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Bed form characteristics in a live bed alluvial channel

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KEYWORDS Bed form; Live bed condition; Flow intensity; Dune celerity; Bed roughness. **Abstract.** Bed forms are generated in alluvial streams due to mutual interaction between flow and erodible bed material. Among the variety of possible bed forms, dunes are the most important feature and much attention has been paid to them in the literature. In the present study, an experimental approach was carried out to investigate the geometry of dunes and their celerity in an erodible sand bed. The tests were conducted under live bed conditions in an experimental flume capable of re-circulating both water and sediment. Present experiments showed that the Shields number had a considerable effect on dune height and celerity, while the effect of this parameter on the dune length was not significant. Furthermore, dimensional analysis is used to present the relationships between dune height and length, as well as celerity. These relationships were also compared with previous empirical equations and experimental data, which showed their acceptable accuracy. Bed roughness related to bed forms was also analyzed based on all available experimental and field data. Results demonstrated that by increasing the Shields number, the ratio of Manning coefficient, related to bed forms, to total Manning coefficient increased with a logarithmic trend.

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1. Introduction

In an alluvial stream, the mobile bed formed by sand is seldom flat, and the streambed is usually covered by periodic bed deformations, known as bed forms. These bed forms change in type and size depending on flow conditions. Bed forms initiate with ripples at low shear stress, with progressive development of dunes, washed-out dunes (transition), flat bed, antidunes, and standing waves, with increasing shear stress or velocity [1]. The prediction of bed form geometry is an essential component for estimating overall flow resistance and water levels during floods in rivers. The Einstein and Barbarossa [2] relationship was the first to consider the rule of bed forms in total bed friction

 Corresponding author. Tel.: +98 21 64543002; Fax: +98 21 66414213 E-mail addresses: H. Heydari@gmail.com (H. Heydari); Zarrati@aut.ac.ir (A.R. Zarrati); M.Karimaee@aut.ac.ir (M. Karimaee Tabarestani) directly by dividing the hydraulic radius into two parts; one due to grain roughness, and the other due to bed forms.

Among the variety of possible bed forms in an alluvial stream, dunes are the most important feature and much attention has been paid to them in the literature [1,3-6] This feature is more frequently encountered in alluvial streams, which have large dimensions and, consequently, resistance to flow (Figure 1). The flow over a dune is characterized by a circulating zone, whose length depends on dune height, and a long fetch of attached flow up to the dune crest [7]. The triangular sections of the dunes are not symmetric. The upstream face is inclined at about 10 to 20 degrees and the downstream face is at an angle of about 30 to 40 degrees to the horizontal. These bed forms are not static and gradually move forward with time, of course at a very slow and creeping velocity much less than that of the flow (Figure 1).

Extensive research has been undertaken in the past by various investigators into the prediction of bed

| | | Equation | | | | | |
|-----|----------------------------|-----------------------------------|---|--|--|--|--|
| No. | Reference | Dune length | Dune height | | | | |
| | | For $\eta < 0.15$ | | | | | |
| 1 | | $Log(\lambda) = 1.0517 Log(\eta)$ | | | | | |
| | Allen (1978) | +1.1534 | T () | | | | |
| | | For $n > 0.15$ | $Log(\eta) = 1.2091 \ Log(h)$ | | | | |
| | | $Log(\lambda) = 1.3543 Log(n)$ | -1.0762 | | | | |
| | | +1.4553 | | | | | |
| 2 | Yalin (1964) | $\lambda = 5h$ | $\frac{n}{h} = \frac{1}{6} \left(\frac{\tau_0 - \tau_c}{\tau_0} \right)$ | | | | |
| 3 | Van Riin (1984) | $\lambda = 7.3h$ | $\frac{\eta}{h} = 0.11 (d_{50}/h)^{0.3} (1 - 1)^{0$ | | | | |
| | van 1ajii (1904) | N = 1.0h | $e^{-0.5T})(25-T)$ | | | | |
| 4 | Julien and Klaassen (1995) | $\lambda = 6.25 h$ | $\frac{\eta}{h} = 2.5 \left(\frac{d_{50}}{h}\right)^{0.3}$ | | | | |

Table 1. Empirical equation for predicting dune dimensions.

