Modelling Multi Tour Inventory Routing Problem for Deteriorating Items with Time Windows

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Abstract

In recent decades, there are intensive researches on deteriorating inventory. However, only a few researchers focus on the inventory routing problem for deteriorating item. There are many items such as foods, electronic products that deteriorate with time, and many other products in the market also have perishable characteristic. The items not only decay during the stockpiling period but they also deteriorate throughout transportation time. Since deteriorated rate and time is necessary, in this paper, an inventory routing problem with time windows for deteriorating items is developed. Particle Swarm Optimization (PSO) is used to solve the problem since PSO can solve problems in a reasonable period with near optimal solutions. We use two examples to illustrate the model. In a sensitivity analysis, way parameters that impact costs are demonstrated. Our results show that the deteriorating rate in inventory has bigger effects than deteriorating rate in the vehicle, so this research has a significant contribution and managers can give more effort to reduce deteriorating in inventory than the deteriorating rate in vehicles.

Keywords: Inventory, IRP, Time Windows, Deteriorating items, PSO

1. Introduction

A large number organizations face products distribution issue starting with the producer or merchant to a significant number retailers. That goal of the organizations will be how on streamline those items transportation and stock holding expense also. This issue is called as an inventory routing problem (IRP).

The IRP problem has been introduced for more than 30 years ago, and many

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variations of models and solutions have been studied [1]. The model variations include IRP with continuous move [2], vehicle multi-tours [3], stochastic demand [4] and time windows [5].

The time windows would a demand will a chance to be satisfied the place vehicles must land at the retailers within the time windows defined by the earliest and latest times. There are two types of time windows which are hard time windows and soft time windows. For the hard time windows, that vehicle must touch base on time, and for the soft time windows, vehicles can come before the most initial time or after the most recent time with penalty cost as a consequence.

The cyclic inventory routing problem is a variant of IRP where customer demand rates are stable, and the planning horizon is infinite [6]. Similar to the other IRP problem, in this issue, the objective function is to minimize total transportation and inventory cost for the long term. Banos [7] solved vehicle routing problems with time windows using a combination of evolutionary algorithm and simulated annealing. Shen [8] developed an IRP model with multiple customer, time period and transportation mode. The model was applied in the crude oil transportation problem. Qin [9] proposed a local search for solving a periodic inventory routing problem where replenishment period, delivery quantity and vehicle routing remains the same for every replenishment time. Raa and Aghezzaf [10] proposed solid modeling and solution approach for solving cyclic IRP by considering driving time restrictions. Osvald and Stirn [11] developed a vehicle routing problem with perishable food. They assumed food has limited life span and divide into three periods. In the first period, the food quality is stable, in the second-period quality decrease and in the last period the product is unacceptable. Chen et al. [12] developed vehicle routing problem with perishable products. They considered that value of the product decayed in particular
value compare to its original price. They did not consider product decay in term of product’s volume. Amorim and Alamada-Lobo [13] developed a routing problem model by considering two objectives which are distribution cost and freshness of food. They concluded that time windows have high impacts on the freshness level of product.

