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Uncertainty analysis using fuzzy randomness method towards development of fragility curves for moment-resisting steel structures

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Monte Carlo method.

Abstract. Seismic fragility analysis is one of the main steps of consequence based earthquake engineering process. Accurate uncertainties modeling involved in this methodology, affects the final results of seismic fragility analysis and hence assessment of decision variables which are the final products of performance-based seismic analysis. One aspect of such efforts is to incorporate the sources of uncertainties associated with various factors controlling seismic loads on the buildings as well as structural responses to such excitations. Probabilistic approach is usually used to model quantitative sources of such uncertainties, however, there are other factors with descriptive nature which probabilistic approach may not well incorporate them. In this paper a fuzzy randomness approach is used to model epistemic uncertainties as an alternative to the conventional probabilistic method. The approach is used to model those uncertainties which have not been addressed by the others, in particular the definition of the collapse limit state. To illustrate the efficiency of the proposed approach, fragility curves for a sample moment-resisting steel frame are developed. The results demonstrate the superiority of fuzzy solution in comparison with excising probabilistic methods to incorporate epistemic uncertainty in view of much less computational effort.

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1. Introduction

There are four major components and sources of uncertainties involved in the consequence-based evaluation of structural seismic performance. Such evaluation of building performance should be able to incorporate variability associated with seismic hazard, structural performance, structural damage, and decision variables. These uncertainties should be taken into account in order to provide solutions which could assist all stakeholders to make reasonable decision with regard to desired performance of new buildings or those under retrofitting process. PEER [1] summarizes the decision variables as number of casualties, down time and economic loss, and presents a consistent probabilistic framework to incorporate sources of uncertainties towards assessment of these decision variables [2].

Variability in the Intensity Measure (IM) is usually expressed by probabilistic strong ground motion values with various return periods or probability of exceedance. As intensity measure various parameters, such as Peak Ground Acceleration (PGA), Spectral Acceleration of first-mode natural period (SA) [3], or vector valued SA and epsilon may be used [4]. The variability in the estimated ground motion proposed

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by conventional seismic hazard methods project the likelihood of seismic events, as well as variability of induced ground motions modeled by empirical attenuation functions. Seismic hazard variability may also address other sources of uncertainties associated with earthquake source mechanism, ground motion frequency content and duration and even record-to-record variability [5].

The results of performing multiple nonlinear dynamic analyses of structural model subjected to a selected set of ground motion records (each scaled to increasing level of intensity) can be presented in terms of IDA curves. Each curve represents intensity measure (scalar/vector) versus Engineering Demand Parameter (EDP). The EDPs are commonly measured in terms of maximum story drift, floor acceleration or plastic hinge rotation. The contribution of EDP given IM (EDP|IM), in PEER probabilistic framework, is calculated based on results of IDA curves [3]. Probabilistic damage analysis, which relates the EDP to Damage Measurements (DMs), is the next step in DV assessment. The DMs quantitate descriptions of damage to components of a building which affects DVs, and may be categorized damages to structural elements, nonstructural elements and contents. Probabilistic aspect of DM given EDP (DM|EDP) is another contribution to DVs uncertainties. Successful evaluation of assumed DVs, in terms of necessary repairs, downtime estimation or number of casualties, requires DMs to be estimated relevantly. The seismic fragility functions, defined as the probability of a structural system or element reaching or exceeding a damage state, given a seismic intensity measure or EDP for equal seismic hazard, constitute probability context of damage. The convolution of these conditional probabilities is presented in the following form by PEER [6]:

$$\lambda(\mathrm{DV})=\iiint G(\mathrm{GV}|\mathrm{DM})\,dG(\mathrm{DM}|\mathrm{EDP})\,dG(\mathrm{EDP}|\mathrm{IM})\,d\lambda(\mathrm{IM}), \tag{1}$$

where, λ(DV) is the annual rate of exceeding a decision variable(s), G(DV|DM) is the complimentary cumulative distribution function of DV conditioned on the engineering damage measure(s), G(DM|EDP) is the complimentary cumulative distribution function of DM conditioned on engineering demand parameter(s), G(EDP|IM) is the complimentary cumulative distribution function of EDP conditioned on intensity measure(s), and $d\lambda$(IM) is the mean annual frequency of intensity measure(s). In this paper, uncertainties of a sideway collapse damage measure in moment-resisting steel structures are investigated. Sideway collapse is the consequence of successive reduction of load-carrying capacity of structural components that are part of building's lateral load-resisting system to the extent that second-order ($P-\Delta$) effects, accelerated by component deterioration, overcome gravity load resistance [7]. Two distinct uncertainty sources, entitled as aleatory, due to randomness, and epistemic, due to lack of knowledge or inaccuracy of analytical model, affect the characteristics of collapse fragility curves [5,8].

Probability theory has been applied to involve effects of these uncertainty sources and their combinations by several researches [5]. A Monte Carlo simulation, based on several realizations of probable parameters according to their probability distributions as input data for dynamic analyses, has been applied to structures resulted in variation of collapse performance of buildings, presented by lognormal probability distributions [9,10].

Variations associated with some parameters, such as construction quality and human errors, may not well be presented by probability distributions. On the other hand, the limit states, which are defined linguistically as EDP or IM thresholds, corresponds to initiation of various damage states [11]. Probabilistic methods must be enhanced to involve these sources of uncertainties. In this paper, the fuzzy randomness method is proposed to consider epistemic uncertainties effects on collapse fragility curves of steel moment-resisting frames. Variability of applied strong ground motions is considered as aleatory uncertainty, while connection moment-rotation modeling parameters, entitled as plastic hinge rotation capacity (θ_P), post capping rotation capacity (θ_{pc}), and rate of cyclic deterioration (Λ), of steel connections based on the moment rotation model proposed by Ibarra and Krawinkler [9], are considered as epistemic uncertainties. In previous researches, effects of these parameters on collapse performance of structures have been considered by First Order Second Moment (FOSM) and Monte Carlo simulation Methods [12]. In this paper, fuzzy randomness method is applied to involve modeling parameters uncertainties. Further application of the proposed method may be involving uncertainties of parameters which may not be presented by probability distributions.

