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Modelling multi-tour inventory routing problem for deteriorating items with time windows

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Abstract. In the recent past, deteriorating inventory has been intensively studied. However, only a few researchers have focused on the inventory routing problem for deteriorating items. There are many items, such as food and electronic products, which deteriorate over time, and many other products in the market have perishable characteristics. The items not only decay during the stockpiling period, but also deteriorate throughout transportation time. To determine deterioration rate and time, this study develops an inventory routing problem with time windows for deteriorating items. Particle Swarm Optimization (PSO) is used to solve the problem since PSO can solve problems in a reasonable amount of time with near-optimum solutions. Two examples are used to illustrate the model. In sensitivity analysis, the parameters that impact costs are demonstrated. Our results show that the deterioration rate in inventory has larger effects than that in the vehicle; therefore, this research study makes a significant contribution, and managers can devote more effort to reducing deterioration rate in inventory rather than the deterioration rate in vehicles.

1. Introduction

The problem of product distribution is prevalent among a large number of organizations, including producers or merchants to a significant number of retailers. The goal of organizations is to determine how such items are transported and what the stock holding expense is. This problem is called an Inventory Routing Problem (IRP).

The IRP problem was introduced more than 30 years ago, and many variations of related models and solutions have been studied since then [1]. The model variations include IRP with continuous move [2], multi-vehicle tours [3], stochastic demand [4], and time windows [5].

The time windows provides a chance to satisfy a demand and to deliver products by vehicles to retailers at the earliest and latest times. There are two types of time windows: hard time windows and soft time windows. For the hard time windows, vehicles must touch the base on time; for the soft time windows, vehicles can arrive before the initial set time or after the most recent time with penalty cost included.

The cyclic inventory routing problem is a variant of IRP where customer demand rates are stable and planning horizon is infinite [6]. Similar to the other IRP problems, in this issue, the objective function is to minimize total transportation and inventory cost for the long term. Banos et al. [7] solved vehicle routing problems with time windows using a combination of evolutionary algorithm and simulated annealing. Shen [8] developed an IRP model with

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multiple customers, time periods, and transportation modes. The model was applied to the crude oil transportation problem. Qin et al. [9] proposed a local search for solving a periodic inventory routing problem where replenishment period, delivery quantity, and vehicle routing remain the same for every replenishment time. Raa and Aghezzaf [10] proposed solid modeling and solution approach for solving cyclic IRP by considering driving time restrictions. Osvald and Stirn [11] developed a vehicle routing problem with perishable food. They assumed that food had limited life span and divided it into three periods. In the first period, the food quality is stable; in the second period, quality decrease; in the last period, the product is unacceptable. Chen et al. [12] developed a vehicle routing problem with perishable products. Accordingly, the value of the product decayed at a particular time limit is comparable to its original price. They did not consider product decay in terms of product's volume. Amorim and Alamada-Lobo [13] developed a routing problem model by considering two objectives: distribution cost and freshness of food. They concluded that time windows had high impact on the freshness level of product.

According to the authors' extensive literature research, only few papers have discussed the inventory routing problem for deteriorating items. A deteriorating item are defined as a commodity exposed to decay, evaporation, obsolescence, and loss of quality marginal value, resulting in a decrease in its usefulness as compared to its original condition. Things like vegetables and fuels are the two examples of deteriorating items. There is intensive research on deteriorating items such as a deteriorating inventory model with a permissible delay of payment [14], deteriorating inventory items considering quality, inspection, and maintenance, varying demands and production rates [15], and deteriorating items with preventive maintenance and rework [16]. Some researchers, such as Sarkar et al. [17], Chung et al. [18], and Yang et al. [19], have developed EOQ models for deteriorating items by simultaneously considering two important variables including pricing and replenishment policy. They concluded that solving pricing and replenishment policy problems simultaneously results in higher total profit instead of only considering one of them, especially for price-dependent items. Therefore, deteriorating inventory item models should not only consider replenishment policy, but also should find other relevant costs or strategies. Baker et al. [20] mentioned some research studies concerning deteriorating inventory models; however, they did not discuss any investigation into the deteriorating inventory model considering transportation cost. Taleizadeh et al. [21] developed a deteriorating inventory model considering quantity discount, prepayment, and transportation costs. However, the shipping cost is only from one vendor to one buyer. This paper introduces an inventory routing problem for deteriorating items, and the model considers not only inventory cost, but also transportation cost and the items that started to deteriorate when the items are loaded onto the truck. The shipping cost involves the total cost of transporting goods from one depot to retailers and returning them to the depot. Time is an important parameter to consider for deteriorating items; therefore, time windows is an important element of the model. This is the development of our previous research, which did not consider time windows [22]. Since the model is an NP-hard model, a Particle Swarm Optimization method (PSO) is used to solve the problem. PSO can solve vehicle routing problems as discussed by [23-26]. This paper is divided into five sections. The first section discusses research motivation and research gap. An inventory deteriorating item model is developed in Section 2, and Section 3 shows the PSO method for solving the model. A numerical example is exhibited in Section 4 to obtain the best PSO parameters, and a sensitivity analysis is conducted to get some insights into the model. Conclusions are presented in Section 5.

