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A seismic slope stability probabilistic model based on Bishop's method using analytical approach

A. Johari*, **S. Mousavi and A. Hooshmand Nejad**

Department of Civil and Environmental Engineering, Shiraz University of Technology, Shiraz, Iran.

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Limit equilibrium method.

Abstract. Probabilistic seismic slope stability analysis provides a tool for considering uncertainty of the soil parameters and earthquake characteristics. In this paper, the Jointly Distributed Random Variables (JDRV) method is used as an analytical method to develop a probabilistic model of seismic slope stability based on Bishop's method. The selected stochastic parameters are internal friction angle, cohesion and unit weight of soil, which are modeled using a truncated normal probability density function (pdf) and the horizontal seismic coefficient which is considered to have a truncated exponential probability density function. Comparison of the probability density functions of slope safety factor with the Monte Carlo simulation (MCs) indicates superior performance of the proposed approach. However, the required time to reach the same probability of failure is greater for the MCs than the JDRV method. It is shown that internal friction angle is the most influential parameter in the slope stability analysis of finite slopes. To assess the effect of seismic loading, the slope stability reliability analysis is made based on total stresses without seismic loading and with seismic loading. As a result, two probabilistic models are proposed.

1. Introduction

Seismic stability analysis of natural and man-made slopes is an important topic of research for the safe design in the seismic zone. Natural slopes are usually stable when they are not disturbed by any external force, such as seismic forces. For assessing the seismic responses of slopes, considering the characteristics of input ground motions and the properties of soil media are needed. There are four main methods for seismic stability analysis of slopes: pseudo-static [1], Newmark sliding block analysis [2], numerical analysis [3] and testing method using shaking table and centrifuge apparatus. In these methods, the pseudo-static method is the simplest at the earliest applications [4].

The pseudo-static method is modified from Limit Equilibrium Method (LEM) in seismic stability analysis. In fact, the seismic load is equivalent to invariably horizontal and vertical forces, and the problem of seismic stability of slopes is simplified into a static problem in the pseudo-static method. In the next step, the ratio of resistant forces to driving forces on a potential sliding surface is defined as Factor of Safety (FS). The slope is considered safe only if the calculated safety factor clearly exceeds unity. However, due to the model and parameter uncertainties, even a factor of safety greater than one does not confirm the safety against failure of slope. Therefore, it is important to calibrate the deterministic method considering the effect of different sources of model and parameter uncertainties. Reliability analyses provide a rational framework for dealing with uncertainties and decision making under uncertainty.

In general, the source of uncertainty in stability of a slope is divided into three distinctive categories: soil parameters uncertainty, model uncertainty, and human uncertainty [5]. Parameter uncertainty is the

*. *Corresponding author.*
E-mail address: johari@sutech.ac.ir (A. Johari)

uncertainty in the inputs parameters for analysis [6-9]. Model uncertainty is due to the limitation of the theories and models used in performance prediction [10], while human uncertainty is due to human errors and mistakes [11]. In this research, parameter uncertainty is assessed to seismic stability of infinite slope.

The reliability analysis of slope stability has attracted considerable research attention in the past few decades. Many probabilistic methods have been used for slopes stability analysis. These methods can be grouped into four categories: analytical methods, approximate methods, Monte Carlo simulation and random finite element method:

- In analytical methods, the probability density functions of input variables are joined together to derive a mathematical expression for density function of the factor of safety. These approaches can be grouped into JDRV method [12-14] and First Order Reliability Method (FORM) [15] categories. Many researches have been made to apply FORM method in slope stability (e.g., [16-20]). Limited attempts have been made to apply JDRV method (e.g., [21-23]).
- Most of the approximate methods are modified version of two methods namely: First Order Second Moment (FOSM) method [24] and Point Estimate Method (PEM) [25]. The approaches require knowledge of the mean and variance of all input variables as well as the performance function that defines safety factor (e.g., Bishop's equation). Many attempts have been made to apply the PEM (e.g., [26-29]) and by FOSM method (e.g. [19,27-33]) in slope stability.
- The MCs is based on repeated random sampling to address risk and uncertainty in quantitative analysis and decision making [34]. Recent attempts have been made to analyze the stability of slopes using this method (e.g., [28,31,35-47]).
- Random finite element method combines elasto-plastic finite-element analysis with random fields generated using the local average subdivision method. Several new slope stability researches are done by this method (e.g., [48-51]).

In this paper, a complete analytical procedure using JDRV for developing a probabilistic model of seismic slope stability based on Bishop's method is proposed. For this purpose, the horizontal seismic coefficient and soil parameters are considered stochastically, and the probability density function relationship of safety, factor is derived. For determining the critical surface with the minimum factor of safety, the Particle Swarm Optimization (PSO) algorithm is used. Furthermore, the steps are repeated for total stress condition and the results are compared to seismic slope stability analysis by their reliability index. At the end, two probabilistic models based on total stresses without seismic loading and with seismic loading are proposed.

2. Limit equilibrium method

The limit equilibrium method is the most popular approach in slope stability analysis. This method is well known to be a statically indeterminate problem, and assumptions on the inter-slice shear forces are required to render the problem statically determinate.

In the limit equilibrium method of slices [52-55], the soil mass above the slip surface is subdivided into a number of vertical slices. The actual number of slices used depends on the slope geometry and soil profile. Some procedures of slices assume a circular slip surface while others assume an arbitrary (noncircular) slip surface. Procedures that assume a circular slip surface consider equilibrium of moments about the center of the circle for the entire free body composed of all slices. In contrast, the procedures that assume an arbitrary shape for the slip surface usually consider equilibrium in terms of the individual slices.

In this study the Bishop's method is used as a typical efficient method. The Bishop approach is a simplified method rather than Morgenstern-Price and Spencer's methods. Although Figure 1 shows that the Spencer and Morgenstern-Price solutions are in good agreement with the simplified Bishop results [56]. In this figure Lambda λ is a ratio of inter-slice forces for slices.

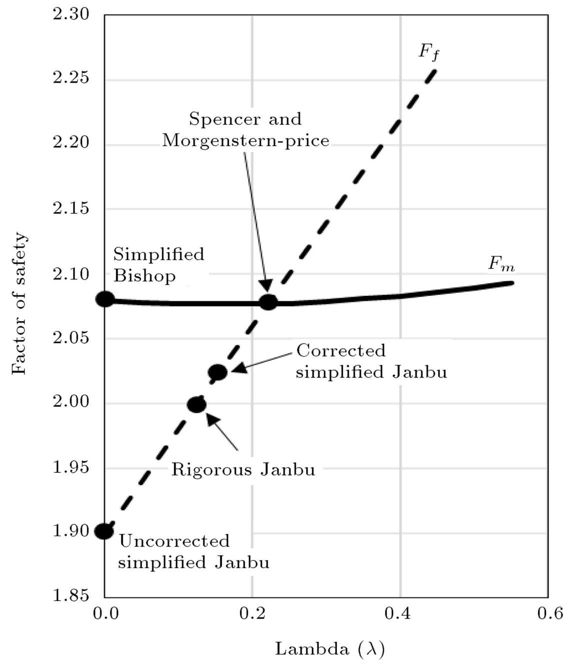

Figure 1. Variation of factor of safety versus Lambda for various methods [56].

