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Research Note

An investigation of friction angle correlation with geotechnical properties for granular soils using GMDH type neural networks

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Abstract. The Standard Penetration Test (SPT) is one of the most effective tests for quick and inexpensive evaluation of the mechanical properties of soil layers. Numerous studies have been conducted to evaluate correlations between SPT blow counts (N_{SPT}) and soil properties such as friction angle (φ'). In this paper, the relation between and in situ parameters of soil, including N_{SPT} , effective stress and fine content, is investigated for granular soils. In order to demonstrate the relevancy of φ' and corrected SPT blow count (N_{60}), a new polynomial model, based on the Group Method of Data Handling (GMDH) type Neural Network (NN), was used based on 195 data sets including three soil parameters. These were recorded after two major earthquakes in Turkey and Taiwan in 1999. This study addresses the question of whether GMDH-type NN is capable of estimating φ' based on specified variables. Results confirm that GMDH-type NN provide an effective way to recognize data patterns and predict performance over granular soils accurately. Finally, the effect of fine content and effective overburden stress on the correlation of N_{60} and φ' has been studied using sensitivity analysis.

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1. Introduction

One of the important parameters considered a typical characteristic for reconnaissance of granular soils is the friction angle (φ'). Owing to difficulties in soil sampling, and the high costs of representative undisturbed specimens, in-situ investigations, in lieu of laboratory element testing, are preferred to determine φ' directly. Therefore, it is necessary to determine φ' indirectly through methods such as the

SPT and Cone Penetration Test (CPT), which are commonly used for conventional geotechnical site investigations [1].

Conducting a SPT test during boring is widely used in geotechnical investigation projects. Local correlations and vast practical equations are available that relate SPT blow counts or N_{SPT} to the engineering behavior of earthworks and foundations [2].

In geotechnical engineering, many design parameters of soil are associated with the SPT. SPT blow count is significant in site investigation, along with other geotechnical parameters such as φ' . To the author's knowledge, there is no established theoretical relation between N_{SPT} and φ' .

Hence, their dependency, and evaluation of geotechnical properties, requires empirical correlations,

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statistical analysis and system identification techniques.

The interdependency of factors involved in such problems prevents the use of regression analysis and demands a more extensive and sophisticated approach. The Group Method of Data Handling (GMDH) type Neural Networks (NN) can be used to model complex systems, where unknown relationships exist between variables, without having specific knowledge of the process. In recent years, the use of such self-organizing networks has led to successful application of the GMDH-type NN in geotechnical science (e.g. [3–5]).

This treatment aims to develop a GMDH-type NN for the prediction friction angle (φ'), based on various soil parameters, such as corrected SPT blow count (N_{60}), fine content and effective overburden stress for granular soils. To this end, the paper first reviews previous efforts in correlating N_{SPT} and φ' . Then, a brief explanation of the case histories under consideration and the procedure of modeling with GMDH are presented. Finally, the developed GMDH model is described and its accuracy is assessed through sensitivity analysis.

2. Background to previously proposed correlations

The literature presents a portfolio of research regarding application of N_{SPT} for geotechnical characterization. A correlation between shear strength parameter, φ' ,

standard penetration resistance and effective overburden pressure was published by Schmertman, based on the previous work by De Mello. It must be noted that the mentioned chart provides only a rough estimate of the φ' value and should not be used for very shallow depths [6].

Also, other researchers have proposed correlations between N_{SPT} and φ' for different types of soil. Shioi and Fukui [7] proposed empirical relationships between φ' and energy corrected N_{SPT} (N_{70}). It is obtained from the Japanese Railway Standards [8] for roads, bridges and buildings. Zekkos et al. [9] presented an equation, based on best fit on the resulted equation suggested by Hatanaka and Uchida [10].

