Research Note

Numerical simulation of flow over labyrinth spillways

Y. Sangsefidia, M. Mehraeinf,b,* and M. Ghodsianc

a. Faculty of Civil and Environmental Engineering, Tarbut Modares University, Tehran, Iran.

b. Faculty of Engineering, Khurram University, Tehran, Iran.

c. Water Engineering Research Institute, Faculty of Civil and Environmental Engineering, Tarbut Modares University, Tehran, Iran.

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KEYWORDS
Discharge; Hydraulic performance; Labyrinth spillway; Local submergence; Numerical simulation.

Abstract. Spillways play an important role in the safety of dams. To ensure the safety of dams, spillways should be designed considering floods with long return periods. To increase the discharge capacity of a spillway, a designer can increase the crest length using the labyrinth spillway. For high head conditions, the local submergence may decrease the efficiency of a labyrinth spillway. In this study, the hydraulic characteristics of linear and arced labyrinth spillways are compared, and the effect of the downstream bed level on the discharge coefficient of labyrinth spillways is studied numerically. The numerical simulations are done using Flow-3D software. RNG k-ε model is used for turbulence simulations and the free-surface profiles are calculated by VOF method. Experimental data of previous researchers are used for validation of the proposed numerical model. The results show that in the high head conditions, lowering the downstream bed level of a labyrinth spillway increases its efficiency, especially in the case of an arced labyrinth spillway.

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1. Introduction

Spillways possess an essential role in the safety of dams and should spill floods with high return periods. Designers of spillways can increase their width to increase the discharge capacity. However, this approach is usually limited to some topography and economic constraints. In such cases, labyrinth spillways are another alternative. Labyrinth spillways can be categorized as linear spillways folded in the plan-view. These spillways provide an increase in the crest length for a given channel width, which causes an increase in the flow capacity for the same head.

One of the first experimental studies on the hydraulic of labyrinth spillways is carried out by Hay and Taylor [1]. They introduced a method for the discharge calculation and the design of triangular and trapezoidal linear labyrinth spillways. Darvas [2] reported laboratory results on trapezoidal linear labyrinth spillway models, which are in close agreement with Hay and Taylor’s results. Labyrinth spillways are also studied by research institutions like Utah Water Research Laboratory (UWRL), University of Georgia, and United States Bureau of Reclamation (USBR). Lux and Hinchliff [3], Megalhaes and Loaena [4], Tullis et al. [5], and Khode et al. [6] introduced practical methods for designing linear trapezoidal labyrinth spillways. Falvey [7] summarized a comprehensive review of the previous researches on the labyrinth spillway. Tullis et al. [8] obtained the head-discharge relationships for submerged linear labyrinth spillways. Ghodsian [9] used a dimensional analysis and proposed an equation to estimate the discharge of linear triangular labyrinth spillways. Cerullo et al. [10] modified the proposed equation by Ghodsian [9].
More recently, Crookston and Tullis [11,12] studied the hydraulic performance of a trapezoidal arced labyrinth spillway in reservoirs. They reported that the arced labyrinth spillways have higher discharge efficiency (∼5-11%) higher than an in-channel labyrinth spillway orientation. Kumar et al. [13,14] showed that the discharge coefficient of an arced spillway located in a channel is smaller than that of a linear spillway. They also reported that by increasing the spillway arc angle (θ), the discharge coefficient of an arced spillway increases when θ < 90°. The opposite behavior was observed for θ > 90°. Christensen [15] indicated that if the number of cycles of an arced labyrinth weir is equal or greater than 5, the discharge coefficient of the weir is independent of the number of cycles.

According to the best knowledge of the authors, most of the previous researches were conducted to study the hydraulic characteristic of a linear labyrinth spillway. Traditionally, labyrinth spillway cycles follow a linear configuration (e.g., Ute Dam in USA); however, arced labyrinth spillways have also been constructed (e.g., Avoc Dam in Australia). In this paper, the hydraulic characteristics of labyrinth spillways are numerically studied using Flow-3D software. The hydraulic performance of the arced labyrinth spillway is compared with in-channel labyrinth configuration, including the effects of approach flow conditions, nappe interference, and local submergence. This paper also studies the effects of the downstream bed level on the discharge coefficient of the labyrinth spillways.