2. Mathematical model

In this problem, a depot with a boundless limit serves some retailers with distinctive and constant demand rates. As a Vendor Managed Inventory (VMI) scheme, the depot might control inventory in his warehouse and inventory in all retailers store. Since the items are deteriorating ones, suppliers should consider the amount of deteriorated items in vehicles and retailer's warehouse quantity of items conveyed to retailers. The IRP uses cyclic IRP where the planning horizon is infinite. However, retailers set specific time windows for the delivered items. When vehicles arrive at the retailer's warehouse before the earliest time window, the retailers will charge additional costs to the depot. In another case, when a vehicle lands after the latest time window, the retailer has to allocate extra time for workers to unload the truck. Those additional time installments provide a chance to be charged in the depot. When optimal cycle time and routing are set for every vehicle, the same decisions are applied to every cycle time. One vehicle may have a chance to deliver in consecutive days, and there is more than one vehicle that might have an opportunity to work. The entire paper uses assumptions, parameters, and decision variables as follows:

Assumptions

1. Vehicle has to return to the depot in 24 hours;
2. One retailer is served by only one vehicle;

3. Every vehicle can have more than one trip in a particular cyclic period;
4. Vehicle capacity is fixed and homogeneous;
5. Vehicle speed is constant;
6. Items are deteriorated when vehicles depart from the depot with a constant deteriorating rate;
7. Deteriorating rate in vehicles is higher than that in the warehouse;
8. Shortage is not allowed.

Parameters

i	Retailers $1, 2, ..., I$;
R	Depot;
v	Vehicles $1, 2, ...V$;
d_i	Demand rate/units/time at retailer i;
Q'_i	Total demand at retailer i;
s_{ij}	Distance between locations i to j where $\{i, j\} \in S^+ = S \cup \{r\}$;
ve_v	Average speed of vehicle v;
t_{ij}	Transportation time between locations i to j where $\{i, j\} \in S^+$, where $t_{ij} = s_{ij}/ve_v$;
γ	Deterioration cost (€/unit);
δ_v	Transportation cost for vehicle v (€/km);
ve_v	Average speed of vehicle v;
φ_i	Handling cost for retailer i (€);
ψ_v	Operational cost of vehicle v (€/unit time);
η_i	Holding cost for retailer i (€/unit/unit time);
P_e	Earliest penalty cost;
P_l	Latest penalty cost;
k_v	Maximum capacity of vehicle v;
E_t	Earliest time window;
L_t	Latest time window;
T^v	Cycle time of vehicle v;
θ_1	Deterioration rate throughout delivery process;
θ_2	Deterioration rate at retailer's warehouse;
t_0	Time of vehicle's start from DC;
t_{1i}	Time of vehicle's arrival to retailer i;
t_{2i}	Time when inventory stock is zero at retailer i;
I_i	Average inventory at retailer i;
$T^v_{\min}$	Minimum cycle time of vehicle v;
$T^v_{\max}$	Maximum cycle time of vehicle v.

