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# Designing granular layers for railway tracks using ray optimization algorithm

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Minimum thickness of ballast layer.

**Abstract.** In this paper, the recently developed meta-heuristic algorithm, named Ray Optimization (RO), was applied to optimize the thickness of granular layers in railway tracks. RO is a multi-agent algorithm, each agent of which is modeled as a ray of light that moves in the search space in order to find global or near-global optimum solutions. To utilize RO, considering structural and serviceability constraints, first, hypothetical values of three thicknesses were assigned to the layers. Then, stresses under the sleeper and on the subgrade layer were compared with the allowable values. The total minimum thickness of the layers was also compared with possible minimum thickness (serviceability constraint) and then minimum thickness values were selected. Optimization results showed that the moment of inertia of the rail had no significant impact on the minimum total thickness of the ballast and sub-ballast layers. On the other hand, train speed and axle load were the parameters that had a considerable effect on the minimum total thickness of granular layers in the railway track.



## 1. Introduction

Ballast and sub-ballast layers in ballasted railway tracks are layers consisting of aggregate (crushed stone) and granular (gravel and sand) materials, respectively. These materials are used as intermediate layers between sleepers and subgrade materials. For designing granular layers in ballasted railway tracks, some important points should be considered as follows [1-3]:

1. Structural considerations (i.e. ability of granular layers to transfer loads from sleeper to subgrade);
2. Operational considerations (i.e. providing proper drainage for the railroad as well as granting longitudinal and lateral stability to tracks).

In order to obviate the above-mentioned requirements, a suitable thickness should be adopted for ballast and sub-ballast layers based on the following four important criteria [1,2,4,5]:

(a) Minimum required thickness of ballast for the possibility of a tamping operation.
(b) Minimum thickness of the sub-ballast layer to play the role of the transition zone between ballast and subgrade layers.
(c) Minimum required thickness of granular layers, including ballast and sub-ballast layers to decrease intensity under sleeper pressure to allowable sub-

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grade bearing capacity and to avoid shear failure and excessive subsidence in subgrade.

(d) Providing a suitable thickness of granular materials for drainage purposes.

Based on the above criteria, many minimum thicknesses have been introduced by domestic and universal codes and regulations, including Leaflet 301 of the Management and Planning Organization of Iran (MPOI) [6], the American Railway Engineering and Maintenance of Way Association, the Manual for Railway Engineering (the so-called AREMA Manual) [7] and Standard 719 of the International Union of Railways, entitled "Earthworks and Trackbed Construction for Railway Lines" [8].

For instance, Leaflet 301 by MPOI has provided the minimum thickness of a ballast layer under different conditions. In the main lines of categories A, B, C and D, the minimum thickness of the ballast layer should not be selected less than 300 mm. In the accessory lines, the minimum thickness of the ballast under sleepers has been considered equal to 250 mm. Moreover, in order to provide the stability of railway lines, the minimum thickness of ballast layers should not exceed 500 mm. According to the leaflet's conditions, the ballast layer thickness should be high enough to decrease the stress on the surface of the ballast layer to an allowable value for subgrade. Moreover, this leaflet recommends using linear analysis, finite element modeling or any other exact method for evaluating stress distribution through the ballast layer. On the other hand, in the mentioned leaflet, no criterion is presented for minimum sub-ballast thickness. Among domestic standards, only Leaflet 394 of MPOI, entitled "Superstructure Design and Monitoring Instruction of High-speed Railway Tracks" [9], has proposed a value of 300 mm as the minimum thickness of sub-ballast.

The Railway Engineering Manual of American Railway Engineering and Maintenance of Way Association (AREMA) has presented two minimum thicknesses for ballast and sub-ballast layers. In this manual, by limiting stress on the subgrade layer, a number of computational formulas have been suggested to determine the total thickness of ballast and sub-ballast layers. This regulation has explicitly expressed that the minimum ballast layer thickness should not be less than 300 mm. Accordingly, the thickness of the compacted sub-ballast layer is considered equal to 300 mm. It does, however, emphasize the existence of a 150 mm compacted sub-ballast layer for separation purposes of the ballast and subgrade.

Another guideline that presents a methodology for evaluating the thickness of ballast and sub-ballast layers is the UIC code, 719 R, entitled "Earthworks and Track Bed Construction for Railway Lines" [8]. Based on the mentioned standard, an equation has been established for calculating minimum thicknesses of ballast layers, protective layers (sub-ballast) against frost and layers of filter material.

In this paper, ray optimization was applied to optimize the thickness of granular layers in railway tracks. In order to utilize RO considering structural and serviceability constraints, first, hypothetical values for three thicknesses were assigned to the layers. Then, the stresses under the sleeper and on the subgrade layer were compared with the allowable values. The total minimum thickness of the layers was then compared with the possible minimum thickness (serviceability constraint) and, finally, minimum thickness values were selected.

## 2. Reviewing optimization applications in railway engineering

Despite the significance of optimization in the railway industry, little optimization work has been so far performed. For example, Nielsen [10] optimized the acoustic of a railway sleeper using a mathematical approach. Qiao et al. [11] applied a commercial program, IDESIGN, to propose optimized wood railroad crossties encased with Glass Fiber-Reinforced Plastic (GFRP). Markine et al. [12] performed multi-criteria optimization on Embedded Rail Structure (ERS) by a numerical technique, using multipoint approximations based on a response surface fitting (MARS) method. Sadeghi and Babaee [13] investigated 40 pre-stressed concrete sleepers with different dimensions and selected the best one in comparison to the concrete sleeper type B70 commonly used in Iranian railways. As shown, in spite of the importance of determining the optimum thickness of granular materials in conventional railway tracks, no work has been conducted in this regard. So, the present study was devoted to this issue. Details of the research methodology are described in the following section.

## 3. Research methodology

In this paper, the recently developed optimization algorithm, called Ray Optimization (RO), was used for optimizing the thickness of granular layers in ballasted tracks. The RO algorithm is categorized among multi-agent meta-heuristic methods, which uses the Snell light refraction law for navigating its agents in the search space.