2. Study methods

2.1. Governing equations and computational scheme

Flow-3D solves the modified form of the Reynolds Average Navier-Stokes (RANS) equations. The governing equations are [16]:

\[
\frac{\partial}{\partial x}(uAx) + \frac{\partial}{\partial y}(vAy) + \frac{\partial}{\partial z}(wAz) = 0,
\]

\[
\frac{\partial u}{\partial t} + \frac{1}{V_F} \left( U_j A_j \frac{\partial u_j}{\partial x_j} \right) = \frac{1}{\rho} \frac{\partial P}{\partial x} + g_i + f_i,
\]

where \( V_F \) is the fractional volume open to the flow; \( u, v, \) and \( w \) are the velocities in the \( x, y, \) and \( z \) directions, respectively; \( Ax, Ay, \) and \( Az \) are fractional areas open to the flow in the subscript directions; \( U_j \) and \( A_j \) are the velocity and the face area of the cell in \( j \) direction; \( g_i \) is the gravitational force in \( i \) direction; and \( f_i \) represents the Reynolds stresses to close the turbulence model. For the cells full of fluids, \( V_F \) and \( A_j \) are equal to 1, and the equations are reduced to the incompressible RANS equations.

2.2. Turbulence models

Recently the Renormalization-Group (RNG) is used in turbulence models. These methods use some statistical methods to derive the averaged equations of the turbulence quantities such as dissipation rate and turbulent kinetic energy. The equations of the RNG model are similar to those of the \( k-\varepsilon \) model. However, the empirical constants of the RNG model are derived explicitly. The transport equation can be expressed as [16]:

\[
\frac{\partial k_T}{\partial t} + \frac{1}{V_F} \left( uAx \frac{\partial k_T}{\partial x} + vAy \frac{\partial k_T}{\partial y} + wAz \frac{\partial k_T}{\partial z} \right) = P_T + G_T + \text{Diff}_T - \varepsilon_T,
\]

where \( k_T \) is the turbulent kinetic energy, and \( P_T \) is the production of turbulent kinetic energy, which can be determined as:

\[
P_T = \text{CSPRO} \left( \frac{\mu}{\rho V_F} \right) \left( 2Ax \left( \frac{\partial u}{\partial x} \right)^2 + 2Ay \left( \frac{\partial u}{\partial y} \right)^2 + 2Az \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{A_x \frac{\partial u}{\partial x} + A_y \frac{\partial u}{\partial y}}{A_z} + \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \frac{A_x \frac{\partial u}{\partial x} + A_y \frac{\partial u}{\partial y}}{A_z} \right),
\]

where \( \mu \) is the dynamic viscosity, and \( \text{CSPRO} \) is a turbulence parameter with a default value equal to 1. In Eq. (3), the value of the buoyancy production term, \( G_T \), can be determined as:

\[
G_T = -\text{CRHO} \left( \frac{\mu}{\rho^2} \right) \left( \frac{\partial \rho \frac{\partial \rho}{\partial x}}{\partial x} + \frac{\partial \rho \frac{\partial \rho}{\partial y}}{\partial y} + \frac{\partial \rho \frac{\partial \rho}{\partial z}}{\partial z} \right).
\]

In Eq. (5), \( \text{CRHO} \) is another turbulence parameter with a default value equal to 0. In Eq. (3) the diffusion term \( \text{Diff}_T \) can be expressed as:

\[
\text{Diff}_T = \frac{1}{V_F} \left( \frac{\partial}{\partial x} \left( v_k Ax \frac{\partial k_T}{\partial x} \right) + \frac{\partial}{\partial y} \left( v_k Ay \frac{\partial k_T}{\partial y} \right) + \frac{\partial}{\partial z} \left( v_k Az \frac{\partial k_T}{\partial z} \right) \right),
\]

where \( v_k \) is the coefficient of turbulent diffusion, which is computed based on the local value of the turbulent viscosity. The turbulent energy diffusion coefficient
multiplier RMTE (with a default value equal to 1.39 for the RNG model and 1 for other models) is a user-defined parameter. In the Prandtl mixing length model, the turbulent kinetic energy $k_T$ affects the dissipation rate. In Flow-3D, the default value of the turbulent mixing length (TLEN) is 7% of the smallest domain dimension. Shojaei Fard and Boyaghi [17] recommended to select this value equal to 7% of the hydraulic diameter. The default values of TLEN are recommended to provide a value for all turbulence models (except the large eddy model). Rodi [18] stated that the $k - \varepsilon$ model provides reasonable approximations for many types of flows. The turbulent dissipation $\varepsilon_T$ should be estimated using an additional transport equation as [16]:

$$\frac{\partial \varepsilon_T}{\partial t} + \frac{1}{V_F} \left( u_A \frac{\partial \varepsilon_T}{\partial x} + v_A \frac{\partial \varepsilon_T}{\partial y} + w_A \frac{\partial \varepsilon_T}{\partial z} \right) = C_1 \frac{\varepsilon_T}{k} (P_T + C_3 G_T) + \text{Diff}_\varepsilon - C_2 \frac{\varepsilon_T^2}{k},$$

(7)

where $C_1$, $C_2$, and $C_3$ are the user defined coefficients. In the $k - \varepsilon$ model, the default values of these coefficients are $C_1 = 1.44$, $C_2 = 1.92$, and $C_3 = 0.2$. In the RNG model, $C_2$ is computed based on the turbulent production ($P_T$) and the turbulent kinetic energy ($k_T$) terms. The diffusion of dissipation $\text{Diff}_\varepsilon$ is computed as:

$$\text{Diff}_\varepsilon = \frac{1}{V_F} \left( \frac{\partial}{\partial x} \left( v_c A_x \frac{\partial \varepsilon_T}{\partial x} \right) + \frac{\partial}{\partial y} \left( v_c A_y \frac{\partial \varepsilon_T}{\partial y} \right) + \frac{\partial}{\partial z} \left( v_c A_z \frac{\partial \varepsilon_T}{\partial z} \right) \right),$$

(8)

where RMDTKE is the coefficient of turbulent dissipation diffusion. The default values of this parameter for $k - \varepsilon$ and RNG models are 0.77 and 1.39, respectively.

### 2.3. Numerical methodology

The finite-volume method was used by the CFD package Flow-3D to solve the RANS equations. The computational domain is subdivided into a grid of variable-sized hexahedral cells by Cartesian coordinates. The modeling of free-surface flow over an obstacle using Flow-3D limits the cell conditions within the grid to one of the following conditions: partially solid and fluid, completely solid, partially fluid, completely fluid, and completely empty. Using the Fractional Area/Volume Obstacle Representation (FAVOR) method [19], the spillway is defined as an obstacle in the rectangular domain. The free-surface computation was done using a modified Volume-Of-Fluid (VOF) method [20]. A function of fraction of fluid $F(x, y, z, t)$ is used to reveal the fluid configuration. In a single fluid problem, $F$ reveals the volume fraction occupied by the fluid, where $F = 0$, $F = 1$, and $0 < F < 1$ correspond to empty cell, full cells, and partially filled cells, respectively.

### 3. Description of the model

#### 3.1. Effective parameters

The following equation was used as the head-discharge relationship for a labyrinth spillway [5]:

$$Q = \frac{2}{3}C_d L \sqrt{2gH_0^{1.5}}.$$  

(9)

In Eq. (9), $Q$ = discharge, $C_d$ = discharge coefficient, $L$ = total length of labyrinth spillway, $g$ = acceleration due to gravity, and $H_0$ = total upstream head = $h + V^2/2g$ ($V$ is the approach velocity of flow and $h$ is the approach depth of flow). The numerical models were used to determine $Q$ and $H_0$ for different geometries of the labyrinth spillways, and $C_d$ data were calculated using Eq. (9). The parameters affecting the discharge of an arced labyrinth spillway can be presented as:

$$Q = f (H_0, P, D, L, t', \Theta, w, N, A, \alpha, S_0, g, S_e).$$  

(10)

where $P$ is the height of spillway; $D$ is the vertical distance from the crest level to the downstream bed level; $t'$ is the thickness of spillway wall; $w$ is the width of a single labyrinth spillway cycle; $N$ is the number of labyrinth spillway cycles; $A$ is the inside apex width; $\alpha$ is the downstream sidewall angle; $S_0$ is the longitudinal slope of bed; and $S_e$ is the representation of the crest shape (see Figure 1). Using the dimensional analysis and some simplifications, Eq. (10) can be rewritten as:

$$f \left( \frac{Q}{L \sqrt{gH_0^{1.5}}} \right) = f \left( \frac{H_0}{P}, \frac{w}{D}, \frac{P}{D}, \frac{A}{P}, \frac{N}{P}, \Theta, \alpha, S_0, S_e \right) = 0.$$  