Decision variables

$$x^v_{ij} \quad \begin{cases} 1 & \text{if there is delivery from retailer } i \\ & \text{to retailer } j \text{ using vehicle } v \\ 0 & \text{otherwise} \end{cases}$$

$$y^v \quad \begin{cases} 1 & \text{if vehicle } v \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

T^v_{EOQ}	The optimal cycle time of vehicle v;
A^i_v	Time of vehicle v arrival to retailer i;
z^v_{ij}	Quantity of items loaded by vehicle v from retailer i to retailer j;
L^{nv}_{rr}	Total vehicle capacity at sub-tour n in multi-tour v starting from depot r back to depot r.

Since there is no production in retailers, the inventory level in each retailer decreases by constant customer demand and deterioration rate. The inventory level for deteriorating items at a particular time is:

$$\frac{dI(t)}{dt} + \theta_2 I(t) = -d_i. \tag{1}$$

Through some calculation processes and simplifications, one has:

$$I(t) = \left(e^{\theta_2 t} - 1\right)\frac{d_i}{\theta_2}. \tag{2}$$

The order quantity at retailer i depends on demand and deterioration rate during a cycle time. The order quantity for retailer i is:

$$Q_i = \left(e^{\theta_2 T} - 1\right)\frac{d_i}{\theta_2}. \tag{3}$$

Item quantity during the transportation time decreases with a constant deterioration rate. Therefore, quantity of items at a particular time throughout transportation can be modeled as follows:

$$\frac{dI(t)}{dt} + \theta_1 I(t) = 0. \tag{4}$$

Through some simplifications, one has:

$$I(t) = \frac{Q'}{e^{\theta_1 t}}. \tag{5}$$

When t_1 is the transportation time and Q is the quantity of items required by each retailer, the quantity of items carried by each vehicle (Q') is determined as follows:

$$Q'_i = Q_i e^{\theta t_{1i}}. \tag{6}$$

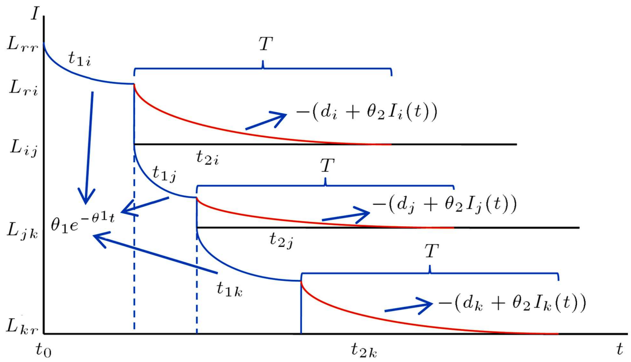

Figure 1. Inventory level of vehicle and warehouse of every sub-tour and of multiple tours.

From Eqs. (3) and (6), one can derive the total quantities that have been delivered by each vehicle to each retailer as follows:

$$Q_i' = \frac{d_i(e^{\theta_2 T} - 1)e^{\theta_1 t_{1i}}}{\theta_2}. \tag{7}$$

Figure 1 shows the total quantity loaded by each vehicle. The total amount depends on transportation time, replenishment time, deterioration rate, and demand rate.

According to Figure 1, the total quantity delivered by each vehicle for a single tour can be modeled as follows:

$$L_{rr} = \sum_{i \in ST_n} \frac{(e^{\theta_2 T} - 1)e^{\theta_1 t_{1i}} d_i}{\theta_2}. \tag{8}$$

The total cost of the problem consists of transportation cost, handling cost, holding cost, deterioration cost, and penalty time cost. Transportation cost/unit time can be modeled as follows:

$$C_T = \frac{1}{T^v}\left(\sum_{i \in S+}\sum_{j \in S+}(\delta_v t_{ij} x_{ij}^v)\right). \tag{9}$$

Handling cost/unit time can be formulated as follows:

$$C_H = \sum_{i \in S}\left(\frac{\varphi_i}{T^v}\right)\left(\sum_{j \in S+} x_{ij}^v\right). \tag{10}$$