Sarat Kumar and Prabir Kumar [11] and Goktepe et al. [12] used artificial neural networks to correlate the friction angle of clays based on the index properties of soil. Mahdavian and Lalerokh [13] stated that a fuzzy algorithm can predict friction angle based on geotechnical soil properties. Esmaeilzadeh et al. [14] presented a detailed historical review and reported statistical correlations between N_{70} and φ' for sandy soil of Babolsar city, Iran, based on 46 standard penetration tests and direct shear tests.

Others have developed correlating equations accounting for corrected SPT blow counts (N_{60}) (e.g. [15–17]) and effective overburden stress (e.g. [18,19]). However, almost all studies have focused on relationships between N_{SPT} and φ' . Table 1 summarizes an inventory of antecedent research and empirical correlations.

Table 1. Inventory of the proposed correlations between uncorrected N_{SPT} and φ .

Ref.	Proposed relations	Remark
Dunham (1954)	$\varphi = (12N_{60})^{0.5} + 25$	Angular and well-grained soil particles
	$\varphi = (12N_{60})^{0.5} + 15$	Round and uniform-grained soil particles
	$\varphi = (12N_{60})^{0.5} + 20$	Round and well-grained or angular and uniform-grained soil particles
Pek et al. (1974)	$\varphi' = 53.881 - 27.6034\exp(0.0147N_{60})$	- - -
Shioi and Fukui (1982)	$\varphi = \sqrt{18N_{70}'} + 15$	Roads
	$\varphi = 0.36N_{70}' + 27$	Bridges
	$\varphi = 0.45N_{70}' + 20$	Buildings
Wolff (1986)	$\varphi = 27.1 + 0.3N_{60} - 0.00054N_{60}^2$	Sand
Kulhawy and Mayne (1990)	$\varphi = \tan^{-1} \{ N_{60} [12.2 + 20.3(\sigma'/p_a)] \}^{0.34}$	Sand
Hatanaka and Uchida (1996)	$\varphi' = \sqrt{20C_N N_{60}} + 20$	Sand
Zekkos et al. (2004)	$\varphi' = 3.5\sqrt{N_{1,60}} + 22.3 \pm \varepsilon$	- - -
Hettiarachchi and Brown (2009)	$\varphi' = \beta' \tan^{-1} \left[\frac{0.2N_{60}}{K(\sigma'/p_a)} - 0.68B \right]$	Sand
Esmaeilzadeh et al. (2012)	a : $\varphi = 0.66(N_{70}') + 8.52$	Sand
	b : $\varphi = 13.56 \ln(N_{70}') - 18.20$	

3. Overview of database and case histories

The field test results of the two earthquakes, i.e. Chi-Chi and Kocaeli, were used in this study to develop a GMDH model. Hanna et al. [20] synthesized the results of both site investigation programs. The database consists of 195 case records; 120, 50 and 25 data sets were used, respectively, for training, testing and the validation phase. A sample database given in Table 2 covers a wide range of soil parameters, as well as corrected SPT blow counts (N_{60}), FC (Fine Content% $\leq 75\mu\text{m}$), effective overburden stresses (σ'_{v0}) and φ' . Further details regarding the measurement and interpretation of the foregoing parameters are available in Hanna et al. [20].

Figure 1 illustrates the distribution of descriptive variable characteristics for all case histories.

4. Principles of modeling using GMDH type NN

The GMDH algorithm is a self-organizing approach by which, gradually, complicated models are gener-

ated, based on evaluation of their performances on a set of multi-input, single-output data pairs (x_i, y_i) ($i = 1, 2, \dots, M$). The GMDH was first developed by Ivakhnenko [21] as a multivariate analysis method for complex system modeling and identification. The main idea of GMDH is to build an analytical function in a feed forward network based on a quadratic node transfer function, whose coefficients are obtained using a regression technique [22].

