Design an analog CMOS fuzzy logic controller for the inverted pendulum with novel triangular membership function

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Abstract

In this paper, a fuzzy analog controller circuit is provided for the inverted pendulum problem which resulted in a simple analog circuit simply does the act of controlling without requiring to any processing structure. In other words, in case of constructing the proposed circuit, a small analog chip controls the inverted pendulum. For this purpose, the first step is the study of the dynamic model of the inverted pendulum and then a fuzzy controller is designed systematically. In the following, the number of membership functions (MFs) and the formation of them are designed. In addition, the different MFs and different numbers are examined for each variable and the most efficient structure is selected as fuzzy controller. To assess efficiency of designed fuzzy controller, the controller is simulated in Simulink and then this design is implemented at transistor level in TSMC 0.18µm CMOS technology. In this work, the MFs, the circuits for realization of the knowledge-based and defuzzification circuits are designed in current mode. The proposed circuits are simulated and evaluated in Advanced Design System (ADS) software based on CMOS technology. Simulation results show that the inverted pendulum is controlled with high accuracy and high speed, meanwhile the controllers have low power consumption and good robustness to outer large and fast disturbance rather than the previous works.

Keywords: Fuzzy Logic, Fuzzy Rules, Fuzzy Controller, Fuzzy Membership Function, PID controller, The Inverted Pendulum.

1. Introduction

The implementation of systems in analog [1] and integrated circuits causes reduction of area, decrease in power and simplicity of the circuit. Using fuzzy controller is one of the most effective ways to control complex, unstable and nonlinear systems [2-3-4] without processor, while other methods generally entail to have a processing system for implementation. One of the important issues for the evaluation of control systems is the inverted pendulum control problem or rotary inverted pendulum [5]. The control technology derived from the inverted pendulum is applied in many industrial and engineering products such as high-precision control of manipulator, stabilization and tracking control of launching rocket, and attitude control of satellite [6]. Some studies have been done to implement fuzzy controllers to solve this problem [7-11]. These circuits are usually a combination of analog and digital circuits [12] [13] [14] or digital [15], and completely analog implementation have not been done yet. The Fig. 1 shows an inverted pendulum system.

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The inverted pendulum is defined as a rod located on the chariot that moves horizontally. The inverted pendulum inherently is unstable in upright position and it needs a controller in order to keep it vertically. The inverted pendulum is a classic problem in dynamics and control theory, and is used as a benchmark for testing control strategies. The inverted pendulum which is shown in Fig. 1, is affected by an impulse force, F. This condition is essential to control the pendulum vertically.

As can be seen in Fig. 1, the system includes a rod with mass Mp and length l located on a chariot with mass Mc affected by force F and can move along the horizontal axis, thus the pendulum diverges from vertical condition. The purpose of control of this system is balancing the pendulum (i.e. \( \theta = 0 \)). For more precise definition of the mechanical system of inverted pendulum, this system's parameters are defined in Table 1.

Fuzzy controller methods are particularly one of the most impressive and simple ways to control nonlinear systems. In many references are shown that fuzzy controllers operate more efficiently than standard controllers [7-11]. For this reason, in this research has been tried to design fuzzy controller for inverted pendulum issue and is implemented as an analog circuit. As noted above, some studies have been done to design fuzzy control system and other controlling strategies to control inverted pendulum. In this section, a few of them will be noted.

In [7] two methods of PID conventional controller and the rule-based fuzzy logic controller are designed and compared. The proposed fuzzy circuit in [7] has two controllers which are used for chariot position control and pendulum control. Eventually, resultant of these two forces and disturbance force apply to the inverted pendulum to control it. In reference [7], Ziegler-Nichols criteria are used for tuning the control parameters of the PID controller. Due to the simulations in [7], can conclude that fuzzy logic control works more effectively in comparison to the PID Control. These results are achieved in Matlab.