The total quantity delivered by each vehicle to each retailer minus aggregate demand is equal to the total deteriorating items for each retailer. The total deteriorating items for each retailer can be modeled as follows:

$$D_i = \frac{(e^{\theta_2 T^v} - 1)e^{\theta_1 t_{1i}} d_i}{\theta_2} - d_i T^v. \tag{11}$$

The total deteriorating items' cost/unit time can be modeled as follows:

$$C_D = \sum_{i \in S}\left(\frac{\gamma_i D_i}{T^v}\right)\sum_{j \in S+} x_{ij}^v. \tag{12}$$

The total inventory cost/unit time can be formulated as follows:

$$C_s = \sum_{i \in S}\left(\frac{\eta_i d_i}{T^v \theta_2^2}(-1-\theta_2 T^v + e^{\theta_2 T^v})\right)\left(\sum_{j \in S+} x_{ij}^v\right). \tag{13}$$

When a vehicle arrives before the earliest time windows or after the latest time windows, penalty costs are charged by the retailer to the vendor. The penalty costs per unit time for retailer i can be modeled as follows:

$$CP_i = \frac{1}{T^v}\left(\sum_{i \in S} P_e \max(E_t - A_i^v, 0) + \sum_{i \in S} P_L \max(A_i^v - L_t, 0)\right). \tag{14}$$

The objective function and the constraints of the model can be modeled as follows:

IRP_{IP} : Minimize

$$Z = \sum_{v \in m}\left[\frac{1}{T^v}\left(\sum_{i \in S}\sum_{j \in S+}(\delta_v v e_v t_{ij} x_{ij}^v)\right) + \sum_{i \in S}\left(\left(\frac{\varphi_i}{T^v} + \frac{\eta_i d_i}{T^v \theta_2^2}(-1 - \theta_2 T^v + e^{\theta_2 T^v}) + \left(\frac{\gamma D_i}{T^v}\right)\right)\right)\left(\sum_{j \in S+} x_{ij}^v\right) + \frac{1}{T^v}\left(\sum_{i \in S} P_e \max(E_t - A_i^v, 0) + \sum_{i \in S} P_L \max(A_i^v - L_t, 0)\right)\right], \tag{15}$$

subject to:

$$\sum_{v \in V}\sum_{i \in S+} x_{ij}^v = 1 \qquad j \in S, \tag{16}$$

$$\sum_{i \in S+} x_{ij}^v - \sum_{k \in S+} x_{jk}^v = 0 \qquad j \in S^+,\ v \in V, \tag{17}$$

$$\sum_{i\in S}\sum_{j\in S+} t_{ij}^v x_{ij}^v - T^v \leq 0 \qquad v\in V, \tag{18}$$

$$\sum_{v\in V}\sum_{i\in S+} z_{ij}^v - \sum_{v\in V}\sum_{k\in S+} z_{jk}^v = \frac{d_j(e^{\theta_2 T^v}-1)e^{\theta_1 t_{ij} x_{ij}^v}}{\theta_2}, \quad i,j\in S, \tag{19}$$

$$x_{rj}^v - y^v \leq 0, \qquad v\in V,\ j\in S, \tag{20}$$

$$z_{rj}^v \leq k(v), \qquad v\in V,\ j\in S, \tag{21}$$

$$A_i^v + t_{ij}X_{ij}^v \leq A_j^v, \qquad v\in V,\ i,j\in S^+, \tag{22}$$

$$A_d^v \leq 24, \qquad v\in V \tag{23}$$

$$x_{ij}^v \in \{0,1\},\ z_{ij}^v \geq 0,\ y^v\in\{0,1\},\ T^v\geq 0 \quad \text{for all } v\in V,\ i,j\in S^+.$$

