

Prediction of Compression Index of saturated clays (C_c) using polynomial models

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Abstract:

Settlement based design for shallow foundation realizing consolidation aspect is a major task of geotechnical engineer. Compression index (C_c) from the oedometer test is used to estimate the consolidation settlement of clays. Since the determination of C_c from oedometer tests is relatively time-consuming, empirical equations based on index properties can be useful for settlement estimation. Empirical correlations have been proposed to relate the C_c of clay deposits to other soil parameters. New polynomial models are proposed for correlation. In order to assess the merits of the proposed approach, a database containing 352 data points have been compiled from case histories via geotechnical investigation sites in the province of Mazandaran, along southern shoreline of the Caspian Sea, Iran. We compare our results involving polynomial fitting with earlier results of statistical correlation relations among C_c with other geotechnical soil properties. The predicted values using our model are checked with the measured ones to evaluate the performance of the polynomial model. The results suggest that the newly proposed approach of correlation provides a means for recognizing more efficiently the patterns in the data and reliably predicting the C_c .

Keywords: compression index, shallow foundations, saturated clays, consolidation settlement, Polynomial model.

1. Introduction

Geomaterials are extremely complex in terms of their stress-strain-time dependent behavior. These are due to soil non-linear stress-strain relationships, time dependent response to loading, elasto-plastic performance under loading and unloading situation and effects of stress history (pre-consolidation) [1-3]. For any earthen structure, a transition element is used to carry the loads from super structure to substructure or naturally deposited materials [4-6]. Bearing capacity, settlement and structural design are major aspects for foundation engineering practice. Among three common occurrence settlement components, i.e. immediate, creep and consolidation time dependents, the latter plays an important role in geotechnical engineering [7-9].

Settlement prediction especially the time dependent one called consolidation in saturated clays is an important issue in geotechnical engineering. Several researchers have predicted settlement by probabilistic measurements, analytical methods, regression analysis and simplified methods [10].

To calculate settlement for clays, laboratory consolidation tests which depict one-dimensional compression behavior need to be performed on samples taken from representative locations [11].

In settlement calculation for clays, in case of normally consolidated (NC) condition, only the compression index (C_c) from the conventional oedometer test is required. If over consolidated (OC), then both compression and recompression (C_r) indices are necessary. C_r must be obtained to compute the level of settlement for OC clays as opposed to NC clays (Figure 1).

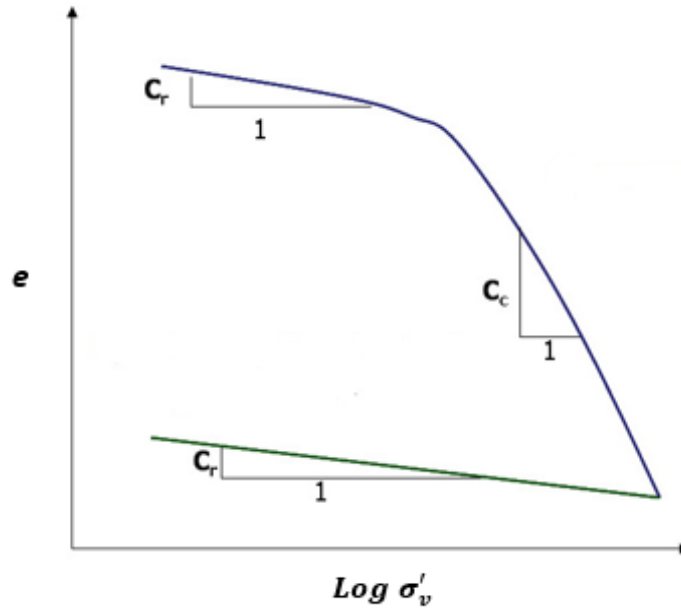


Figure 1. Definition of C_c and C_r indices

For NC clay deposit, the settlement due to an increase in load can be determined from the following equation:

$$S_c = \frac{C_c H}{1+e_0} \log\left(\frac{\sigma'_{v0} + \Delta\sigma_v}{\sigma'_{v0}}\right) \quad (1)$$