Fuzzy numbers and fuzzy logic, introduced by Zadeh in 1965 [13], are powerful tools to incorporate epistemic uncertainties effects of linguistic variables in various problems [14]. Fuzzy numbers are represented by a membership function, which shows the membership degree of the variable belonging to a set. Since the experimental results, used by Lignos [15], have large values of dispersion, as the first step, in this study the modeling parameters are considered fuzzily and the results are verified by previous probabilistic methodologies. Further application of proposed formulation may be used similarly for the sources of uncertainties, which may not be represented by probability distributions such as human errors and construction quality. Non-probabilistic method of uncertainty treatment, applying fuzzy randomness method, has been used in structural analysis and optimization [16], damage detection, seismic risk analysis [17], safety assessment of structures [18] and structural reliability [19]. This method has not yet been applied to seismic fragility curves derivation, applicable in probabilistic framework of PEER. In this research, considering modeling uncertainties as fuzzy numbers, fuzzy randomness methodology is proposed to consider epistemic uncertainties and to derive 3-dimentional collapse fragility curves, presenting epistemic and aleatory uncertainties, separately.

2. Probabilistic formulation of collapse fragility curve

Based on key variable selection, formulation of collapse fragility curve may be written in IM-based or EDP-based formats [7]. IM-based formulation, which applies IM as controlling variable, is written as:

$$P(\text{Collapse}|\text{IM} = im_i) = P(\text{im}_i > \text{IM}_{\text{LS}}) = F_{\text{IM}_{\text{LS}}}(\text{im}_i), \tag{2}$$

where, $F_{\text{IM}_{\text{LS}}}(\text{im}_i)$ is the cumulative probability distribution, expressed by intensity measure of imposed strong ground motion, which is extracted through application of IDA to considered structure. Derivation of parameters of this probability distribution requires a definition of IM_{LS} and a procedure to propagate the epistemic and aleatory uncertainties involved in IM_c [7]. Applying EDP as intermediate variable, EDP-Based formulation is written as:

$$P(\text{Collapse}|\text{IM}=\text{im}_i)=\Sigma_{\text{edp}_c} P(\text{EDP}_d>\text{EDP}_c|\text{EDP}_c = \text{edp}_{c_i}, \text{IM}=\text{im}_i).P(\text{EDP}_c=\text{edp}_{c_i}). \tag{3}$$

The collapse limit state, considered in this study, is defined as the IM of strong ground motion in which the structure undergoes the dynamic instability. In other words, IM_c is defined as the last-converged result on an IDA curve at which the nonlinear response history analysis has converged [7]. Estimation of collapse capacity of structures by EDP-based formulation results in underestimation of EDP_c, since the remaining capacity left after IDA curve passes the collapse criterion of 20% of initial slope; furthermore EDP_c is a function of IM, which adds another approximation of EDP-based method [12]. In this paper, IM-based formulation is applied to evaluate collapse fragility curve of structures. Applying this approach, for each IDA curve a point will be the representative of $\text{IM}_{\text{Collapse}}$ and the exceedance probability distribution of collapse limit state will be achieved [12]. If fixed values of modeling parameters are considered, only aleatory uncertainties will be involved.

The collapse fragility curves are represented by lognormal probability distributions [9]. The fragility curves obtained from IDA analysis is:

$$P(C|\text{IM} = \text{im}_i) = \Phi\left(\frac{\text{Ln}(\text{im}_i) - \mu}{\sigma}\right), \tag{4}$$

in which $\Phi(.)$ is the standard Gaussian distribution function and μ and σ are mean and standard deviation of collapse fragility curve, respectively.

3. Epistemic uncertainty treatment

There are varieties of methods for considering epistemic uncertainties effects, such as sensitivity analysis, First-Order-Second-Moment Method (FOSM) and Monte Carlo simulation methods. In sensitivity analysis, the effect of each random variable on structural response is determined by varying a single modeling parameter and re-evaluating the structure's performance. This method has been applied to select the most important parameter in performance assessment of structures [20-23].

In FOSM method, collapse capacity limit state $g(x)$ is linearized applying a Taylor series expansion about the mean values of modeling parameters at $x = \mu$. So that, mean of collapse fragility function is unchanged, that is $\mu_g = g(x = \mu)$, while variance of collapse fragility function is computed from gradients of $g(x)$. Gradients of $g(x)$ are calculated by perturbation of modeling parameters in a series of sensitivity analysis, which can be done by one-side or two-side methods [4,9]. Calculation of $g(x)$ derivatives by one-side and two-side methods, respectively, are given by:

$$\frac{\partial g}{\partial Q} = \frac{g(\mu_Q) - g(\mu_Q \pm n\sigma_Q)}{\pm n\sigma_Q}, \tag{5}$$

$$\frac{\partial g}{\partial Q} = \frac{g(\mu_Q - n\sigma_Q) - g(\mu_Q + n\sigma_Q)}{2n\sigma_Q}. \tag{6}$$

Changes in mean values of collapse fragility curves cannot be predicted applying FOSM method. In Monte Carlo methods, thousands of input random variable realizations and collapse capacity calculation for each realization results in probability distribution of collapse capacity of assumed structure [24-27]. The crude Monte Carlo method is very elaborate in implementation and is not practical for collapse prediction of structures with probable modeling parameters [5]. Response surface based Monte Carlo method has been applied to reduce computational efforts by several researches [5,24].

The methods for combination of epistemic and aleatory uncertainties are categorized into mean estimate method [28] and Confidence Interval Method [29,30]. Applying mean estimate method, mean value of collapse fragility curve is remained unchanged, and epistemic uncertainties affect standard deviation of collapse fragility curve. On the other hand, in Confidence Interval Method, epistemic uncertainties affect mean value of collapse fragility curves and standard deviation remains unchanged.

Applying hysteretic models which are capable of modeling deterioration of components is important aspect of accurate estimation of collapse limit state in structures and has been investigated in a numerous studies [7,9,31-33]. According to advantages, such as capability of modeling various modes of component deterioration, refinement of parameters definition and consistency with experimental tests of steel and concrete components [15], modified Ibarra-Krawinkelr model is applied here [5]. Modeling parameters of steel moment resisting connections are considered as epistemic uncertainties, and their effects on collapse fragility curves are investigated in this paper. The backbone curve of considered moment-rotation model, referred to as modified Ibarra-Krawinkler model is shown in Figure 1.

Definition of modeling parameters (Figure 1) is as follows:

θ_c : Cap rotation
M_y : Effective yield moment
θ_y : Effective yield rotation
θ_c : Ultimate rotation Capacity
θ_p : Plastic rotation Capacity
θ_{pc} : Post-Capping rotation Capacity.

The hysteretic behavior of the connection is defined based on deterioration rules which are defined according to hysteretic energy dissipated in each hysteretic cycle. The deterioration of basic strength, post capping strength, unloading stiffness and reloading stiffness can be considered in this model [22]. Comparison of considering and neglecting cyclic deterioration of component behavior is shown in Figure 2 [12].