Note: λ is dune length; η is dune height; h is flow depth; τ_0 and τ_c are available and criticcal shear stress, respectively, and T is transport stage parameter defined as $(\tau_0 - \tau_c)/\tau_c$.



(b) Dune migration

Figure 1. Schematic view of dune characteristics and migration.

form geometry, specifically, dunes in alluvial channels, using field [6] or experimental data [8,9,10], or both [5]. Table 1 summarizes some different equations for estimation of dune dimensions. Each of these contributions is applicable to problems similar to the specific physical conditions under which the empirical equations were developed. Significant early contributions, in this respect, were made by Yalin [3], Ranga Raju and Soni [4], and Allen [11] who developed dune height relationships as a function of bed shear stress and other variables. Van Rijn [5] analyzed a large number of experimental and field data to develop a relationship for dune dimensions as a function of flow depth, sediment size and a transport stage parameter (function of shear stress and critical shear stress). However, Julien and Klaassen [6] suggested that the Van Rijn [5] approach underestimates the dune height in large rivers. They found that dune characteristics are independent of transport stage parameters for large rivers and are only a function of relative particle diameter. They also modified the Van Rijn [5] method and presented new relationships for dune characteristics. Karim [1] proposed a new method for predicting relative dune height in a sand-bed stream based on the concept of relating energy loss due to form drag to the head loss across a sudden expansion in open channel flows. There is no study in the literature on dune celerity.

Basically, two approaches are suggested to estimate the bed roughness in alluvial streams: 1) methods based on bed forms and grain related parameters, such as bed form length and height and material size [5], and 2) methods based on integral parameters, such as mean flow depth and velocity and bed-material size [12]. However, since the first method conforms to the physics of the phenomenon, it is more universal and popular. According to this method, the bed shear stress (τ) in an alluvial stream can be divided into two parts: i) shear stress due to grain roughness or skin friction (τ'), and ii) form related bed shear stress (τ'') [13]. The latter is related to the difference in pressure distribution between upstream and downstream of the bed form crest.

Results of previous relationships for dune characteristics and bed resistance related to bed forms differ drastically from each other. This may be due to the complexity of the underlying physical process, which can be attributed to several factors, e.g., a large number of governing variables and interaction between them, the 3D nature of bed form development and the lag in bed form adjustment in response to changing flow conditions [1]. Therefore, additional research is needed and more data is required for the accurate formulation of this complex process. The main scope of the present work is to study, experimentally, dune characteristics and their effect on flow resistance. In addition, the celerity of dunes, which had not been addressed by previous studies, is also investigated and a relationship is suggested for it. An approach for estimating bed roughness is also presented.

2. Experimental setup

Experiments were conducted in a horizontal flume, with an erodible bed 14 m long, 0.75 m wide and 0.6 m deep, at the hydraulic laboratory of Amirkabir University of Technology, Iran. Water and sediment were circulated in the channel by a centrifugal pump with maximum capacity of 120 (Lit/s) and a sludge pump with maximum capacity of 5 (Lit/s), respectively. The flow rate in the flume was controlled and preset by a speed control unit attached to the pump system. An electrical flow-meter was installed in the supply conduit to measure the water discharge through the channel. Figure 1 shows the longitudinal profile of the laboratory flume and its different components. The channel bed was filled with uniform sand about 20 cm thick, with $d_{10} = 0.65$ mm, $d_{50} = 0.85$ mm, as median bed material size, $d_{90} = 1.05$ mm, and a density of 2650 (kg/m^3) , starting at a distance 2 m downstream of the entrance. The geometric standard deviation of sediment grading, $\sigma_g = \sqrt{d_{84}/d_{16}}$, was 1.22, where d_a is the size of sediment for which a percentage of material by weight is finer, implying that the sample is uniform. In order to reduce the flow disturbance and turbulence, a honeycomb (flow straightener) was used at the upstream section of the channel). Velocity profiles measured by ADV (Acoustic Doppler Velocitimeter), when the flume bed was fixed, showed that the flow was fully developed after 5 m from the flume intake.