In over consolidated, if $\sigma'_{v0} + \Delta\sigma_v \leq \sigma'_c$:

$$S_c = \frac{C_r H}{1+e_0} \log\left(\frac{\sigma'_{v0} + \Delta\sigma_v}{\sigma'_{v0}}\right) \quad (2)$$

and $\sigma'_{v0} + \Delta\sigma_v > \sigma'_c$ then:

$$S_c = \frac{C_r H}{1+e_0} \log\left(\frac{\sigma'_c}{\sigma'_{v0}}\right) + \frac{C_c H}{1+e_0} \log\left(\frac{\sigma'_{v0} + \Delta\sigma_v}{\sigma'_c}\right) \quad (3)$$

where:

e_0 Initial void ratio

$\Delta\sigma_v$ Stress increment

σ'_c Pre-consolidation pressure

σ'_{v0} Initial vertical effective stress

C_c Compression index

C_r Recompression index

As the oedometer test in laboratory takes a much longer time than simpler index property tests, various attempts have been made to estimate the C_c from other geotechnical tests, these carried out more easily. Many researchers have used single parameter models for estimation of the compression and recompression indexes, i.e. liquid limit (LL %), natural water content (ω_n %) or in-situ void ratio (e_0) [12-19]. However, others recommend multiple soil parameter models [12-14, 20-24] for the estimation of C_c .

As presented in Table 1, several types of empirical correlations (one and multi-variable equations) are selected. Moreover, easily obtainable fundamental characteristics of soils, which are of the same origin and/or from the same area, can be used to find the C_c indices of fine grained soils by these formulas.

Table 1- Some widely used compression index equations

Independent variable	Equation	References	
Single variable equations	ω_n	$C_c = 0.01\omega_n - 0.05$	Azzouz et al [13]
		$C_c = 0.01\omega_n$	Koppula [14]
		$C_c = 0.01\omega_n - 0.075$	Herrero et al [15]
		$C_c = 0.013\omega_n - 0.115$	Park and Lee [12]
	e_0	$C_c = 0.49e_0 - 0.11$	Park and Lee [12]
		$C_c = 0.4(e_0 - 0.25)$	Azzouz et al [13]
		$C_c = 0.287e_0 - 0.015$	Ahadiyan et al [16]
		$C_c = 1.02 - 0.95e_0$	Gunduz [17]
	LL	$C_c = 0.006(LL - 9)$	Azzouz et al [13]
$C_c = (LL - 13)/109$		Mayne [18]	
$C_c = 0.009(LL - 10)$		Terzaghi and Peck [19]	
$C_c = 0.014LL - 0.168$		Park and Lee [12]	

Multi variable equations	LL, G_s	$C_c = 0.2926 \left(\frac{LL}{100} \right) \cdot G_s$	Park and Lee [12]
	ω_n, LL	$C_c = 0.009\omega_n + 0.005LL$ $C_c = 0.009\omega_n + 0.002LL - 0.1$	Koppula [14] Azzouz et al [13]
	e_0, ω_n	$C_c = 0.4(e_0 + 0.001\omega_n - 0.25)$	Azzouz et al [13]
	e_0, LL	$C_c = -0.156 + 0.411e_0 + 0.00058LL$ $C_c = -0.023 + 0.271e_0 + 0.001L$	Al-Khafaji and Andersland [20] Ahadiyan et al [21]
	e_0, ω_n, LL	$C_c = 0.37(e_0 + 0.003LL + 0.0004\omega_n - 0.34)$ $C_c = -0.404 + 0.341e_0 + 0.006\omega_n + 0.004LL$	Azzouz et al [13] Yoon and Kim [22]
	G_s, e_0	$C_c = 0.141G_s^{1.2}[(1 + e_0)/G_s]^{2.38}$	Herrero [23]
	$\omega_n, LL, e_0, \gamma_d$	$C_c = 0.1597(\omega_n^{-0.0187})(1 + e_0)^{1.592}(LL^{-0.0638})(\gamma_d^{-0.8276})$ $C_c = 0.151 + 0.001225\omega_n + 0.193e_0 - 0.000258LL - 0.0699\gamma_d$	Ozer [24] Ozer [24]

The aim of this study is to propose and test a polynomial model for the prediction of C_c from measured Geotechnical soil parameters, w_n, LL, e_0, G_s and γ_d . In this paper, first, current method is discussed briefly, then a field database is presented and the polynomial model suggested is presented and followed by a validation of this model on the field database.