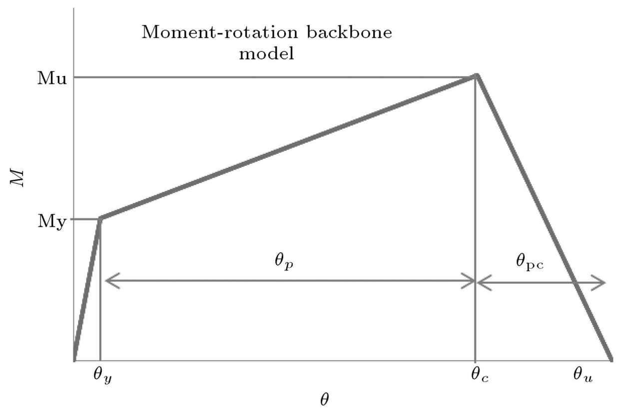

Figure 1. Back-bone curve of moment rotation model based on modified Ibarra-Krawinkler model (Ibarra et al., 2005 [9]).

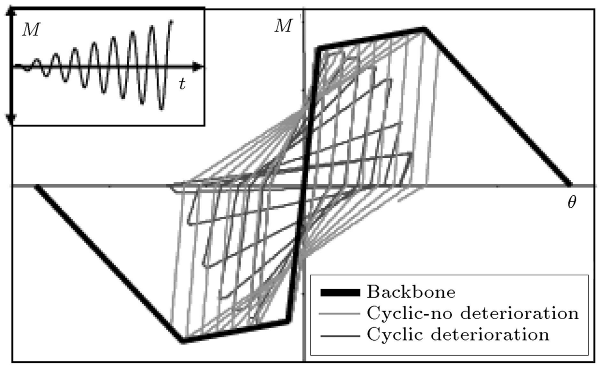

Figure 2. Effects of cyclic deterioration modeling on M-θ backbone curves (Zareian et al., 2009 [12]).

Capacity of energy dissipation of the component is defined by:

$$E_t = \Lambda M_y, \tag{7}$$

where Λ is rate of cyclic deterioration and is evaluated according to calibration of experimental results, which is considered as a modeling parameter in this study. It has been shown that θ_P, θ_{pc} and Λ have the most appreciable effects compared to other modeling parameters on collapse performance of structures [34]. In this study these three parameters have been chosen as modeling parameters including inherent epistemic uncertainty.

4. Proposed method to propagate epistemic uncertainties

In this paper, the fuzzy randomness approach is proposed for propagation of epistemic uncertainties. Application of fuzzy randomness method, in treatment of epistemic uncertainty, is summarized in [18]. In this method, the parameters with epistemic uncertainties are presented by fuzzy numbers, defined by Zadeh in 1965 [13]. For a given membership function value, α, a crisp set, $A\alpha$, is obtained as shown in Figure 3. This is referred to as a "α-cut" set and expressed by:

$$A_\alpha = \{X|\text{MF}(X) \geq \alpha\}, \tag{8}$$

in which MF(X) is membership function value of parameter X. A range of values for α and implementing interval analysis methods developed by Rao and Berke, in 1969 [35] of input variables provides fragility curves with fuzzy means and standard deviations.

The flowchart of the proposed method is shown in Figure 4. The first step of the proposed approach is to

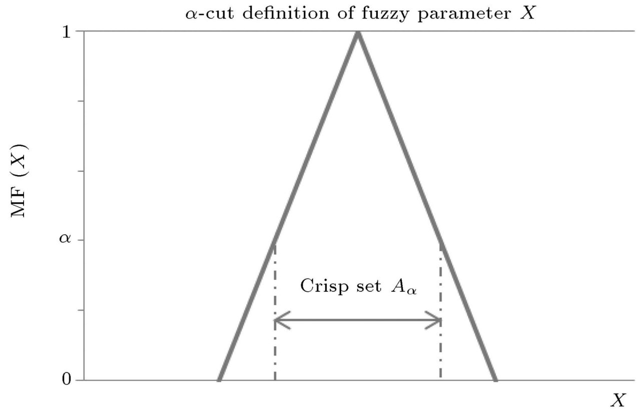

Figure 3. α-cut definition of X.

consider limited numbers of values for input random variables. In this study, mean minus one standard deviation, mean plus one standard deviation and mean values are considered for each variable (totally 27 values). Firstly, mean and standard deviation of equivalent uncorrelated variable are calculated, and the perturbation of input variables is done based on achieved means and standard deviations. The transformation of uncorrelated realization to correlated values is implemented later in the following. Means and standard deviations of collapse fragility curves are calculated based on Incremental Dynamic Analysis of structure considering modeling parameters as perturbed values. Quadratic surfaces are applied to derive analytical relationship between predictor variables (θ_p, θ_{pc} and Λ) and response parameters (collapse fragility curve mean and standard deviation). Formulations of applied quadratic functions are also discussed later.

In the next step, to involve epistemic uncertainties, the modeling variables are considered as fuzzy numbers with triangular membership functions, between mean minus and mean plus one standard deviation; the same interval in which the perturbation of variables is implemented. Several intervals of variables are considered by taking various vales for α-cuts of membership functions. For each interval, minimum and maximum of mean and standard deviation of collapse fragility curves are calculated considering constructed response surfaces, and then solving constrained nonlinear optimization problem. Extreme values of mean and standard deviation are correspondent to α-cut values of derived membership functions of mean and standard deviation of collapse fragility curves, which involve the effects of epistemic uncertainties. Presentation of mean and standard deviation of collapse fragility curves by fuzzy numbers may be interpreted as the bounds of validity for collapse fragility curves.