According to the authors’ extensive literature research, only a few papers discuss the inventory routing problem for deteriorating items. Deteriorating items are defined as decay, evaporation, obsolescence, and loss of quality marginal value of a commodity that results in decreasing usefulness from its original condition. Things like vegetables and fuels are two examples of deteriorating items. There is intensive research on deteriorating items such as a deteriorating inventory model with a permissible delay of payment [14], deteriorating inventory items by considering quality, inspection and maintenance, varying demand and production rates [15] and deteriorating items with preventive maintenance and rework [16]. Some researchers developed EOQ models for deteriorating items by considering two important variables which are pricing and replenishment policy simultaneously such as Sarkar et al. [17], Chung et al. [18], Yang et al. [19]. They concluded that solve pricing and replenishment policy simultaneously result in higher total profit than only considering one of them, especially for price dependent items. Therefore deteriorating inventory item models should not only consider replenishment policy but should find other relevant costs or strategies. Baker et al. [20] mentioned about some research in deteriorating inventory models, but they did not discuss any investigation on the deteriorating inventory model by considering transportation cost. Taleizadeh et al. [21] developed a deteriorating inventory model considering quantity discount, prepayment and transportation costs. However, the shipping cost is only from one vendor to one
This paper introduces inventory routing problem for deteriorating items where the model does not only consider inventory cost but also transportation cost and the items started to deteriorate when the items are loaded onto the truck. The shipping cost is seen from one depot to retailers and return to the depot. Time is an important parameter to be considered for deteriorating items, so the model is also considered time windows. This is the development of our previous research which is not considered time windows [22]. Since the model is an NP-hard model, a particle swarm optimization method (PSO) is used to solve the problem. PSO is a good way to solve vehicle routing problems such as discussed by [23], [24], [25], and [26]. This paper is divided into five sections. The first section discusses research motivation and research gap. An inventory deteriorating item model is developed in Section 2, and Section 3 shows the PSO method for solving the model. A numerical example is exhibited in Section 4 to get the best PSO parameters, and a sensitivity analysis is conducted to get some insights of the model. We derive conclusions in Section 5.

2. Mathematical Model

In this problem, a depot with boundless limit serves some retailers with distinctive and constant demand rate. As a vendor managed inventory (VMI) scheme, the depot might control inventory in his warehouse and inventory in all retailers store. Since the items are deteriorating items, items quantity to be conveyed to retailers should consider the amount of deteriorated items in vehicles and retailer’s warehouse. The IRP using cyclic IRP where the planning horizon is infinite. However, retailers set specific time windows for items delivered. When vehicles arrive at the retailer’s warehouse before the earliest time window, the retailers will charge the additional cost
to the depot. In the other case, when a vehicle lands after the latest time window, the retailer has to pay extra time for unloading workers. Those additional time installment will a chance to be charged to the depot. When an optimal cycle time and routing are set for every vehicle, the same decisions are applied for every cycle time. One vehicle could have a chance to deliver in consecutive days, and there is more than one vehicle might have an opportunity to be worked. The entire paper using assumptions, parameters and decision variables as below:

**Assumptions**

1. Vehicle have to return to the depot in 24 hours
2. One retailer only be served by one vehicle
3. Every vehicle could have more than one trip in particula cycle period
4. Vehicle capacity is fixed and homogeneous
5. Vehicle speed is constant
6. Items are deteriorated when vehicles depart from the depot with the constant deteriorating rate.
7. Deteriorating rate in vehicles is higher than the deteriorating rate in the warehouse.
8. Shortage is not allowed

**Parameters**

\( i \) : Retailers 1, 2, ..., \( I \)

\( R \) : Depot

\( v \) : vehicles 1, 2, ... \( V \)

\( d_i \) : Demand rate/units/time at retailer \( i \)
\( Q'_i \): Total demand at retailer \( i \)
\( s_{ij} \): Distance between location \( i \) to \( j \) where \( \{i, j\} \in S^+ = S \cup \{r\} \)
\( v e_v \): Average speed of vehicle \( v \)
\( t_{ij} \): Transportation time between location \( i \) to \( j \) where \( \{i, j\} \in S^+ \), where \( t_{ij} = s_{ij} / v e_v \)
\( \gamma \): Deteriorating cost (€/unit)
\( \delta_v \): Transportation cost for vehicle \( v \) (€/km)
\( v e_v \): Average speed of vehicle \( v \)
\( \varphi_i \): Handling cost at retailer \( i \) (€)
\( \psi_v \): Operational cost of vehicle \( v \) (€/unit time)
\( \eta_i \): Holding cost for retailer \( i \) (€/unit/unit time)
\( P_e \): Earliest penalty cost
\( P_l \): Latest penalty cost
\( k_v \): Maximum capacity of vehicle \( v \)
\( E_t \): Earliest time window
\( L_t \): Latest time window
\( T^v \): Cycle time of vehicle \( v \)
\( \theta_1 \): Deteriorating rate throughout delivery process
\( \theta_2 \): Deteriorating rate at retailer’s warehouse
\( t_0 \): Time of vehicle start from DC
\( t_{1i} \): Time of vehicle arrive at retailer \( i \)
\( t_{2i} \): Time when inventory stock is zero at retailer \( i \)
\( I_i \): Average inventory at retailer \( i \)
\( T^v_{min} \): Minimum cycle time of vehicle \( v \)
\( T^v_{max} \): Maximum cycle time of vehicle \( v \)

