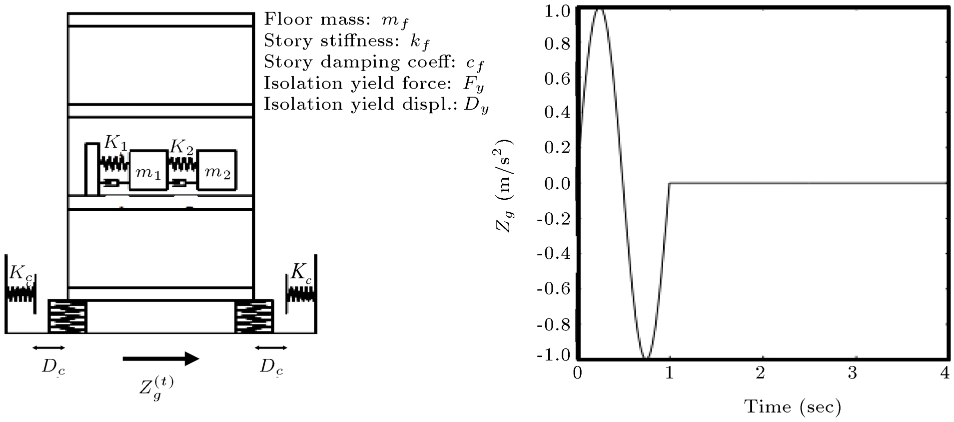

Figure 5. Base-isolated structure with an equipment isolation system on the 2nd floor, including the effects of isolation displacement limits [47].

Table 3. Reliability results for Example 2.

Method			$\boldsymbol{Pf}$	**Reliability index**	**No. of function evaluations**
FORM			$7.4883.10^{5}$	3.791	17
MCS			$1.1900.10^{4}$	3.675	10^{6}
UDRM	**Interpolation Method**	#1	$1.1900.10^{4}$	3.675	13
		#3	$1.1400.10^{4}$	3.686	13
		#2	$1.0800.10^{4}$	3.700	13
		#5	$1.3400.10^{4}$	3.644	13
		#4	$6.2000.10^{5}$	3.838	13
		#6	$4.3000.10^{5}$	3.927	13

Table 4. Statistical properties of random variables for Example 3.

Variable	**Description**	**Units**	**Mean value**	**COV**
$\boldsymbol{m_f}$	Floor mass	Kg	6000	0.1
$\boldsymbol{k_f}$	Floor stiffness	N/m	30000000	0.1
$\boldsymbol{c_f}$	Floor damping coefficient	N/m/s	60000	0.2
$\boldsymbol{d_y}$	Isolation yield displacement	m	0.05	0.2
$\boldsymbol{f_y}$	Isolation yield force	N	20000	0.2
$\boldsymbol{d_c}$	Isolation contact displacement	m	0.5	0
$\boldsymbol{k_c}$	Isolation contact stiffness	N/m	30000000	0.3
$\boldsymbol{m_1}$	Mass of block 1	Kg	500	0
$\boldsymbol{m_2}$	Mass of block 2	Kg	100	0
$\boldsymbol{k_1}$	Stiffness of spring 1	N/m	2500	0
$\boldsymbol{k_2}$	Stiffness of spring 2	N/m	100000	0
$\boldsymbol{c_1}$	Damping coefficient of damper 1	N/m/s	350	0
$\boldsymbol{c_2}$	Damping coefficient of damper 2	N/m/s	200	0
$\boldsymbol{T}$	Pulse excitation period	s	1.0	0.2
$\boldsymbol{A}$	Pulse amplitude	m/s/s	1.0	0.5

2. The peak acceleration of the smaller mass block (the equipment) be less than 0.5 m/s^2;
3. The displacement across the equipment isolation system be less than 0.25 m.

Failing to meet one or two of the conditions does not necessarily lead to a failure in the limit state function, e.g., $g(X<0)$, but will decrease the value of the limit state function. In these simulations, the system fails mainly because of the large acceleration of the smaller mass. The estimation of P_f with direct full-scale MCS of 10^5 sample size is 0.196 [48].

The problem has been solved by the three methods and the results are presented in Table 5. Results show the fail of FORM to converge to a proper solution. The reason is the high nonlinearity of performance function and the dimension size of the problem. However, it is noteworthy that by employing the proposed method, an approximation of failure probability is achievable by 65 times function evaluation with the results in agreement with MCS with 10^5 function evaluations. Result shows that the proposed approach provides accurate solution when the kriging (interpolation #3) method is employed.