In order to define optimum thickness, the first step is to calculate the subgrade modulus using the pyramid method developed by Ahlbeck et al. [14]. This method is capable of estimating the modulus of a railway track considering a series of springs, which correspond to substructure layers. In the next

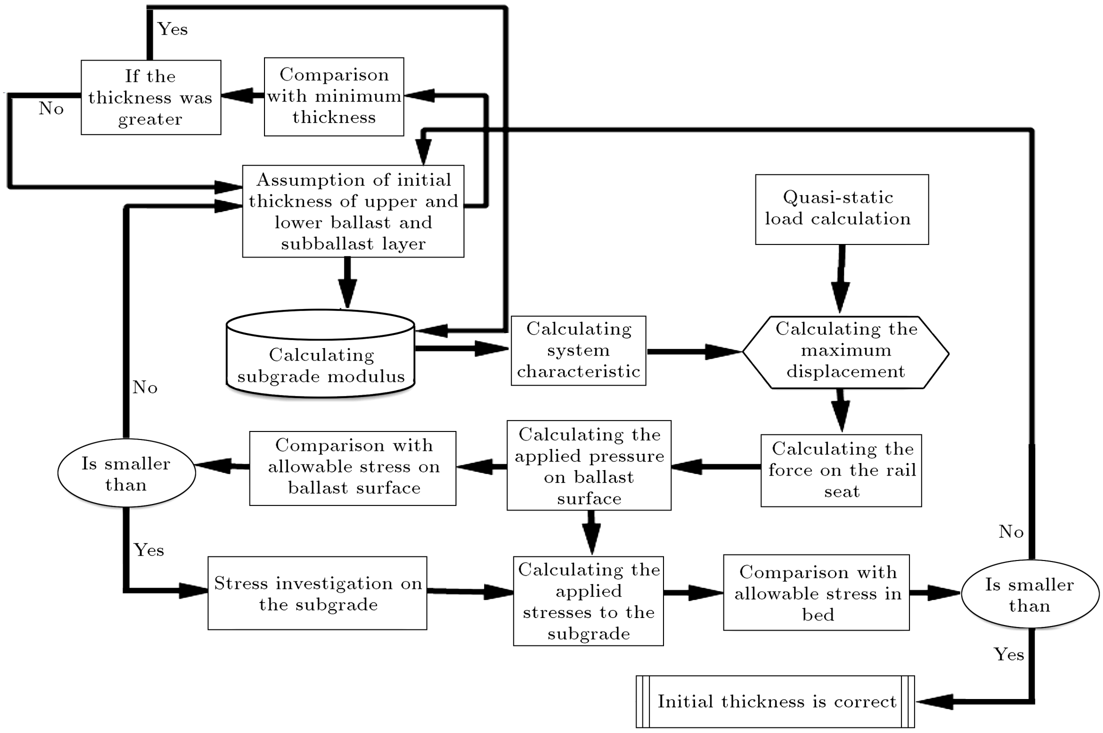



**Figure 1.** Flowchart of calculation of ballast layer thickness.

**Table 1.** Mechanical and geometrical characteristics of components of ballasted railway track.

| Track parameters | $E$ (pa) | $I$ ($m^4$) | $L$ (m) | $B$ (m) | $\theta_b$ (°) | $\theta_{sb}$ (°) | $E_b$ (pa) | $E_{sb}$ (pa) | $C_0$ ($\frac{N}{m^3}$) |
|---|---|---|---|---|---|---|---|---|---|
| **Rail** | $207 \times 10^9$ | $30.55 \times 10^{-6}$ | – | – | – | – | – | – | – |
| **Sleeper** | – | – | 2.6 | 0.22 | – | – | – | – | – |
| **Ballast** | – | – | – | – | 65.2 | 65.2 | $250 \times 10^6$ | – | – |
| **Sub-ballast** | – | – | – | – | 65.2 | 65.2 | – | $150 \times 10^6$ | – |
| **Subgrade** | – | – | – | – | – | – | – | – | $10^6$ |

step, using the Beam On Elastic Foundation (BOEF) formulation developed by Winkler [15], maximum rail deflection was calculated considering the superposition of loads of quasi-static train wheels. Afterward, knowing the subgrade modulus and maximum rail deflection, stress distribution was calculated under the sleepers. In order to determine stress at the subgrade level, Love's equation [1], based on the theory of elasticity, was used. A summary of the flowchart of minimum thickness calculation is shown in Figure 1. Details of the calculation methods are described in the next section.

## 4. Design principles of ballast layer thickness

In this section, the calculation process of granular material thickness is presented considering specifications of a formal ballasted track. Specifications of each component are presented in subsequent sections. Mechanical and geometrical characteristics of ballasted railway track components are demonstrated in Table 1.

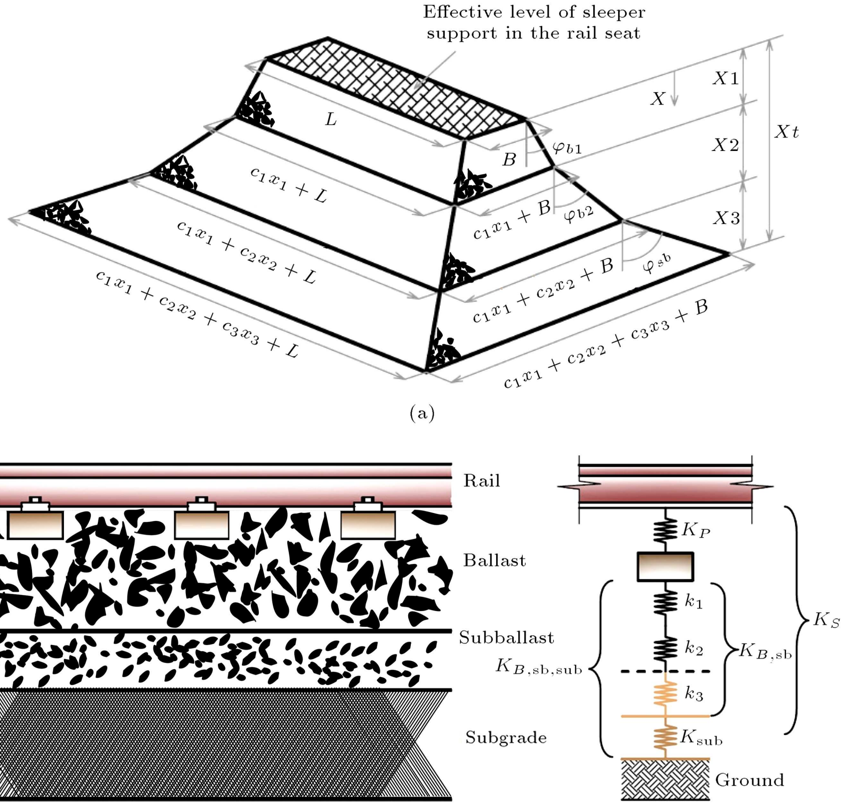
To simplify, by applying impact factor ($\phi$) (calculated by Eq. (1)), static load became quasi-static [16].