Having Fuzzy input variables and resultant fuzzy probability distribution of collapse allows us to calculate the intervals of collapse probability. The interval of the probability of collapse between IM_1 and IM_2 values is calculated based on fuzzy probability theory [14]. The interval of the probability, or possibility according to fuzzy randomness method, shows the effects of epistemic uncertainties (modeling parameters), and the probability itself shows the effects of aleatory uncertainties. Probability density function of collapse limit state, evaluated based on fuzzy input variables, has a fuzzy mean and standard deviation. The PDF is written as:

$$f_{\text{IM}_c}(\text{im}) = \frac{1}{(\text{im})\tilde{\sigma}\sqrt{2\pi}}\exp\left(\frac{-(\text{Ln}(\text{im}) - \tilde{\mu})}{2\hat{\sigma}^2}\right), \qquad (9)$$

in which $\tilde{\mu}$ and $\tilde{\sigma}$ are the fuzzy mean and fuzzy standard deviation of collapse probability distribution. If $z_1 = \frac{\text{IM}_1 - \tilde{\mu}}{\tilde{\sigma}}$ and $z_2 = \frac{\text{IM}_2 - \tilde{\mu}}{\tilde{\sigma}}$, collapse probability

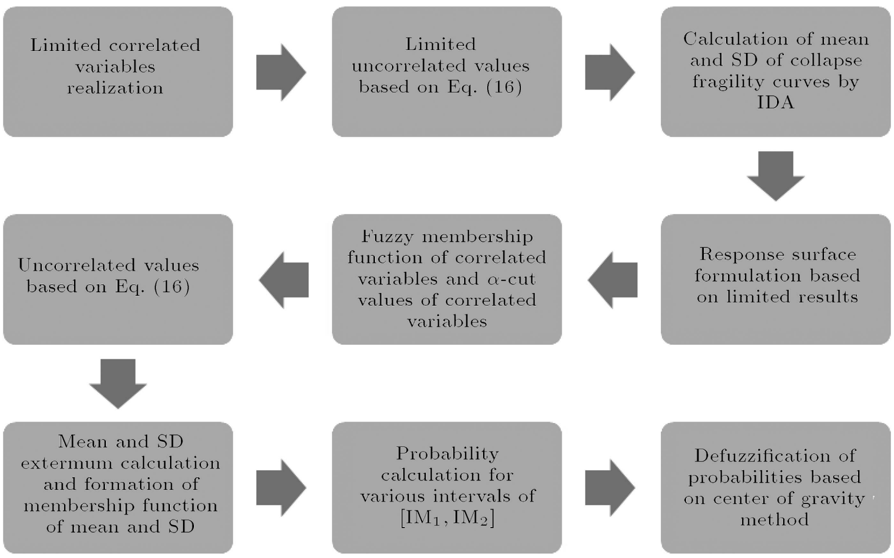

Figure 4. The proposed approach flowchart for incorporating epistemic uncertainty associated with fuzzy randomness.

in the interval $[\mathrm{IM}_1, \mathrm{IM}_2]$ is calculated based on fuzzy probability theory [14], and is written as:

$$\tilde{P}[\mathrm{IM}_1, \mathrm{IM}_2][\alpha]=\left\{\int_{z_1}^{z_2} f_{\mathrm{IM}_c}(\mathrm{IM})d(\mathrm{im})|\mu \in \tilde{\mu}, \sigma \in \tilde{\sigma}\right\}. \quad (10)$$

It is proved that the probability, which is presented by Eq. (7), is a fuzzy number itself. The α-cut values of this fuzzy number are presented by [36]:

$$\tilde{P}[\mathrm{IM}_1, \mathrm{IM}_2][\alpha] = [P_1(\alpha), P_2(\alpha)], \quad (11)$$

in which $P_1(\alpha)$ and $P_2(\alpha)$ are calculated based on:

$$P_1(\alpha)=\mathrm{Min}\left\{\int_{z_1}^{z_2} f_{\mathrm{IM}_c}(\mathrm{im})d(\mathrm{im})|\mu \in \tilde{\mu}, \sigma \in \tilde{\sigma}\right\}, \quad (12)$$

$$P_2(\alpha)=\mathrm{Max}\left\{\int_{z_1}^{z_2} f_{\mathrm{IM}_c}(\mathrm{im})d(\mathrm{im})|\mu \in \tilde{\mu}, \sigma \in \tilde{\sigma}\right\}. \quad (13)$$

While the intervals of probability for different values of IM_1 and IM_2 are calculated, defuzzification of fuzzy probabilities may be done by center of mass method. The collapse probability involving both effects of aleatory and epistemic uncertainties, which is based on most common used center of gravity defuzzification method [14], is given by:

$$\mu = \frac{\Sigma\mu_i \mathrm{MF}(\mu_i)}{\Sigma \mathrm{MF}(\mu_i)}, \qquad \sigma = \frac{\Sigma\sigma_i \mathrm{MF}(\sigma_i)}{\Sigma \mathrm{MF}(\sigma_i)}. \quad (14)$$

5. Correlation of input variables

To generate realizations of correlated variables $\{X\}$, we have to first generate samples of uncorrelated variables $\{Y\}$. Then $\{X\}$ values are calculated based on the variable transformation:

$$\{X\} = [T]\{Y\}. \quad (15)$$

Consideration of correlation between input variables necessitates solving an eigenvalue problem of covariance matrix of input variables $[\Sigma_X]$ [37]. For n input variables, covariance matrix is defined in Eq. (14). Eigenvalues of covariance matrix are calculated by the determinant equation:

$$\det([\Sigma_X] - \lambda[I]) = 0, \quad (16)$$

in which $[I]$ is the unit matrix; det(.) shows the determinant; and $[\Sigma_X]$ is covariance matrix for input variables $\{X\}$ given by:

$$[\Sigma_X]=\begin{bmatrix} \mathrm{COV}(X_1, X_1) & \cdots & \mathrm{COV}(X_1, X_N) \\ . & \cdots & . \\ . & \cdots & . \\ \mathrm{COV}(X_N, X_N) & \cdots & \mathrm{COV}(X_N, X_N) \end{bmatrix}. \quad (17)$$

Solving Eq. (15) produces n values for λ, each being an eigenvalue of covariance matrix. Covariance matrix and mean values of new variables, named $\{Y\}$, which are uncorrelated can be calculated by [37]:

$$[\Sigma_Y] = [T]^T [\Sigma_X] [T] = \begin{bmatrix} \sigma_{Y_1}^2 & 0 & 0 \\ 0 & \sigma_{Y_2}^2 & 0 \\ 0 & 0 & \sigma_{Y_3}^2 \end{bmatrix}, \quad (18)$$

$$\{\mu_Y\} = [T]^T \{\mu_X\}. \quad (19)$$

where $[\Sigma_Y]$ and $\{\mu_Y\}$ are covariance matrix and mean vector of variables $\{Y\}$, respectively. Superscript T denotes transpose. Transformation matrix consists of the eigenvectors corresponding to eigenvalues and is calculated based on:

$$[T] = [\Sigma_X] - \lambda[I]. \quad (20)$$

While sample values of variables $\{Y\}$ are obtained, main-values $\{X\}$, is calculated based on the transformation (Eq. (15)).