2. Modelling Using a Polynomial Function

The basic assumption is that a pair of input parameters can be connected through a polynomial function to outputs. The task is to find a function \hat{f} that can approximate to an observed function f in order to produce the value of the output \hat{y} for a given value of the

input vector, $X = (x_1, x_2, x_3, \dots, x_n)$ such that the difference between \hat{y} and y is minimum. Therefore, for a given M observations of multi-input, single output data pairs,

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad \text{where, } (i = 1, 2, \dots, M). \quad (1)$$

It is possible to use a polynomial function to predict the output values \hat{y}_i for any given input vector, $X = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$ such that,

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad \text{where, } (i = 1, 2, \dots, M). \quad (2)$$

The challenge is to define a polynomial function such that the square of the differences between the observed output and predicted one is minimum:

$$\sum_{i=1}^M [\hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_i) - y_i]^2 \rightarrow \min \quad (3)$$

The general connection between input and output variables can be expressed by a discrete form of the Volterra functional series, known as Kolmogorov-Gabor polynomial (Ivakhnenko 1971). Hence:

$$y = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots \quad (4)$$

This mathematical description can be represented by a system of quadratic polynomials consisting of only two variables in the form:

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2. \quad (5)$$

The coefficients, a_i in Equation (5) are calculated using regression analysis, so that the difference between the observed output, y and the calculated one, \hat{y} for each pair of x_i and x_j as input variables is minimum:

$$E = \frac{1}{M} \sum_{i=1}^M (y_i - G_i)^2 \rightarrow \min \quad (6)$$

Using the quadratic expression in Equation (5) for each of the M rows, the following matrix can be obtained:

$$Aa = Y, \quad (7)$$

where a is the vector of unknown coefficients for the quadratic polynomial function in Equation (5) and Y is the vector of output values from observation. Then A takes the form:

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix}_{M \times 6} \quad (8)$$

A least-squares optimization approach for multiple regression analysis leads to the solution of the normal equations:

$$a = (A^T A)^{-1} A^T Y. \quad (9)$$

This gives the vector of the best-fit coefficients for Equation (5) for the whole set of M data triplets.

3. Database Compilation

Databases have collected the data from 352 consolidation tests for soils sampled at 95 construction sites in province of Mazandaran, Iran [25]. Following the previous trend of studies, in this study C_c of the soils was assumed to be affected by the void ratio (e_o), natural water content (ω_n), liquid limit (LL), plastic index (PI), and specific gravity (G_s). The compiled database contains 352 data produced by the Technical and Soil Laboratory of Mazandaran Province of Iran which is one of the most experienced consultants in the country as summarized in Figure 2 and 3. The samples were all collected using a standard procedure and tests were carried out using ASTM D 2435-96.