$$\left.\begin{aligned} &\varphi = 1 + 5.21\tfrac{V}{D} \\ &P_d = \varphi \times P \end{aligned}\right\} \qquad (1)$$

### 4.1. *Evaluating track modulus using pyramid model*

The pyramid approach is an approximate analytical method for evaluating track vertical stiffness. The main assumption in the method pertains to pyramidal distribution of stress under the sleeper to subgrade (Figure 2(a)). In this model, each component of the track substructure is replaced with an elastic spring; so, total track stiffness is evaluated as the sum of these springs in a series form (Figure 2(a)).

How to calculate the modulus of subgrade using the pyramid method is summarized below.




(b)

**Figure 2.** Track modulus using pyramid model: a) Stress distribution pattern beneath the rail; and b) series springs as representatives of track components.

**Table 2.** Results of plate load test on the pre-consolidated clay.

| Clay quality | Hard | Very dry | Dry |
|---|---|---|---|
| $q_u$ values (kPa) | > 428 | 214-428 | 107-214 |
| $C_0$ ranges ($10^{-6}$kN/mm$^3$) | > 68 | 34-68 | 17-34 |
| Proposed values of $C_0$ ($10^{-6}$kN/mm$^3$) | 102 | 51 | 25.5 |

Since types of ballast and sub-ballast materials were known, the internal distribution angle was calculated according to Table 1.

If the subgrade layer is made of over-consolidated clay, the reaction modulus could be related to undrained compressive strength, $q_u$. Terzaghi proposed Table 2 for different types of clay [18].

In this study, the reaction modulus of subgrade material was selected based on over-consolidated clays under dry conditions.

### *4.2. Loading model configuration*

In this section, the car body used for introducing the loading model is presented [17]. As shown in Figure 3, all dimensions of the car body are in meters, while its wheel diameter is 860 mm. The critical wheel in the figure points to the calculation location of rail deflection.

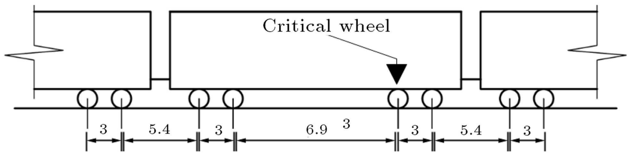



**Figure 3.** Body dimensions of the used car.

### *4.3. Calculating under-sleeper pressure and its distribution on subgrade*

After calculating maximum rail deformation under the benchmarked wheel location, rail seat load ($Q_m$) was calculated according to Eqs. (2) and (3), knowing parameter $S$ as the distance between sleepers. Then, using Eq. (4), under-sleeper pressure ($P_m$) was obtained, where $A_b$ is the effective under-sleeper seating area, which is equal to $B \times L$ and $\frac{2}{3} \times (B \times L)$ for concrete and wooden sleepers, respectively [1].

To calculate distributed pressure on the subgrade level ($P_C$), Love's Eq. (5) can be utilized:

$$F_m = U \times Y_{\max}, \tag{2}$$

$$Q_m = F_m \times S, \tag{3}$$

$$P_m = \frac{2 \times Q_m}{A_b}, \tag{4}$$

$$P_c = P_m \left[ 1 - \left( \frac{1}{1 + \frac{r^2}{h^2}} \right)^{\frac{3}{2}} \right]. \tag{5}$$

In the above equations, $r$ is the radius of a circle with the seating area equivalent sleepers, and $h$ is the thickness of the granular (ballast and sub-ballast) layers.

## 5. Optimizing granular material thickness using RO algorithm

RO is a multi-agent meta-heuristic algorithm, which was proposed by Kaveh and Khayatazad [18] for finding near-global optimum in optimization problems. Similar to other multi-agent methods, RO has a number of particles consisting of variables of the problem, which are considered rays of light. Based on the Snell light refraction law, when light passes from a lighter medium to a denser one, it refracts and its direction changes. Using this phenomenon, the solution vector can be led to a global or near-global optimum solution. Kaveh and Khayatazad showed the efficiency of this algorithm by solving some benchmarks and well-studied engineering problems in 2012 [19] and also used RO for optimizing the size and shape of truss structures [19]. Finally, they optimized cantilever retaining walls under pseudo-dynamic analysis by the RO method [20]. Afterwards, the RO method was utilized for finding the optimum thickness of granular materials in ballasted railway tracks.

### *5.1. Defining constraints*

In order to determine the optimum thickness of granular layers, two separate constraints could be defined as follows:

- Serviceability constraint;
- Structural constraint.

In order to express the serviceability constraint, Eq. (6) can be used. By limiting compressive stress in separating the boundary between ballast and subgrade, ballast penetration into the subgrade layer was prevented.

Due to imposing the above-mentioned limitation, the minimum thickness of the ballast layer could be obtained by the following equation:

$$Z_{\min} = \frac{S - B}{2 \tan \theta}. \tag{6}$$

In the above equation, $S$, $B$ and $\theta$ are distance between sleepers, width of sleepers and angle of internal stress distribution through the ballast layer, respectively.