All experiments were conducted under live bed conditions $(u * / u *_c > 1)$; where u * is bed shear velocity and $u*_c$ is critical shear velocity for the bed material). The range of flow intensity $(u * / u *_c)$ was between 1.2 to 4.7. The flow regime in all experiments was subcritical with maximum Froude number of 0.8. Shear velocity (u^*) in the channel was determined by calculating the water surface profile and the slope of the energy line. Thereby, the water depth at upstream and downstream of the channel was measured by a point gage with accuracy of ± 0.1 mm after the bed form characteristics in the channel were developed, and similar with time. To calculate water surface profile, a value for Chezy coefficient was initially assumed. Then, the calculated water surface elevation upstream of the flume was compared with the measurement results and, if different, the bed roughness coefficient was slightly modified. However, at higher flow intensities $(u * / u *_c \ge 2.35)$, with large dunes, the water surface was undulated and accurate measurements were not possible. From experiments in flow intensity lower than 2 $(u * / u *_c < 2)$, it was concluded that the measured Chezy coefficient for the flow conforms well to the empirical equation presented by Van Rijn [13], which is:

$$C = 18 \text{Log}\left(\frac{12R_h}{3d_{90} + 1.1\eta(1 - e^{\eta/\lambda})}\right),$$
 (1)

where C is Chezy coefficient, R_h is hydraulic radius, λ is dune length, η is dune height and d_{90} is the size of sediment for which 90% of the material by weight is finer. Therefore, at higher flow intensities, the Van Rijn [13] equation was used for the estimation of the Chezy coefficient. Shear velocity was then calculated from the Chezy coefficient. In addition, critical shear velocity $(u*_c)$ was found by the Shields' diagram, knowing the bed shear velocity.

In each experiment, after filling the channel with water and adjusting the downstream tailgate for the desired flow depth, the flow and sludge pumps were turned on. The flow discharge was gradually increased to the considered value. Initially, the bed was in a nonequilibrium condition and was degraded by the flow. The transported sediment was then deposited in the reservoir at the downstream end of the channel and was pumped continuously by the sludge pump to the channel inlet (Figure 2). Gradually the bed forms were developed in the channel and migrated downstream with increasing dimensions, temporally. However, after a couple of hours, the bed forms (dune wave) reached equilibrium condition, where their length and height were similar along the channel.

The longest time for development of dunes was about 5 hours for the smallest flow intensity, which was decreased by increasing flow intensity. After the equilibrium state was reached, the dimensions of dunes and their celerity were measured by plastic rulers attached to the flume side walls, both horizontally and vertically. Each experiment was repeated at least twice for verification. Figure 3 shows the dune under equilibrium condition.

3. Dimensional analysis

Important parameters that affect dune dimensions, including its length, height, and also dune celerity in a steady and quasi-uniform flow, can be expressed as [5,10]:

$$G = f(h, d_{50}, u_*, \rho, \rho_s, \mu, g), \qquad (2)$$



Figure 2. Side view of laboratory flume and its different components.