**Decision variables**

\( x^v_{ij} \):
\[
\begin{cases}
1, & \text{if there is delivery from retailer } i \text{ to retailer } j \text{ using vehicle } v \\
0, & \text{otherwise}
\end{cases}
\]
\( y^v \):
\[
\begin{cases}
1, & \text{if vehicle } v \text{ is used} \\
0, & \text{otherwise}
\end{cases}
\]
\( T^v_{EOQ} \): The optimal cycle time of vehicle \( v \)
\( A^v_i \): Time of vehicle \( v \) arrive at retailer \( i \)
$z_{ij}^v$: Quantity of items loaded by vehicle $v$ from retailer $i$ to retailer $j$

$L_{rr}^{nv}$: Total vehicle capacity at sub-tour $n$ in multi-tour $v$ start from depot $r$ back to depot $r$

Since there is no production in retailers, the inventory level in each retailer decrease by constant customer demand and deteriorating rate. The inventory level for deteriorating items at a particular time is:

$$\frac{dI(t)}{dt} + \theta_2 I(t) = -d_i$$  \hspace{1cm} (1)

Through some calculation processes and simplifications, one has:

$$I(t) = \left(e^{\theta_2 t} - 1\right)\frac{d_i}{\theta_2}$$  \hspace{1cm} (2)

The order quantity at retailer $i$ depend on demand and deteriorating rate during a cycle time. The order quantity for retailer $i$ is:

$$Q_i = \left(e^{\theta_2 T} - 1\right)\frac{d_i}{\theta_2}$$  \hspace{1cm} (3)

Item quantity during the transportation time is decreasing with a constant deteriorating rate. So items amount at a particular time throughout transportation can be modeled as:

$$\frac{dI(t)}{dt} + \theta_1 I(t) = 0$$  \hspace{1cm} (4)

Through some simplifications one has:

$$I(t) = \frac{Q'}{e^{\theta_1 t}}$$  \hspace{1cm} (5)

When $t_1$ is a transportation time and quantity of item needed at each retailer $Q$, then quantity of items that has been brought by each vehicle ($Q'$) is

$$Q'_i = Q_i e^{\theta t_1}$$  \hspace{1cm} (6)

From (3) and (6) can be derived the total quantities that have been delivered by
each vehicle to each retailer as:

$$Q_i' = \frac{d_i(e^{\theta_2 T} - 1)e^{\theta_1 t_1}}{\theta_2} \quad (7)$$

Figure 1 shows the total quantity loaded by each vehicle. The total amount is depending on transportation time, replenishment time, deteriorating rate and demand rate.

Using Figure 1, the total quantity that is delivered by each vehicle for a single tour can be modeled as:

$$L_{rr} = \sum_{i \in S_T} \left( e^{\theta_2 T} - 1 \right) e^{\theta_1 t_1} d_i \quad (8)$$

The total cost of the problem consists of transportation cost, handling cost, holding cost, deteriorating cost and penalty time cost. Transportation cost/unit time can be modeled as:

$$C_T = \frac{1}{T^v} \left( \sum_{i \in S^+} \sum_{j \in S^+} \left( \delta_v t_{ij} x_{ij}^v \right) \right) \quad (9)$$

Handling cost/unit time can be formulated as:

$$C_H = \sum_{i \in S} \left( \frac{\varphi_i}{T^v} \right) \left( \sum_{j \in S^+} x_{ij}^v \right) \quad (10)$$

The total quantity delivered by each vehicle at each retailer minus aggregate demand is equal to total deteriorating items at each retailer. The total deteriorating
items at each retailer can be modeled as:

$$D_i = \frac{(e^{\theta_2 T^v} - 1)e^{\theta_1 t_{il}d_i}}{\theta_2} - d_i T^v$$  \hspace{1cm} (11)$$