2.1. Slope stability analysis by simplified Bishop's method

In the simplified Bishop's method [53] the forces on the sides of the slice are assumed to be horizontal (i.e., there are no shear stresses between slices). This method considers equilibrium of moments about the center of the circle therefore, the sliding surface is assumed to be circular. The Bishop's equation for factor of safety in terms of total stresses is as follow [53,57]:

$$\mathrm{FS}=\frac{\sum_{i=1}^{n}\left[\frac{c.\Delta l_i.\cos\alpha_i+w_i.\tan\varphi}{\cos\alpha_i+(\sin\alpha_i.\tan\varphi)/\mathrm{FS}}\right]}{\sum_{i=1}^{n}w_i.\sin\alpha_i}, \tag{1}$$

where:

w_i Weight of slice $=\gamma.b_i.h_i$;
Δl_i Area of the base of the slice for a slice of unit thickness;
c Cohesion;
α_i Angle of the base of slice;
γ Unit weight of soil;
b_i The width of the slice;
h_i The height of the slice at the centerline;
φ Internal friction angle;
FS Factor of safety.

A slope with circular slip surface has been shown in Figure 2. The soil mass above the slip surface is subdivided into a number of vertical slices. A sample slice with its forces is shown in Figure 3. Where N_i is the normal force applied at center of the base of the slice, S_i is soil strength, and E_i and E_{i+1} are the normal inter-slice force.

2.2. Seismic slope stability analysis by Bishop's method

Bishop developed a pseudo-static method for seismic analysis of natural slopes. In this approach, the factor of safety against sliding is obtained by including horizontal and vertical forces in the static analysis. These forces are usually expressed as a product of the seismic coefficients and the weight of the potential sliding mass. Slope model and a typical vertical slice

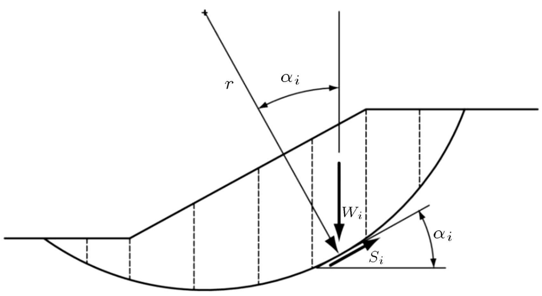

Figure 2. Slope with circular slip surface and subdivided slices.

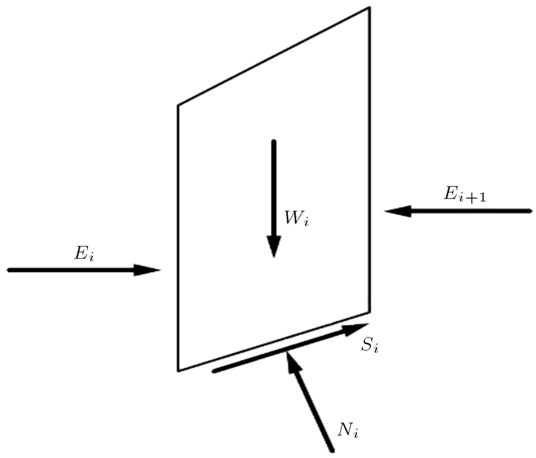

Figure 3. Forces on a sample slice.

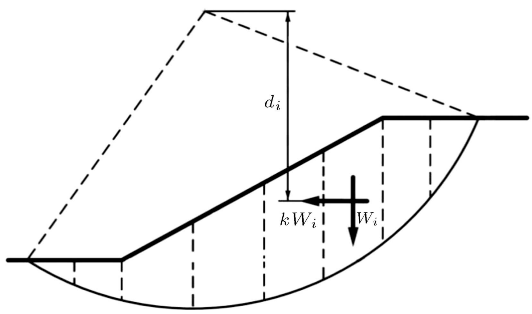

Figure 4. Slope with weight and seismic forces.

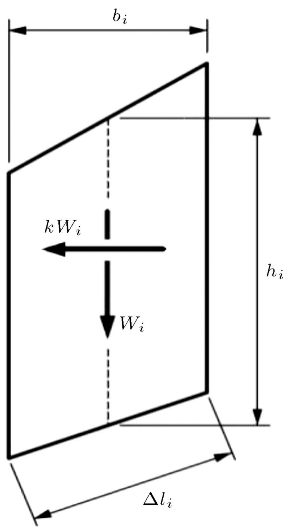

Figure 5. Dimensions for an individual slice.

for seismic stability analysis are shown in Figures 4 and 5, respectively. The proposed Bishop's equation for seismic slope analysis is as follow [57]:

$$\mathrm{FS}=\frac{\sum_{i=1}^{n}\left[\frac{c.\Delta l_i.\cos\alpha_i+w_i.\tan\varphi}{\cos\alpha_i+(\sin\alpha_i.\tan\varphi)/\mathrm{FS}}\right]}{\sum_{i=1}^{n}w_i.\sin\alpha_i+\left(\sum_{i=1}^{n}k_h.w_i.d_i\right)/r}, \tag{2}$$

where:

k_h Horizontal seismic coefficient;

d_i The vertical distance between the center of the circle and the center of gravity of the slice;

r Radius of circular slip surface.

It is evident that it would be too conservative to select for this purpose the peak value of the strong motion record, $a_{\max}$, because it lasts for a very short time and appears only once in the record. Therefore, instead of $a_{\max}$, a fraction of it, $K_h = \xi \times a_{\max}/g$, is used, where k_h is called the seismic coefficient [58]. Different magnitudes of ξ, between 0.2-0.65 have been proposed by various authors [59-61].

3. The JDRV method

Jointly distributed random variables method is an analytical probabilistic method. In this method, probability density functions of independent input variables are expressed mathematically and jointed together by statistical relations. The adopted slope model integrated analytically to derive a mathematical expression of the density function of the factor of safety. Failure probability is obtained by multiple integrals of that expression over the entire failure domain.