(11)

Since $w/P = 2$, $P/t' = 8$, $A/P = 1$, $N = 5$, $\alpha = 6^\circ$, $S_0 = 0$, and the crest shape was considered the same for all the runs (half-round), these parameters are omitted from Eq. (11). Thus:

$$Q = \frac{L \sqrt{gH_0^{1.5}}}{f \left( \frac{H_0}{P}, \Theta, \frac{D}{P} \right)}.$$  

(12)

From Eqs. (9) and (12) the following is obtained:

$$C_d = f \left( \frac{H_0}{P}, \Theta, \frac{D}{P} \right).$$  

(13)

The effects of the headwater ratio ($H_0/P$), the arc angle ($\Theta$), and the relative height ratio ($D/P$) on the discharge coefficient of labyrinth spillways are presented in the following sections.
3.2. Experimental data
To validate the proposed numerical model, the laboratory data of Crookston and Tullis [11] is used. Crookston and Tullis implemented a physical model of labyrinth spillways in an elevated head box (dimensions: 7.3×6.7×1.5 m deep) to emulate the conditions of a reservoir. The model characteristics are $\alpha = 6^\circ$, $P = 0.203$ m, $N = 5$, $A = 0.023$ m, $d = 0.025$ m, $w = 407$ mm, half-round crest shape, and trapezoidal cycles. In addition, data were collected under steady-state conditions without artificial nappe aeration.

3.3. Specification of the numerical model
In this work, the geometry of the labyrinth spillways was defined as an STL (stereo lithography) file using the Catia software, and then was transferred to Flow-3D. As shown in Figure 2, the arced and linear labyrinth spillways were located inside a reservoir and a channel, respectively. The details of the geometries of numerical model are summarized in Table 1.

In the numerical model, the number of computational volumes were increased until the computation time was increased disproportionately compared to the achieved accuracy. This was chosen as a criterion and then used for further runs. Flow-3D calculates the time step automatically. The time step was controlled with respect to the criteria of stability and convergence (in the range of 0.0005-0.003 s). The computational domains were extended at least up to a distance of 4P upstream of the spillway crest to avoid the curvature effect of water surface [14,21]. The flow domain is defined as a hexahedral Cartesian coordinates; therefore, the software includes six different boundaries [22]. The upstream boundary condition was introduced as $F = 1$ with a specified water height. The total head and discharge at this section were calculated by Flow-3D.
and were used for further analysis. The meshing and boundary conditions are shown in Table 2. It should be noted that the computational time was between 12 and 72 hours for the models using a personal computer with eight cores of a CPU (Intel E5620 2.4 GHz).

4. Results and discussion

4.1. Numerical model validation
Table 3 shows the numerical simulation error using Flow-3D software. In this table, \( Q_{\text{measured}} \) represents the measured discharge, \( Q_{\text{numerical}} \) represents the simulated discharge, and the error is the percentage difference between the measured and simulated values. This table shows that there is a good agreement between the numerical simulation and experimental results. The maximum difference is 3.2%, which confirms the ability of the numerical simulation to predict the discharge capacity of linear and arc flow spillway.

![Figure 3](image-url) Discharge coefficient variations with \( H_0/P \) for different values of \( \Theta \) and \( D/P \).

Table 1. Details of the modeled spillways.

<table>
<thead>
<tr>
<th>Model no.</th>
<th>( \alpha ) (°)</th>
<th>( \Theta ) (°)</th>
<th>( L ) (m)</th>
<th>( P ) (m)</th>
<th>( w ) (m)</th>
<th>( t' ) (m)</th>
<th>( A ) (m)</th>
<th>( N )</th>
<th>( D/P )</th>
<th>Cycle type</th>
<th>Crest shape</th>
<th>( H_0/P )</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>0</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>1</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.05 - 0.7</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>2</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.05 - 0.7</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>150</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>1</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.05 - 0.7</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>150</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>1.5</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.5</td>
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<tr>
<td>5</td>
<td>6</td>
<td>150</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
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<td>Trapezoidal</td>
<td>half-round</td>
<td>0.05 - 0.7</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>150</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>2.5</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>150</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>3</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>150</td>
<td>15.376</td>
<td>0.203</td>
<td>0.407</td>
<td>0.025</td>
<td>0.025</td>
<td>5</td>
<td>4</td>
<td>Trapezoidal</td>
<td>half-round</td>
<td>0.5</td>
</tr>
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Table 2. Meshing and boundary conditions.