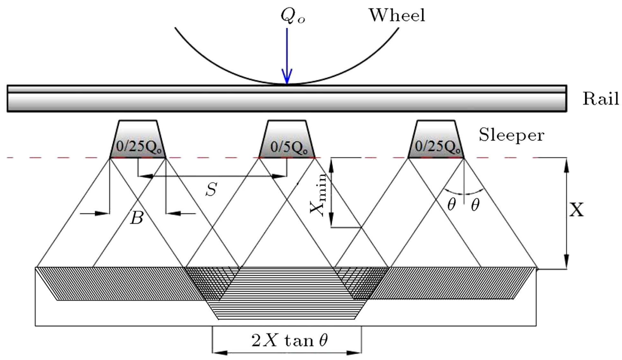



**Figure 4.** Minimum thickness of granular materials.

Figure 4 graphically illustrates the philosophy of minimum thickness based on the above criteria [2].

In order to define the structural constraint, the value of the applied pressure beneath the concrete sleeper on the ballast surface had to be limited to 590 kPa, according to the AREMA manual [1]. From another point of view, the transferred pressure to the subgrade through the ballast layer had to be limited to the allowable bearing capacity of the subgrade. For over-consolidated clay in a dry condition, the allowable bearing capacity of subgrade ($q_c$) could be evaluated by the following equation [21]:

$$q_c = 40 \times \text{F.S.} \times C_0, \tag{7}$$

where F.S. is the factor of safety equal to 1.5, and $C_0$ is the reaction modulus of the subgrade.

### *5.2. Defining optimization variables and goal function*

The vector of optimization variables can be introduced as follows:

$$X = (x_1, x_2, x_3). \tag{8}$$

In this vector, $x_1$, $x_2$ and $x_3$ are thicknesses of upper ballast, lower ballast and sub-ballast layers, respectively. In each new step of the calculation, based on the introduced algorithm, all vector components were assumed to have thicknesses in the range of 0 to 500 mm. Consequently, according to Figure 1, the modified thicknesses were evaluated.

The selected goal function of the problem was minimized by the following equation:

$$H = x_1 + x_2 + x_3,$$

where $H$ is minimum total thickness of granular materials in the ballasted track.

### *5.3. Concepts of RO optimization algorithm*

Consider a light ray which is crossing transparent medium $K$ with $\mathbf{V}_i^k$ (Figure 5). When this ray reaches $\mathbf{X}_i^k$, after refraction, it enters a darker medium, $K+1$, and continues its path with $\mathbf{V}_i^{k+1}$. The direction of

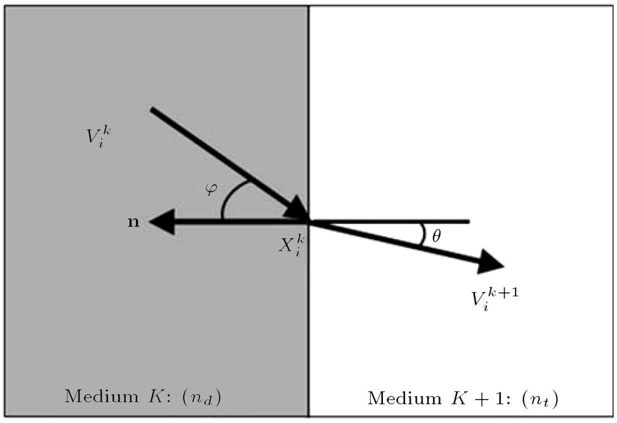



**Figure 5.** Incident and refracted rays and their specifications.

$\mathbf{V}_i^{k+1}$ depends on the direction of $\mathbf{n}$ and the refraction index ratio, $(\frac{n_d}{n_t})$.

RO method is a multi-agent optimization method which is inspired by the light refraction concept [18]. In this method, by moving the agents to new positions, an optimal solution is found. Thus, if the movement vector for the $i$th agent in the $k$th iteration is $\mathbf{V}_i^k$ and the current position of this agent is $\mathbf{X}_i^k$, it can be moved to its new position by $\mathbf{V}_i^{k+1}$. The refraction index ratio for this method is selected as 0.45. The direction of $n$ passes through two points: the beginning point of $\mathbf{O}_i^k$ and the final one of $\mathbf{X}_i^k$. $\mathbf{O}_i^k$ is defined as follows:

$$\mathbf{O}_i^K = \frac{(\text{ite}+k).\mathbf{GB}+(\text{ite}-k).\mathbf{LB}_i}{2.\text{ite}}, \tag{9}$$

where $\mathbf{GB}$ and $\mathbf{LB}_i$ are the so-far best position and goal function values obtained by all the agents and the $i$th agent, respectively.

If the number of variables is greater than 3 ($N > 3$), for using the ray tracing concept, the search space can be divided to a number of 2D and or 3D spaces. In general, if $N$ is an even number, the search space is divided into $\frac{N}{2}$ for 2D spaces; if $N$ is an odd number, the search space is divided into $\frac{N-3}{2}$ for 2D space(s) and a one 3D space. Each of these 2D or 3D spaces is named a sub-space. With this description, $\mathbf{V}_{i,l}^k$ is the movement vector of the $l$th sub-space which belongs to the $i$th agent in the $k$th iteration, and $v_{i,j,l}^k$ is the $j$th component of movement vector of the $l$th sub-space, which belongs to the $i$th agent in the $k$th iteration.

The steps of the RO algorithm are described below and its flowchart is shown in Figure 6.

The steps of the calculations based on the RO algorithm are as introduced below.

#### *5.3.1. Scattering and evaluation steps*

Based on Eq. (10), randomly scatter the agents in the search space:

$$x_{i,j}^0 = x_{j,\min} + \text{rand} \times (x_{j,\max} - x_{j,\min}), \tag{10}$$

where $x_{i,j}^0$ is the $j$th component of the $i$th agent. $x_{j,\min}$ and $x_{j,\max}$ are allowable minimum and maximum values of the $j$th component. *rand* is a random number distributed 0 through 1. After scattering, evaluate the value of the goal function for each agent. Then, save the position and goal function values of each agent and the best position and goal function values of the best agent, as $\mathbf{LB}_i$ and $\mathbf{GB}$, respectively.