The total deteriorating items cost/unit time can be modeled as:

$$C_D = \sum_{i \in S} \left( \frac{\gamma D_i}{T^v} \right) \sum_{j \in S^+} x_{ij}^v$$  \hspace{1cm} (12)$$

The total inventory cost/unit time can be formulated as follows:

$$C_s = \sum_{i \in S} \left( \frac{\eta d_i}{T^v \theta_2} \left(-1 - \theta_2 T^v + e^{\theta_2 T^v}\right) \right) \left( \sum_{j \in S^+} x_{ij}^v \right)$$  \hspace{1cm} (13)$$

When a vehicle arrives before the earliest time windows or after the latest time windows, penalty costs are charged by the retailer to the vendor. The penalty costs per unit time at retailer $i$ can be modeled as:

$$CP_i = \frac{1}{T^v} \left( \sum_{i \in S} P_e \max(E_t - A_i^v, 0) + \sum_{i \in S} P_L \max(A_i^v - L_t, 0) \right)$$  \hspace{1cm} (14)$$

The objective function and the constraints of the model can be modeled as follows:

**IRP_{IP}: Minimize**

$$Z = \sum_{v \in m} \left[ \frac{1}{T^v} \left( \sum_{i \in S} \sum_{j \in S^+} \left( \delta_v v e_{vij} x_{ij}^v \right) \right) \right. \hspace{1cm} (15)$$

$$+ \sum_{i \in S} \left( \frac{\phi_i}{T^v} + \frac{\eta_i d_i}{T^v \theta_2^2} (-1 - \theta_2 T^v + e^{\theta_2 T^v}) \right) \left( \sum_{j \in S^+} x_{ij}^v \right)$$

$$+ \left( \frac{\gamma D_i}{T^v} \right) \left( \sum_{j \in S^+} x_{ij}^v \right)$$

$$+ \frac{1}{T^v} \left( \sum_{i \in S} P_e \max(E_t - A_i^v, 0) + \sum_{i \in S} P_L \max(A_i^v - L_t, 0) \right) \left. \right]$$

Subject to:
The objective function is represented by equation (15). The objective function consists of transportation cost, handling the cost, inventory holding costs, deteriorating cost and earliest and latest penalty cost. It is derived from the summation of the equation (9-14). Equation (16) assures that one retailer is served by one and only one vehicle. A first routing equation that ensures that once a vehicle enters a retailer, it will leave the retailer is shown in equation (17). Equation (18) state that the total transportation time of one vehicle cannot be higher than the cycle time. Equation (19) guarantees that volume of the items load in one vehicle is equal to total demand during one cycle, deteriorated items in a retailer’s warehouse during one period and deteriorated items during transportation time. When there is a delivery from the depot, the same vehicle
must be used. This condition is shown in Equation (20). Equation (21) assures that total demand and deteriorated items loaded in one vehicle cannot bigger than vehicle’s capacity. Equation (22) ensures that time of a vehicle arrive at retailers must be larger than vehicle comes at previous retailers and transportation time between retailers. A vehicle should return to the depot before a day so that it can be used in the next day is shown in equation (23). Since the model is a non-linear model and it is an NP-hard model, PSO algorithm is used to solve the model.

3. Particle Swarm Optimization for solving IRPDITW

This section describes a PSO algorithm to address an inventory routing problem for deteriorating items with time windows (IRPDITW). In this section, a PSO algorithm will be recommended to solve inventory routing problem for deteriorating items with time windows (IRPDITW). This section is isolated unde three parts. The first part examines the PSO framework; the second shows the decoding method and the final section discusses the routing for one day.

3.1. PSO framework

Particle swarm optimization is a population-based computation technique where each particle moves according to its best position and the best position of the other
particle. It is like a flock of birds collectively foraging for food, where the fitness function represents the food location. Detail of the PSO algorithm for solving multi tour inventory routing problem for deteriorating items is presented as Algorithm 1.