4. Stochastic parameters

To account for the uncertainties in slope stability, four input parameters have been defined as stochastic variables. The distribution curve of these stochastic parameters have been studied by numerous researchers (e.g., [62,63]). Generally normal distribution for soil parameters and exponential distribution for seismic parameters are accepted.

Therefore, internal friction angle (φ), cohesion (c) and unit weight (γ) are modeled using a normal probability density function and horizontal seismic coefficient (k_h) which is considered to have exponential probability density function. The parameters related to geometry are regarded as constant parameters. The probability density functions for the stochastic parameters are as follows:

$$f_c(c) = \frac{1}{\sigma_c\sqrt{2\pi}}\exp\left(-0.5\left(\frac{c-c_{\text{mean}}}{\sigma_c}\right)^2\right)$$

$$c_{\min} \leq c \leq c_{\max}, \tag{3}$$

$$f_\varphi(\varphi) = \frac{1}{\sigma_\varphi\sqrt{2\pi}}\exp\left(-0.5\left(\frac{\varphi-\varphi_{\text{mean}}}{\sigma_\varphi}\right)^2\right)$$

$$\varphi_{\min} \leq \varphi \leq \varphi_{\max}, \tag{4}$$

$$f_\gamma(\gamma) = \frac{1}{\sigma_\gamma\sqrt{2\pi}}\exp\left(-0.5\left(\frac{\gamma-\gamma_{\text{mean}}}{\sigma_\gamma}\right)^2\right)$$

$$\gamma_{\min} \leq \gamma \leq \gamma_{\max}, \tag{5}$$

$$f_{k_h}(k_h) = \frac{\exp(-k_h/k_{h_{\text{mean}}})}{k_{h_{\text{mean}}}.\left(\exp(-k_{h_{\min}}/k_{h_{\text{mean}}}) - \exp(-k_{h_{\max}}/k_{h_{\text{mean}}})\right)}$$

$$k_{h_{\min}} \leq k_h \leq k_{h_{\max}}, \tag{6}$$

where:

$$\begin{cases}\varphi_{\min} = \varphi_{\text{mean}} - 4\sigma_\varphi\\ \varphi_{\max} = \varphi_{\text{mean}} + 4\sigma_\varphi\\ c_{\min} = c_{\text{mean}} - 4\sigma_c\\ c_{\max} = c_{\text{mean}} + 4\sigma_c\\ \gamma_{\min} = \gamma_{\text{mean}} - 4\sigma_\gamma\\ \gamma_{\text{man}} = \gamma_{\text{mean}} + 4\sigma_\gamma\\ k_{h_{\min}} = 0.01\end{cases} \tag{7}$$

By considering the stochastic variables within the range of their mean plus or minus 4 times standard deviation as in Eq. (7), 99.994% of the area beneath the normal density curve is covered. For choosing the initial data, the following conditions must be observed for soil parameters of slope:

$$\begin{cases}\varphi_{\text{mean}} - 4\sigma_\varphi > 0\\ c_{\text{mean}} - 4\sigma_c > 0\\ \gamma_{\text{mean}} - 4\sigma_\gamma > 0\end{cases} \tag{8}$$

5. Slope stability stochastic analysis

For reliability assessment of safety factor of slope using JDRV method, the suggested safety factor equations of Bishop's method are rewritten into terms of K_1 to K_4 and u_1 to u_4 as given in Eqs. (A.1) to (A.14). These equations are presented in the Appendix. The probability density functions of safety factor for each method are derived separately. The terms K_1 to K_4, are:

$$\begin{cases}k_1 = c\\ k_2 = \tan\varphi\\ k_3 = \gamma\\ k_4 = k_h\end{cases} \tag{9}$$

and u_1 to u_4 are presented in Eqs. (A.2) and (A.11); derivations of these equations are presented in the Appendix.

Using the new form of input independent parameters, the probability density functions of k_1 to k_4 have been obtained by Eqs. (3) to (6) directly. Therefore the probability density functions of k_1 to k_4 can be written as:

$$f_{k_1}(k_1) = \frac{1}{\sigma_c\sqrt{2\pi}}\exp\left(-0.5\left(\frac{k_1 - c_{\text{mean}}}{\sigma_c}\right)^2\right)$$

$$c_{\min} \leq k_1 \leq c_{\max}, \quad (10)$$

$$f_{k_2}(k_2) = \frac{1}{(1+k_2^2)\sigma_\varphi\sqrt{2\pi}}$$

$$\exp\left(-0.5\left(\frac{\tan^{-1}(k_2) - \varphi_{\text{mean}}}{\sigma_\varphi}\right)^2\right)$$

$$\tan\varphi_{\min} \leq k_2 \leq \tan\varphi_{\max}, \quad (11)$$

$$f_{k_3}(k_3) = \frac{1}{\sigma_\gamma\sqrt{2\pi}}\exp\left(-0.5\left(\frac{k_3 - \gamma_{\text{mean}}}{\sigma_\gamma}\right)^2\right)$$

$$\gamma_{\min} \leq k_3 \leq \gamma_{\max}, \quad (12)$$

$$f_{k_4}(k_4)=\frac{\exp\left(-k_4/k_{4_{\text{mean}}}\right)}{k_{4_{\text{mean}}}.(\exp(-k_{4_{\min}}/k_{4_{\text{mean}}})-\exp(-k_{4_{\max}}/k_{4_{\text{mean}}}))}$$

$$k_{h_{\min}} \leq k_h \leq k_{h_{\max}}. \quad (13)$$

Using Eqs. (A.1) to (A.14), a computer program was developed (coded in MATLAB) to determine the probability density functions of the safety factor based on total stresses without seismic loading and with seismic loading.

6. Illustrative example

To examine the accuracy of the proposed method in determining the probability density function of the safety factor, an illustrative example with arbitrary parameter values is demonstrated. A typical slope shape for this example is shown in Figure 6. The stochastic parameters with truncated normal and exponential distributions are given in Tables 1 and 2, respectively. Furthermore, the selected deterministic parameters are given in Table 3.

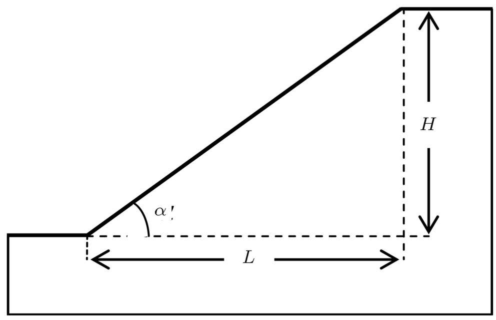

Figure 6. A typical slope.

Table 3. Deterministic parameters.