<table>
<thead>
<tr>
<th>Meshing</th>
<th>Number of computational blocks</th>
<th>1-2</th>
</tr>
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<tr>
<td>Boundary conditions</td>
<td>Number of computational volumes</td>
<td>300000-1200000</td>
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<tr>
<td>Spillway body</td>
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<td></td>
</tr>
<tr>
<td>Lateral boundaries</td>
<td>Wall</td>
<td></td>
</tr>
<tr>
<td>Inlet</td>
<td>Specific pressure (( F = 1 ))</td>
<td></td>
</tr>
<tr>
<td>Outlet</td>
<td>Specific pressure (( F = 0 ))</td>
<td></td>
</tr>
<tr>
<td>Between the blocks</td>
<td>Symmetry</td>
<td></td>
</tr>
<tr>
<td>Equations</td>
<td>Turbulence model</td>
<td>RNG</td>
</tr>
<tr>
<td>Free surface model</td>
<td>VOF</td>
<td></td>
</tr>
<tr>
<td>Time interval</td>
<td>0.01 s</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. A comparison between the numerical and experimental values of head-discharge.

<table>
<thead>
<tr>
<th>Model no.</th>
<th>( H_0/P ) (m³/s)</th>
<th>( Q_{\text{measured}} ) (m³/s)</th>
<th>( Q_{\text{numerical}} ) (m³/s)</th>
<th>Error (%)</th>
</tr>
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<td>1</td>
<td>0.2</td>
<td>0.178</td>
<td>0.182</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.259</td>
<td>0.263</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.535</td>
<td>0.517</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0.030</td>
<td>0.031</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.093</td>
<td>0.095</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.272</td>
<td>0.278</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.360</td>
<td>0.372</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.433</td>
<td>0.446</td>
<td>3</td>
</tr>
</tbody>
</table>
conditions, a labyrinth spillway behaves similar to a linear spillway \((\alpha = 90^\circ)\), so that \(C_d\) is an increasing function of \(H_0/P\) [23]. However, by increasing \(H_0/P\), \(C_d\) reaches a maximum value, then it becomes a monotonic decreasing function of \(H_0/P\). To justify the decreasing trend of the discharge coefficient of the labyrinth spillways, it should be noted that in the vicinity of the upstream apices, due to the interaction of the flow over adjacent sidewalls (thereafter called nappe interference), the discharge over the spillway decreases. Depending on the labyrinth spillway geometry and the flow conditions, the nappe interference region may include a nappe collision region, a local submergence region, or both (see Figure 4). At lower values of \(H_0/P\), nappe interference includes only a nappe collision region. However, at higher values of \(H_0/P\), if the labyrinth spillway flow becomes larger than the outlet cycles free-flow capacity, the portion of adjacent to the upstream apex becomes submerged [11]. As a result, the discharge capacity decreases.

Figure 5 represents a typical fraction of fluid contours at the crest elevation of the spillway \((z = 0.203 \text{ m})\). According to this figure, an increase in the headwater ratio leads to an increase in the size of the local submergence region. Therefore, at higher values of \(H_0/P\), the labyrinth spillway may be entirely submerged. Overall, it can be concluded that an increase in the headwater ratio increases the nappe interference and hence the size of the local submergence region. As a result, decreasing trend of \(C_d\) occurs. For the trend lines fitted to the data of Figure 3, the following equation is derived to represent the discharge coefficient:

\[
C_d = \frac{A_1 \left( \frac{H_0}{P} \right)^3 + A_2 \left( \frac{H_0}{P} \right)^2 + A_3 \left( \frac{H_0}{P} \right) + A_4}{\left( \frac{H_0}{P} \right)^2 + A_5 \left( \frac{H_0}{P} \right) + A_6},
\]

where \(A_1\) to \(A_6\) are the equation coefficients. By using the least squares method, the values of coefficients were obtained as shown in Table 4. The correlation coefficients \((R^2)\), due to Eq. (14), are also indicated in this table. The values of \(R^2\) confirm that there is a good agreement between the proposed equation and the data presented in Figure 3. Eq. (14) is valid for the range of \(0.05 \leq H_0/P \leq 0.7\).