**Algorithm 1.**

1. Initialize particle by setting particles \( pr \), some iterations \( \alpha \) and some initial parameters. Set \( \overline{v}_0 = 0 \), personal best (Pbest) \( \overline{x}_{ps} = \overline{x}_{ps} \) and iteration \( i=1 \).

2. For \( i=1, \ldots, p \) decode \( \overline{x}_{ps} \) to a set vehicle route \( R_i \).

3. For \( i=1, \ldots, p \).calculate the performance measurement of \( R_i \) as \( Z_i \)

   Calculate optimal economic period using (24). The solution can be found by using Bisection method.

\[
\text{Minimize: } Z_i =
\]

\[
R_i \left( \sum_{s \in S} \sum_{e \in E} \delta_{e} v_{s,e} e_{i} \right) + \sum_{s \in S} \left( \frac{\varphi}{T_{EOQ}(C_v)} + \eta T_i + \frac{\gamma (T_{EOQ}(C_v))}{T_{EOQ}(C_v)} \right) +
\]

\[
\frac{1}{T_{EOQ}(C_v)} \sum_{s \in S} P_e \max (E_t - A^v_i, 0) + \sum_{s \in S} P_L \max (A^v_i - L_t, 0) \right) \]

\[ v \in V \]

subject to

\[
T_{min}^v \leq T_{EOQ}^v \leq T_{max}^v \quad v \in V \]

(25)

4. Update Pbest by setting \( \overline{x}_{ps} = \overline{x}_{ps} \) if \( Z_{x_{ps}} < Z_{x_{ps}} \)
5. Update Gbest by setting $\bar{x}_{G_s} = \bar{x}_{l_{ps}}$ if $Z_{xl_{ps}} < Z_{xG_s}$

6. Update the velocity and the position of each particle

$$
\bar{v}_{ps}(i + 1) = w(i) \times \bar{v}_{ps}(i) + u[0,1] \times c1(i) \times \left( \bar{x}_{G_s} - \bar{x}_{ps}(i) \right) + u[0,1] \\
\times c2(i) \times \left( \bar{x}_{l_{ps}} - \bar{x}_{ps}(i) \right)
$$

Update of the moment inertia using fitness distance ratio (FDR), and it can be shown as:

$$
w(i) = w(F) + \left( \frac{i-F}{1-F} \right) (w(1)0 - w(F))
$$

Calculate the new position using (28)

$$
\bar{x}_{ps}(i + 1) = \bar{x}_{ps}(i) + \bar{v}_{ps}(i + 1)
$$

7. If the generation meets the stopping criteria, stop. Otherwise, add generation by one and return to step 2.

8. Set Gbest from the last solution as the best option for multi-route inventory routing problem for deteriorating items.

3.2. The decoding method

Three parts represent a particle. A first part is some retailers, the second part is a constant value from 0 to 1, and the third part is the order of each retailer. The sequencing procedure using Algorithm 2.

Algorithm 2. Decoding method

1. Generate random numbers from 0 to 1 for the $x_{ps}$ values.
2. Sort in ascending order the value of $x_{ps}$ and set the sequence of the retailers

3. Particle representation for nine retailers can be represented in Table 1.

Once a global route has been established, the next step is allocating the path to vehicles by one day schedule considering vehicles capacity and time constraint. Since items are deteriorating, a quantity that should be brought by each vehicle consists of retailer demand and the amount of deteriorated items during delivery time and stock period in the warehouse.

3.3. The routing schedule in one day

Routing is set to get the balance of quantity in one day to be delivered and the distance. The routing method is thoroughly described in Algorithm 3. We calculate it using weights as shown in equation (27).

**Algorithm 3. Routing method**

1. For all $i$, calculate $we_i$ using equation (29)
   \[ we_i = d_i(t_{i-1,i}) \quad i \in S \]  

2. Calculate $W = \sum_{i \in S} we_i$

3. Set $i=1, ws_0=0, j = 1$

4. Set $ws_j = we_i$ If $we_i > W$, go to 7

5. Set $i=i+1$

6. Calculate $ws_j=ws_{j-1} + we_i$, if $ws_j<W$ go to 5 otherwise go to 7
7. Calculate $\lvert ws_{i-1} - W \rvert$ and $\lvert ws_i - W \rvert$. If $\lvert ws_{i-1} - W \rvert < \lvert ws_i - W \rvert$, allocate 1 to $i-1$ into route $j$, otherwise allocate 1 to $i$ into route $j$

8. Set $i=0$, $ws_0=0$, $j = j+1$ and go to 5.

9. If all retailers have been allocated then finish

The next step is setting how many days the routing for the determined cycle time should be assigned to one vehicle. The solution is done by using Algorithm 4.