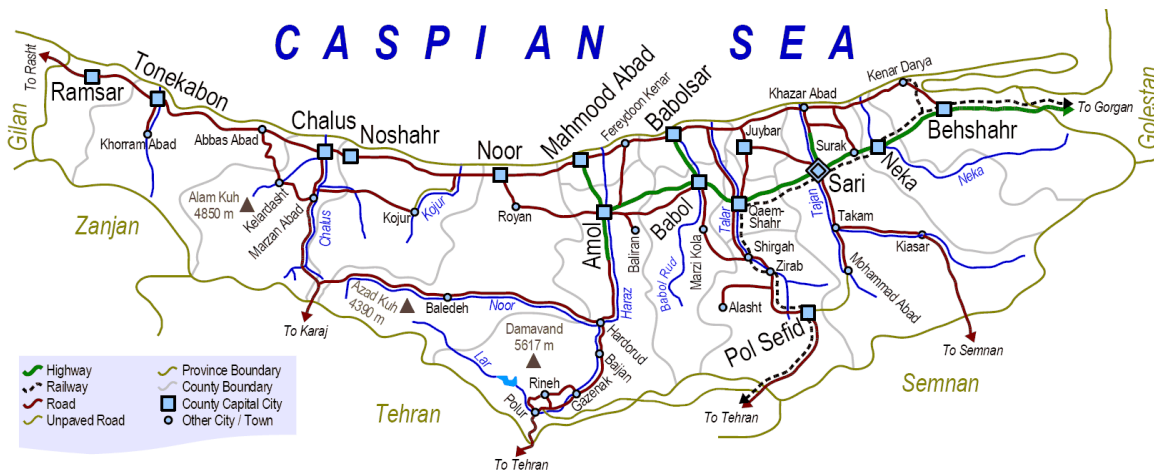


Figure 2. Descriptive data collection location

The samples were all collected using a standard procedure and tests were carried out using ASTM D 2435-96.

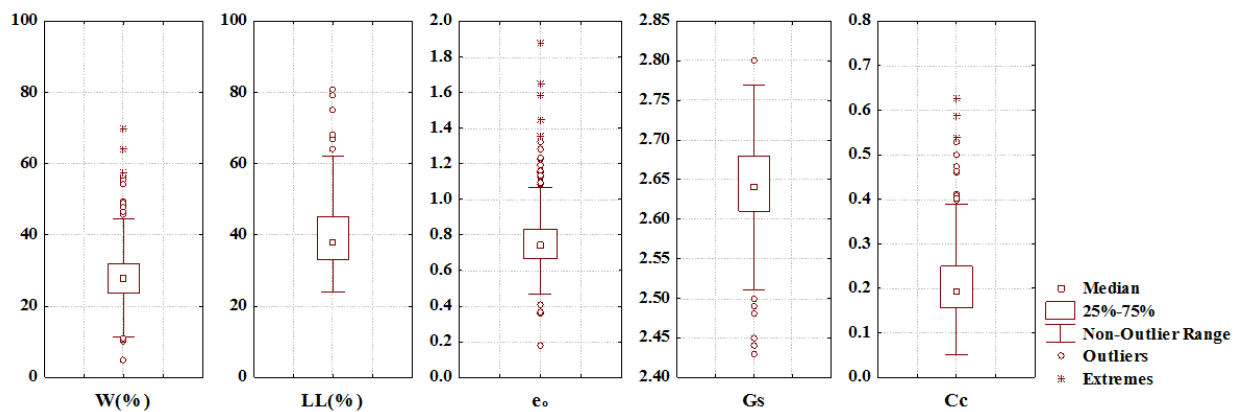


Figure 3. Descriptive statistics of variables

4. Modelling Compression Index Using a Polynomial Function

Based on Table 1 which shows previous effort in predicting C_c and using polynomial model, 8 functions introduced as following:

Function 1: $C_c = a_1 + a_2w_n + a_3LL + a_4w_{n2} + a_5LL_2 + a_6w_nLL$

Function 2: $C_c = a_1 + a_2G_s + a_3LL + a_4G_{s2} + a_5LL_2 + a_6G_sLL$

Function 3: $C_c = a_1 + a_2e_0 + a_3LL + a_4e_{02} + a_5LL_2 + a_6e_0LL$

Function 4: $C_c = a_1 + a_2LL + a_3e_0 + a_4LL_2 + a_5e_{02} + a_6e_0LLw_n + a_7w_n + a_8w_{n2}$

Function 5: $C_c = a_1 + a_2G_s + a_3e_0 + a_4G_{s2} + a_5e_{02} + a_6e_0G_sw_n + a_7w_n + a_8w_{n2}$

Function 6: $C_c = a_1 + a_2\gamma_d + a_3LL + a_4\gamma_{d2} + a_5LL_2 + a_6\gamma_dLL$