The objective function is represented by Eq. (15). The objective function consists of transportation cost, handling cost, inventory holding costs, deterioration cost, and earliest and latest penalty costs. It is derived from the summation of Eqs. (9)-(14). Eq. (16) assures that one retailer is served by one and only one vehicle. The first routing equation ensures that once a vehicle enters a retailer, it will leave the retailer, as shown in Eq. (17). Based on Eq. (18), the total transportation time of one vehicle cannot be higher than the cycle time. Eq. (19) guarantees that the volume of the items load in one vehicle is equal to total demand during one cycle, deteriorated items in a retailer's warehouse during one period, and deteriorated items during transportation time. When there is a delivery from the depot, the same vehicle must be used. This condition is shown in Eq. (20). Eq. (21) assures that the total demand and deteriorated items loaded in one vehicle cannot exceed the vehicle's capacity. Eq. (22) ensures that the time a vehicle arrives at retailers must be more than the time a vehicle comes at previous retailers and the transportation time between retailers. A vehicle should return to the depot before a day so that it can be used in the next day, as shown in Eq. (23). Since the model is a non-linear one and is an NP-hard model, PSO algorithm is used to solve the model.

3. Particle swarm optimization for solving IRPDITW

This section describes a PSO algorithm to address an Inventory Routing Problem for Deteriorating Items with Time Windows (IRPDITW). In this section, a PSO algorithm will be recommended to solve inventory routing problem for deteriorating items with time windows (IRPDITW). This section is divided into three parts. The first part examines the PSO framework; the second shows the decoding method, and the final section discusses the routing for one day.

3.1. PSO framework

Particle swarm optimization is a population-based computation technique by which each particle moves according to its best position and the best position of the other particle. It is like a flock of birds collectively foraging for food, where the fitness function represents the food location. Details of the PSO algorithm for solving multi-tour inventory routing problem for deteriorating items are presented as Algorithm 1.

Algorithm 1

1. Initialize particle by setting particles (pr), some iterations (α), and some initial parameters. Set $\overrightarrow{v_0}=0$, personal best (Pbest) $\overrightarrow{xl_{ps}}=\overrightarrow{x_{ps}}$, and iteration $i=1$.
2. For $i=1,...,p$, decode $\overrightarrow{x_{ps}}$ to a set of vehicle routes R_i.
3. For $i=1...,p$, calculate the performance measurement of R_i as Z_i.

   Calculate the optimal economic period using Eq. (24). The solution can be found using Bisection method:

   Minimize:

$$Z_i = R_i\Bigg(\frac{\sum_{i\in S}\sum_{j\in S^+}\delta_v v e_v t_{ij}}{T_{EOQ}(C_v)} + \sum_{i\in S}\bigg(\frac{\varphi}{T_{EOQ}(C_v)} + \eta\bar{I}_l + \frac{\gamma(L_r^{iv}-d_i T_{EOQ}(C_v))}{T_{EOQ}(C_v)}\bigg) + \frac{1}{T_{EOQ}(C_v)}\sum_{i\in S} P_e\max(E_t - A_i^v,0) + \sum_{i\in S} P_L\max(A_i^v - L_t,0)\Bigg), \qquad v\in V. \tag{24}$$

   subject to:

$$T_{\min}^v \leq T_{EOQ}^v \leq T_{\max}^v, \qquad v\in V. \tag{25}$$

4. Update Pbest by setting $\overrightarrow{xl_{ps}}=\overrightarrow{x_{ps}}$ if $Z_{x_{ps}} < Z_{xl_{ps}}$.
5. Update Gbest by setting $\overrightarrow{xg_s}=\overrightarrow{xl_{ps}}$ if $Z_{xl_{ps}} < Z_{xg_s}$.

6. Update the velocity and the position of each particle:

$$\overrightarrow{v_{ps}}(i+1) = w(i) \times \overrightarrow{v_{ps}}(i) + u[0,1] \times c1(i) \times (\overrightarrow{xg_s} - \overrightarrow{x_{ps}}(i)) + u[0,1] \times c2(i) \times \left(\overrightarrow{xl_{ps}} - \overrightarrow{x_{ps}}(i)\right). \tag{26}$$

   Update the moment inertia using Fitness Distance Ratio (FDR), as shown in the following:

$$w(i) = w(F) + \left(\frac{i-F}{1-F}\right)(w(1)0 - w(F)). \tag{27}$$

   Calculate the new position using Eq. (28):

$$\overrightarrow{x_{ps}}(i+1) = \overrightarrow{x_{ps}}(i) + \overrightarrow{v_{ps}}(i+1). \tag{28}$$

7. If the generation meets the stopping criterion, stop. Otherwise, add generation by one and return to step 2.
8. Set Gbest from the last solution as the best option for multi-route inventory routing problem for deteriorating items.