**Algorithm 4. Routing allocation in one day**

1. Set $sr=1$, $k=1$

2. Calculate $T_{min} = \sum_k T_k$

3. Calculate $T_{max} = \frac{K_v}{\sum_{k \in ST_n} \left( e^{\theta_1 t_{1k}} d_k \right)}$, if $T_{min} > T_{max}$ go to 5

4. Set $k=k+1$ and go to 2

5. Put 1 to $k-1$ into sub routing $sr$. If all retailers in routing have been allocated or the $T_{min}$ violating 24 hours for routing time limitation for one day then go to 6, otherwise, go to 1.

6. Calculate the fitness function

7. Set $K$ as number of vehicles and $k = 1$

8. Set discrete random variable from 1 to some retailers ($n=U(1..N)$).

9. Allocate the first $n$ retailers to vehicle $k$

10. If $k<K$, then $k=k+1$ and go to 9, otherwise go to 11
11. Calculate the fitness function. If $K_v < \sum_{k \in ST_n} \left( \frac{e^{\theta_2 T - 1}}{\theta_2} \right) e^{\theta_1 t_k d_k}$ then fitness function = fitness function + infeasible penalty cost, where infeasible penalty cost is a big value.

12. Choose days allocating vehicles with the best fitness function

4. **Numerical Example and sensitivity analysis**

A numerical example is conducted to show how the model works and to get the best PSO solution and a sensitivity analysis is carried out to get management insight of the model.

4.1. **A numerical example**

A set data from Aghezzaf (2006) is used for the first example, where 15 retailers are supplied from one depot. Table 2 shows demand rate at each retailer where each retailer has different demand rate. The other parameters used in this numerical example are vehicle capacity equal to 100 units, average vehicle speed 50 km/hour, fixed operating cost €50/hour, transportation time €1/km, inventory holding cost €0.1/unit/hour and fixed handling cost is equal to €50. There are two vehicles available.
The best solution is derived when Pbest weight = 0.5 and Gbest weight = 2 with a total cost per unit time is equal to € 92.2715 with route schedules for every vehicle in every day as shown in Table 3.

4.2. Sensitivity analysis

Sensitivity analysis is conducted by changing one particular parameter and keep the other parameters with the same value. Different values of the deteriorating rate of the vehicle, deteriorating rate at the warehouse, latest time windows and inventory cost are used to analysis the impact of the individual's parameters also provide a few management insight. The deteriorating rate at vehicle and store are necessary to be analyzed since we need to recognize which impact is more significant to the other. The latest time windows need to be analyzed to show the significance of time windows to the decision. We only analyzed the latest time windows since most solutions show the problem has a tendency to be late delivery than previous delivery. The inventory cost is examined since in many research inventory cost significantly influences the total cost of the inventory issue model. The sensitivity analysis is conducted by decreasing and increasing the parameter value to 20% and 40%. The
sensitivity analysis results are shown in Table 4 -Table 7.

From Table 4-7 we can figure out a sensitivity analysis for the four parameters as shown in Figure 2.