Height of slope (m)	Horizontal length of slope (m)	γ_w (kN/m^3)
12.0	12.0	10.0

7. Reliability assessment of slope stability

Using the selected deterministic and mean of stochastic parameters, the slip surface with minimum safety factor is determined by PSO algorithm [64-66]. Using Eqs. (A.1) to (A.14), the pdf and cumulative density function (cdf) curve of safety factor based on total stresses without seismic loading and with seismic loading are determined. In order to verify the results of the presented methods with those of the MCs, the final probability density functions for the safety factor are determined, using the same data by this simulation technique. For this purpose, 1,000,000 generations are used for the MCs.

The results are shown in Figures 7 to 10 for simplified Bishop's method. Figures 7 and 8 show the pdf and cdf of the safety factor based on total stresses, and Figures 9 and 10 show them for seismic slope stability analysis, respectively. It can be seen that the obtained results using the proposed methods are very close to those of the MCs.

To compare the pdf and cdf of safety factor of two stability analysis approaches (i.e. total stress and seismic), the prediction by proposed method are plotted in Figures 11 and 12, respectively. It can

Table 1. Stochastic parameters with truncated normal distribution.

Parameters	Mean	Standard deviation	Minimum	Maximum
c (kPa)	10.0	$0.20^* c_{\text{mean}}$	2.0	18.0
φ (degree)	28.0	$0.10^* \varphi_{\text{mean}}$	16.0	40.0
γ (kN/m^3)	18.0	$0.05^* \gamma_{\text{mean}}$	14.0	24.0

Table 2. Stochastic truncated exponential parameters.

Parameters	λ	Minimum	Maximum	Mean	Standard deviation
k_h	10	0.01	0.35	0.10	0.10

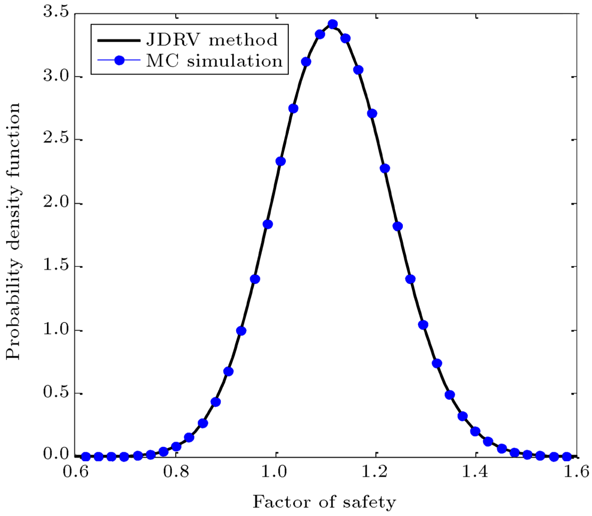

Figure 7. Probability density function of safety factor based on total stresses.

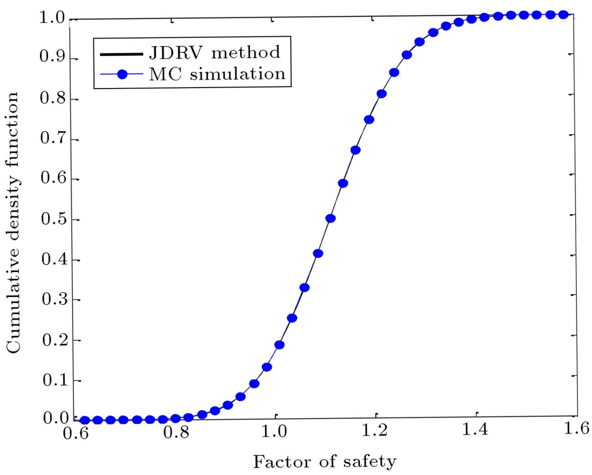

Figure 8. Cumulative density function of safety factor based on total stresses.

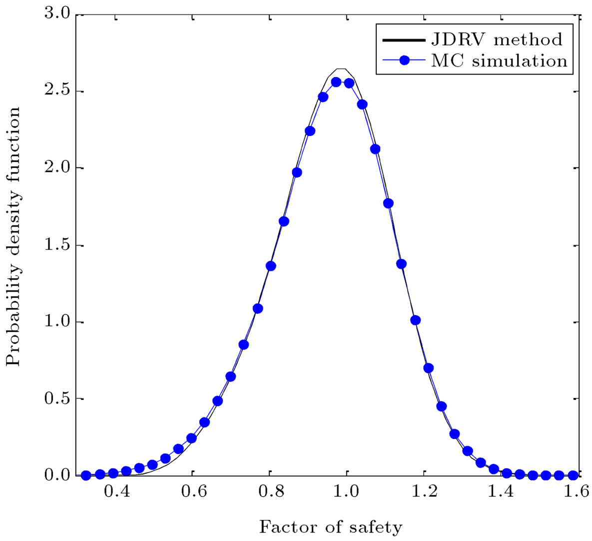

Figure 9. Probability density function of safety factor for seismic stability analysis.

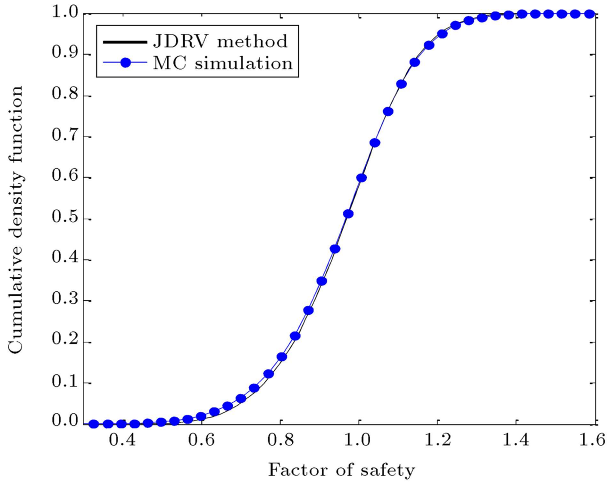

Figure 10. Cumulative density function of safety factor for seismic slope stability analysis.