### 4.3. Effects of arc angle

As shown in Figure 3, the arced configuration in the low head conditions can increase the hydraulic efficiency of the labyrinth spillway. Figure 6 shows the approach flow velocity field for an arced labyrinth spillway when \(H_0/P = 0.3\) and \(0.6\). According to this figure, the hydraulic efficiency increment of the arced labyrinth spillway is caused by the improvement in the alignment of the cycles with respect to the approach

<table>
<thead>
<tr>
<th>Model no.</th>
<th>(A_1)</th>
<th>(A_2)</th>
<th>(A_3)</th>
<th>(A_4)</th>
<th>(A_5)</th>
<th>(A_6)</th>
<th>(R^2)</th>
</tr>
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<td>1</td>
<td>-0.0146</td>
<td>0.08413</td>
<td>0.11181</td>
<td>0.00356</td>
<td>0.05353</td>
<td>0.01118</td>
<td>0.998</td>
</tr>
<tr>
<td>2</td>
<td>0.34998</td>
<td>-0.40385</td>
<td>0.38426</td>
<td>-0.00253</td>
<td>0.40078</td>
<td>0.00438</td>
<td>0.999</td>
</tr>
<tr>
<td>3</td>
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<td>0.41802</td>
<td>-0.0975</td>
<td>0.01803</td>
<td>-0.24191</td>
<td>0.03108</td>
<td>0.998</td>
</tr>
<tr>
<td>4</td>
<td>-0.18372</td>
<td>0.49036</td>
<td>-0.0667</td>
<td>0.01383</td>
<td>-0.18521</td>
<td>0.02416</td>
<td>0.997</td>
</tr>
</tbody>
</table>

**Figure 4.** Flow features due to the convergence of flow passing over an labyrinth spillway \((H_0/P = 0.3, \Theta = 150^\circ, \text{ and } D/P = 1)\).

**Figure 5.** Contours of fraction of fluid \((f)\) at the crest elevation of the labyrinth spillways \((D/P = 1)\): (a) \(\Theta = 0^\circ\) at \(H_0/P = 0.3\); (b) \(\Theta = 0^\circ\) at \(H_0/P = 0.7\); and (c) \(\Theta = 150^\circ\) at \(H_0/P = 0.3\).

\[
C_d = \frac{A_1 \left( \frac{H_0}{P} \right)^3 + A_2 \left( \frac{H_0}{P} \right)^2 + A_3 \left( \frac{H_0}{P} \right) + A_4}{\left( \frac{H_0}{P} \right)^2 + A_5 \left( \frac{H_0}{P} \right) + A_6},
\]
flow. However, the orientation of the velocity vectors change with $H_0/P$ and deviate more from the ideal condition (perpendicular to the crest) by increasing this parameter. These results well match with those reported by Christensen [15].

Figure 3 also shows that for $D/P = 1$, the trend of variations of the discharge coefficient with $H_0/P$ is more for the arced labyrinth spillway in such a way that in higher head conditions, the discharge coefficient of the arced labyrinth spillway is smaller than that of a linear labyrinth spillway. The reason is that for a specific $a$, $a'$ increases when $\Theta$ increases. As a result, a larger inlet cycle flow area occurs. Therefore, compared to a linear labyrinth spillway, an arced labyrinth spillway spills more flow into the downstream cycles and leads to the larger local submergence area for a given head (see Figure 5). In addition, in an arced labyrinth spillway, the convergence of the flow exiting the labyrinth cycles forms a mound in the downstream channel, which may act as an obstacle against the spilling flow (see Figures 4 and 5(c)).

4.4. Effects of the downstream bed level
According to the results presented in previous sections, the local submergence condition has a significant role on the discharge coefficient of the labyrinth spillways. For increasing the discharge capacity of the outlet cycles, the downstream bed level is lowered. According to Figure 3, the discharge coefficient of the labyrinth spillways increases when $D/P$ increases. This may be due to the reductions of the local submergence region area and the mound height (see Figure 7).