By means of the GMDH algorithm, a model can be represented as a set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial, and thus, produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs. The formal definition of the identification problem is to find a function, \hat{f} , that can be used to approximate instead of the observed one, \hat{f} , in order to predict output, \hat{y} , for a given input vector, $X = (x_1, x_2, x_3, \dots, x_n)$, as close as possible to its observed output, \hat{y} . Therefore, given M observations of multi-input, single output data pairs, so that:

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad (i = 1, 2, 3, \dots, M). \quad (1)$$

The next step is training a GMDH type neural network to predict the output values, \hat{y} , for any given input vector, $X = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$; the predicted output is defined as:

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad (i = 1, 2, 3, \dots, M). \quad (2)$$

The problem is now to determine a GMDH type neural network, such that the square of differences between the observed and predicted outputs is minimized, that is:

$$\sum_{i=1}^M [\hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) - y_i]^2 \rightarrow \min. \quad (3)$$

The general connection between input and output variables can be defined by a complicated discrete form of the Volterra functional series, known as the

Table 2. A sample of the database extracted from Hanna et al. [19].

N_{60}	FC (%)	σ'_{v0} (kPa)	φ'
35	8	79	40.78
31	41	67.6	40.2
31	18	163.8	36.97
23	45	83	35.1
48	35	176.2	41.07
28	10	181.4	36.25
34	4	96.6	44.29
8	42	67.6	29.17
6	43	20.5	29.17
20	12	162.4	33.86
16	25	76.1	32.96
13	23	77	31.49
49	19	188.2	41.35
44	11	96.3	43.24
24	20	179.9	35.1
18	10	71.9	33.42

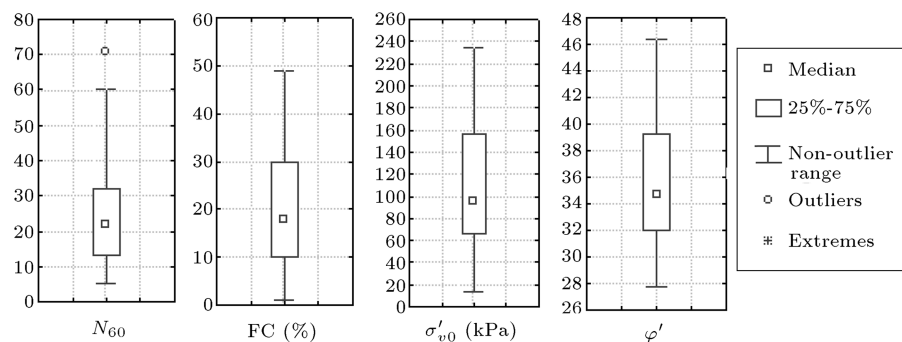


Figure 1. Distribution of descriptive variable characteristics for all case histories.

Kolmogorov-Gabor polynomial; Eq. (4) represents the polynomial series:

$$y = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots \quad (4)$$

The full form mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of:

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2. \quad (5)$$

By this means, the partial quadratic description is used in a network of connected neurons during a back calculation procedure to build the general mathematical relation between inputs and outputs given in Eq. (4). The coefficients, a_i , in Eq. (5) are calculated using regression techniques, so that the difference between the observed output, y , and the calculated one, \hat{y} , for each pair of x_i and y_i as input variables is minimized. Apparently, a tree of polynomials is constructed using the quadratic form given in Eq. (5), whose coefficients are obtained according to the least squares rule. In this way, the coefficients of each quadratic function, G_i , are derived to fit, optimally, the output in the whole set of input-output data pairs, that is:

$$E = \frac{\sum_{i=1}^M (y_i - G_i())^2}{M} \rightarrow \min. \quad (6)$$

In the basic GMDH algorithm, all possibilities of two independent variables out of the total n input variables are taken to construct the regression polynomial in the form of Eq. (5) that best fits the dependent observations ($y_i = 1, 2, \dots, M$) in a least squares sense.

Consequently, $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons will be built up in the first hidden layer of the feed forward network from the observations $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \dots, M)\}$ for different $p, q \in \{1, 2, \dots, n\}$.