Make a movement vector for each agent as follows:

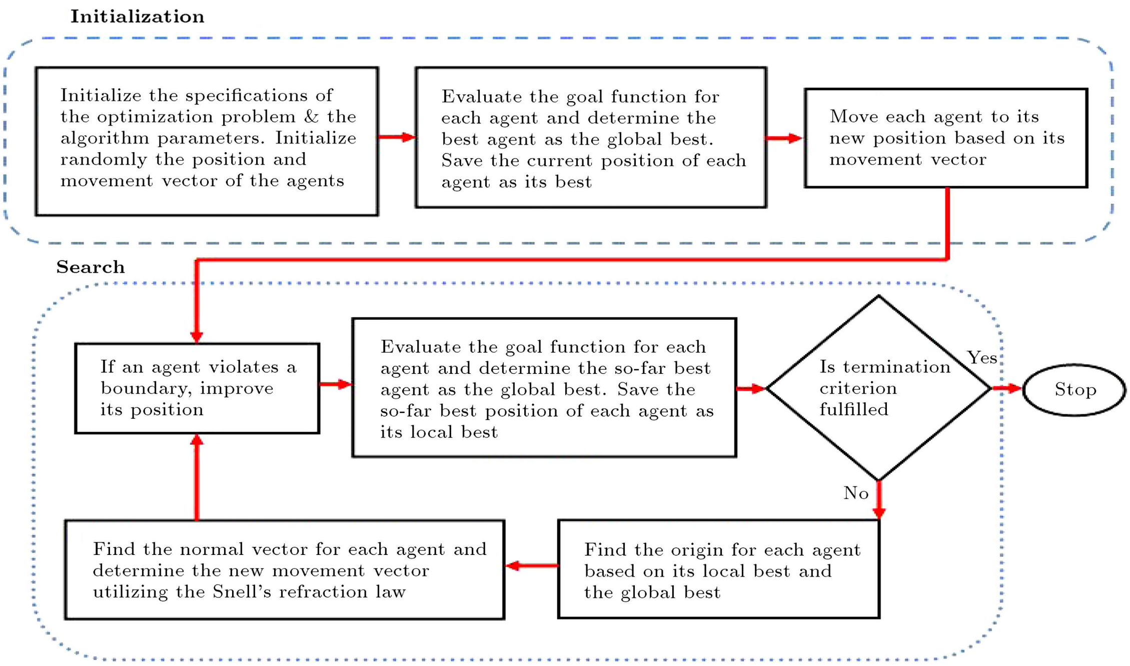



**Figure 6.** Flowchart of RO algorithm.

$$v_{i,j}^0 = -1 + 2 \times \text{rand}, \qquad (11)$$

where $v_{i,j}^0$ is the $j$th component of the $i$th agent. Finally, based on the sub-space grouping, convert 2D and 3D movement vectors into normalized ones.

*5.3.2. Movement vector and motion refinement steps*
Move the agents to their new positions based on their movement vectors. If an agent violates the allowable boundaries, modify the length of its movement vector. The new length of the movement vector equals 0.9 times the distance of the current agent position and the intersection of the boundary. After modifying the movement vector, evaluate the goal function of each agent and update $\mathbf{GB}$ and $\mathbf{LB}_i$.

*5.3.3. Cockshy point making and convergent step*
Determine $\mathbf{O}_i^k$ for each agent. Then, based on Eq. (12), obtain the new movement vector, where stoch and $d$ are 0.35 and 7.5, respectively.

$$\mathbf{X}_{i,l}^k \neq \mathbf{O}_{i,l}^k \rightarrow \begin{cases} \text{w.p.}(1-\text{stoch}) \rightarrow \mathbf{V}_{i,l}^{k+1} = \mathbf{V}_{i,l}^{(k+1)} \\ \quad \times \text{norm}(\mathbf{X}_{i,l}^k - \mathbf{O}_{i,l}^k) \\ \quad \mathbf{V}_{i,l}^{(k+1)} \text{ is determined by} \\ \quad \text{ray tracing.} \\ \text{w.p. stoch} \rightarrow \mathbf{V}_{i,l}^{(k+1)} = \frac{\mathbf{V}_{i,l}^{(k+1)}}{\text{norm}(\mathbf{V}_{i,l}^{(k+1)})} \\ \quad \times \frac{a}{d} \times \text{rand} \\ \quad \text{Each component of } \mathbf{V}_{i,l}^{(k+1)} \\ \quad \text{is calculated as blow:} \\ \quad v_{i,j,l}^{(k+1)} = -1 + 2 \times \text{rand} \\ \quad a = \sqrt{\sum_{j=1}^{n} (x_{j,\max} - x_{j,\min})^2} \\ \quad n = \begin{cases} 2 & \text{for two variable groups} \\ 3 & \text{for three variable groups} \end{cases} \end{cases} \qquad (12)$$

$$\mathbf{X}_{i,l}^k = \mathbf{O}_{i,l}^k \rightarrow \mathbf{V}_{i,l}^{(k+1)} = \frac{\mathbf{V}_{i,l}^k}{\text{norm}(\mathbf{V}_{i,l}^k)} \times \text{rand} \times 0.001.$$

*5.3.4. Finish or redoing step*
If the finishing criterion of the algorithm is obtained, the search procedure is terminated; otherwise, the algorithm returns to the second step and continues the searching. The finishing criterion can be a specific number of iterations for obtaining the optimal solution.

## 6. Numerical results and discussion

In this section, the results extracted from the optimization algorithm are presented in the framework of some numeric examples. To demonstrate the practical application of the described algorithm, the influence of parameters such as rail type, train axle load, wheel diameter of train, width of sleeper and distance between sleepers on the minimum total thickness of ballast and sub-ballast layers is investigated. In all the graphs, axle load was considered constant and other parameters were changed (except for Figure 10).

### *6.1. Effect of rail type on total thickness*

As seen in Figures 7-9, with increasing train speed from 10 to 160 km/hr, minimum total thickness of the ballast and sub-ballast layers increased. Each of these graphs is plotted for different moments of inertia of the rail. For this purpose, four rail profiles (119 RE, 133 RE and 141 RE) were selected, according to AREMA regulations.