6. Sample structure

To evaluate the proposed method, one 3-storey steel moment resisting frames in two ways is considered (Figure 5). The structure is designed based on Iranian Seismic Code 2800 [38]. The soil type is considered as type B. The story height and bay width are assumed to be 3.2 and 5 meters, respectively. A rigid diaphragm is supposed based on usual floor systems existing in common structures. The value of response modification factor is considered as $R = 10$, adopted from [38], corresponding to special moment resisting structures. The designed member sections are depicted in Table 1.

Non-structural and content vulnerability of the building is not considered in this research. Effects of

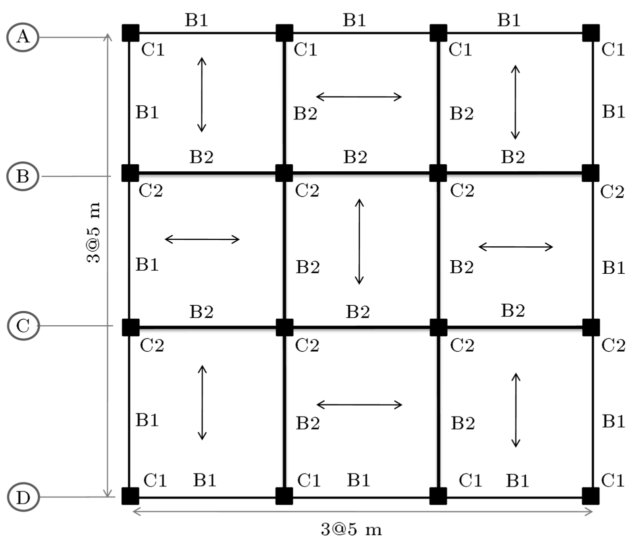

Figure 5. Plan of sample structure.

Table 1. Design sections for considered structure.

Storey	C1	C2	B1	B2
1	BOX 180 × 180 × 16	BOX 200 × 200 × 16	IPE 300	IPE 330
2	BOX 180 × 180 × 16	BOX 200 × 200 × 16	IPE 300	IPE 330
3	BOX 180 × 180 × 16	BOX 200 × 200 × 16	IPE 300	IPE 330

construction quality and human errors in construction are not considered in deriving seismic fragility curves. The assumed structure is symmetric in plan and elevation, which allows two dimensional structural analyses.

To assess the proposed method, a two-dimensional frame (frame NO.B or NO.C) is considered. Moment resisting connections are considered as rotational springs whose behaviors are based on modified Ibarra-Krawinkler model [9]. M2-WO panel zone model is considered since yielding in the beams, columns and panel zones is represented well by this model (Figure 6).

Fundamental parameters which are considered as epistemic uncertainties in this study are θ_p, θ_{pc} and Λ. Estimation of modeling parameters, based on laboratory tests, is shown in Table 2. The correlation coefficients of input variables depicted in this table were presented by Lignos [15].

The 5% linear elastic spectral acceleration at the first-mode period of the structure is considered as intensity measure of strong ground motions. The advantage of this IM is that the seismic hazard data for $S_a(T_1)$ is available. Maximum inter-storey drift is considered as EDP, since this EDP is the main source of sideway instability which causes the sideway collapse of the structure [9]. A set of 40 strong ground motions according to Medina [39] entitled as LMSR records, is considered for investigation of global collapse limit state of the structure.

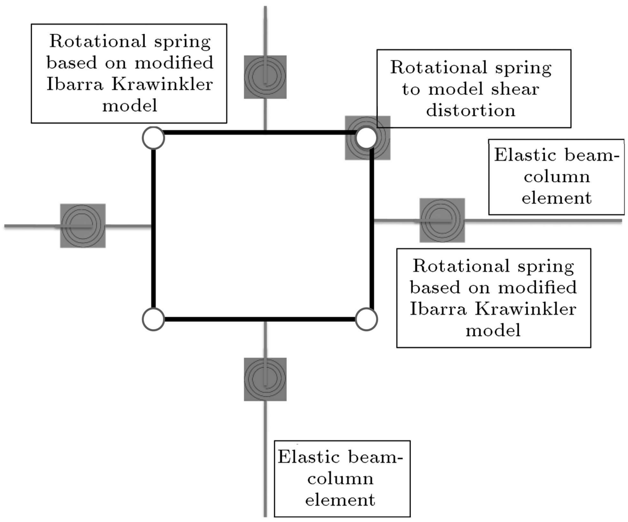

Figure 6. Panel zone model M2-WO (Douglas et al., 2002 [40]).

Covariance matrix of these input variables is:

$$\lambda_{1,2,3} = 0.0407, 0.103, 0.3937,$$

$$[\rho] = \begin{bmatrix} 1 & 0.69 & 0.44 \\ 0.69 & 1 & 0.67 \\ 0.44 & 0.67 & 1 \end{bmatrix}. \tag{21}$$

Three values of input parameters are considered corresponding to $\mu-\sigma, \mu, \mu+\sigma$. Considering the correlation between variables, the realization matrix of input variables is transformed by Eq. (15). The eigenvalues and transformation matrix for input variables are calculated based on Eq. (17).

The input variables considering the correlation are obtained according to:

$$\begin{bmatrix} (\theta_P)_1 & (\theta_P)_2 & (\theta_P)_3 \\ (\theta_{\text{PC}})_1 & (\theta_{\text{PC}})_2 & (\theta_{\text{PC}})_3 \\ (\Lambda)_1 & (\Lambda)_2 & (\Lambda)_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0239 & 0.025 & 0.0161 \\ 0.0941 & 0.16 & 0.2721 \\ 0.6041 & 1.000 & 1.6555 \end{bmatrix}. \tag{22}$$

The tree diagram of realizations for input variables is shown in Figure 7. Each branch of the tree shows a value for one of input variables. For each set of values, the IDA analysis of the building is done and collapse fragility curve is derived based on Eq. (2), considering $S_a(T_1)$ as intensity measure and maximum inter-storey drift as engineering demand parameter. The sample IDA and collapse fragility curves are shown in Figures 8 and 9. Eqs. (21) and (22) show the function forms, and Table 3 shows the derived constant coefficients based on nonlinear regression analysis. The geometric representation of response surfaces for mean and standard deviation of collapse capacity is shown

Table 2. Modeling parameters of mean and dispersion and correlation calibration based on experimental results (Lignos, 2008 [15]).

Median θ_p (rad)	$\sigma_{\theta p}$ (rad)	Median θ_{pc} (rad)	$\sigma_{\theta\text{pc}}$ (rad)	Median Λ	σ_Λ	$\rho_{\theta p,\theta\text{pc}}$	$\rho_{\theta p,\Lambda}$	$\rho_{\theta\text{pc},\Lambda}$
0.025	0.43	0.16	0.41	1.00	0.43	0.69	0.44	0.67

Table 3. Constant coefficients of response surface functions for mean and standard deviation.