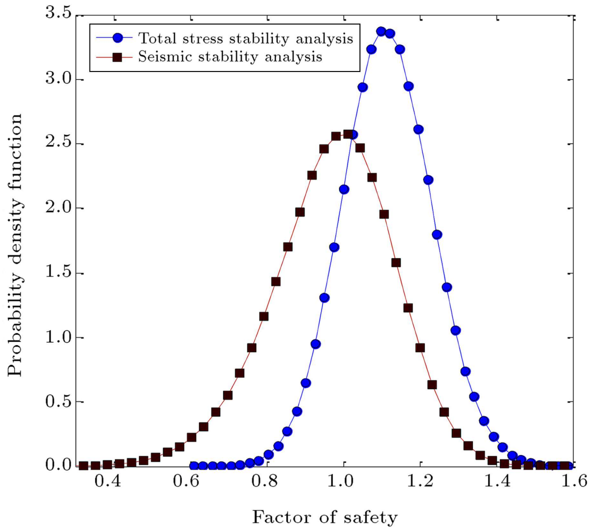

Figure 11. Comparison of probability density functions of safety factor of the methods.

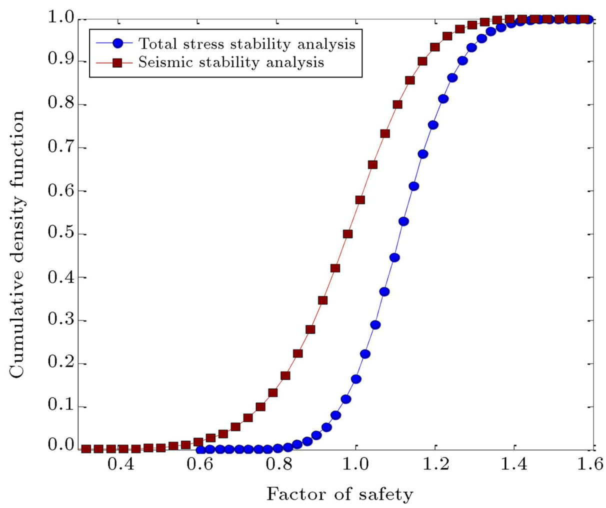

Figure 12. Comparison of cumulative distribution function of safety factor of the methods.

Table 4. Comparison of reliability index of slope safety factor by two methods.

Method	Total stress analysis	Seismic analysis
Reliability index (β)	0.9778	- 0.1759

be seen that the seismic analysis predicted upper probability of failure with respect to analysis based on total stress.

The reliability indices of safety factor of the two methods are determined using their pdf and the equation [67].

$$\beta = \frac{E(\mathrm{FS}) - 1}{\sigma(\mathrm{FS})}, \quad (14)$$

where β is the reliability index, $E(\mathrm{FS})$ is the mean value of safety factor and $\sigma(\mathrm{FS})$ is the standard deviation of the safety factor.

Reliability index of the two methods are given in Table 4. It can be seen the seismic stability analysis shows the lower reliability index or upper probability of failure with respect to stability analysis based on total stress.

8. Sensitivity analysis

To evaluate the response of slope stability to changes in parameters, a sensitivity analysis is carried out using the JDRV method. For this purpose, the seismic method is selected and the sensitivity analysis has been done. In the first approach, the means of the four stochastic input parameters are increased based on their standard deviation (new mean = old mean + 1.0×std). The results of probability density curves are shown in Figure 13. Additionally, the cumulative distribution curves are plotted in Figure 14. To evaluate the influence of changes in each parameter, the parameter is increased while the ranges of the other stochastic input parameters are kept constant. Using Figure 14, the amounts of changes in probability of failure (FS=1), corresponding to 1*std increase in the pdf of the input parameters, are calculated and given in Table 5. It can be seen that the internal friction angle of sliding surface is the most effective parameter in the stability of slopes.

In the second approach, the mean of horizontal seismic coefficient and internal friction angle are changed while the means of the other stochastic input parameters are kept constant. For this purpose, the mean of k_h and φ has been varied between the minimum and maximum. In Figures 15 and 16 the changes in probability of failure corresponding to increase in the pdf of the friction angle and horizontal seismic coefficient are shown, respectively. As expected, with increasing the friction angle, the probability of failure decrease, however, with increasing the horizontal seismic coefficient, the probability of failure increase.

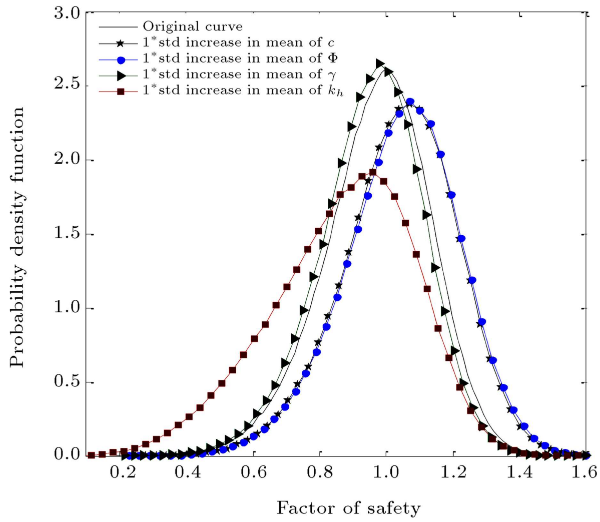

Figure 13. Variation of probability density functions of the safety factor in sensitive analysis.

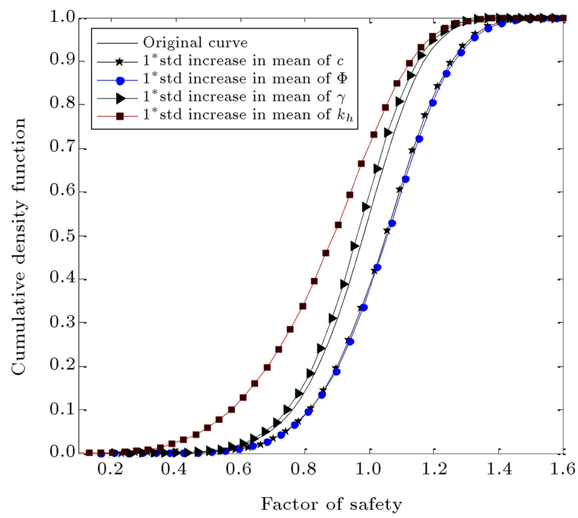

Figure 14. Variation of cumulative density functions of the safety factor in sensitive analysis.

Table 5. Changes in probability of failure corresponding to 1*std increase (shift rightward) in the pdf of input.