In [8] basic performances of fuzzy logic control (FLC) and conventional PID control are contrasted. According to paper [8] fuzzy control has smaller overshoot and shorter settling time; hence, superiority of fuzzy control is brought to attention. Also, according to the results of the study, compared with modern control theory, fuzzy logic is easier to implement because it eliminates the complexities of the process of mathematical modeling, instead a set of control rules is used. Fuzzy control, in comparison with, PID control is more robust against parameters changes. And fuzzy logic also is not based on the mathematical model of the inverted pendulum and is more robust to mass variations. The weakness of the proposed fuzzy controller in [8] is the complexity of fuzzy rules that is used; however, it is not easy to modify or change specifications. Our proposed work is trying to have simple structure of fuzzy rules that can be reset for other characteristic easily.

In reference [9], according to single input rule modules (SIRMs), a fuzzy controller is used for the stabilization of an inverted pendulum. The intended fuzzy controller has a high ability to stabilize a wide range of the inverted pendulum systems [9]. In this procedure, angular control of the pendulum and the chariot control is done in parallel, and both of them, according to control
situations, are switched smoothly by adjusting automatically the dynamic importance degrees. In paper [9], Large deviations occur and then the system will be controlled.

Another architecture of fuzzy controller for the inverted pendulum is studied [10]. Facility and understandable fuzzy rules are the advantages of the intended design, but the circuit consists of A/D blocks, D/A blocks, power amplifier and servo motor, while using analog design causes elimination of these blocks. This control technique relies on the human capability to understand the system’s behavior and is based on qualitative control rules. Paper [10] shows that fuzzy control systems are more robust to variations of parameters (mass and length of the pendulum) than conventional PID control systems.

In reference [11], an 8-bit microprocessor, according to fuzzy control for rotary inverted pendulum, is presented. The hardware of the system includes: user interface, control circuit box and the rotary inverted pendulum system. The fuzzy controller consists of two microprocessors, the master one for fuzzy logic operation and the slave one for user interface [11]. This circuit consists: optical encoder for angle and location detection, counters, keyboard, two microprocessors, clock, programmable peripheral interface, transceiver, DC motor and etc. The proposed design in paper [11], because of using different components, analog and digital parts is more complex.

In reference [16], PID controllers are applied to the stabilization and tracking control of three types of inverted pendulum. when a control system has more than two PID controllers, the adjustment of PID parameters is not an easy problem. Paper [16] is given a new way to design of very simple and effective PID controllers.

Unfortunately, mentioned papers that outlined above did not evaluate proposed controller performance accurately and quantitatively and did not compare with other’s works. Thus, cannot achieve appropriate conclusion of research [7-11], [16]. Perhaps, one reason for this problem is that the desired control issue is multifaceted. As a brief conclusion it can be said that in all last works, the controllers have been implemented on processing systems which Indicating hardware complication. However, in this paper, the controller completely implemented in analog chip. The proposed work is an attempt to design fuzzy rules to control the system fast enough. This particular example may have purely scientific value, but the proposed approach in the design and the implementation of analog chips can be basis of designing larger and more practical control systems. Unlike previous works, in the present study some indicators like; robustness, power consumption, controlling range of disturbance and maximum deviation are considered for evaluating the performance of controllers.

Rest of the paper is organized as follows: the second section is a brief review on fuzzy logic and fuzzy controllers. In the third section, fuzzy control system design is expressed and systematical simulation results are discussed. In the fourth section, the implementation of the intended controller is presented as an analog chip. In the fifth section, ADS simulation results are presented and compared with Matlab simulations. In the sixth section, some conclusions are stated.

2. **Brief review on fuzzy logic and fuzzy controllers**
2.1. Fuzzy Logic

One of the advantages of fuzzy controllers is that the designer does not need to know the exact dynamic model. Hence, the fuzzy controllers can be designed just based on system behavior, and then control the intended system. Lotfi A. Zadeh mentioned, "As the complexity of the system increases, the possibility of describing the system with deterministic terms diminishes" [17]. In the following the fuzzy logic and fuzzy controller will be expressed briefly. One of the most important characteristics of fuzzy logic is the use of linguistic variables, instead of numerical variables. Linguistic variables are defined as variables in natural language, such as small/ large, almost/ certainly and, etc. Fuzzy sets are not as classic sets in which a member belongs or not. Fuzzy sets allow partial membership in a set, which means an element belongs to more than one set [18]. Each fuzzy set is characterized by its MF which each member is allocated a number between zero and one. Based on these MFs, logical operations were redefined among sets.