3.2. The decoding method

Three parts represent a particle. The first part represents some retailers; the second part is a constant value from 0 to 1; the third part is the order of each retailer. The sequencing procedure uses Algorithm 2.

Algorithm 2. Decoding method

1. Generate random numbers from 0 to 1 for the x_{ps} values.
2. Sort the value of x_{ps} and set the sequence of the retailers in ascending order.
3. Particle representation for nine retailers can be represented in Table 1.

Once a global route has been established, the next step is allocating the path to vehicles by one-day schedule considering vehicles capacity and time constraint. Since items are deteriorating, a quantity that should be brought by each vehicle consists of retailer demand and the amount of deteriorated items during delivery time and stock period in the warehouse.

3.3. The routing schedule in one day

Routing is set to obtain the balance between delivered quantity in one day and the distance. The routing method is thoroughly described in Algorithm 3. It is calculated using weights, as shown in Eq. (27).

Algorithm 3. Routing method

1. For all i, calculate we_i using Eq. (29):

$$we_i = d_i\left(t_{(i-1,i)}\right), \qquad i \in S. \tag{29}$$

2. Calculate $W = \sum_{i \in S} we_i$.
3. Set $i = 1$, $ws_0 = 0$, $j = 1$.
4. Set $ws_i = we_i$. If $we_i > W$, go to 7.
5. Set $i = i + 1$.
6. Calculate $ws_i = ws_{i-1} + we_i$. If $ws_i < W$ go to 5, otherwise go to 7.
7. Calculate $|ws_{i-1} - W|$ and $|ws_i - W|$. If $|ws_{i-1} - W| < |ws_i - W|$, allocate 1 to $i-1$ into route j, otherwise allocate 1 to i in route j.
8. Set $i = 0$, $ws_0 = 0$, $j = j + 1$, and go to 5.
9. If all retailers have been allocated, then it is finished.

The next step determine the number of days to assign for one vehicle under the routing at the determined cycle time. The solution is obtained using Algorithm 4.

Algorithm 4. Routing allocation in one day

1. Set $sr = 1$, $k = 1$;
2. Calculate $T_{\min} = \sum_k T_k$;
3. Calculate $T_{\max} = \frac{K_v}{\sum_{k \in ST_n} \frac{(e^{\theta_2 T} - 1)e^{\theta_1 t_1 k} d_k}{\theta_2}}$. If $T_{\min} > T_{\max}$, go to 5;
4. Set $k = k + 1$ and go to 2;
5. Put 1 to $k - 1$ into subrouting sr. If all retailers in routing have been allocated or the $T_{\min}$ violating 24 hours for routing time limitation for one day, then go to 6, otherwise go to 1;
6. Calculate the fitness function;
7. Set K as the number of vehicles and $k = 1$;
8. Set a discrete random variable from 1 to some retailers ($n = U(1...N)$);

Table 1. Particle representation.

Retailers	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
x_{ps}	0.47	0.61	0.29	0.26	0.43	0.66	0.23	0.95	0.9
Sequence	5	6	3	2	4	7	1	9	8

9. Allocate the first n retailers to vehicle k;
10. If $k < K$, then $k = k + 1$ and go to 9, otherwise go to 11;
11. Calculate the fitness function. If $K_v < \sum_{k \in ST_n} \frac{(e^{\theta_2 T}-1)e^{\theta_1 t_{1k}} d_k}{\theta_2}$, then fitness function = fitness function + infeasible penalty cost, where infeasible penalty cost is a big value;
12. Choose days to allocate to vehicles with the best fitness function.

4. Numerical Example and sensitivity analysis

A numerical example is conducted to show how the model works and to obtain the best PSO solution; then, a sensitivity analysis is carried out to obtain a management insight into the model.

4.1. A numerical example

A dataset from Aghezzaf (2006) is used for the first example, where 15 retailers are supplied from one depot. Table 2 shows the demand rate for each retailer where each retailer has different demand rates. The other parameters used in this numerical example include vehicle capacity equal to 100 units, average vehicle speed of 50 km/hour, fixed operating cost of €50/hour, transportation time of €1/km, inventory holding cost of €0.1/unit/hour, and fixed handling cost of €50. There are two vehicles available.