As shown, an increase in the moment of inertia

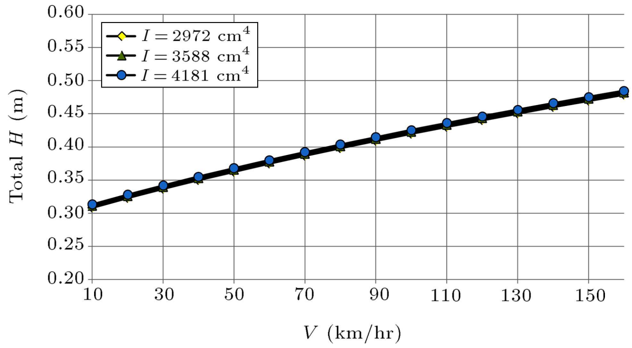



**Figure 7.** Variation of total thickness of granular materials versus train speed for 8 ton axle load.

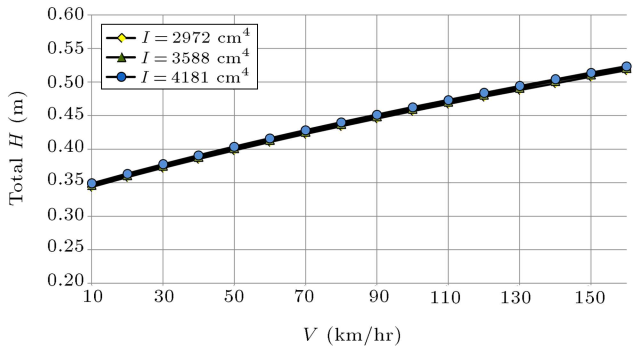



**Figure 8.** Variation of total thickness of granular materials versus train speed for 9 ton axle load.

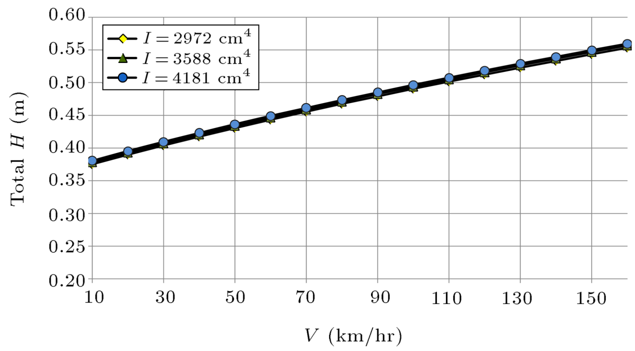



**Figure 9.** Variation of total thickness of granular materials versus train speed for 10 ton axle load.

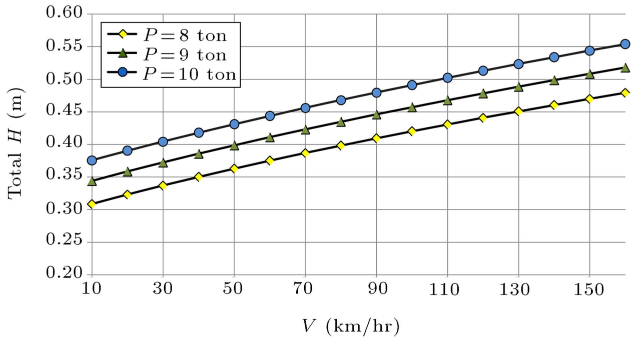



**Figure 10.** Minimum total thickness of granular materials versus train speed for different axle loads.

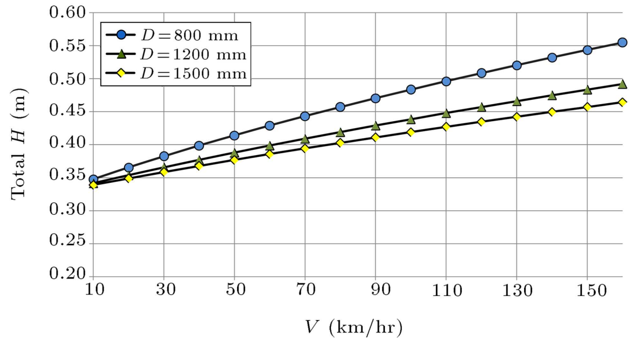



**Figure 12.** Minimum total thickness of ballast layer versus train speed for 9 ton axle load.

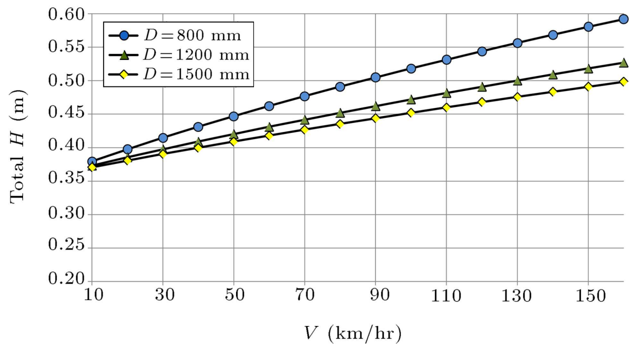



**Figure 13.** Minimum total thickness of granular materials versus train speed for 10 ton axle load.

of the rail, in the case of train axle loads of 8, 9 and 10 tons, caused a reduction in the minimum total thickness of granular materials by about 1.6%, 1.5% and 1.4%, respectively. Overall, an increase in moment of inertia of the rail from 2972 $cm^4$ to 4181 $cm^4$ did not considerably affect the reduction of the total thickness of granular materials.

### *6.2. Effect of axle load on total thickness of granular materials*

The minimum total thickness of granular materials is presented for different train speeds in Figure 10. In this diagram, it is evident that, due to an increase in train speed, minimum total thickness increased. Moreover, with increasing axle load, the minimum total thickness of granular materials was raised.

With increasing train speed from 10 to 160 km/hr, all the diagrams showed a smooth growth with a linear pattern.

### *6.3. Effect of train wheel diameter on minimum total thickness of granular materials*

As illustrated in Figures 11 to 13, due to the effect of wheel diameter on increasing the impact factor of the axle load, an increasing trend was observed in the minimum total thickness of granular materials.

In these figures, with increasing train speed, minimum total thickness was increased. However, decreasing wheel diameter caused an increased minimum total thickness of granular materials for three different values of axle load, i.e. 8, 9 and 10 tons.