In other words, it is now possible to construct M data triples $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \dots, M)\}$ from observations using $p, q \in \{1, 2, \dots, n\}$ in the form of:

$$\begin{bmatrix} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ \vdots & \vdots & \vdots \\ x_{Mp} & x_{Mq} & y_M \end{bmatrix}. \quad (7)$$

Using the quadratic sub-expression in the form of Eq. (5) for each row of M data triples, the following matrix equation can be readily obtained as:

$$Aa = Y, \quad (8)$$

$$a = \{a_0 + a_1 + a_2 + a_3 + a_4 + a_5\}, \quad (9)$$

$$Y = \{y_1, y_2, y_3, \dots, y_M\}^T, \quad (10)$$

where a is the vector of unknown coefficients for the quadratic polynomial in Eq. (5) and (Y) is the vector of output values from observation. It can be readily seen that:

$$\begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix}. \quad (11)$$

The least squares technique from multiple regression analysis leads to solve the normal equations;

$$a = (A^T A)^{-1} A^T Y. \quad (12)$$

Eq. (12) determines the vector of best coefficients of Eq. (5) for the whole set of M data triples. It should be noted that this procedure is repeated for each neuron of the next hidden layer, according to the connectivity topology of the network. However, such a solution directly from normal equations is rather susceptible to round off errors and more important to the singularity of these equations [23].

5. Proposed model

In order to develop the evolved GMDH-type NN, the database was divided into two different sets, namely, training and testing. The training set is consisted of 120 inputs-output data pairs. The testing set, consisting of 50 inputs-output unforeseen data during the training process, is merely used for testing the trained GMDH-type NN models. Of note, the training and testing sets are randomly selected from the data sets with approximately the same statistical information. Figure 2 illustrates the model predictive performance in comparison with the observed data tested for the training dataset. As seen in Figure 2, predicted and measured values are fairly close.

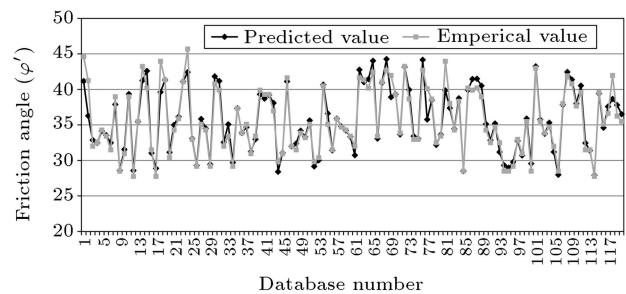


Figure 2. Neural network model predicted performance in comparison with observed data for the training set.

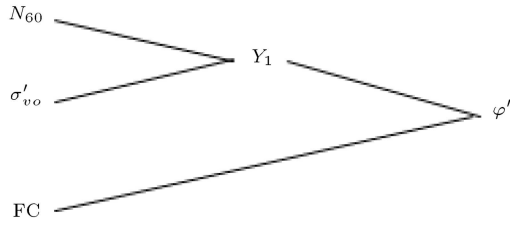


Figure 3. Evolved structure of generalized GMDH neural network for φ' .

Table 3. Statistical information for this study model in predicting friction angle.

Statistic	R^2	MSE	MAD	RMSE
Neural training	0.998	1.57	0.8	1.25
Neural testing	0.997	2.2	0.94	1.48

The corresponding polynomial representation of such a model for the friction angle is as follows:

$$\begin{aligned}\varphi' = & -3.574 - 0.183FC + 1.346Y_1 + 0.00111FC^2 \\ & - 0.0056Y_1^2 + 0.00127Y_1FC \\ Y_1 = & 27.298 + 0.638N_{60} - 0.0422\sigma'_{v0} - 0.00327N_{60}^2 \\ & + 0.000146\sigma'_{v0} - 0.000696\sigma'_{v0}N_{60}.\end{aligned}\quad (13)$$

The architecture of the evolved GMDH type neural networks is shown in Figure 3, corresponding to the genome representations.