| Exp. number | $ \begin{array}{c} Flow \\ depth \\ (m) \end{array} $ | ${f Discharge}\ ({f m^3/s})$ | <i>d</i> ₅₀ (mm) | <i>u</i> * (m/s) | $u * / u *_c$ | $egin{array}{c} { m Shields} \ { m number} \ (heta) \end{array}$ | Sediment Reynolds number (Re*) | Dune height (cm) | ${f Dune}\ length\ (cm)$ | Dune celerity (cm/s) | Investigation |
|----------------|---|------------------------------|-----------------------------|---------------------|---------------|---|--------------------------------------|------------------------|--------------------------|----------------------------|---|
| 1 | 0.20 | 0.05 | 0.85 | 0.025 | 1.20 | 0.045 | 19.86 | 2.5 | 98 | 0.014 | |
| 2 | 0.20 | 0.06 | 0.85 | 0.032 | 1.52 | 0.074 | 25.42 | 3 | 95 | 0.051 | |
| 3 | 0.20 | 0.07 | 0.85 | 0.039 | 1.85 | 0.111 | 30.98 | 3.5 | 93 | 0.064 | |
| 4 | 0.20 | 0.08 | 0.85 | 0.049 | 2.35 | 0.175 | 38.93 | 5 | 95.5 | 0.108 | |
| 5 | 0.20 | 0.10 | 0.85 | 0.068 | 3.28 | 0.336 | 54.02 | 7 | 94 | 0.192 | |
| 6 | 0.17 | 0.05 | 0.85 | 0.033 | 1.56 | 0.079 | 26.21 | 3 | 81 | 0.092 | |
| 7 | 0.17 | 0.06 | 0.85 | 0.041 | 1.95 | 0.122 | 32.57 | 3.5 | 84 | 0.108 | |
| 8 | 0.17 | 0.07 | 0.85 | 0.053 | 2.54 | 0.204 | 42.10 | 5 | 84.5 | 0.151 | Present study |
| 9 | 0.17 | 0.08 | 0.85 | 0.063 | 3.01 | 0.288 | 50.05 | 5.5 | 82 | 0.238 | |
| 10 | 0.17 | 0.10 | 0.85 | 0.085 | 4.07 | 0.525 | 67.52 | 7 | 80 | 0.277 | |
| 11 | 0.15 | 0.05 | 0.85 | 0.037 | 1.75 | 0.100 | 29.39 | 2.5 | 70 | 0.128 | |
| 12 | 0.15 | 0.06 | 0.85 | 0.052 | 2.52 | 0.197 | 41.31 | 4.5 | 68 | 0.121 | |
| 13 | 0.15 | 0.07 | 0.85 | 0.065 | 3.12 | 0.307 | 51.64 | 5.5 | 70 | 0.178 | |
| 14 | 0.15 | 0.08 | 0.85 | 0.076 | 3.65 | 0.420 | 60.37 | 6 | 73.5 | 0.277 | |
| 15 | 0.15 | 0.10 | 0.85 | 0.097 | 4.67 | 0.684 | 77.06 | 6.5 | 79 | 0.5 | |
| 16-30 | 0.085 \sim | 0.01 \sim | $0.23 \sim$ | $0.025 \sim$ | $1.28 \sim$ | 0.053 \sim | $8.17 \sim$ | $1.1 \sim$ | $10.5 \sim$ | | Colman et al. |
| | 0.017 | 0.21 | 0.82 | 0.073 | 5.02 | 1.159 | 55.56 | 4.3 | 71.8 | | (2005) [14] |
| $31 \sim$ | 0.108 \sim | $0.08~\sim$ | 0.4 \sim | $0.052~\sim$ | $2.38 \sim$ | 0.188 \sim | 19.55 \sim | $1.5 \sim$ | $44.7~\sim$ | | Tallebbydokhti |
| 84 | 0.272 | 0.14 | 1 | 0.112 | 6.04 | 1.157 | 104.91 | 6.8 | 121 | | et al. (2006) [10] |
| 85 | 0.305 | _ | 0.28 | 0.043 | 0.03 | 0.416 | 11.36 | 1.8 | 36.6 | _ | Guy et al. (1966) from Karim (1999) [1] |

Table 2. Results of experiments.



Figure 3. Dune waves in equilibrium condition.

where h is flow depth, d_{50} is the median diameter of the bed sediment, u^* is friction velocity, ρ and ρ_s are fluid and sediment density, μ is fluid dynamic viscosity, and g is gravitational acceleration. The parameter, G, in the above equation stands for dune length (λ), its height (η) or its celerity (V_d). By applying Buckingham theory with repeatable variables of ρ , d_{50} and g, Eq. (2) can be written as:

$$\bar{G} = f\left(\frac{h}{d_{50}}, \frac{\rho u_* d_{50}}{\mu}, \frac{\rho u_*^2}{d_{50} g \Delta \rho}\right),\tag{3}$$

where G is a dimensionless parameter and is defined as dune effective length (λ/d_{50}) or dune effective height (η/h) or dune effective celerity, $V_s = V_d/\sqrt{g.d_{50}.\Delta\rho/\rho}$, where $\Delta\rho$ is the difference between sediment and flow densities. The first parameter in the right hand side of Eq. (3) is relative bed roughness. The second and third parameters are defined as particles Reynolds number (Re*) and Shields parameter (θ) , respectively.