2.2. Fuzzy Logic Controller

Fuzzy systems are knowledge-based systems. The heart of a fuzzy system is a knowledge base that consists of if-then rules. The if-then rule is an expression that some words have been specified by continuous MFs. As some of the fuzzy rules applications can be noted to fuzzy controllers, fuzzy expert systems, fuzzy approximate arguments, pattern recognition and fuzzy decision. In daily life the if-then statements are being used frequently.

Computational complexity of fuzzy rules is expressed by several parameters: the number of inputs, the number of outputs, the numbers and shapes of the MF I/O, the number of rules, method of rules inference, defuzzification algorithm and accuracy. Typically, a fuzzy controller is composed by a set of if-then rules involving three basic operations: (Fig. 2)

Fuzzification: it is an operation that translates crisp input data into a membership degree by means of the MFs. Fuzzy inference: this operation uses the membership degrees to deduce a fuzzy output for each rule and a final fuzzy output of the controller. Defuzzification: it is an operation that translates the final fuzzy output into a crisp value, compatible with the deterministic external environment [7].

3. Fuzzy Control System Design and System Simulation Results

It is practically impossible to control the inverted pendulum because of extreme nonlinear behavior with the linear conventional control method, or is applicable in very small ranges. This system is actually a one-input and two-output system. Position of chariot and angle of the pendulum simultaneously will be under control with a control signal.

3.1. Inverted Pendulum Modeling

With writing dynamic equations of chariot and pendulum, categories of equations obtained. Indeed, these equations are dynamic model of system. Equations of the dynamics of the pendulum and the chariot can be described by different ways [7] [19-20]. Summary of the equations which is describing the system is as Table 2 [7].
Variable N is an interaction force of pendulum and chariot along the x axis and P is an interaction force of these two along the y axis. The rest of the variables are in the Table 1.

3.2. The design of fuzzy controller for the inverted pendulum

General exponent of a simultaneously chariot position and angle deviation of pendulum controller is shown in Fig. 3.

In this design the inverted pendulum has been controlled with two controllers [7]: angular controller and position controller. Each controller has two input variables. The pendulum angle error and its derivative have been used in the pendulum angular controller. The position controller uses the position error and its derivative. Due to the dynamic equations of this system, there are two dynamic objects in inverted pendulum-chariot system: the pendulum and the chariot. The rule-base mechanism of controller, for separately controlling of pendulum and chariot is easier. However, there is only one control action to control the inverted pendulum-chariot system. Thus, control force Fp for the pendulum subsystem and control force Fc for chariot subsystem require to be combined into a control force.

It can be seen that driving the chariot to the left side causes the pendulum goes to the right. The goals of the inverted pendulum control system are balancing the pendulum in vertical situation and fixing the chariot in the reference point. Combination of Fp and Fc is force F = Fp - Fc. In this study, all six universe of discourse are divided into five overlapping fuzzy set values labeled Negative Big (NB), Negative Medium (NM), Zero (Z), Positive Medium (PM), Positive Big (PB). The fuzzification in a fuzzy set is determined by different types of MFs like: trapezoidal, triangular, z-shape, s-shape, Gaussian-shape [21] [22] [23] [24] [25] and bell-shaped MF. The shapes of MFs are less important for fuzzy inference system and the number and the extent of changes in MFs are more important. Simplicity in design and desired precision cause to use triangular MFs in this work. Mamdani's technique is the most commonly used inference strategies in the existing fuzzy control systems owing to its simplicity. In this method, the minimum operation is adopted for computing a fuzzy implication relation, and the max-min as the compositional rule of inference [26]. The proposed fuzzy controller is used this procedure. The output of a fuzzy inference engine is a fuzzy set which represents the possible distribution of control action. For practical use, a crisp control output is usually required. Thus, a defuzzification interface is necessary to convert the inferred fuzzy control action into a nonfuzzy (crisp) value. Among the suggested defuzzication strategy, the center of area (COA) method is the most commonly used [12].