Stochastic parameter	Cohesion (c)	Friction angle (φ)	Unit weight (γ)	Seismic coefficient (k_h)
Change (%)	-30.36	-32.07	+8.38	+29.27

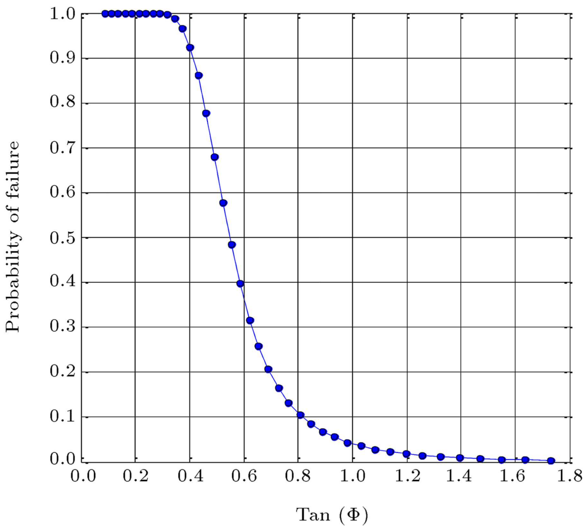

Figure 15. Sensitivity analysis of tan (φ) versus probability of failure.

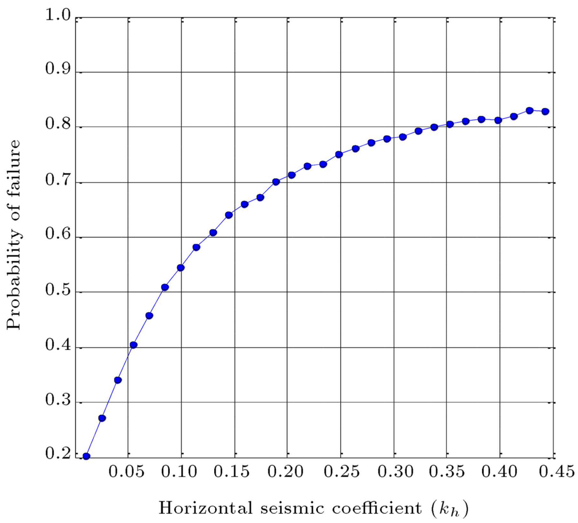

Figure 16. Sensitivity analysis of horizontal seismic coefficient versus probability of failure.

9. Parametric analysis

For further verification of the proposed model, a parametric analysis is performed based on Bishop's seismic method. The main goal is to determine how each parameter affects the stability of slopes. Figure 17 presents the predicted values of the probability of failure (instability) as a function of each parameter where others being constant. For this purpose, in six steps, the probability density function of each stochastic input parameter is increased based on their standard deviation (new pdf = old pdf + 1/3×std). The results of the parametric analysis indicate that, as expected, the probability of failure (instability) continuously increases due to increasing in unit weight and horizontal seismic coefficient. The probability of failure decreases with increase in internal friction angle

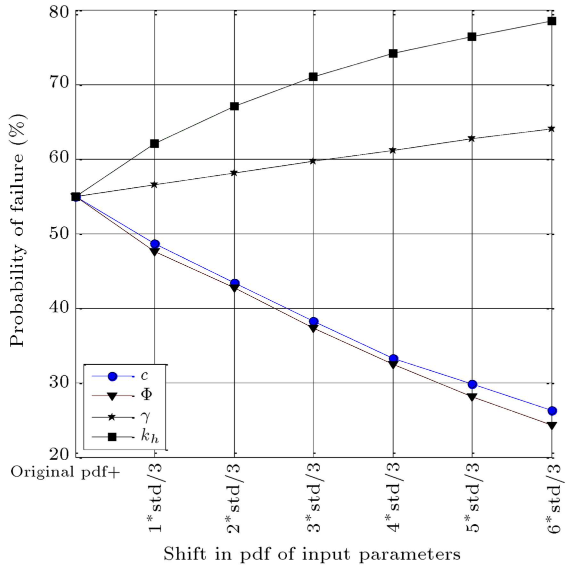

Figure 17. Parametric analysis of probability of failure with respect to change of probability density functions of input parameters.

and cohesion. Also it can be seen that the curve of change in internal friction angle with respect to probability of failure has a steeper slope than others indicating that it is the most influential parameter.

10. Probabilistic model

To develop probabilistic slope stability models based on total stresses without seismic loading and with seismic loading by JDRV, the following procedures were followed:

- Several random series of input parameters, c, φ and γ (including cross correlation -0.5 between c and φ) were considered as mean value.
- A finite slope with arbitrary $L = 10$ m was selected.
- The critical slip surface for each data series has been obtained by the PSO algorithm for the slope.
- The uncertainty in the input parameters used in the calculation of safety factor of slope for each series of database was assessed (including cross correlation -0.5 between c and φ).
- The cumulative distribution function of each data series was determined using the JDRV method as described in Section 7.
- The probability of failure was computed from the cumulative distribution function for each series of data.
- The safety factor of each data series was calculated using the deterministic approach described in Section 3.
- The probability of failure (P_f) and the related factor of safety from two previous steps were plotted with

Table 6. Parameters which used to probabilistic models evaluation.

Parameters	Cohesion (c) (kPa)	Friction angle (φ) (degree)	Unit weight (γ) (kN/m^3)	Slope angle (α) (degree)	Horizontal length (L) (m)
Values	12.0	35.0	17.0	50.0	10.0

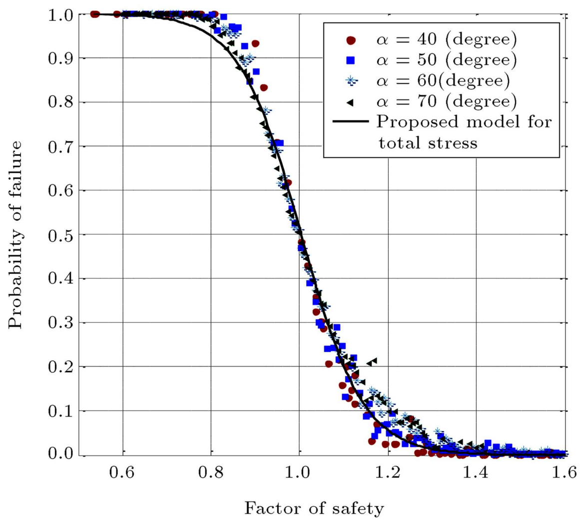

Figure 18. Probability of failure of slopes by different angles based on total stresses stability analysis.

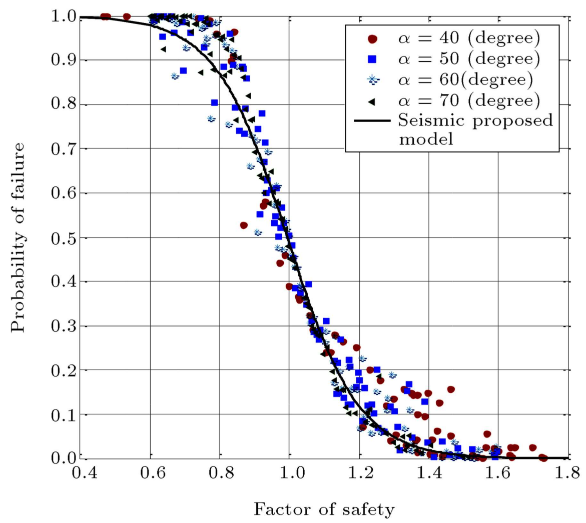

Figure 19. Seismic slope stability analysis probability of failure by different angles.

respect to each other for all data series (as shown in Figures 18 and 19).