6.4. Example 4

Consider a roof structure subjected to a uniformly distributed vertical load q, as shown in Figure 6. The example is adapted from [49]. The top cords and the compression bars are concrete, and the bottom cords and the tension bars are steel. In structural analysis, the uniformly distributed load q is transformed into three nodal loads with each being $P={}^{ql}\!/_{4}$. The serviceability limit state of the structure with respect to its maximum vertical displacement is considered. The limit state function is given by:

$$g=u_a-\frac{ql^2}{2}\left(\frac{3.81}{A_cE_c}+\frac{1.13}{A_sE_s}\right), \tag{21}$$

where u_a is the allowable displacement and set to

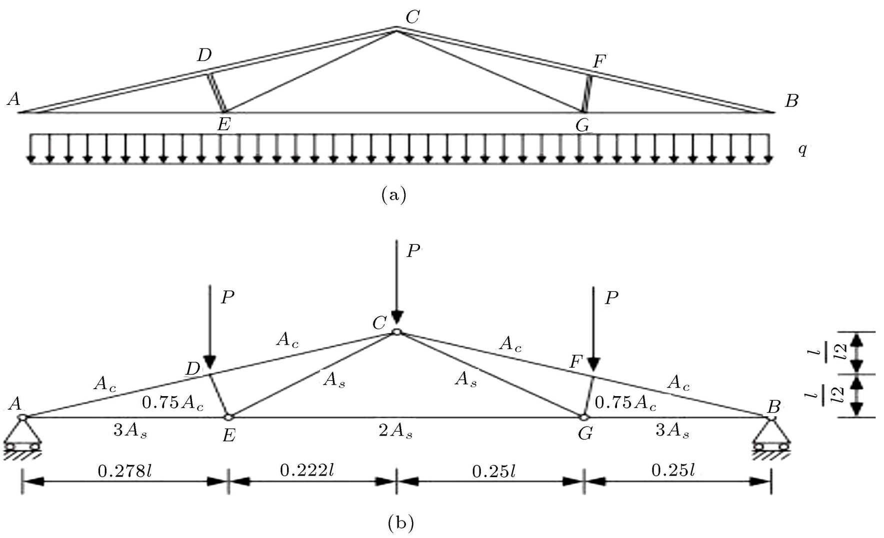

Figure 6. A roof structure (redrawn from [48]).

Table 5. Reliability results for Example 3.

Method			P_f	Reliability index	No. of function evaluations
FORM			–	–	–
MCS			0.196	0.856	10^5
UDRM	Interpolation method	#3	0.1848	0.897	65
		#2	0.2243	0.758	65
		#5	0.2399	0.707	65
		#1	0.2402	0.706	65
		#6	$8.3800.10^4$	3.142	65
		#4	$7.6900.10^4$	3.167	65

Table 6. Random variables for Example 4.

Variable	Mean	COV
q (N/m)	20,000	0.07
l (m)	12	0.01
A_s (m^2)	9.82×10^{-4}	0.06
A_c (m^2)	400×10^{-4}	0.12
E_s (N/m^2)	1×10^{11}	0.06
E_c (N/m^2)	2×10^{10}	0.06

0.03 m, E and A denote the modulus of elasticity and cross-sectional area, respectively, and the subscripts s and c indicate the steel and concrete material, respectively. Table 6 summarizes the statistical information of the random variables. All random variables are assumed to be independent normal. The probability of failure is found to be 9.37×10^{-3} after direct 5×10^7 Monte Carlo simulations.

Table 7 presents the results of the MCS, FORM, and proposed method. The results of the proposed method agree reasonably well with the Monte Carlo results. The relative error is 5% after 49 times function evaluation.

6.5. Example 5

In automobile engineering, the front axle beam is used to carry the weight of the front part of the vehicle (Figure 7) [50]. As the entire front part of the automobile rests on the front axle beam, it must be robust enough in construction to ensure its reliability. An I-beam is often used in the design of front axle due to its high bend strength and light weight. In this example, as shown in Figure 7, a dangerous cross section happens in the I-beam part. The maximum normal stress and shear stress are $\sigma = M/W_x$ and $\tau = T/W_p$, respectively, in which M and T are bending moment and torque, respectively, and W_x and W_P are section factor and polar section factor, respectively, given as:

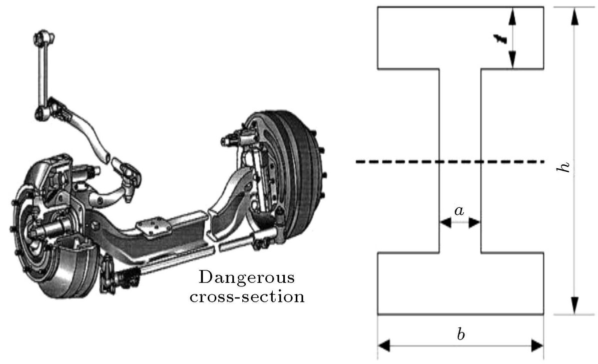

Figure 7. Schematic diagram of automobile front axle [48].