4. Results

4.1. Dune characteristics

Experiments were conducted with different flow depths and discharge, so that the Shields number was increased up to 0.68. Table 2 summarizes all experiments conducted in the present study, and the measurement results. In addition to the present work, experimental and field data for dune dimensions under a wide range of flow conditions and sediment sizes presented by previous investigations are also shown in Table 2. Figure 4 shows the variations of dune different characteristics with Shields number, and no correlation can be observed between the dune effective length (λ/d_{50}) and θ . Nevertheless, present experiments show that Shields number is important on dune height (η/h) . For example, by increasing θ about 15 times, the dune effective height (η/h) is increased 3.5 times (Figure 4(b)). An interesting conclusion from this figure is that the increasing rate of dune relative height decreased at higher Shields number (θ) , i.e. $\theta > 0.4$. This may be due to the change in the flow regime, as the height of dunes in the low flow regime decreased in the high flow regime, where the dune crest is washed away towards transition.

Finally, as shown in Figure 4(c), by increasing the Shields number, dunes effective celerity (V_s) increased



Figure 4. Variations of dune dimensions and celerity with Shields number: a) Dune relative length; b) dune relative height; and c) dimensionless celerity of dunes.

considerably. From experiments, by increasing θ from 0.045 to 0.68 (for about 15 times), parameter V_s increased about 35.7 times. This shows the substantial effect of flow velocity and bed shear stress on the celerity of the dunes.

Based on the present experimental data and Eq. (2), the following equations were derived for dune characteristics:

$$\frac{\eta}{h} = 0.05 \times \left(\frac{d_{50}}{h}\right)^{0.15} \times \left(\frac{\rho u_* d_{50}}{\mu}\right)^{0.7} \times \left(\frac{\rho u_*^2}{d_{50} g \Delta \rho}\right)^{0.1},\tag{4}$$

$$\lambda = 4.8 \times h,\tag{5}$$

$$V_s = 0.05 \times \left(\frac{d_{50}}{h}\right)^{0.95} \times \left(\frac{\rho u_* d_{50}}{\mu}\right)^{1.1} \times \left(\frac{\rho u_*^2}{d_{50} g \Delta \rho}\right)^{0.3}$$
(6)

The regression coefficients for Eqs. (4) and (5) are about $R^2 = 0.96$, and, for Eq. (6), is about $R^2 = 0.93$.



Figure 5. Comparison of measured and calculated relative dune height.



Figure 6. Comparison of measured and calculated dune length.

Eq. (5) shows that the most effective parameter on dune length is the flow depth. This is in agreement with previous studies (see Table 1). Figures 5 to 7 show a comparison of Eqs. (4) to (6) predictions with the present experimental data. The 45° line and the dashed lines in these figures show complete agreement and error bounds of prediction, respectively.

Furthermore, Figures 8 and 9 show the comparison of Eqs. (3) and (4) with previous empirical equations using all existing data presented in Table 2. As shown in Figure 8, nearly all clouds of experimental data for dune length are between the two lines of empirical equations presented by Allen [11] and Van Rijn [5]. However, the results of Eq. (4) are nearly in the middle of the regions between these two lines.

Figure 9 compares the prediction of different empirical equations for dune effective height with all available experimental data. The line of best agreement is also shown in this figure. The range of each equation prediction is shown by a polygon in this figure. Obviously, the closer this polygon is to the line of perfect agreement, the more accurate the equation is. As shown in this figure, the lowest and highest polygons (under prediction and over prediction) are related to the empirical equations presented by Allen [11] and



Figure 7. Comparison of measured and calculated dune celerity.



Figure 8. Comparison of different equations in predicting dune length.



Figure 9. Comparison of different equations in predicting relative dune height considering all available data.

Julien and Klaassen [6], respectively. However, the data region of Eq. (4) is between them and encompasses data regions for the equations presented by Julien and Klaassen [6], VanRijn [5] and Yalin [3].