The best solution is derived when Pbest weight = 0.5 and Gbest weight = 2 with a total cost per unit time equal to € 92.2715 with route schedules for every vehicle in every day, as shown in Table 3.

Table 2. Demand rate for retailers.

Retailer	Demand rate (units/hour)
1	0.109
2	0.326
3	0.322
4	0.478
5	0.134
6	0.429
7	0.381
8	0.503
9	0.187
10	0.123
11	0.953
12	0.638
13	0.247
14	0.188
15	0.441

4.2. Sensitivity analysis

Sensitivity analysis is conducted by changing one particular parameter and keeping the other parameters with the same value. Different values of the deteriorating rate of the vehicle, deteriorating rate at the warehouse, latest time windows, and inventory cost are used to analyze the impact of the individual's parameters and, also, provide a few management insight. It is necessary to analyze the deterioration rate for vehicle and store, since we need to recognize which impact is more significant than the others are. The latest time windows need to be analyzed to show the significance of time windows to the decision-making. We only analyzed the latest time windows since most solutions show that the problem has a tendency towards late delivery than early delivery. The inventory cost is examined since, in many research, inventory cost significantly influences the total cost of the inventory issue model. The sensitivity analysis is conducted by decreasing and increasing the parameter value to 20% and 40%, respectively. The sensitivity analysis results are shown in Tables 4-7.

From Tables 4-7, we can figure out a sensitivity analysis for the four parameters as shown in Figure 2.

Figure 2 shows that the total cost per unit time is significantly sensitive to the varying values of inventory cost, and the total cost increases as the inventory cost increases. This result is consistent with the other research in inventory modeling. The total cost per unit time has an opposite trend with the latest time windows. Total cost decreases as the most recent time windows increase. However, the effect of varying latest time windows on the total cost is lower than that on the inventory cost. In practice, it is better to dedicate effort to reduce inventory cost than ask retailers for an increase in their latest time windows. Even though the effect of deteriorating rates on the total cost is lower than that on latest time windows and inventory cost, deteriorating rates affect the total cost. The total

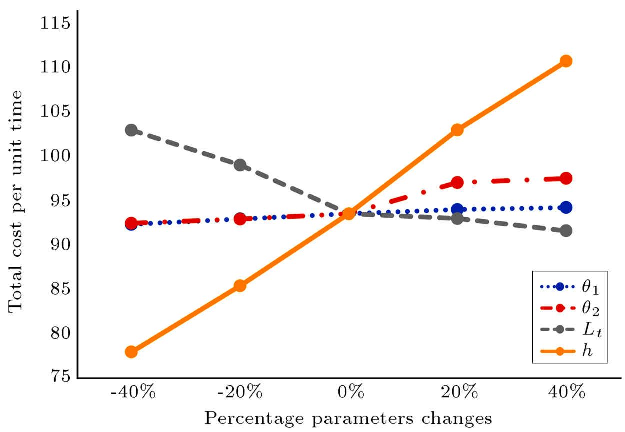

Figure 2. Sensitivity analysis of the total cost per unit time.

Table 3. The route schedules for every vehicle.

Vehicle 1		Vehicle 2	
T optimal = 120 hours (5 days)		T optimal = 192 hours (8 days)	
Day	**Route**	**Day**	**Route**
1	D – 13 – D	1	D – 14 – 3 – D
2	D – 15 – 12 – D	2	D – 1 – D
3	D – 9 – D	3	D – 5 – 6 – D
4	D – 4 – 11 – D	4	D – 2 – D
5	D – 8 – D	5	D – 7 – 10 – D

Table 4. The total cost per unit time for various values of the latest time windows.