This increase was more evident at a speed of 160 km/hr, so that, in axle loads of 8 tons, 9 tons and 10 tons, decreasing wheel diameter from 1200 to 800, in comparison to 1500 to 1200, caused a 212%, 214% and 215% increase in minimum total thickness of granular materials, respectively.

### *6.4. Effect of width of sleeper on minimum total thickness of granular materials*

In Figures 14-16, variations of minimum total thickness are illustrated for three different sleeper widths (0.15 m, 0.22 m and 0.3 m). Due to an increase in the

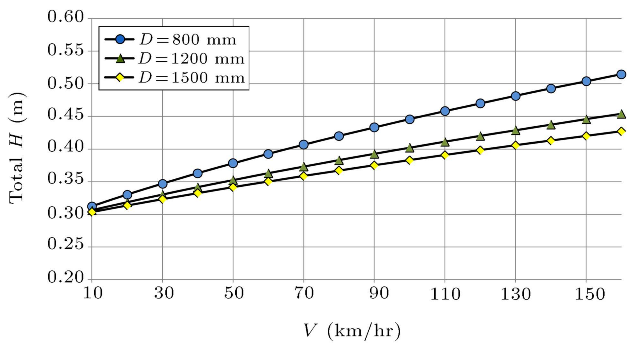



**Figure 11.** Minimum total thickness of granular materials versus train speed for 8 ton axle load.

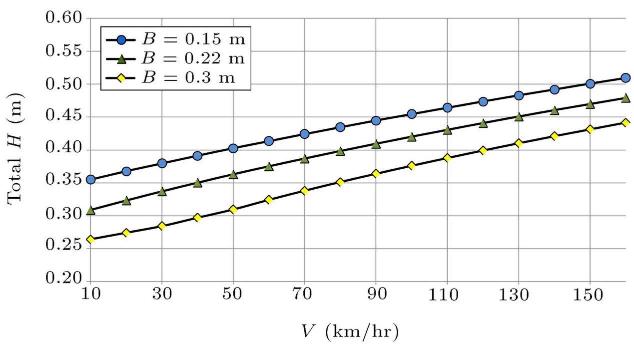



**Figure 14.** Minimum total thickness of granular materials versus train speed for 8 ton axle load.

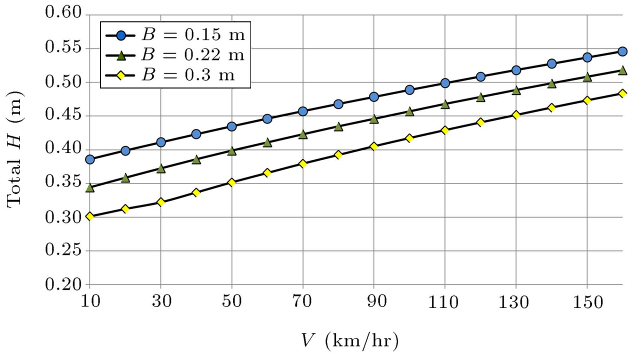



**Figure 15.** Minimum total thickness of granular materials versus train speed for 9 ton axle load.

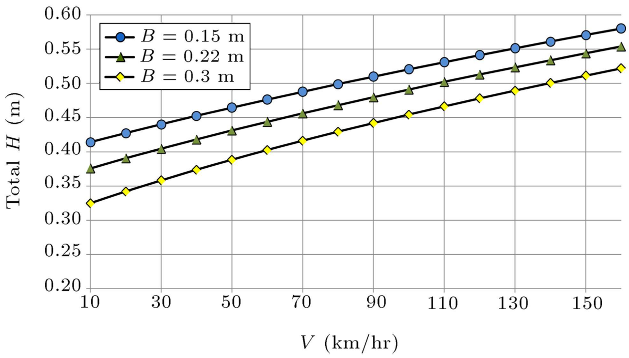



**Figure 16.** Minimum total thickness of granular materials versus train speed for 10 ton axle load.

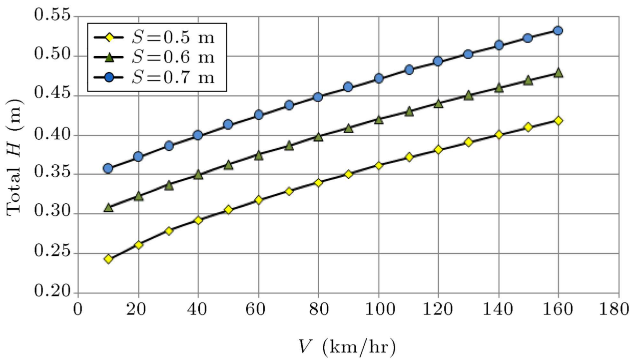



**Figure 17.** Minimum total thickness of granular materials versus train speed for 8 ton axle load.

running speed of the train, the calculated minimum total thickness decreased with increasing the sleeper width.

### 6.5. *Effect of sleeper distance on total thickness*

In Figures 17-19, a minimum total thickness variation is shown for three different distances of sleepers (0.5 m, 0.6 m and 0.7 m).

The presented figures evidently prove the increasing trend of the minimum total thickness of granular materials due to the increase in sleeper distance for the whole range of train speed.