Table 8. Random variables for Example 5.

Variable	Mean	COV
a (mm)	12	0.06
b (mm)	65	0.325
t (mm)	14	0.07
h (mm)	85	0.425
M (N.mm)	3.5×10^6	1.75×10^5
T (N.mm)	3.1×10^6	1.55×10^5

$$W_x = \frac{a(h-2t)^3}{6h} + \frac{b}{6h}\left[h^3 - (h-2t)^3\right], \tag{22}$$

$$W_p = 0.8bt^2 + 0.4\left[a^3(h-2t)/t\right]. \tag{23}$$

To test the static strength of the front axle, the limit-state function can be expressed as:

$$g = \sigma_s - \sqrt{\sigma^2 + 3\tau^2}, \tag{24}$$

where σ_s is limit-state stress of yielding. Considering the characteristic of material in the front axle, the limit stress of yielding σ_s is 460 MPa. The geometry variables of I-beam, namely a, b, t, h, the local M, and T, are independent normal; they are listed with their distribution parameters in Table 8.

Table 7. Reliability results for Example 4.

Method			P_f	Reliability index	No. of function evaluations
FORM			–	–	–
MCS			0.00479	2.59	10^5
UDRM	Interpolation method	#1	0.00307	2.74	49
		#2	0.00297	2.75	49
		#6	0.00297	2.75	49
		#5	0.00280	2.77	49
		#3	0.00225	2.84	49
		#4	0.00002605	4.046	49

Table 9. Reliability results for Example 5.

Method		P_f	Reliability index	No. of function evaluations
FORM		0.0194	2.06	49
MCS		0.0195	2.05	10^5
UDRM (Interpolation method)	#1	0.0195	2.05	49
	#2	0.0195	2.05	49
	#5	0.0195	2.05	49
	#6	0.0195	2.05	49
	#3	0.0192	2.07	49
	#4	0.0000145	4.18	49

Table 9 presents the estimated values of failure probability with different methods. The number of samples used for each method is also listed in Table 9. The table shows that the proposed method can achieved good results with the lowest number of samples. According to Table 9, among various interpolation methods used in the proposed approach, the methods #1, #2, #5, and #6 present suitable approximations in such a manner that by 49 times function evaluation, they provide acceptable results in comparison with MCS with 10^5 function evaluations.

Efficiently solving these examples involving highly nonlinear and implicit LSFs confirms the high potential of the method to be applied to the real-world engineering problems.

7. Conclusions

In this study, an adapted DOE was presented for decomposition-based meta-models in structural reliability. The idea of the proposed approach was based on simulation approaches that separated the design space into two safe and unsafe regions. The employed DRM-based meta-model required a one-dimensional interpolation method to approximate the LSF; hence, this study also investigated the efficiency and accuracy of various interpolation techniques in applicability of the proposed method. Solving numerical and engineering problems with 6 different interpolation techniques proved that among the investigated interpolation approaches, using the spline and kriging method in the proposed approach provided results with acceptable accuracy. By using the proposed framework, we found out that efficiency of the method was similar to that of FORM, while its accuracy was close to that of MCS.