- This procedure is repeated for α=40, 50, 60 and 70. The results are shown in Figures 18 and 19.
- The probabilistic slope stability models was developed using MATLAB curve fitting toolbox using all data. The probabilistic models have the following form:

Model based on total stresses slope stability analysis:

$$P_f(\mathrm{FS}) = \frac{1}{1+\exp\left(14.34(\mathrm{FS}-1.00)\right)}. \qquad (15)$$

Seismic slope stability analysis model:

$$P_f(\mathrm{FS} = \frac{1}{1+\exp\left(9.72(\mathrm{FS}-1.00)\right)}. \qquad (16)$$

In Eqs. (15) and (16), FS is computed using deterministic methods.

To evaluate the proposed probabilistic models, an illustrative example is presented. The selected parameters are given in Table 6. The deterministic minimum safety factors are calculated using Eqs. (1) and (2) and PSO algorithm. The probabilities of failure are determined by substituting appropriate calculated safety factors in Eqs. (15) and (16). The results are given in Table 7. It can be seen that the values of P_f for seismic slope stability is greater than stability of slope based on total stresses. Table 8 indicates that including seismic force causes the slope condition to change from below average to hazardous level.

Table 7. Determined probability of failures by proposed models.

Method	Factor of safety	Probability of failure
Total stress analysis	1.31	0.0114
Seismic analysis	1.14	0.2041

Table 8. Target reliability indices [68].

Expected performance level	Reliability index (β)	Probability of failure (P_f)
High	5.0	0.30 E -6.0
Good	4.0	0.30 E -4.0
Above average	3.0	0.10 E -2.0
Below average	2.5	0.60 E -2.0
Poor	2.0	0.23 E -1.0
Unsatisfactory	1.5	0.07
Hazardous	1.0	0.16

The proposed models are typical which have some limitations such as:

- The soil is assumed wet and the influence of water table and saturation is neglected.
- The models are proposed for finite slopes.
- Due to using Bishop's method, the sliding surface is assumed to be circular.

- In order to develop the model, a finite slope with arbitrary horizontal length of 10 m was selected. Therefore, the proposed model is recommended for horizontal length of 10 m and any arbitrary heights.

11. Conclusion

In this paper, the jointly distributed random variables method was used to develop a probabilistic model of seismic slope stability based on Bishop's method. The selected stochastic parameters are internal friction angle, cohesion and unit weight of soil, which are modeled using a truncated normal probability density function and the horizontal seismic coefficient which is considered to have a truncated exponential probability density function. The parameters related to geometry, height and angle of slope were regarded as constant parameters.

The safety factor relationships for probability density functions of the Bishop's method were derived analytically for total stress and seismic conditions and for an arbitrary slope. For this purpose, first using the mean value of the stochastic parameters, the critical surface with the minimum factor of safety was determined by the PSO algorithm. Then, by considering the soil parameters' uncertainty, the probability density functions of safety factor of the methods were obtained by JDRV. The results showed that the pdf has a nearly normal distribution and is compared favorably with the output of MCs. However, the required time to reach the same probability of failure is greater for the MCs than the JDRV method.

Sensitivity and parametric analyses were conducted to verify the results. It is shown that the friction angle is the most influential parameter in stability of slopes. At the end two probabilistic models for the seismic and total stresses methods are proposed.

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Appendix

Derivations of mathematical probability density functions of FS based on total stresses without seismic loading and with seismic loading have been presented in this appendix.

Slope stability analysis based on total stresses

The simplified Bishop's relationship can be rewritten by changed variables:

$$\mathrm{FS}=\frac{\sum_{i=1}^{n}\left[\frac{k_1.\Delta l_i.\cos\alpha_i+k_3.b_i.h_i.k_2}{\cos\alpha_i+(\sin\alpha_i.k_2)/\mathrm{FS}}\right]}{\sum_{i=1}^{n}k_3.b_i.h_i.\sin\alpha_i}. \tag{A.1}$$

For developing the probability density function of FS, the variables u_1, u_2 and u_3 are selected as an independent arbitrary function of k_1, k_2 and k_3 as follows:

$$\begin{cases} u_1=g_1(k_1,k_2,k_3)=\mathrm{FS} \\ \quad=\frac{\sum_{i=1}^{n}\left[\frac{k_1.\Delta l_i.\cos\alpha_i+k_3.b_i.h_i.k_2}{\cos\alpha_i+(\sin\alpha_i.k_2)/\mathrm{FS}}\right]}{\sum_{i=1}^{n}k_3.b_i.h_i.\sin\alpha_i} \\ u_2=g_2(k_1,k_2,k_3)=k_2 \\ u_3=g_3(k_1,k_2,k_3)=k_3 \end{cases} \tag{A.2}$$

Mapping from (k_1,k_2,k_3) to (u_1,u_2,u_3) is one-to-one, Hence, the functions of k_1, k_2 and k_3 can be written based on u_1, u_2 and u_3 as the following form:

$$\begin{cases} k_1=h_1(u_1,u_2,u_3)=c \\ \quad=\frac{u_1.\sum_{i=1}^{n}(u_3.b_i.h_i.\sin\alpha_i)-\sum_{i=1}^{n}\left[\frac{u_3.b_i.h_i.u_2}{\cos\alpha_i+(\sin\alpha_i.u_2)/u_1}\right]}{\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i}{\cos\alpha_i+(\sin\alpha_i.u_2)/u_1}\right]} \\ k_2=h_2(u_1,u_2,u_3)=u_2 \\ k_3=h_3(u_1,u_2,u_3)=u_3 \end{cases} \tag{A.3}$$