4.2. Bed roughness

In a uniform flow, the bed shear stress (τ) can be correlated to the square of the mean velocity with the following relationship:

$$\tau = \frac{1}{8} f \rho V^2, \tag{7}$$

where f is the Darcy-Weisbach roughness coefficient. The following relationship can be written for total roughness coefficient in a live bed alluvial stream [13]:

$$f = f' + f'',\tag{8}$$

where f' and f'' are roughness coefficients related to grains and bed forms, respectively. Based on the relationship between the Darcy-Weisbach roughness coefficient and the Manning roughness coefficient, neglecting the effect of hydraulic radius on Manning coefficient, one can write:

$$n^2 = n^{\prime 2} + n^{\prime \prime 2}. (9)$$

The Manning coefficient related to grains in a plane bed (n') can be estimated from [13]:

$$n' = \frac{R_h^{\frac{1}{6}}}{18 \text{Log}\left(\frac{12R_h}{d_{50} + \frac{3.3\nu}{u_*}}\right)},\tag{10}$$

where R_h is hydraulic radius and ν is the kinematic viscosity of water. By estimating total Manning coefficient (n) from the Manning equation and also the Manning coefficient related to grains (n') from Eq. (10), one can predict the Manning coefficient related to bed forms (n'') from Eq. (9). Analyzing experimental data showed that the ratio of n''/n is between 0.5 and 0.92



Figure 10. Variation of n''/n calculated by log. law with Shields number.

for Shields number between 0.045 to 1.16, equivalent to Froude number (F) between 0.24 and 0.76 (Figure 10). By regression analysis, including all data in Table 2, the following logarithmic relationship was developed for the relative Manning coefficient related to the bed form as:

$$\frac{n''}{n} = 0.11Ln\theta + 0.88.$$
(11)

5. Conclusion

An experimental study was carried out to investigate dune characteristics in sand beds. The experiments were conducted in a water and sediment recirculating flume. The flow and deposited sediment in the reservoir at the downstream of the flume were pumped to its upstream inlet. Experiments were performed under live bed conditions, with a flow intensity parameter $(u * / u *_c)$ range of 1.2 to 4.7. Each test was continued until equilibrium condition was achieved, where the bed features (dune wave) were similar in length and height along the channel. Under this condition, dune dimensions, including height and length, as well as celerity, were recorded. Present experiments showed that the Shields number has a considerable effect on dune height and celerity. For example, by increasing the Shields number from 0.045 to 0.68, the dune effective height and its effective celerity increased by 3.5 and 35.7 times, respectively. However, the effect of this parameter on dune length was negligible. Based on dimensional analysis and experimental data, empirical equations were developed for dune characteristics by regression analysis. Comparison of the present relationship with the available experimental and field data, as well as empirical equations presented by previous investigations, showed an acceptable accuracy.

Furthermore, in addition to dune characteristics, bed roughness related to bed forms was analyzed based on all available experimental and field data. Results of this part showed that by increasing the Shields number from 0.045 to 1.16, the ratio of Manning coefficient, related to dune height, to total Manning coefficient increased from 0.5 to 0.92. An empirical equation was also developed for the Manning coefficient related to bed forms.

Abbreviations

- λ Dune length
- η Dune height
- V_d Dune celerity
- V_s Dimensionless dune celerity
- h Flow depth

- τ_0 Available shear stress
- τ_c Critical shear stress
- T Transport stage parameter
- σ_g Geometric standard deviation of sediment grading
- d_a Size of sediment for which a percent of material by weight are finer
- u* Bed shear velocity
- $u*_c$ Critical shear velocity for the bed material
- C Chezy coefficient
- R_h Hydraulic radius
- ρ Fluid density
- ρ_s Sediment density
- μ Fluid dynamic viscosity
- ν Kinematic viscosity of water
- g Gravitational acceleration
- Re* Sediment Reynolds number
- θ Shields parameter
- f Darcy-Weisbach roughness coefficient
- f' Darcy-Weisbach roughness coefficient related to grains
- f'' Darcy-Weisbach roughness related to bed forms
- *n* Manning roughness coefficient
- n' Manning roughness coefficient related to grains
- n'' Manning roughness coefficient related to bed forms
- F Froude number

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