As presented in Table 3, the statistically assessed accuracy of the model is evaluated via R^2 (absolute fraction of variance), RMSE (root-mean squared error), MSE (mean squared error), and MAD (mean absolute deviation), defined as follows:

$$R^2 = 1 - \left[\frac{\sum_{i=0}^M (Y_{i(\text{Model})} - Y_{i(\text{Actual})})^2}{\sum_{i=0}^M (Y_{i(\text{Actual})})^2} \right], \quad (14)$$

$$\text{RMSE} = \left[\frac{\sum_{i=0}^M (Y_{i(\text{Model})} - Y_{i(\text{Actual})})^2}{M} \right]^{1/2}, \quad (15)$$

$$\text{MSE} = \left[\frac{\sum_{i=0}^M (Y_{i(\text{Model})} - Y_{i(\text{Actual})})^2}{M} \right], \quad (16)$$

$$\text{MAD} = \left[\frac{\sum_{i=0}^M |Y_{i(\text{Model})} - Y_{i(\text{Actual})}|}{M} \right]. \quad (17)$$

The ability of the polynomial model to predict unforeseen data is conducted for the testing dataset. As seen in Figure 4, results from the model agree well with measured values.

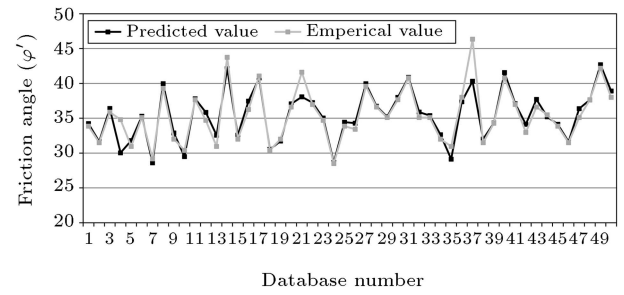


Figure 4. Neural network model performance in comparison with observed data for the testing set.

6. Validation and sensitivity analysis

The accuracy of the proposed model in predicting φ' was compared with correlations presented previously by Pek et al. [16], Shioi and Fukui [7], Wolff [17], Kulhawy and Mayne [18], Hatanaka and Uchida [10], Hettiarachch and Brown [19] and Esmaeilzadeh et al. [14]. The comparison was performed for all 25 validation cases that not included in training and testing sets. Figure 5 illustrates the scattering of predicted (calculated by different methods) versus observed φ' values.

As seen, the correlations of Kulhawy and Mayne [18] and Hatanaka and Uchida [10] overestimate measured values. Other researchers, such as Esmaeilzadeh et al. [14], Hettiarachch and Brown [19], Wolff [17], Shioi and Fukui [7] and Pek et al. [16], estimate φ' lower than measured values. But, according to this study, the GMDH model predicts the fiction angle accurately.

A sensitivity analysis is performed by a technique similar to the Partial Derivative or PaD method [24]. The basis for this method [25] is that the partial derivative of output Y_j , with respect to input x_i is a mathematical definition, reflecting the sensitivity of Y_j to x_i . Therefore, the higher the sensitivity of a function to a specific input variable, the greater its partial derivative with respect to that variable will be. This method can also reveal the effect of an input change on the network's final output value. For example, if the numerical value of the partial derivative, with respect to a variable, at a specific point, is positive, then, a slight increase in input variable will cause an increase in network output. Results of sensitivity analysis for the GMDH model of φ' are shown in Figure 6.

Overall, N_{60} has positive effect on φ' (Figure 6(a)). That is to say, an increase in N_{60} leads to an increase in φ' . The observed change trend is ascending with an increase in N_{60} . The main reason for this issue is N_{SPT} dependency on soil strength and density, which, as density increases, shear strength increases and results in a higher φ' [6].