3.3. Fuzzy control rules

The rule base is constructed using the If-then statements. The most difficult aspect in designing fuzzy controllers is the design and construction of a rule base. The rule base mechanism is more convenient to design for separate control of the pendulum and the chariot. Rules design are based on innovative knowledge of the behavior of the system and its theory. Also, the designer uses the experience and trial to discover the best system rule base. For example, in this application if the pendulum is falling in one direction, then with pushing the chariot in the same direction can have inverse movement of the pendulum. For example, to control the chariot can Evolve following
rules. In this analysis, displacement $\Delta x$ is the difference of the starting location minus current chariot position. Therefore, when the chariot shifts toward positive position, displacement is negative and in negative position, displacement is positive. If displacement is negative big and the pace of change is negative big, this mean the chariot is in positive part of the axis and moving away from the starting location. In this case, a negative big force should apply to control the chariot to return it to starting location. This descriptive rule is expressed with a gray background in Table 3. Likewise, the rest of the rules can be described. The fuzzy rules can be seen in Table 3.

Fuzzy rule base is completed and next step is implementation of controller based on this knowledge base. First, fuzzy variables must be described. In other words, each MF should be defined. How to choose the details of MFs depend on behavior recognition and design knowledge. In this example, MFs are designed in a relatively small range to control action done more appropriately. In this study, assuming that the system is fully controlled and in practice chariot could not get away from the starting location. Therefore, under perturbation, if the system is out of the control, the chariot position will have small changes around its equilibrium point. MFs of chariot position ($\Delta x$) and its derivative ($\Delta \dot{x}$) is shown in Fig. 4.

Force MFs are as Fig. 5. The force membership degree should be obtained to any possible states to implement controller for each mode of force. To get the possibility of NB force for position controller, the following system diagram uses (Fig. 6). Pendulum angular controller is designed in the similar way. The triangular MFs of $\Delta \theta$ and $\Delta \dot{\theta}$ is defined as: $\Delta \theta [-0.1,0.1]$, $\Delta \dot{\theta} [-0.2,0.2]$. The Fig. 7 is shown MFs of $\Delta \theta$ and $\Delta \dot{\theta}$.

The inverted pendulum specifications are mc=0.5Kg, mp=0.2Kg, l=0.3m, b=0.1N/m/sec, inertia=0.006 Kg.m², g=9.8m/s². Various disturbances are applied in MATLAB software and controller operation is assessed. If the input with 1N amplitude in a short period of time be applied to the inverted pendulum-chariot system, the chariot and the angle of the pendulum, be controlled about four seconds. See Fig. 8. The input actions which is ranged from two to five seconds with the amount of 1N will have output as shown in Fig. 9.

4. Implementation of the proposed controller for analog chip

To appraise the performance of designed fuzzy controller in ADS, the inverted pendulum model is implemented. An inverted pendulum model includes: op-amp circuits, resistors, capacitors and math blocks (e.g. SDD blocks is used to implement mathematical operations). First, it is necessary that MFs should be designed as analog circuits to implement analog designed controller. The minimum (Min) and maximum (Max) circuits are used to fulfill the MFs.

4.1. Min-max circuits

According to the application of min and max circuits, diverse circuits are designed in current and voltage-mode [27] [28] [29] [30] [31]. The circuits would act in different ways. One of these methods is use of equations to acquire minimum or maximum current [30]. A current-current MFG is presented in [30] to display the min and max circuit performance. Comparing with other MFG circuits in current-mode, input and output ranges for this MFG circuit are quite wide and
each side Slope can independently vary to achieve triangular or trapezoidal MFG. Another large part of the circuits are designed based on current mirrors [27].

In paper [27] simultaneously minimum and maximum current are obtained, but in this study, the modified circuits are used (Fig. 10). In this case, the number of min and max circuit transistors will be eight and twelve transistors respectively.

Considering that the controller design is in current mode, it is preferred that the circuits input impedance be small. Therefore, the proposed min-max circuits have been selected on this basis. It is explicit that with increasing aspect ratio or increasing DC current bias, input impedance is improved with sacrificing die chip area and speed of circuit and output impedance is large enough to transferring all of output currents to the load also to ameliorate the output impedance circuit, the DC bias current should be diminished [27].

Suppose that $I_1 > I_2$ for min circuit; thus voltage at node A increases than node B, accordingly M8 turns on and M7 is off. Then, the current $I_2$ is transferred to current mirror and difference of this two current pass through M8, but to get the maximum current this difference current be transferred to other current mirrors and will be added to the minimum current to create maximum current.