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	**Error**
μ_{Ln} (Sac)	-3.4871	244.1685	3.2836	1.0601	-45.2769	-5.1520	1.0558	-465 3.2599	-6.5875	-0.3707	0.0191
	C'_0	C'_1	C'_2	C'_3	C'_4	C'_5	C'_6	C'_7	C'_8	C'_9	Error
σ_{Ln} (Sac)	11.38863	-814.422	-5.4950	-0.8624	175.1236	32.5449	0.346 7	15162.9477	2.6023	0.0016	0.0320

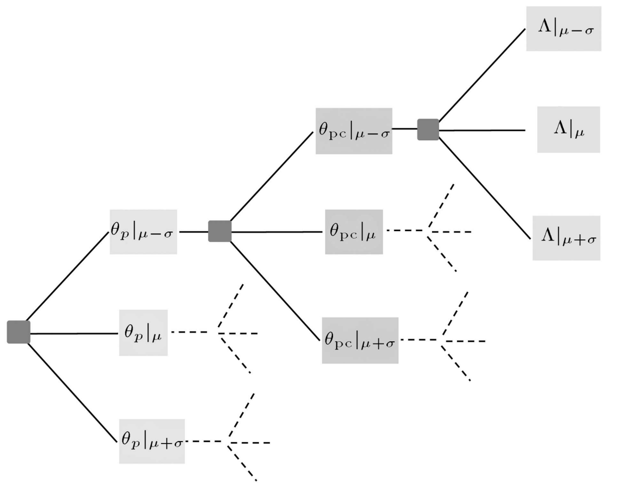

Figure 7. Tree diagram for preassumed values of modelling parameters.

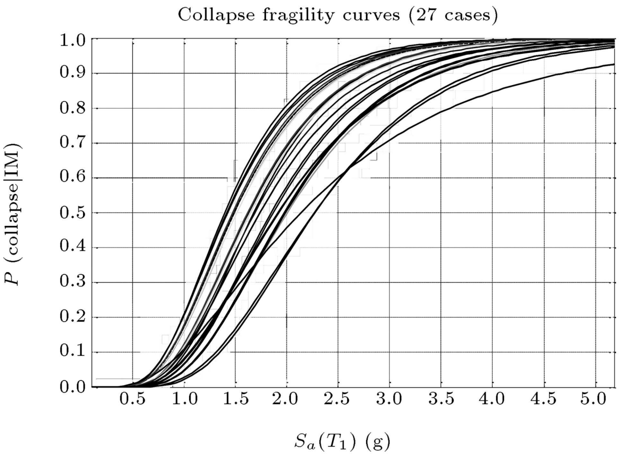

Figure 9. Sample IDA curves, and collapse fragility curves for 27 cases.

in Figure 10 in which x_1 is θ_p, x_2 is θ_{pc} and x_3 is Λ. The response surface functions are used for estimation of minimum and maximum values of collapse fragility curve mean and standard deviation. Membership functions considered for modeling variables are shown in Figure 11. Four cases for α-cut values are considered, $\alpha = 0.2$, 0.4, 0.6, 0.8. For values of α, bounds of input variables are calculated. Maximum and minimum values for mean and standard deviation of collapse capacities are presented in Table 4. These values are corresponding to α-cut of moments of collapse fragility curves. The derived membership functions for mean and standard deviation of collapse capacity are shown in Figure 12 and fuzzy collapse fragility curves are presented in Figure 13. In Figures 14 and 15 the results of FOSM method, the proposed method and Monte Carlo simulation based on quadratic response surface are shown.

The perspective view of three-dimensional fragility curves resulted from fuzzy randomness method can be seen in Figure 13. In this figure, the

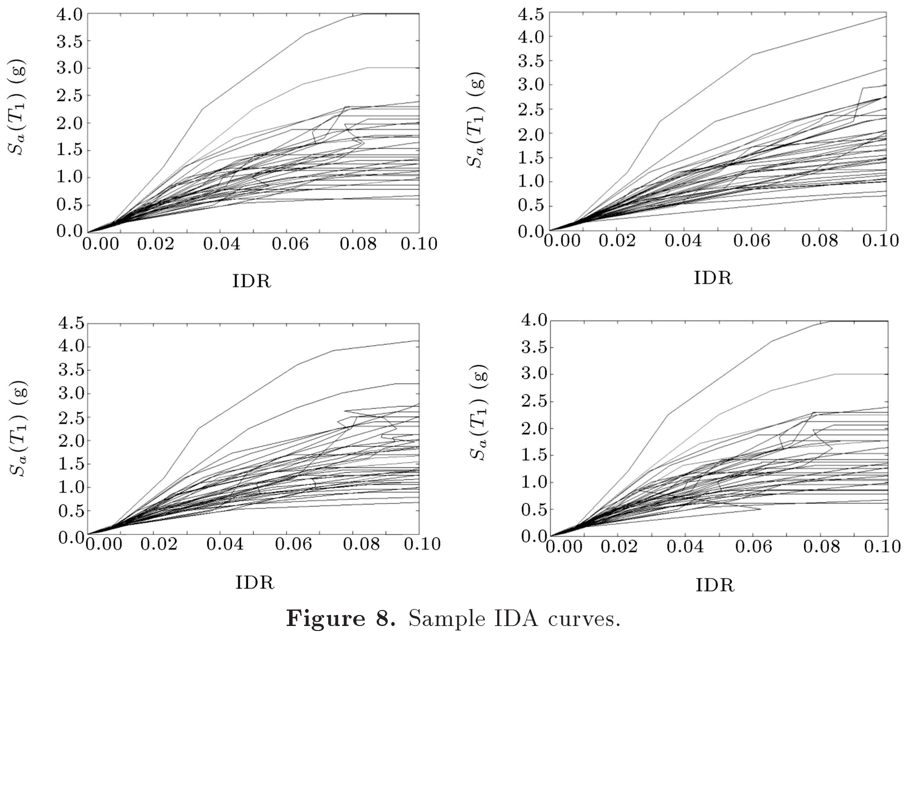

Figure 8. Sample IDA curves.

Table 4. Values of modeling and collapse fragility curves parameters for several values of α-cuts.