As seen in Figure 6(b), Fine Content (FC) has a decreasing effect on φ' . This negative effect is less at

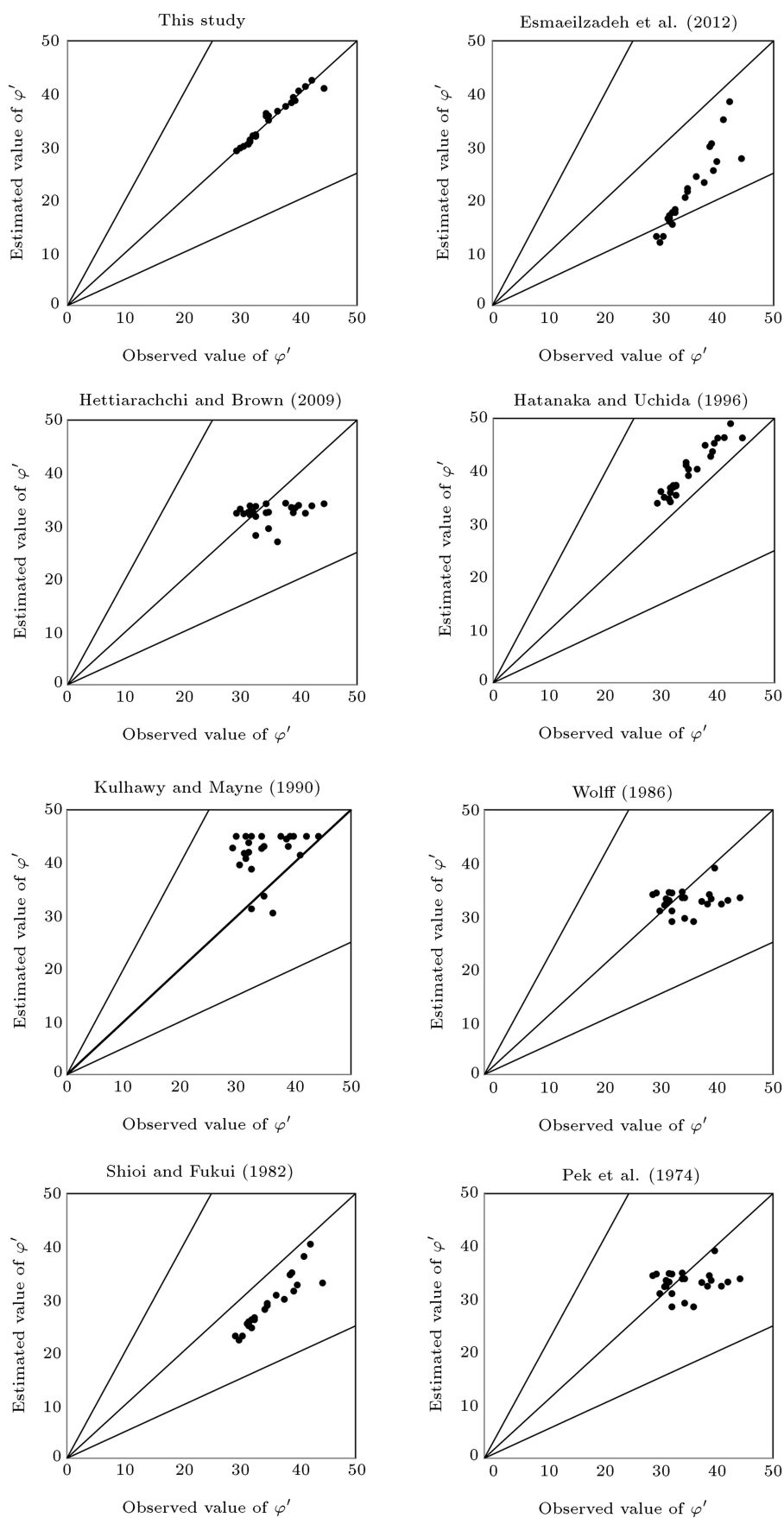


Figure 5. Estimated versus measured friction angle by different methods.