### 4.2. Novel MF generator

One method of the implementation of triangular and trapezoidal MFs is presented in [30]. In this manner, MF is made by three min circuits which occupy high area. The proposed MFG circuit (Fig. 11) is presented in this paper uses only one min circuit so that the size and power consumption are decreased considerably. In the proposed triangular MF a and c are the minimum and maximum input current respectively (Fig. 12). The highest membership degree is $I_{\text{max}}$ which happen when input current is b. The input current is compared with the lowest current and maximum current, if $I_{\text{in}} < a$ and $I_{\text{in}} > c$; output current value will be zero, otherwise will have two currents according to left Slope and right slope of triangle. Then, will compare these two currents, and the lowest value of these two currents will be output current.

The inverted pendulum-chariot controller system, which uses the proposed MF has 480 transistors lower than the previous structure of MF. Therefore, the occupied area of controller significantly is reduced. And also power consumption is improved than the prior circuits.

Since the MFs which are implemented in an analog circuit are based on current. Currents are values about a few tens or hundreds of micro amperes in 0.18µm technology, the MFs in the analog circuit have been rewritten (from Matlab values Fig. 4 and 6) in Fig. 13 and Fig. 14. In other words, real variables such as position and angle in centimeter and radian scale become current times of Micro. Therefore, the used MFs are as follows.

### 5. ADS simulation results

In this section the force of 1N on the brief moment (0.001s) (Fig. 15.a) is applied to an inverted pendulum. The pendulum will be also unstable and chariot displaces from reference location and after a few seconds both of them will be returned to the reference situation (Fig. 15.b, 15.c).
the next examination, disturbance force is 1N step for three seconds (Fig. 16.a). The controller performs the desired control action (Fig. 16.b, 16.c).

In the comparison, the systematical simulation results (Fig. 8 and 9) and analog design results (Fig. 15 and 16), both behaviors were properly similar. There are minor differences in overshoot and settling time, due to the use of non-ideal circuit components (capacitors, transistors, etc.) the differences are not unexpected.

5.1. Controlling range of disturbance

About controlling ranges, when the disturbance force which is equal to 34N in 0.001s is applied to the inverted pendulum-chariot system, the system has been controlled (Fig. 17). And if more force be applied to the system, the controller cannot control them (Fig. 18, shows results with force=35N). Also, it can be seen that the system controls negative disturbance force until -35N. (Fig. 19). And the controller cannot control disturbance force lower than -36N (Fig. 20). As a result, the controlling ranges for a given fixed specifications of inverted pendulum is [-35,34] N. This design can be used to control a wide range of perturbations force.

5.2. Robustness

Robustness is one of the parameters is concerned in control systems. Parameters defining the inverted pendulum, will be considered fixed and changes in the mass of the pendulum and cart is investigated. The results of this system is summarized in Table 4.

In the past sections, this factor is mentioned as one of the important control assays. By analyzing the system, the robustness of proposed controller was seen.

5.3. Comparison of the designed fuzzy system and PID control system

The inverted pendulum is intended as mc=0.5(Kg), mp=0.2(Kg), l=0.3(m), b=0.1(N/m/sec), inertia=0.006(Kg.m²), g=9.8(m/s²) and PID control parameters are as Table 5. The results of comparing these two control methods for turbulence with the same specification of inverted pendulum are presented in Tables 6 and 7. As shown in Table 6 and 7, the proposed fuzzy controller is more effective and has lower deviation.

6. Conclusions

One of the most important issues of control, is the inverted pendulum control. In this paper, analog fuzzy controller as an Efficient control method is implemented for controlling the inverted pendulum. Using this method causes to remove processing systems, remove the digital circuits, converters and also other equipment. Analog chips with very small area and high speed without any external processor, can control a wide range of disturbances. The architectures have been designed and implemented in a TSMC 0.18 µm CMOS technology with a 1.8 V supply voltage. According to the simulation results, the average current of controller is 13µA. As a result, the power consumption of each chip is 23.4µW. The main characteristics of the intended fuzzy controller is summarized as follows:

- Simple circuitry
- High speed controlling
- Low die area
- Control a wide range of disturbances
- Low power consumption than other circuits. Table 8 compares proposed FLC with other works.