Function 7: $C_c = a_1 + a_2\gamma_d + a_3w_n + a_4\gamma_{d2} + a_5w_{n2} + a_6\gamma_dw_n$

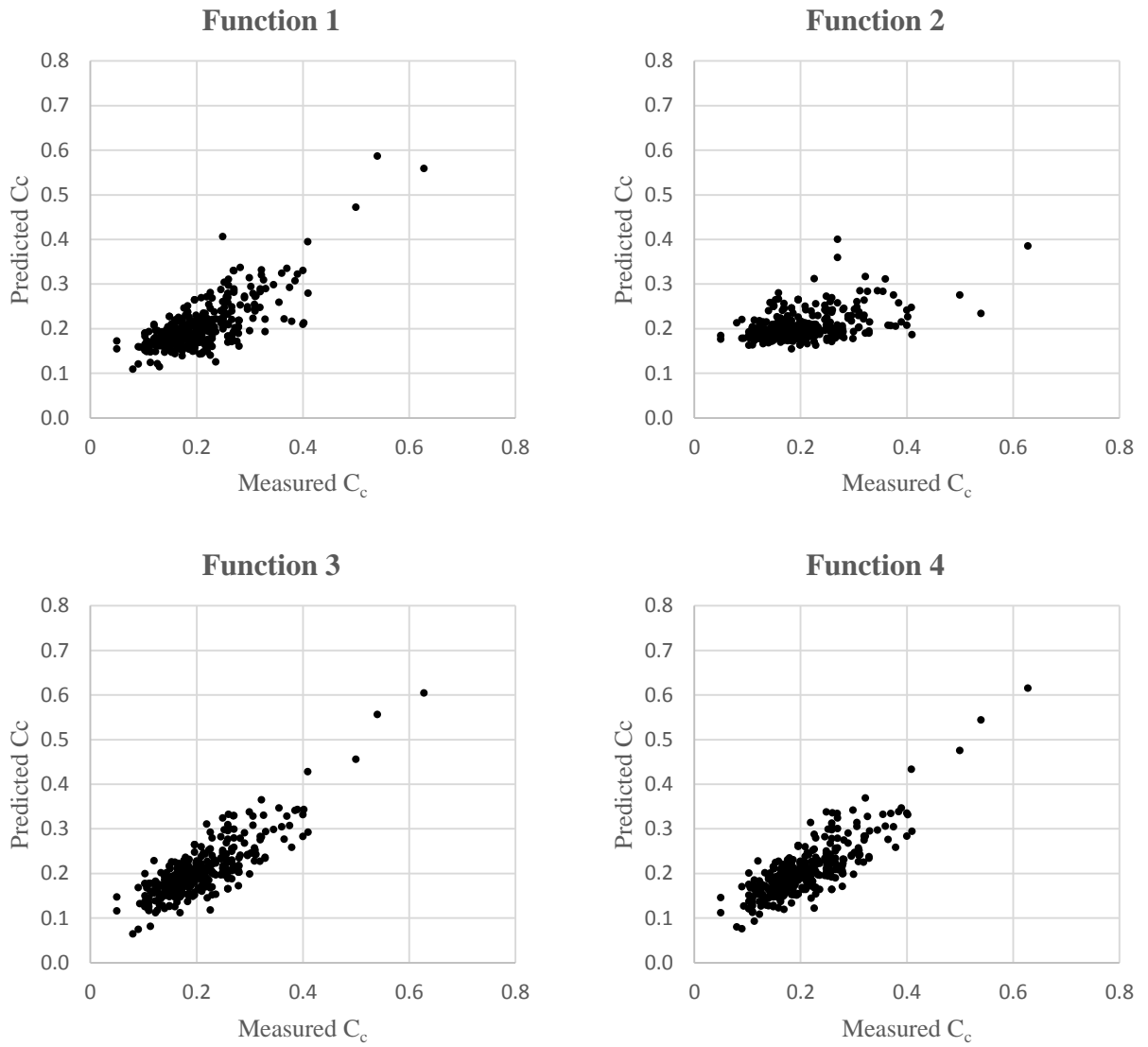
Function 8: $C_c = a_1 + a_2\gamma_d + a_3e_0 + a_4\gamma_{d2} + a_5e_{02} + a_6\gamma_de_0$

Where a_i are constant coefficients. The corresponding coefficients are shown in Table 2 for different combination of soil condition.