Figure 8 shows the variation of the ratio of the discharge coefficient of the linear and arced labyrinth spillways for $D/P = 2$ to the discharge coefficient for $D/P = 1$. It can be observed that the hydraulic benefits of lowering the downstream bed level are more significant for higher values of $H_0/P$ and $\Theta$. By increasing these two parameters, the area of local submergence region and its negative effect on the hydraulic performance of the labyrinth spillway increases (see Figure 5). Therefore, the advantages gained from the increase in $D/P$ are more remarkable in these circumstances. For example at $H_0/P = 0.7$, the discharge coefficient of the arced labyrinth spillway with $D/P = 2$ is about twice as compared to $D/P = 1$. Figure 9 shows a typical variation of the discharge coefficient for an arced labyrinth spillway with respect to $D/P$ when $H_0/P = 0.5$ and $\Theta = 150^\circ$. Based on Figure 9, the discharge coefficient of the arced labyrinth spillway increases with an increase in $D/P$. It then, remains almost constant for higher values of $D/P$. Figure 9 also indicates that for the mentioned conditions, the discharge coefficient of the arced labyrinth spillway

![Figure 6](image1.png)

**Figure 6.** Approach flow velocity vectors at 0.6th depth of flow for an arced labyrinth spillway ($\Theta = 150^\circ$, $D/P = 1$, $H_0/P = 0.3$ (black), and $H_0/P = 0.6$ (grey)).

![Figure 8](image2.png)

**Figure 8.** Improvement of the hydraulic performance of labyrinth spillways by lowering the downstream bed level.

![Figure 7](image3.png)

**Figure 7.** Contours of fraction of fluid ($F$) at the crest elevation of the labyrinth spillway ($H_0/P = 0.7$, $\Theta = 150^\circ$): (a) $D/P = 1$; and (b) $D/P = 2$. 
Figure 9. $C_d$ versus $D/P$ for the arced labyrinth spillway ($H_0/P = 0.5$, $\Theta = 150^\circ$).

increases up to about 100% by increasing $D/P$. However, the downstream bed level does not propose a significant effect when $D/P$ exceeds 3, because, the local submergence is not large enough to influence the discharge coefficient. It should be noted that although lowering the downstream bed level eliminates local submergence, it does not eliminate the nappe interference.

5. Conclusions

This paper studies the hydraulic performance of the labyrinth spillways using the Flow-3D software. Comparison between the presented numerical simulations with previously reported experimental results validates the ability of Flow-3D to simulate the flow over labyrinth spillways with an acceptable accuracy. The simulation results show that nappe interference and local submergence have important roles in the spillway performance. In the low head conditions, the discharge coefficient of a labyrinth spillway is an increasing function of $H_0/P$. However, $C_d$ reaches to a maximum value and, then, becomes a monotonic decreasing function of $H_0/P$. The comparison between the discharge coefficients of the arced and linear labyrinth spillways shows that this coefficient is greater for the arced spillway in the low head conditions. However, in the high head conditions, the opposite behavior is observed due to the formation of local submergence region and flow mound. Also, the downstream bed level has an important role on the performance of the labyrinth spillways. By lowering the downstream bed level, the discharge coefficient increases, especially for arced labyrinth spillways. This behavior continues until the discharge coefficient reaches to a constant value. The mentioned approach can be used to decrease the effect of local submergence on the performance of labyrinth spillways.

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References


**Biographies**

**Yousef Sangsefidi** received his MSc degree in Hydraulic Structures from Tarbiat Modares University (TMU), Tehran, Iran, in 2014. He is currently a PhD Student at Tarbiat Modares University. His research interests include flow control structures, flow and sediment transport in rivers, energy dissipation structures, and response surface methodology.

**Mojtaba Mehraein** received PhD degree in 2011, from Tarbiat Modares University. His research interests include experimental and numerical investigation of scour and flow field around the hydraulic structures. He is currently Assistant Professor at Kharazmi University of Tehran.

**Masoud Ghodsian** obtained his PhD in 1992 from Indian Institute of Technology-Roorkee, India. He started his academic and research activities when he joined Tarbiat Modares University, Tehran, Iran, in 1992. He has published about 80 peer reviewed journal papers and about 160 papers in international and national conferences. His field of research activities includes: river engineering, hydraulic structure, scouring and sediment transport.