References

1. Ditlevsen, O. and Madsen, H.O., *Structural Reliability Methods*, New York, Wiley, **178**, pp. 18-25 (1996).
2. Hasofer, A.M. and Lind, N.C. "Exact and invariant second-moment code format", *Journal of the Engineering Mechanics Division*, **100**(1), pp. 111-121 (1974).
3. Rackwitz, R. and Flessler, B. "Structural reliability under combined random load sequences", *Computers & Structures*, **9**(5), pp. 489-494 (1978).
4. Der Kiureghian, A., Lin, H.Z., and Hwang, S.J. "Second-order reliability approximations", *Journal of Engineering Mechanics*, **113**(8), pp. 1208-1225 (1987).
5. Liu, P.L. and Der Kiureghian, A. "Optimization algorithms for structural reliability", *Structural Safety*, **9**(3), pp. 161-177 (1991).
6. Wang, L. and Grandhi, R.V. "Safety index calculation using intervening variables for structural reliability analysis", *Computers & Structures*, **59**(6), pp. 1139-1148 (1996).
7. Metropolis, N. and Ulam, S. "The Monte Carlo method", *Journal of the American Statistical Association*, **44**(247), pp. 335-341 (1949).
8. Yazdani, A., Nicknam, A., Khanzadi, M., and Motaghed, S. "An artificial statistical method to estimate seismicity parameter from incomplete earthquake catalogs: A case study in metropolitan Tehran, Iran", *Scientia Iranica, Transactions A, Civil Engineering*, **22**(2), p. 400 (2015).
9. Zuev, K. "Subset simulation method for rare event estimation: An Introduction", ArXiv preprint arXiv: 1505.03506 (2015).
10. Wang, Z. and Song, J. "Cross-entropy-based adaptive importance sampling using von Mises- Fisher mixture for high dimensional reliability analysis", *Struct Saf*, **59**, pp. 42-52 (2016).
11. Fan, H. and Liang, R. "Importance sampling based algorithm for efficient reliability analysis of axially loaded piles", *Computers and Geotechnics*, **65**, pp. 278-284 (2015).
12. Ibrahim, Y. "Observations on applications of importance sampling in structural reliability analysis", *Struct Safety*, **9**(4), pp. 269-281 (1991).

13. Nie, J. and Ellingwood, B.R. "Directional methods for structural reliability analysis", *Struct Safety*, **22**(3), pp. 233-249 (2000).

14. Depina, I., Le, T.M.H., Fenton, G., and Erikson, G. "Reliability analysis with metamodel line sampling", *Struct Saf*, **60**, pp. 1-15 (2016).

15. Pradlwarter, H.J., Schueller, G.I., Koutsourelakis, P.S., and Charmpis, D.C. "Application of line sampling simulation method to reliability benchmark problems", *Struct Safety*, **29**(3), pp. 208-221 (2007).

16. Rashki, M., Miri, M., and Moghaddam, M.A. "A new efficient simulation method to approximate the probability of failure and most probable point", *Structural Safety*, **39**, pp. 22-29 (2012).

17. Au, S.K. and Beck, J.L. "Estimation of small failure probabilities in high dimensions by subset simulation", *Probabilistic Engineering Mechanics*, **16**(4), pp. 263-277 (2001).

18. Ghanem, R. and Spanos, P.D. "Polynomial chaos in stochastic finite elements", *Journal of Applied Mechanics*, **57**(1), pp. 197-202 (1990).

19. Kamiński, M. and Świta, P. "Structural stability and reliability of the underground steel tanks with the stochastic finite element method", *Archives of Civil and Mechanical Engineering*, **15**(2), pp. 593-602 (2015).

20. Ghavidel, A., Mousavi, S.R., and Rashki, M. "The effect of FEM mesh density on the failure probability analysis of structures", *KSCE Journal of Civil Engineering*, **22**(7), pp. 2370-2383 (2018).

21. Gong, J.X. and Yi, P. "A robust iterative algorithm for structural reliability analysis", *Structural and Multidisciplinary Optimization*, **43**(4), pp. 519-527 (2011).

22. Johari, A., Momeni, M., and Javadi, A.A. "An analytical solution for reliability assessment of pseudo-static stability of rock slopes using jointly distributed random variables method", *Iranian Journal of Science and Technology Transactions of Civil Engineering*, **39**, pp. 351-363 (2015).

23. Hasofer, A.M. and Lind, N.C. "Exact and invariant second-moment code format", *Journal of the Engineering Mechanics Division*, **100**(1), pp. 111-121 (1974).

24. Keshtegar, B. and Miri, M. "Reliability analysis of corroded pipes using conjugate HL-RF algorithm based on average shear stress yield criterion", *Engineering Failure Analysis*, **46**, pp. 104-117 (2014).

25. Johari, A., Mousavi, S., and Nejad, A.H. "A seismic slope stability probabilistic model based on Bishop's method using analytical approach", *Scientia Iranica. Transactions A, Civil Engineering,* **22**(3), p. 728 (2015).

26. Rackwitz, R. and Flessler, B. "Structural reliability under combined random load sequences", *Computers & Structures*, **9**(5), pp. 489-494 (1978).

27. Santosh, T.V., Saraf, R.K., Ghosh, A.K., and Kushwaha, H.S. "Optimum step length selection rule in modified HL-RF method for structural reliability", *International Journal of Pressure Vessels and Piping*, **83**(10), pp. 742-748 (2006).

28. Yang, D. "Chaos control for numerical instability of first order reliability method", *Communications in Nonlinear Science and Numerical Simulation*, **15**(10), pp. 3131-3141 (2010).