The probability density function of safety factor can be obtained as follow:

$$f_{X_i}(x_i)=\iint_{R_{Xi}}\cdots\int f_{X_1,X_2,\cdots,X_n}(x_1,x_2,...,x_n)\, dx_1dx_2...dx_{i-1}dx_{i+1}...dx_n, \tag{A.4}$$

where:

$$f_{X_1,X_2,...,X_n}(x_1,x_2,...,x_n) = |J(x_1,x_2,...,x_n)| .f_{X_1,X_2,...,X_n}\Big(h_1(x_1,x_2,...,x_n), ..., h_n(x_1,x_2,...,x_n)\Big), \quad (A.5)$$

and:

$$f_{X_1,X_2,...,X_n}(x_1,x_2,...,x_n) = f_{X_1}(x_1) .f_{X_2}(x_2)...f_{X_n}(x_n), \quad (A.6)$$

$$J(u_1,u_2,u_3) = \begin{vmatrix} \frac{\partial k_1}{\partial u_1} & \frac{\partial k_1}{\partial u_2} & \frac{\partial k_1}{\partial u_3} \\ \frac{\partial k_2}{\partial u_1} & \frac{\partial k_2}{\partial u_2} & \frac{\partial k_2}{\partial u_3} \\ \frac{\partial k_3}{\partial u_1} & \frac{\partial k_3}{\partial u_2} & \frac{\partial k_3}{\partial u_3} \end{vmatrix}. \quad (A.7)$$

Eq. (A.8) is shown in Box I.

$$B_1 = \cos\alpha_i + (\sin\alpha_i.u_2)/u_1. \quad (A.9)$$

Seismic slope stability analysis

The simplified Bishop's relationship can be rewritten by changed variables:

$$\mathrm{FS} = \frac{\sum_{i=1}^{n}\left[\frac{k_1.\Delta l_i.\cos\alpha_i + k_3.b_i.h_i.k_2}{\cos\alpha_i + (\sin\alpha_i.k_2)/\mathrm{FS}}\right]}{\sum_{i=1}^{n} k_3.b_i.h_i.\sin\alpha_i + \left(\sum_{i=1}^{n} k_4.k_3.b_i.h_i.d_i\right)/r}, \quad (A.10)$$

$$\begin{cases} u_1 = g_1(k_1,k_2,k_3,k_4) = \mathrm{FS} \\ \quad = \frac{\sum_{i=1}^{n}\left[\frac{k_1.\Delta l_i.\cos\alpha_i + k_3.b_i.h_i.k_2}{\cos\alpha_i + (\sin\alpha_i.k_2)/\mathrm{FS}}\right]}{\sum_{i=1}^{n} k_3.b_i.h_i.\sin\alpha_i + \left(\sum_{i=1}^{n} k_4.k_3.b_i.h_i.d_i\right)/r} \\ u_2 = g_2(k_1,k_2,k_3,k_4) = k_2 \\ u_3 = g_3(k_1,k_2,k_3,k_4) = k_3 \\ u_4 = g_4(k_1,k_2,k_3,k_4) = k_4 \end{cases} \quad (A.11)$$

Eqs. (A.12) and (A.13) are shown in Box II.

$$B_1 = \cos\alpha_i + (\sin\alpha_i.u_2)/u_1. \quad (A.14)$$

$$J = \frac{\sum_{i=1}^{n} u_3.b_i.h_i.\sin\alpha_i - \sum_{i=1}^{n}\left[\frac{u_3.b_i.h_i.\sin\alpha_i.u_2^2}{u_1^2.(B_1)^2}\right]}{\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i}{B_1}\right]} + \frac{\left(\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i.\sin\alpha_i.u_2}{u_1^2.(B_1)^2}\right]\right) \times \left(\sum_{i=1}^{n}\left[\frac{u_3.b_i.h_i.u_2}{B_1}\right] - u_1.\sum_{i=1}^{n}(u_3.b_i.h_i.\sin\alpha_i)\right)}{\left(\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i}{B_1}\right]\right)^2} \quad (A.8)$$

Box I

$$\begin{cases} k_1 = h_1(u_1,u_2,u_3,u_4) = c = \frac{u_1.\left(\sum_{i=1}^{n}(u_3,b_i,h_i.\sin\alpha_i) + (\sum_{i=1}^{n} u_4.u_3.b_i.h_i.d_i)/r\right) - \sum_{i=1}^{n}\left[\frac{u_3.b_i.h_i.u_2}{\cos\alpha_i + (\sin\alpha_i.u_2)/u_1}\right]}{\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i}{\cos\alpha_i + (\sin\alpha_i.u_2)/u_1}\right]} \\ k_2 = h_2(u_1,u_2,u_3,u_4) = u_2 \\ k_3 = h_3(u_1,u_2,u_3,u_4) = u_3 \\ k_4 = h_4(u_1,u_2,u_3,u_4) = u_4 \end{cases} \quad (A.12)$$

$$J = \frac{\sum_{i=1}^{n} u_3.b_i.h_i.\sin\alpha_i + \left(\sum_{i=1}^{n} u_4.u_3.b_i.h_i.d_i\right)/r - \sum_{i=1}^{n}\left[\frac{u_3.b_i.h_i.\sin\alpha_i.u_2^2}{u_1^2.(B_1)^2}\right]}{\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i}{B_1}\right]} + \frac{\left(\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i.\sin\alpha_i.u_2}{u_1^2.(B_1)^2}\right]\right) \times \left(\sum_{i=1}^{n}\left[\frac{u_3.b_i.h_i.u_2}{B_1}\right] - u_1.\left(\sum_{i=1}^{n}(u_3.b_i.h_i.\sin\alpha_i) + \left(\sum_{i=1}^{n} u_4.u_3.b_i.h_i.d_i\right)/r\right)\right)}{\left(\sum_{i=1}^{n}\left[\frac{\Delta l_i.\cos\alpha_i}{B_1}\right]\right)^2} \quad (A.13)$$

Box II

Biographies

Ali Johari obtained his BS, MS and PhD degrees in 1995, 1999 and 2006, respectively, from Shiraz University, Iran, where he is currently Assistant Professor in the Civil and Environmental Engineering Department. He was a Post-Doctoral Researcher at Exeter University in 2008, where he is also a member of the research staff of the Computational Geomechanics Group. His research interests include unsaturated soil mechanics, application of intelligent systems in geotechnical engineering, probabilistic models and reliability assessment. He has also consulted and supervised numerous geotechnical projects.

Sobhan Mousavi was born in Behbahan, Iran, in 1989. He received his BS degree in civil Engineering from Yasuj University, Yasuj, Iran, in 2012. He is studying geotechnical engineering at Shiraz University of Technology at MS degree. His current main interests of research are about soil slope stability analysis and reliability analysis in geotechnical engineering.

Ahmad Hooshmand Nejad was born in 1988 in Shiraz, Iran. He graduated from Shahid Bahonar University of Kerman in 2011 with a BS degree in Civil Engineering and, currently, he is furthering his geotechnical engineering education at Shiraz University of Technology as an MS student. His main research interests include: Artificial intelligence and reliability analysis in geotechnical engineering.