Table 2. Coefficients of different functions.

a_i	Function1	Function2	Function3	Function4	Function5	Function6	Function7	Function8
a_1	0.124401	-1.53507	-0.021526	-0.106924	2.801158	1.286989	1.183080	-4.16759
a_2	-0.001284	1.51988	0.233135	0.001286	-0.003871	-0.119995	-0.095982	0.41618
a_3	-0.000616	-0.00854	-0.000644	0.584375	-0.414050	0.006046	0.011880	4.61053
a_4	0.000045	-0.32568	-0.050804	-0.000015	-0.000009	0.002985	0.002083	-0.00998
a_5	-0.000015	0.00006	-0.000017	-0.195431	0.024656	-0.000007	-0.000050	-0.82259
a_6	0.000111	0.00222	0.004298	0.000054	0.000460	-0.000312	-0.000724	-0.23311
a_7	-	-	-	-0.004482	-0.242307	-	-	-
a_8	-	-	-	0.000040	0.005753	-	-	-

The performances of polynomial models are shown in Figure 4.



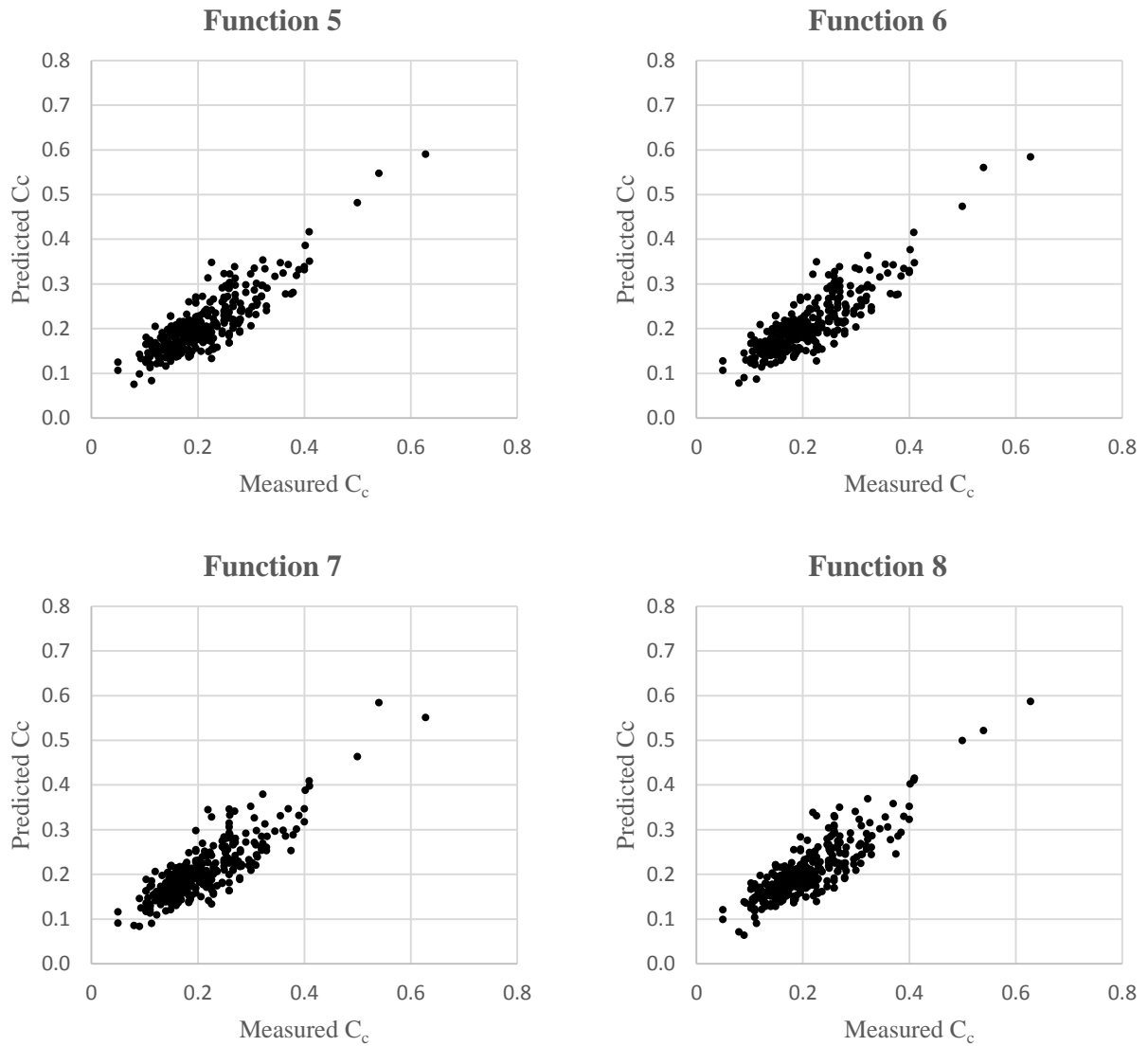


Figure 4. The performance of this study functions.

The goodness of the fit between observation and model is evaluated through estimation of the variance (R^2), root-mean-square error ($RMSE$), mean-square-error (MSE) and mean absolute deviation (MAD):

$$R^2 = 1 - \left[\frac{\sum_1^M (C_{mi} - C_{pi})^2}{\sum_1^M (C_{mi})^2} \right] \quad (10)$$

$$RMSE = \sqrt{\frac{1}{M} \sum_1^M (C_{mi} - C_{pi})^2} \quad (11)$$

$$MAPE = \frac{\sum_1^M |C_{mi} - C_{pi}|}{\sum_1^M C_{mi}} \times 100 \quad (12)$$

$$MAD = \frac{\sum_1^M |C_{mi} - C_{pi}|}{M} \quad (13)$$

Table 3. Statistical information for polynomial model for predicting C_c .