29. Zhou, W., Gong, C., and Hong, H.P. "New perspective on application of first-order reliability method for estimating system reliability", *Journal of Engineering Mechanics*, **143**(9), p. 04017074 (2017).

30. Yun, W., Lu, Z., Jiang, X., and Zhao, L.F. "Maximum probable life time analysis under the required time-dependent failure probability constraint and its meta-model estimation", *Structural and Multidisciplinary Optimization*, **55**(4), pp. 1439-1451 (2017).

31. Bhadra, A. and Ionides, E.L. "Adaptive particle allocation in iterated sequential Monte Carlo via approximating meta-models", *Statistics and Computing*, **26**(1-2), pp. 393-407 (2016).

32. Dengiz, B., İç, Y.T., and Belgin, O. "A meta-model based simulation optimization using hybrid simulation-analytical modeling to increase the productivity in automotive industry", *Mathematics and Computers in Simulation*, **120**, pp. 120-128 (2016).

33. Sudret, B. "Meta-models for structural reliability and uncertainty quantification", arXiv preprint arXiv: 1203.2062 (2012).

34. Cadini, F., Santos, F., and Zio, E. "An improved adaptive kriging-based importance technique for sampling multiple failure regions of low probability", *Reliability Engineering & System Safety*, **131**, pp. 109-117 (2014).

35. Echard, B., Gayton, N., and Lemaire, M. "AK-MCS: an active learning reliability method combining kriging and Monte Carlo simulation", *Structural Safety*, **33**(2), pp. 145-154 (2011).

36. Bucher, C.G. and Bourgund, U. "A fast and efficient response surface approach for structural reliability problems", *Structural Safety*, **7**(1), pp. 57-66 (1990).

37. Nguyen, X.S., Sellier, A., Duprat, F., and Pons, G. "Adaptive response surface method based on a double weighted regression technique", *Probabilistic Engineering Mechanics*, **24**(2), pp. 135-143 (2009).

38. Kang, S.C., Koh, H.M., and Choo, J.F. "An efficient response surface method using moving least squares approximation for structural reliability analysis", *Probabilistic Engineering Mechanics*, **25**(4), pp. 365-371 (2010).

39. Allaix, D.L. and Carbone, V.I. "An improvement of the response surface method", *Structural Safety*, **33**(2), pp. 165-172 (2011).

40. Cai, W. "A dimension reduction algorithm preserving both global and local clustering structure", *Knowledge-Based Systems*, **118**, pp. 191-203 (2017).

41. Xu, H. and Rahman, S. "A moment-based stochastic method for response moment and reliability analysis",

In *Proceedings of 2nd MIT Conference on Computational Fluid and Solid Mechanics*, pp. 17-20 (2003).

42. Xu, H. and Rahman, S. "A generalized dimension-reduction method for multidimensional integration in stochastic mechanics", *International Journal for Numerical Methods in Engineering*, **61**(12), pp. 1992-2019 (2004).
43. Huang, B. and Du, X. "Uncertainty analysis by dimension reduction integration and saddle point approximations", *Journal of Mechanical Design*, **128**(1), pp. 26-33 (2006).
44. Lee, I., Choi, K.K., and Du, L. "Alternative methods for reliability-based robust design optimization including dimension reduction method", In *ASME 2006 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pp. 1235-1246 (2006).
45. GhohaniArab, H. and Ghasemi, M.R. "A fast and robust method for estimating the failure probability of structures", *P I Civil Eng Str B*, **168**(4), pp. 298-309 (2015).
46. Lv, Z., Lu, Z., and Wang, P. "A new learning function for Kriging and its applications to solve reliability problems in engineering", *Computers & Mathematics with Applications*, **70**(5), pp. 1182-1197 (2015).
47. Zou, T., Mahadevan, S., Mourelatos, Z., and Meernik, P. "Reliability analysis of automotive body-door subsystem", *Reliability Engineering & System Safety*, **78**(3), pp. 315-324 (2002).
48. Gavin, H.P. and Yau, S.C. "High-order limit state functions in the response surface method for structural reliability analysis", *Structural Safety*, **30**(2), pp. 162-179 (2008).
49. Dubourg, V. and Sudret, B. "Meta-model-based importance sampling for reliability sensitivity", *Structural Safety*, **49**, pp. 27-36 (2014).
50. Wang, P., Lu, Z., and Tang, Z. "An application of the Kriging method in global sensitivity analysis with parameter uncertainty", *Applied Mathematical Modelling*, **37**, pp. 6543-6555 (2013).

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