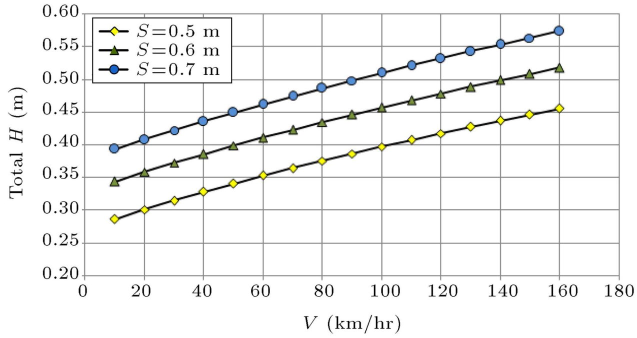



**Figure 18.** Minimum total thickness of granular materials versus train speed for 9 ton axle load.

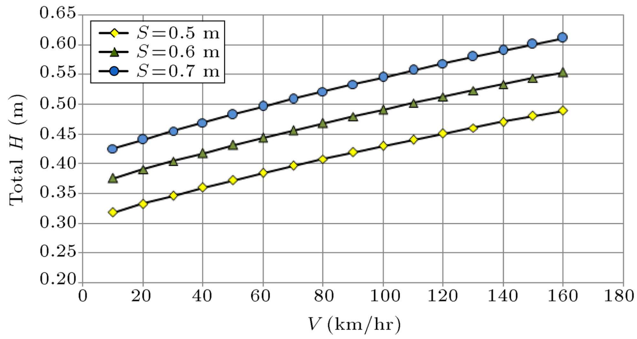



**Figure 19.** Minimum total thickness of granular materials versus train speed for 10 ton axle load.

## 7. Validation of the proposed method to determine the optimum thickness of granular material

In order to validate the optimization method used in the present research for evaluating the thickness of granular material in ballasted tracks, a comparison is made with the proposed method of British Railway. This method is usually used for determining the whole thickness of the ballast and sub-ballast layer as a granular layer. The base of the mentioned method has been established by British Railway and the Office for Research and Experiments of the International Union of Railways (UIC) in parallel with extensive field and laboratory experiments. The main inputs of the method are: Threshold stress of subgrade material and design axle load, which is the real axle load multiplied to impact factor. In this section, recalling the used input data in the previous sections, the comparison is fulfilled with the optimized values of granular material thickness proposed in this research.

By assuming the value of the threshold stress for over-consolidated clay in dry condition (the subgrade material) as 53.5 kPa, and adopting the design axle load as static axle load multiplied to the impact factor of Eq. (1), selecting the wheel diameter as shown in Figure 3 and train speed as in Figure 10, three different thicknesses are obtained with respect to design axle loads 78, 118 and 1157 kN, as illustrated in Figure 20.

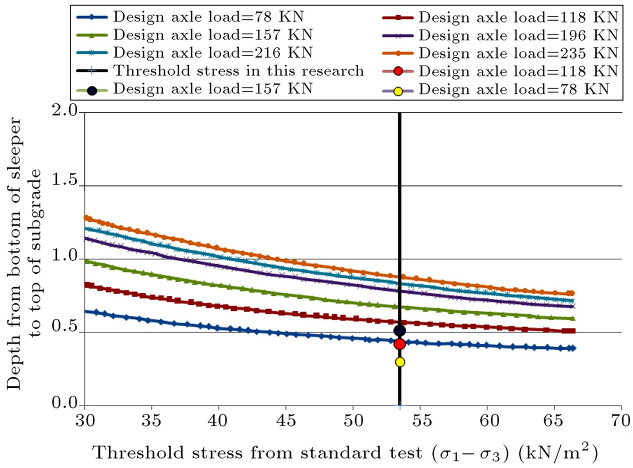



**Figure 20.** Total thickness of granular materials versus threshold stress from British Railway method.

As seen in these points, for the design axle load equal to 78 kN, the calculated optimized thickness is less than the recommended amount by British Railway (about 14.4 cm) [1]. For the design axle load equal to 118 kN and 157 kN, the calculated optimized thickness has fallen about 15.1 cm and 16.7 cm, respectively. These results prove the efficiency of the proposed optimization process in the present study in determining the whole thickness of granular material in ballasted tracks.

## 8. Conclusions

In this paper, a ray optimization algorithm was used for finding the optimal thickness of granular materials in traditional ballasted railway tracks. In this regard, a step-by-step calculation process was introduced and two different structural and serviceability constraints were imposed. Consequently, by limiting the under-sleeper pressure and distributed stress on the subgrade to the allowable values, the optimum thickness of granular layers, including ballast and sub-ballast layers, was evaluated. As part of the study, sensitivity analyses were performed to evaluate the effect of train speed and axle load, wheel diameter, sleeper width and sleeper distance on the total optimized thickness of granular materials. The main findings of this research can be summarized as follows:

1. An increase in the moment of inertia of the rail from 2972 cm$^4$ to 4181 cm$^4$ did not have a considerable effect on reducing the total thickness of granular materials. Thus, with increasing the moment of inertia of the rail from 2972 cm$^4$ to 4181 cm$^4$, maximum and minimum thickness ratios of granular materials were related to the speed of 10 km/hr with a value of 1.016, and the speed of 160 km/hr with a value of 1.012. Also, with increasing the moment of inertia of the rail from 2972 cm$^4$ to 3588 cm$^4$, maximum and minimum thickness ratios of granular materials were related to the speed of 10 km/hr with a value of 1.009, and the speed of 160 km/hr with a value of 1.007.

2. With increasing the train speed from 10 to 160 km/hr and at various axle loads, smooth growth with a linear pattern occurred in the total thickness of granular materials. Thus, the thickness ratio of granular materials at a load equivalent to 10 tons to another equivalent to 8 tons at a speed of 10 km/hr was reduced from 1.22 to 1.16 at the speed of 160 km/hr, indicating the reduced effect of axle load on granular material thickness.

3. Increasing impact factor from 1.043 (equivalent speed of 10 km/hr and wheel diameter of 1200 mm) to 1.695 (equivalent speed of 160 km/hr and wheel diameter of 1200 mm) caused an increase in the minimum total thickness of granular materials from 31 cm to 45 cm, showing that the minimum thickness of granular materials decreased with increasing the impact factor at constant axle load.

4. In an 8 tons equivalent axle load, the minimum total thickness of granular materials in 22 cm width concrete sleepers increased from 31 cm to 48 cm at speeds of 10 km/hr and 160 km/hr, respectively. However, in the same axle load and 30 cm width concrete sleepers, these thicknesses were 26 cm and 44 cm at speeds of 10 km/hr and 160 km/hr, respectively, representing a decrease of 19% and 9% in the minimum total thickness of granular materials.

5. With increasing sleeper distance from each other, the minimum thickness of granular materials was increased. As an example, in an 8 ton axle load, with increasing distance between the sleepers from 50 cm to 70 cm, the highest and lowest increases in thickness were related to the speeds of 10 km/hr and 160 km/hr, with values of 47% and 27%, respectively.

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