α-cut		θ_P	θ_{PC}	Λ	Mean of fragility curve	Standard deviation of fragility curve
0.2	Min	0.0241	0.1046	0.6681	0.3859	0.3973
	Max	0.0259	0.2446	1.4968	0.7939	0.4348
0.4	Min	0.0243	0.1163	0.7390	0.4429	0.3984
	Max	0.0257	0.2200	1.3533	0.7607	0.4193
0.6	Min	0.0245	0.1294	0.8174	0.5	0.3993
	Max	0.0254	0.1978	1.2235	0.7175	0.4079
0.8	Min	0.0247	0.1439	0.9041	0.5824	0.4001
	Max	0.0252	0.1779	1.1061	0.6432	0.4015

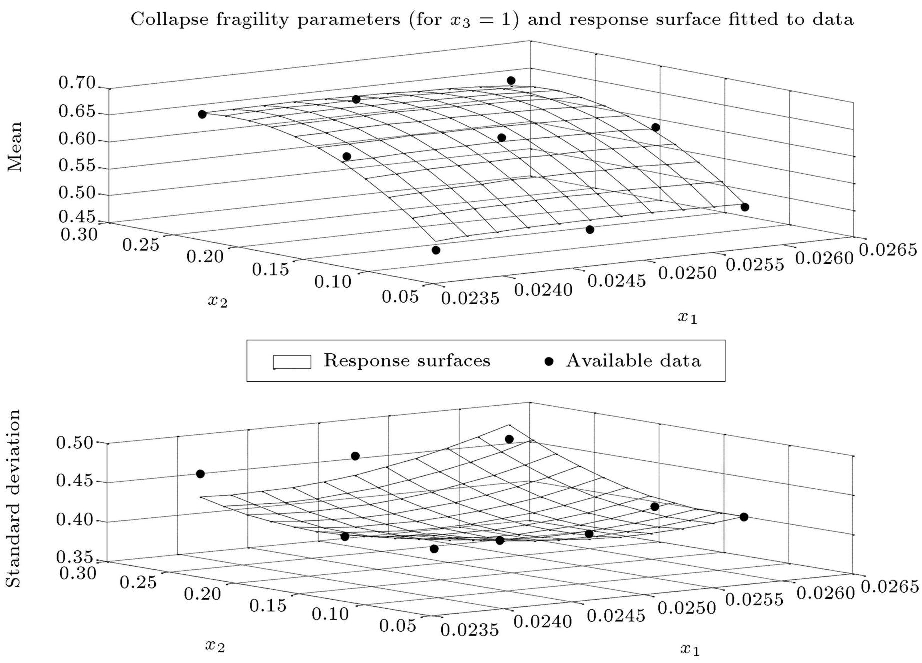

Figure 10. Response surface fitted to calculated means and standard deviations.

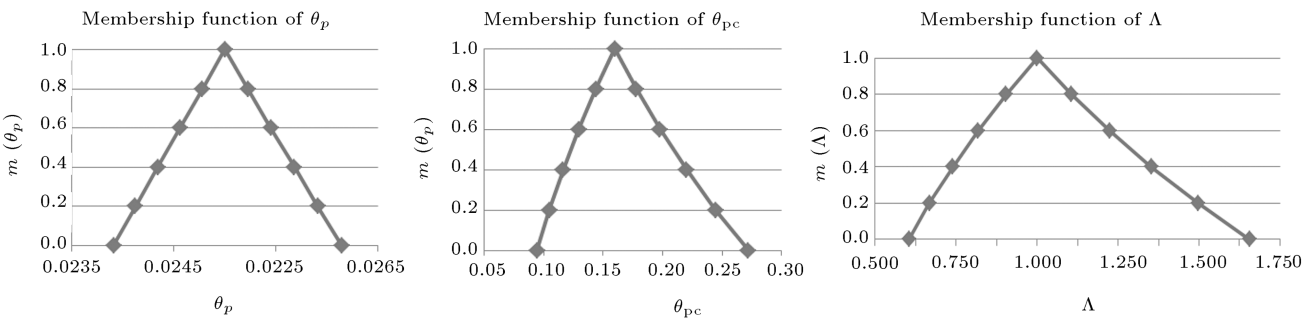

Figure 11. Membership functions considered for modeling parameters.

probability of reaching or exceeding collapse capacity as a function of both IM and membership values of mean and standard deviation of collapse fragility curves are shown.

The membership function of achieved fuzzy mean and standard deviation show the variation of response parameters due to uncertainty of modeling parameters. These membership functions may be interpreted as effects of epistemic uncertainties of modeling parameters on response parameters. These effects both make changes to mean and standard deviation of collapse fragility curves. To compare the proposed method with

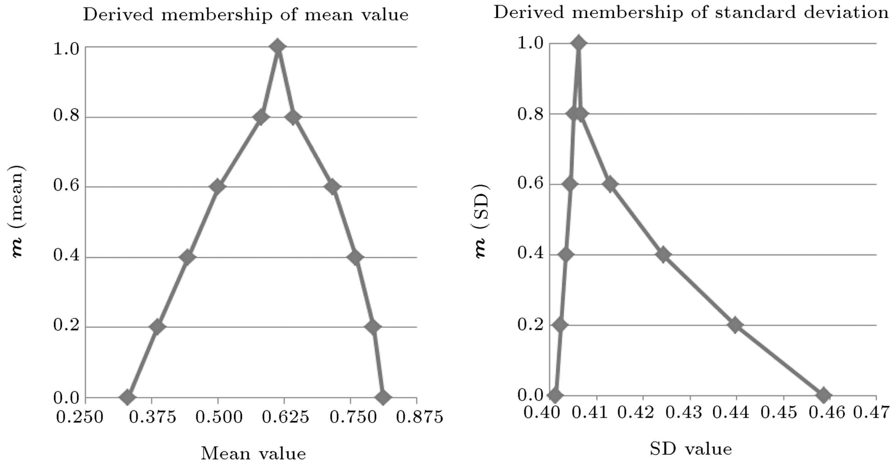

Figure 12. Fuzzy results for mean and standard deviation of collapse capacity.

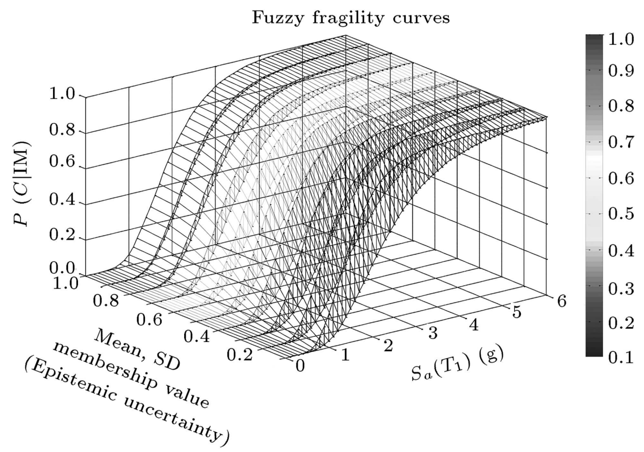

Figure 13. Fuzzy collapse fragility curves; mean and SD membership value spectrum.