	Function1	Function2	Function3	Function4	Function5	Function6	Function7	Function8
R²	0.949792	0.905944	0.962176	0.962372	0.968529	0.967752	0.96691	0.967406
RMSE	0.04895	0.066998	0.042486	0.042376	0.038755	0.03923	0.039739	0.03944
MSE	0.002396	0.004489	0.001805	0.001796	0.001502	0.001539	0.001579	0.001555
MAD	0.038256	0.051587	0.03405	0.033769	0.031158	0.031258	0.031306	0.031198

The accuracy of the proposed models (see Table 3) for predicting C_c compared with correlations presented earlier by Azzouz et al [13], Koppula [14], Herrero et al [15], Park and Lee [12], Ahadiyan et al [16], Gunduz [17], Mayne [18], Terzaghi and Peck [19], Al-Khafaji and Andersland[20], Ahadiyan et al [21], Yoon and Kim [22] and Ozer [24] (see Table 1) which shown in Table 4.

Table 4. Statistical information for previous models for predicting C_c .

Equation	R²	RMSE	MSE	MAD
$C_c = 0.01\omega_n - 0.05$	0.914504	0.063876	0.00408	0.05223
$C_c = 0.01\omega_n$	0.799978	0.097703	0.009546	0.085753
$C_c = 0.01\omega_n - 0.075$	0.932479	0.056766	0.003222	0.044957
$C_c = 0.013\omega_n - 0.115$	0.849831	0.084656	0.007167	0.069448
$C_c = 0.49e_0 - 0.11$	0.870633	0.078574	0.006174	0.065142
$C_c = 0.4(e_0 - 0.25)$	0.956251	0.045693	0.002088	0.03666
$C_c = 0.287e_0 - 0.015$	0.954599	0.046548	0.002167	0.036931
$C_c = 1.02 - 0.95e_0$	0.17737	0.237041	0.056189	0.186696
$C_c = 0.006(LL - 9)$	0.876973	0.076625	0.005871	0.059162