	Total cost per unit time (€)				
Trial	**-40%**	**-20%**	**0%**	**20%**	**40%**
1	99.6382	99.4641	100.8938	89.7916	90.5262
2	102.079	95.7855	92.5685	91.8985	90.8656
3	106.4597	99.1407	95.2522	93.7587	95.7397
4	106.3438	99.0277	90.0606	92.9678	91.1201
5	100.6295	101.9542	89.1333	97	90.0207
Average	103.03	99.0744	93.5817	93.0374	91.6545

Table 5. The total cost per unit time for various values of deterioration rate for a vehicle.

	Total cost per unit time (€)				
Trial	**-40%**	**-20%**	**0%**	**20%**	**40%**
1	89.7673	89.103	100.8938	89.7579	89.6748
2	90.7791	96.7518	92.5685	94.0219	89.9219
3	97.7465	96	95.2522	99.7857	96.0674
4	90.4186	89.8167	90.0606	90.8412	100.7251
5	93.2059	93.2948	89.1333	95.8621	95.036
Average	92.3835	92.9922	93.5817	94.0538	94.2850

Table 6. The total cost per unit time for various values of deterioration rate at the warehouse.

	Total cost per unit time (€)				
Trial	**-40%**	**-20%**	**0%**	**20%**	**40%**
1	90.3121	92.3688	100.8938	97.9547	97.0158
2	89.9552	94.8818	92.5685	96.9384	96.9922
3	93.3972	87.3266	95.2522	102.6957	99.4056
4	97.6252	93.9552	90.0606	95.7184	98.6285
5	91.2684	96.3813	89.1333	92.2641	95.8131
Average	92.5116	92.9827	93.5817	97.1143	97.5710

cost increases as deteriorating rates increase. These results are consistent with results of some previous research studies on deteriorating inventory items. The effect of deterioration rate in inventory on the total cost is greater than that of the deterioration rate in the vehicle. This finding shows that managers should put more effort into reducing the deterioration rate in inventory than that in the vehicle in order to reduce the total cost. The effort to reduce the deterioration rate in the warehouse is also easier than reducing the

Table 7. The total cost per unit time for various values of inventory cost.

	Total cost per unit time (€)				
Trial	**-40%**	**-20%**	**0%**	**20%**	**40%**
1	77.0046	84.8719	100.8938	105.6132	107.2956
2	82.7451	83.9341	92.5685	103.0243	111.3618
3	79.0822	81.2	95.2522	105.3589	117.0833
4	72.5536	86.2717	90.0606	100.6852	106.1367
5	78.4133	90.9494	89.1333	100.5856	112.1578
Average	72.5536	81.2000	89.1333	100.5856	106.1367

deterioration rate in the vehicle. The effect of varying environmental temperatures is also easier to handle in a warehouse as compared to such an effect during transportation time. Temperature stability is easier to control in the warehouse than during shipping time. The result also supports the purpose of this research which considers the deterioration rate in inventory and vehicle instead of considering deteriorating items in the vehicle, as shown by previous researches before.

5. Conclusion

This research study attempted to examine the deterioration rate of the cyclic inventory routing problem. Based on our intensive literature study, no research study has examined deteriorating items in the cyclic inventory problem. This study hypothesized that the deterioration rate influences inventory total cost of any items with deteriorating characteristics. A mathematical model was developed to solve the problem. Since the proposed model was an NP-hard problem, a Particle Swarm Optimization (PSO) was drawn up to deal with the problem. A numerical illustration was conducted to show how the model works. Sensitivity analysis was done by changing the parameter of one variable and keeping the same values of the other parameters. The changing parameters included latest time windows, the deterioration rate in the vehicle, deterioration rate in warehouse, and inventory holding cost. The sensitivity analysis shows consistent results with results of previous inventory deteriorating models and gives some management insights. The holding cost exerts the greatest impact on the total cost as compared to the other involved parameters. Therefore, it is better for organizations to devote greater effort to reducing inventory holding cost to reduce the total cost. Even though the effect of deterioration rates is not as large as that of inventory holding cost, deterioration rate influences the total cost. Deterioration rate in inventory results in greater effect on the total cost than deterioration rate in vehicles. Therefore, as a contribution to the study field, this study considered both deterioration rates in inventory and vehicle instead of only considering the deteriorating rate in a vehicle alone. The model can be developed by acknowledging perishable items and price-dependent time.

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