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Research Note

Criterion for an oscillatory charged jet during the bubble spinning process

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Abstract. The oscillatory diameter of the charged jet during bubble electrospinning results in beads on the obtained nanofibers. We demonstrate that the applied voltage and the initial flow rate of the jet are crucial parameters necessary to control the morphology of the nanofibers. We also find that there is a criterion for the production of smooth nanofibers without beads. The theory developed in this paper can be extended to classical electrospinning and blown bubble-spinning.

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1. Introduction

A nonlinear equation governing the motion of the charged jet during the bubble electrospinning process was obtained in [1], which reads as:

$$\frac{d^2U}{dz^2}+a-\frac{b}{U}-cU\frac{dU}{dz}+d\left(\frac{dU}{dz}\right)^{n-1}\frac{d^2U}{dz^2}=0, \quad (1)$$

where $U=r^{-2}=\frac{\pi\rho u}{Q}$, $a=\frac{\pi\rho^2EI}{\mu Q^2}$, $b=\frac{k\pi^2\rho^2E^2}{\mu Q^2}$, $c=\frac{Q}{\mu\pi}$, and $d=\frac{\alpha n}{\mu}\left(\frac{Q}{\pi\rho}\right)^{n-1}$, u is the jet velocity, ρ is the liquid density, σ is the surface charge, r is radius of the jet, E is the applied electric field, I is the current, Q is the flow rate, k is the conductivity of the fluid, μ is viscosity coefficient, and α and n are constants.

A detailed description of the problem and the derivation of the equation are available in [1].

Ignoring the viscous and inertial effects, we can simplify Eq.(1) as:

$$\frac{d^2U}{dz^2}+a-\frac{b}{U}=0. \quad (2)$$

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We re-write Eq. (2) in the form:

$$\frac{d^2U}{dz^2}+\frac{aU^2-b}{U^2}U=0. \quad (3)$$

It is obvious that Eq. (2) admits a periodic solution when $aU^2>b$.

Using a transform:

$$U=\frac{b}{a}+\frac{a}{b}u. \quad (4)$$

Eq. (3) becomes:

$$u''+\omega_0^2u+\alpha u''u=0, \quad (5)$$

where

$$\omega_0^2=\frac{a^2}{b}=\frac{\rho^2I^2}{\mu kQ^2},$$

$$\alpha=\frac{a^2}{b^2}=\frac{I^2}{k^2\pi^2E^2}.$$

Eq. (5) can be solved by various methods, such as variational iteration [2,3], homotopy perturbation[4,5], iteration perturbation [6,7], parameter expansion [8] and others [9,10]. In this paper, we use the amplitude-frequency formulation developed in [9-12] to obtain the frequency of Eq. (5).

2. Amplitude-frequency formulation

Eq. (5) admits periodic solutions as revealed in [1]. Hereby, we suggest a simple approach to the search for an approximate frequency of Eq. (5) using the simple amplitude-frequency formulation [9-12].

We assume that the periodic solution of Eq. (5) has the form:

$$u = A\cos\omega t. \tag{6}$$

Submitting Eq. (6) into Eq. (5) results in the following residual:

$$R(\omega,t) = -A\omega^2\cos\omega t + \omega_0^2 A\cos\omega t - \alpha A^2\omega^2\cos^2\omega t. \tag{7}$$

In order to use the amplitude-frequency formulation [9-12], we choose two trial frequencies and locate at:

$$t = \pi/(4\omega).$$

Setting $\omega_1 = 1$, $\omega_1 t = \frac{\pi}{4}$, and $\omega_1 = 2$, $\omega_2 t = \frac{\pi}{4}$, in Eq. (7), respectively, we have:

$$R_1 = -\frac{\sqrt{2}}{2}A + \frac{\sqrt{2}}{2}\omega_0^2 A - \frac{1}{2}\alpha A^2, \tag{8}$$

$$R_2 = -2\sqrt{2}A + \frac{\sqrt{2}}{2}\omega_0^2 A - 2\alpha A^2. \tag{9}$$

The frequency can be then obtained approximately in the form [9-12]:

$$\omega^2 = \frac{R_1\omega_1^2 - R_2\omega_2^2}{R_1 - R_2}. \tag{10}$$

This formulation has been widely used to solve periodic solutions of various nonlinear oscillators [13-20]. It is often called He's frequency formulation, He's amplitude-frequency formulation or He's frequency-amplitude formulation.

Submitting the trial frequencies and residuals into Eq. (10), we obtain:

$$\omega^2 = \frac{R_1\omega_1^2 - R_2\omega_2^2}{R_1 - R_2} = \frac{15\sqrt{2} - 3\sqrt{2}\omega_0^2 + 15\alpha A}{3\sqrt{2} + 3\alpha A}$$
$$= 5 - \frac{\sqrt{2}\omega_0^2}{\sqrt{2} + \alpha A} = 5 - \frac{\sqrt{2}k\pi^2 E^2\rho^2 I^2}{\mu Q^2(\sqrt{2}k^2\pi^2E^2 + I^2 A)}. \tag{11}$$

A similar result was obtained in [1] (see Eq. (48) in [1]), which was readily verified by the experimental data (see Figure 13 in [1]).

Generally, the pulsation amplitude is very small compared with the jet diameter, which results in $A << 1$. Under these conditions, Eq. (11) can be simplified as:

$$\omega^2 = 5 - \frac{\rho^2 I^2}{\mu Q^2 k}. \tag{12}$$

In case $\omega^2 < 0$, no period solution is admitted.

3. Discussion and conclusion

Beads are predicted when:

$$\frac{\rho^2 I^2}{\mu k Q^2} < 5. \tag{13}$$

When Eq. (13) is satisfied, the diameter of the jet during the spinning process is varied periodically. Under surface tension, the oscillation will finally lead to beads when solidified. The bead morphology can be controlled by the applied voltage (current I depends strongly on voltage) and the flow rate (Q), which keeps unchanged during the spinning process according to mass conservation. The addition of some salts to the polymer solution will greatly enhance conductivity (k), and provides a simple way to control the morphology of the nanofibers. Density (ρ) and viscosity (μ) can be adjusted by the solution's concentration. Bubble electrospinning [21-27] is a newly developed technology for mass-production of nanofibers, and the present theory provides a complete theoretical guideline for controlling the beads in the nanofibers. The theory developed in this paper can be extended to classical electrospinning [24] and blown bubble-spinning [28].

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Biographies

Ji-Huan He is Professor of Textile Engineering and Mathematics. He is the creator of the variational iteration method, the homotopy perturbation method, the exp-function method, and other analytical methods, and is responsible for various inventions concerning mass production of nanofibers, e.g. bubble electrospinning and blown bubble-spinning.

Hai-Yan Kong is a PhD degree candidate working in nanotechnology. Her research interests include optimal design of various bubble electrospinning methods, including portable equipment for experimental use.