$C_c = (LL - 13)/109$	0.789435	0.100245	0.010049	0.07632
$C_c = 0.009(LL - 10)$	0.744839	0.110351	0.012177	0.085795
$C_c = 0.014LL - 0.168$	0.788907	0.10037	0.010074	0.076895
$C_c = 0.2926 \left(\frac{LL}{100} \right) \cdot G_s$	0.641566	0.130789	0.017106	0.110119
$C_c = 0.009\omega_n + 0.005LL$	0.4116	0.259551	0.067367	0.250593
$C_c = 0.009\omega_n + 0.002LL - 0.1$	0.91727	0.062835	0.003948	0.051422
$C_c = 0.4(e_0 + 0.001\omega_n - 0.25)$	0.951911	0.047906	0.002295	0.038758
$C_c = -0.156 + 0.411e_0 + 0.00058LL$	0.946186	0.050678	0.002568	0.040067
$C_c = -0.023 + 0.271e_0 + 0.001L$	0.950249	0.048727	0.002374	0.040886
$C_c = 0.37(e_0 + 0.003LL + 0.0004\omega_n - 0.34)$	0.959833	0.043783	0.001917	0.034929
$C_c = -0.404 + 0.341e_0 + 0.006\omega_n + 0.004LL$	0.878516	0.076143	0.005798	0.060311
$C_c = 0.141G_s^{1.2}[(1 + e_0)/G_s]^{2.38}$	0.924811	0.059902	0.003588	0.044856
$C_c = 0.1597(\omega_n^{-0.0187})(1 + e_0)^{1.592}(LL^{-0.0638})(\gamma_d^{-0.8276})$	0.270614	0.186572	0.034809	0.173611
$C_c = 0.151 + 0.001225\omega_n + 0.193e_0 - 0.000258LL - 0.0699\gamma_d$	17.3718	0.936361	0.876772	0.933046

5. Conclusions

Relatively accurate prediction of time dependent settlement has been a challenge in geotechnical engineering. To achieve this important purpose, considering different soil parameters as compiled in a database can improve the process instead of relying solely on a couple of mini-scale and relatively disturbed oedometer test outputs.

We have proposed a new approach for correlation of C_c and geotechnical soil parameters viz., w_n , LL , e_0 , G_s and γ_d . We have assessed the performance of this approach in C_c prediction. A polynomial function has been used to model C_c as a function of mentioned. A database consisting of 352 consolidation tests, from southern part of the Caspian Sea in Iran was compiled and used to evaluate the performance of the new approach. The polynomial model that we have proposed represents such mixed of parameters.

The proposed models for all combination of parameters except function 2 (combination of G_s and LL) show good performance in predicting the C_c . As seen in Table 3 and 4 in

comparison this study functions and previous studies, the performance of polynomial model is acceptable for all each combination of soil parameters.

Results of this study confirm the conclusion reached by many earlier studies that an empirical correlation between C_c and geotechnical parameters should only be used in a site-specific sense.

Nomenclature

C_c Compression index

C_r Recompression index

e Void ratio

e_0 Initial void ratio

G_s Specific gravity of soil particles

H Initial thickness of the soil layer

LL Liquid limit (%)

PI Plastic index (%)

ω_n Natural water content (%)

$\Delta\sigma_v$ Stress increment

S_c Primary consolidation settlement

σ_c' Pre-consolidation pressure

σ_{v0}' Initial vertical effective stress

σ_v' Vertical effective stress

a_i Constant of empirical equation

$RMSE$ Root mean square error

MAD Mean absolute deviation

- MAPE** Mean absolute percent error
- R^2** Absolute fraction of variance
- M** Total number of dataset
- C_m** Measured C_c from the seismic measurements
- C_p** Calculated C_c from empirical correlations

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