The advantages of the proposed triangular MF are: simple structure, lower power consumption, small area and precise performance than previous works.

References


Fig. 1. The inverted pendulum

<table>
<thead>
<tr>
<th>Table 1. The inverted pendulum-chariot system parameters</th>
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<tbody>
<tr>
<td>Mc(Kg)</td>
</tr>
<tr>
<td>Mp(Kg)</td>
</tr>
<tr>
<td>b(N/m/sec)</td>
</tr>
<tr>
<td>L(m)</td>
</tr>
<tr>
<td>Inertia($Kg\cdot m^2$)</td>
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<tr>
<td>------------------------</td>
</tr>
<tr>
<td>g($m=\bar{s}^2$)</td>
</tr>
<tr>
<td>F(N)</td>
</tr>
<tr>
<td>X(m)</td>
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<tr>
<td>$\theta$(radian)</td>
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</table>

**Fig. 2. Basic configuration of fuzzy logic controller**

**Table 2. Describing equations of the inverted pendulum**

\[
\begin{align*}
\frac{d^2 \theta}{dt^2} &= \frac{1}{I} (NL \cos(\theta) + PL \sin(\theta)) \\
\frac{d^2 x}{dt^2} &= \frac{1}{M} (F - N - b \frac{dx}{dt}) \\

P &= m \left( \frac{d^2 y_p}{dt^2} + g \right) \\
N &= m \frac{d^2 x_p}{dt^2} \\

\frac{d^2 y_p}{dt^2} &= -l \cos(\theta)(\frac{d\theta}{dt})^2 - l \sin(\theta) \frac{d^2 \theta}{dt^2} \\
\frac{d^2 x_p}{dt^2} &= \frac{d^2 x}{dt^2} + l \sin(\theta)(\frac{d\theta}{dt})^2 - l \cos(\theta) \frac{d^2 \theta}{dt^2}
\end{align*}
\]

**Fig. 3. Block diagram of inverted pendulum-chariot controller system**
Fig. 4. Chariot position (a) ($\Delta x$) and (b) its derivative ($\Delta \dot{x}$) MFs

Fig. 5. Force membership functions

Table 3. Chariot position fuzzy control rules

<table>
<thead>
<tr>
<th>$\Delta \dot{x}$</th>
<th>NB</th>
<th>NM</th>
<th>Z</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>Z</td>
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<td>NM</td>
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<td>PB</td>
<td>Z</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
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</table>
Fig. 6. Diagram of position controller (for F=NB) (a) fuzzification (b) fuzzy inference (c) defuzzification
Fig. 7. Membership functions of (a) pendulum angle ($\Delta \theta$) and (b) its derivative ($\Delta \dot{\theta}$)

Fig. 8. First system response

Fig. 9. Second system response
Fig. 10. (a) min circuit (b) max circuit

\[ \frac{I_{\text{max}}}{I_{\text{a}}} \quad \frac{I_{\text{max}}}{I_{\text{b}}} \]

Fig. 11. Novel MFG block

Fig. 12. Triangular MF
Fig. 13. Chariot position (a) \( \Delta x \) and (b) its derivative \( \Delta \dot{x} \) membership functions in analog implementation.

Fig. 14. Pendulum angle (a) \( \Delta \theta \) and (b) its derivative \( \Delta \dot{\theta} \) membership functions in analog implementation.
Fig. 15. System behavior (a. disturbance input b. angle c. position)
**Fig. 16.** System behavior (a. disturbance input b. angle c. position)

(a) ![Graph](image1)
(b) ![Graph](image2)

**Fig. 17.** System response to $F = 34N$ (a. angle b. position)

(a) ![Graph](image3)
(b) ![Graph](image4)

**Fig. 18.** System response to $F = 35N$ (a. angle b. position)

(a) ![Graph](image5)
(b) ![Graph](image6)
Fig. 19. System response to $F = -35N$ (a. angle, b. position)

Fig. 20. System response to $F = -36N$ (a. angle, b. position)

Table 4. The stability investigation of the inverted pendulum system for changes in mass