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Put Figure 2 here

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Figure 2 shows that the total cost per unit time is significantly sensitive to varies the value of inventory cost and the total cost increase as the inventory cost increase. This result is consistent with the other research in inventory modeling. The total cost per unit time has an opposite trend with latest time windows. Total cost decrease as the most recent time windows increase. However, the effect of varying latest time windows to the total cost is smaller than inventory cost. In practice, it is better to give higher effort to reduce inventory cost than asking for retailers to increase their latest time windows. Even though the effect of deteriorating rates to the total cost are smaller to the effect of latest time windows and inventory cost, deteriorating rates affect the total cost. The total cost increase as deteriorating rates
increase. These results are consistent with some previous research on deteriorating inventory items. The effect of deteriorating rate in inventory to the total cost is larger than the deteriorating rate in the vehicle. This finding shows the manager has to put more effort to reduce the deteriorating rate in inventory than the deteriorating rate in the vehicle to reduce the total cost. The effort to reduce the deteriorating rate in the warehouse is also easier than reducing the deteriorating rate in the vehicle. The effect of varying environmental temperature is also more comfortable to be handled in a warehouse than during transportation time. Temperature stability is easier to be controlled in the warehouse than during shipping time. The result also supports the purpose of this research for considering the deteriorating rate in inventory and vehicle instead of considering deteriorating items in the vehicle as shown by previous research before.

5. Conclusion

In this research, we attempt with examine deteriorating rate in the cyclic inventory routing problem. From our intensive literature study, there is no research which has examined deteriorating items in the cyclic inventory problem. We have hypotheses that deteriorating rate influences inventory total cost of any items that have deteriorating characteristic. A mathematical model is developed to solve the problem. Since the model is an NP-hard problem, a Particle Swarm Optimization (PSO) drawn
up to address the problem. A numerical illustration is conducted to show how the model work. Sensitivity analysis has been done by changing the parameter of one variable and keep the same values of the other parameters. The changing parameters are latest time windows, the deteriorating rate in the vehicle, deteriorating rate in warehouse and inventory holding cost. The sensitivity analysis shows consistent results as results of previous inventory deteriorating models and gives some management insights. The holding cost provides for the greatest impact on the total cost comparable to the other parameters. So, it is better to organizations to provide the highest effort for reducing inventory holding cost to get the smaller total cost. Even though the effect of deteriorating rates is not as big as inventory holding cost, deteriorating rate influences the total cost. Deteriorating rate in inventory results in a higher effect on the total cost than deteriorating rate in vehicles. So the contribution of this research by considering both deteriorating rates in inventory and vehicle instead of only considering the deteriorating rate in a vehicle alone is critical. The model can be developed by acknowledging perishable items and price dependent time.
References


List of Captions

Figure 1. Inventory level in vehicle and warehouse every sub-tour and in a multi tour

Figure 2. Sensitivity analysis for the total cost per unit time

Table 1. Particle representation

Table 2. Demand rate at retailers

Table 3. The route schedules for every vehicle

Table 4. The total cost per unit time for varies value of latest time windows

Table 5. The total cost per unit time for varies value of deteriorating rate at vehicle

Table 6. The total cost per unit time for varies value of deteriorating rate at warehouse

Table 7. The total cost per unit time for varies value of inventory cost
Figure 2. Inventory level in vehicle and warehouse every sub-tour and in a multi tour

Figure 2. Sensitivity analysis for the total cost per unit time
### Table 2. Particle representation

<table>
<thead>
<tr>
<th>Retailers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>$x_{ps}$</td>
<td>0.47</td>
<td>0.61</td>
<td>0.29</td>
<td>0.26</td>
<td>0.43</td>
<td>0.66</td>
<td>0.23</td>
<td>0.95</td>
<td>0.9</td>
</tr>
<tr>
<td>Sequence</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>9</td>
<td>8</td>
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### Table 2. Demand rate at retailers

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Demand rate (units/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.109</td>
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<td>2</td>
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<td>3</td>
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<td>5</td>
<td>0.134</td>
</tr>
<tr>
<td>6</td>
<td>0.429</td>
</tr>
<tr>
<td>7</td>
<td>0.381</td>
</tr>
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<td>8</td>
<td>0.503</td>
</tr>
<tr>
<td>9</td>
<td>0.187</td>
</tr>
<tr>
<td>10</td>
<td>0.123</td>
</tr>
<tr>
<td>11</td>
<td>0.953</td>
</tr>
<tr>
<td>12</td>
<td>0.638</td>
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<tr>
<td>13</td>
<td>0.247</td>
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<td>14</td>
<td>0.188</td>
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<tr>
<td>15</td>
<td>0.441</td>
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Table 3. The route schedules for every vehicle