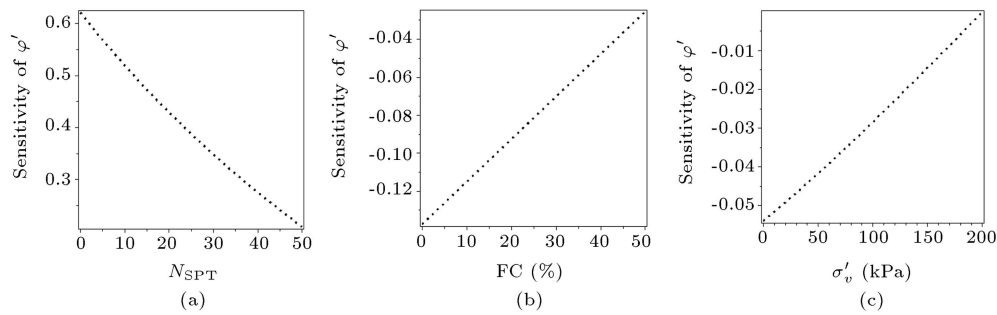


Figure 6. Result of sensitivity analysis based on N_{spt} (a), FC (b) and overburden stresses (c).

high FC values. The trend can be attributed to the low for fine soils in comparison to coarse soils [6].

Figure 6(c), shows the sensitivity of φ' based on effective overburden stresses, σ'_{v0} . It can be concluded that σ'_{v0} , as a representation of confining stress [6], has a negative effect on φ' .

Also, a sensitivity analysis of the obtained model can be done to evaluate the input parameters influence on model output. This is carried out by changing each of the input neurons at a constant rate, one at a time, while other variables are constant. Various constant rates (0.9, 0.95, 1.05, 1.1 and 1.2) were selected in the study. For every input neuron, the percentage change in the output, as a result of the change in the input neuron, is observed. The sensitivity of each input neuron is calculated by the following equation [26]:

$$\text{Sensitivity level of } X(\%) = \frac{1}{M} \sum_{j=1}^M \left(\frac{\% \text{change in output}}{\% \text{change in input}} \right)_j \times 100. \quad (18)$$

Results of the mentioned analysis of the obtained model are shown in Figure 7. It can be noticed that φ is considerably influenced by N_{60} .

7. Conclusions

The main aim of this paper is to deploy an identification system technique to develop φ' correlation with

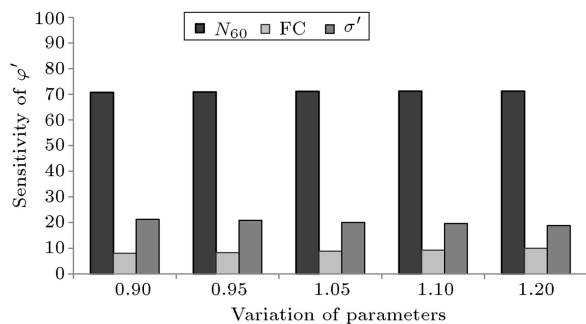


Figure 7. Results of the sensitivity analysis of the obtained model for friction angle.

soil geotechnical properties, and assess their influence on φ' . The evolved GMDH type neural network has been used to obtain a model for friction angle prediction.

A database of historical cases, consisting of 195 datasets from earthquakes in Taiwan and Turkey, was collected. A polynomial model was developed for φ' in terms of N_{60} , effective stress and fine content.

The validation and performance of the new model was assessed, and compared with previous statistical correlations. For all 25 validation case records, including φ' and soil geotechnical properties, predicted and measured φ' values were compared. The results demonstrate that correlations represented by Kulhawy and Mayne [18], and Hatanaka and Uchida [10] are overestimated, and those represented by Esmaeilzadeh et al. [14], Hettiarachch and Brown [19], Wolff [17], Shioi and Fukui [7] and Pek et al. [16] result in lower φ' than the measured values. However, the proposed approach predicts φ' with high accuracy and low variance.

The sensitivity analysis showed the effect and significance of input parameters, i.e. soil properties, on the predicted φ' . The results reveal that φ' is considerably affected by N_{60} , and decreases by increasing σ'_{v0} and fine content.

Results obtained from this study and previous research reveal that empirical correlations derived from a local dataset should not be implemented for different sites with significantly varying properties. Therefore, the study approach should be used while paying attention to geotechnical engineering for the same ranges and conditions.

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Biographies

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