<table>
<thead>
<tr>
<th>Constant parameters</th>
<th>The extent of changes in the mass of the system is controlled with desired speed</th>
</tr>
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<tbody>
<tr>
<td>$Mp= 0.2$ Kg, $L=0.3$ m</td>
<td>$(0.1 \leq Mc \leq 3.5)$ Kg</td>
</tr>
<tr>
<td>$Mc=0.5$ Kg, $L=0.3$ m</td>
<td>$(0.1 \leq Mp \leq 8)$ Kg</td>
</tr>
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Table 5. PID control parameters

<table>
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<tr>
<th>Gain</th>
<th>For pendulum angle controller</th>
<th>For cart position controller</th>
</tr>
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<tr>
<td>$K_P$</td>
<td>41</td>
<td>2</td>
</tr>
<tr>
<td>$K_I$</td>
<td>38</td>
<td>0.02</td>
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<tr>
<td>$K_D$</td>
<td>27</td>
<td>0.4</td>
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Table 6. The evaluation of fuzzy control and PID control methods with 0.001N amplitude of impulse input
<table>
<thead>
<tr>
<th></th>
<th>cart position</th>
<th>pendulum angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proposed fuzzy method</td>
<td>-70 nm</td>
<td>50 n rad</td>
</tr>
<tr>
<td>PID control</td>
<td>0.6 mm</td>
<td>30 µ rad</td>
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</table>

Table 7. The evaluation of fuzzy control and PID control methods with 1N amplitude of impulse input

<table>
<thead>
<tr>
<th></th>
<th>Maximum deviation of cart position</th>
<th>Maximum deviation of pendulum angle</th>
</tr>
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<tbody>
<tr>
<td>The proposed fuzzy method</td>
<td>19 µm</td>
<td>48 µ rad</td>
</tr>
<tr>
<td>PID control</td>
<td>0.6 m</td>
<td>0.03 rad</td>
</tr>
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</table>

Table 8. The comparison of proposed FLC chip with previous recent works

<table>
<thead>
<tr>
<th>References</th>
<th>Technology (µm)</th>
<th>Shape type</th>
<th>Supply voltage (v)</th>
<th>Power (mW)</th>
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<tbody>
<tr>
<td>[3]</td>
<td>0.35</td>
<td>Trapezoidal, Gaussian, Triangular</td>
<td>-</td>
<td>13.4</td>
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<tr>
<td>[5]</td>
<td>0.35</td>
<td>Trapezoidal, Triangular</td>
<td>-</td>
<td>10.49</td>
</tr>
<tr>
<td>[6]</td>
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<td>Trapezoidal, Triangular</td>
<td>-</td>
<td>49</td>
</tr>
<tr>
<td>[26]</td>
<td>0.35</td>
<td>Trapezoidal, Triangular</td>
<td>3.3</td>
<td>8</td>
</tr>
<tr>
<td>[27]</td>
<td>1.2</td>
<td>Trapezoidal, Triangular</td>
<td>2.5</td>
<td>16.3</td>
</tr>
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<tr>
<td>Proposed</td>
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<td>Triangular</td>
<td>1.8</td>
<td>0.0234</td>
</tr>
</tbody>
</table>

Biography

**Seyed Milad Azimi** was born in Sari, Iran in 1992. He received the B.Sc. and the M.Sc. degrees in electrical engineering from Babol Noshirvani University of Technology, Babol, Iran, in 2014 and 2017, respectively.

In 2014, He joined the Integrated Circuit Research Laboratory, Department of Electrical and Computer Engineering, Babol Noshirvani University of Technology, as a Researcher. His current research interests include Cmos Integrated Circuits for Fuzzy controllers and Flip-flop design in nanometer CMOS from high speed to low energy.

**Hossein Miar Naimi** was born in Chalous, Iran, in 1972. He received the B.Sc. degree from Sharif University of Technology, Tehran, Iran, in 1994, the M.Sc. degree from Tarbiat Modares University, Tehran, in 1996 and the Ph.D. degree from Iran University of Science and Technology, Tehran, in 2002, all in electrical engineering. Since 2003 he has been a Member of the Electrical and Electronics Engineering Faculty, Babol University of Technology where he became a Professor in 2014.

His research interests are analog CMOS integrated circuit design, RF and microwave microelectronics.