<table>
<thead>
<tr>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T Optimal = 120 hours (5 days)</strong></td>
<td><strong>T Optimal = 192 hours (8 days)</strong></td>
</tr>
<tr>
<td><strong>Day</strong></td>
<td><strong>Route</strong></td>
</tr>
<tr>
<td>1</td>
<td>D – 13 – D</td>
</tr>
<tr>
<td>2</td>
<td>D – 15 – 12 – D</td>
</tr>
<tr>
<td>3</td>
<td>D – 9 – D</td>
</tr>
<tr>
<td>4</td>
<td>D – 4 – 11 – D</td>
</tr>
<tr>
<td>5</td>
<td>D – 8 – D</td>
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</tbody>
</table>

Table 4. The total cost per unit time for varies value of latest time windows

<table>
<thead>
<tr>
<th>Trial</th>
<th>Total cost per unit time (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-40%</td>
</tr>
<tr>
<td>1</td>
<td>99.6382</td>
</tr>
<tr>
<td>2</td>
<td>102.079</td>
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<tr>
<td>3</td>
<td>106.4597</td>
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<td>4</td>
<td>106.3438</td>
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<td>5</td>
<td>100.6295</td>
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<tr>
<td>Average</td>
<td>103.03</td>
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</table>

Table 5. The total cost per unit time for varies value of deteriorating rate at vehicle

<table>
<thead>
<tr>
<th>Trial</th>
<th>Total cost per unit time (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-40%</td>
</tr>
<tr>
<td>1</td>
<td>89.7673</td>
</tr>
<tr>
<td>2</td>
<td>90.7791</td>
</tr>
<tr>
<td>3</td>
<td>97.7465</td>
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<td>4</td>
<td>90.4186</td>
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<tr>
<td>5</td>
<td>93.2059</td>
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<tr>
<td>Average</td>
<td>92.3835</td>
</tr>
</tbody>
</table>
Table 6. The total cost per unit time for varies value of deteriorating rate at warehouse

<table>
<thead>
<tr>
<th>Trial</th>
<th>Total cost per unit time (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-40%</td>
</tr>
<tr>
<td>1</td>
<td>90.3121</td>
</tr>
<tr>
<td>2</td>
<td>89.9552</td>
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<tr>
<td>3</td>
<td>93.3972</td>
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<td>4</td>
<td>97.6252</td>
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<tr>
<td>5</td>
<td>91.2684</td>
</tr>
<tr>
<td>Average</td>
<td>92.5116</td>
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</tbody>
</table>

Table 7. The total cost per unit time for varies value of inventory cost

<table>
<thead>
<tr>
<th>Trial</th>
<th>Total cost per unit time (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-40%</td>
</tr>
<tr>
<td>1</td>
<td>77.0046</td>
</tr>
<tr>
<td>2</td>
<td>82.7451</td>
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<tr>
<td>3</td>
<td>79.0822</td>
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<tr>
<td>4</td>
<td>72.5536</td>
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<tr>
<td>5</td>
<td>78.4133</td>
</tr>
<tr>
<td>Average</td>
<td>72.5536</td>
</tr>
</tbody>
</table>
Short Biography

Gede Agus Widyadana is a lecturer at Industrial Engineering Department, Faculty of Industrial Technology, Petra Christian University, Surabaya, Indonesia. His research interests include inventory management, supply chain management, and operation research. He has published in several journals such as OMEGA, International Journal of Production Economics, International Journal of Systems Science, International Journal of Production Economics and Applied Mathematics and Modelling.

Takashi Irohara is a Professor in the Department of Information and Communication Sciences, Faculty of Science and Technology, Sophia University, Tokyo, Japan. His research interests include facility logistics (order picking, inbound/outbound truck scheduling in the warehouse, facility layout problem, material handling), supply chain management (inventory control, transportation, and vehicle routing problem), production scheduling and humanitarian relief logistics. He is a board member of Japan Industrial Management Association, Japanese Material Handling Society and APIEMS (Asia Pacific Industrial Engineering and Management Systems Conference).