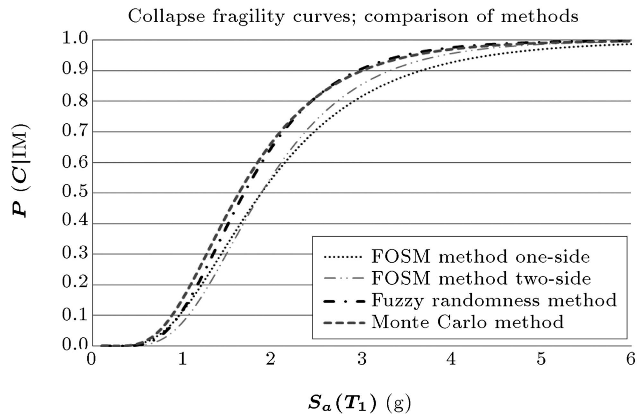

Figure 14. Defuzzification results; comparison of methods.

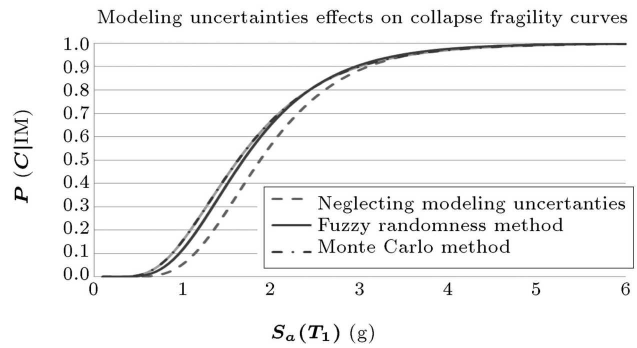

Figure 15. Defuzzification results; fuzzy randomness method efficiency.

other probabilistic methods, defuzzification of fuzzy parameters is implemented by Eq. (16). Comparison of achieved collapse fragility curves, applying various methods, is shown in Figures 14 and 15. Further application of proposed method is to consider effects of parameters which cannot be presented by probability distributions such as human errors and construction quality.

Appling FOSM approximation results in change in standard deviation of collapse fragility curves compared with fragility curve neglecting modeling uncertainties. The mean value does not change applying FOSM approximation. As shown in Table 5, mean and standard deviation of collapse fragility curve of sample structure are 0.6292 and 0.3894, respectively. Application of FOSM method to involve modeling uncertainty remains mean value unchanged, and standard deviation is changed to 0.5191 and 0.4417, for one-side and two-side formulations presented by Eqs. (5) and (6), respectively.

Application of Monte Carlo simulation shows both mean and standard deviation values change, involving effects of modeling uncertainties. As shown in Table 5, mean and standard deviation values are 0.4866 and 0.4762, utilizing Monte Carlo method, respectively. The proposed method, entitled as fuzzy randomness method, predicts changes in mean and standard deviation values. Mean and standard deviation values are 0.5218 and 0.4363, respectively. Figures 14 and 15 show the efficiency of the fuzzy randomness method. The proposed method shows good correlation to Monte

Table 5. Modeling uncertainty effects by FOSM, fuzzy randomness and Monte Carlo method on collapse fragility curves.

Collapse fragility function	No consideration of modeling uncertainties	Consideration of modeling uncertainties (FOSM method)		Consideration of modeling uncertainties (Fuzzy randomness method)	Consideration of modeling uncertainties (Monte Carlo method)
Mean	0.6292	0.6292		0.5218	0.4866
Standard deviation	0.3894	One-side method	0.5191	0.4363	0.4762
		Two-side method	0.4417		
Change in median (%)	–	%0		-%10.74	–%13.7
Change in dispersion (%)	–	One-side method	%12.96	%4.69	%6.86
		Two-side method	%5.23		

Carlo simulation method as more accurate results in comparison with others, while this method takes much less computation efforts than Monte Carlo simulation method.

We finally have made use of the following relationships:

$$\mu_{\mathrm{Ln}(S_{ac})} = C_0 + \Sigma_{i=1}^{N} C_i x_i + \Sigma_{i<j}^{N} C_{ij} x_i x_j + \Sigma_{i=1}^{N} C_{ii} x_i^2 = C_0 + C_1\theta_P + C_2\theta_{\mathrm{PC}} + C_3\Lambda + C_4\theta_P\theta_{\mathrm{PC}} + C_5\theta_P\Lambda + C_6\theta_{\mathrm{PC}}\Lambda + C_7\theta_P^2 + C_8\theta_{\mathrm{PC}}^2 + C_9\Lambda^2, \tag{23}$$

$$\sigma_{\mathrm{Ln}(S_{ac})} = C'_0 + \Sigma_{i=1}^{N} C'_i x_i + \Sigma_{i<j}^{N} C'_{ij} x_i x_j + \Sigma_{i=1}^{N} C'_{ii} x_i^2 = C'_0 + C'_1\theta_P + C'_2\theta_{\mathrm{PC}} + C'_3\Lambda + C'_4\theta_P\theta_{\mathrm{PC}} + C'_5\theta_P\Lambda + C'_6\theta_{\mathrm{PC}}\Lambda + C'_7\theta_P^2 + C'_8\theta_{\mathrm{PC}}^2 + C'_9\Lambda^2. \tag{24}$$

7. Conclusion

In this paper, fuzzy randomness formulation is presented to involve modeling uncertainties effects on collapse fragility curves. The efficiency of the proposed method compared with other existing approaches respected to Monte Carlo simulation results. It is also noteworthy that the proposed method keeps accurate results, regardless of much less computation efforts in comparison with Monte Carlo simulation method. Involvement of modeling uncertainties decreases mean value and increases the standard deviation value, as shown applying probabilistic-based methods. To this end, collapse fragility curves of special moment resisting frame are derived through FOSM, Monte Carlo and fuzzy randomness methods. Moment-rotation modeling parameters of connections, entitled as θ_p, θ_{pc} and Λ are considered as epistemic uncertainties. The effects of epistemic uncertainties, on collapse fragility curves, are evaluated by aforementioned methods. One of the more advantages of the proposed method is that the possibility of involving effects of parameters cannot be presented by probability distributions, or expressed descriptively, such as construction quality and human errors.

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