Scientia Iranica A (2014) 21(1), 82–90

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions A: Civil Engineering
www.scientiairanica.com

Investigating vehicle-slab track interaction considering random track bed stiffness

M. Mehrali, S. Mohammadzadeh*, **M. Esmaeili and M. Nouri**

Department of Railway Egineering, Iran University of Science and Technology, Tehran, Iran.

Received 25 February 2013; received in revised form 1 Junuary 2013; accepted 18 Junuary 2013

KEYWORDS
Vehicle-slab track interaction;
Random stiffness;
Monte Carlo simulation;
Two layer Euler-Bernoulli beam.

Abstract. This paper deals with the modeling of a vertically coupled vehicle-slab track using the finite element method. The slab track system is represented by a two layer Euler-Bernoulli beam model, including a rail and a concrete slab. The vertical stiffness of the track bed is assumed to be a random variable using Monte Carlo simulation. The vehicle is simplified as a multi-body system with 10 degrees of freedom. The accuracy of the simulation is verified by deterministic and random approaches, as well. Sensitivity analyses of the various parameters such as slab thickness, coefficient of track bed stiffness and vehicle velocity are performed. It is demonstrated that the uncertainty in track bed stiffness has a major effect on rail and slab deflections. From a practical point of view, the obtained results of the present study can be utilized efficiently in the analysis and design of slab track systems.

1. Introduction

Railway track systems can be divided into two main categories: ballasted and non-ballasted. In general, a slab track has better stiffness uniformity and higher stability in comparison to a ballasted track. From a Life Cycle Cost (LCC) point of view, for developing new recent railway track networks, the slab track system is more justifiable, aiming to increase train speed, as well as train axle load [1,2].

Conducted research in the field of slab track dynamic analysis is mainly focused on evaluating track component behaviour, assuming a constant pattern for parameters, including moving load or moving mass values, rail corrugation and track subgrade stiffness. This type of analysis is usually called a deterministic approach, which may be in conflict with the stochastic nature of the mentioned parameters. Considering the deterministic nature of relevant parameters of slab track systems, various research work can be pointed out. For instance, Xiang et al. (2008) [3] offered a track segment element to analyze coupled vehicle-slab track dynamics. In the mentioned study, the vertical displacement of the slab was calculated simply by the lateral finite strip method, and the responses of track displacement and wheel-rail force were investigated. Lei and Zhang (2011) [1] proposed a new type of slab track element to calculate the vibration of a vertical coupled vehicle-slab track system. In this regard, the global mass, damping and stiffness matrices were efficiently assembled using this slab track element and this led to assessing the effects of various slab track parameters on the dynamic responses of the vehicle and track. Mazilu (2010) [4] solved the dynamic response of a coupled vehicle-slab track system using the Green function in frequency and time domains. In that research, the vehicle model was simplified using a two-mass oscillator and the slab track was simulated as an infinite two-layer beam. Bitzenbauer and Dinkel (2002) [5] proposed a semi-analytical method, based on

*. *Corresponding author. Tel.: +98 9125570493;*
Fax: +98 21 77451568
E-mail addresses: m_mehrali@rail.iust.ac.ir (M. Mehrali), mohammadz@iust.ac.ir (S. Mohammadzadeh), m_esmaeili@iust.ac.ir (M. Esmaeili), mehrdad_nouri@rail.iust.ac.ir (M. Nouri)

Fourier transforms, to achieve the dynamic response of a coupled vehicle-slab track system. In this matter, the slab track was simulated as a continuously supported model to obtain a closed-form solution in the Fourier domain.

From another standpoint, many researchers have modelled slab track systems using the double beam theory. Hussein and Hunt (2006) [6] modeled tracks with continuous slabs, accounting moving oscillating loads.

One of the most important issues which can considerably affect slab track dynamic behavior is the random characteristic of the subgrade. There are many reasons, such as non-uniformity in the subgrade material, compaction effort, the elasticity modulus and some differences in constructional methods and equipment, for the random distribution of subgrade properties along the track. For this reason, investigating vehicle-slab track coupling behaviour, considering the random specifications of subgrade material, seems to be essential.

The aforementioned investigations in the literature indicate that none of the presented work deals with the interaction of a vehicle-double beam system assuming random track bed stiffness.

In the present study, using the finite element method and Monte Carlo technique, the response analysis of a vertically coupled vehicle-slab track is carried out. The developed model consists of an upper Euler-Bernoulli beam to account for the rail, and a lower Euler-Bernoulli beam to account for the slab. There are two continuous resilient layers in the model: One to account for rail pads between the rails and slab, and another to account for the track foundation underneath the slab. The foundation stiffness is considered to be a random variable using Monte Carlo simulation. The vehicle is assumed to have 10 degrees of freedom. The accuracy of the simulation is verified using two approaches. The first model validation is done by comparing the results with a deterministic model, while the second validation is performed through comparison of the outputs within the exisiting random finite element model. In the last stage of the research, using the validated model, the responses of the double beams are calculated numerically in the framework of the parametric study on many parameters, such as slab thickness, subgrade stiffness, and vehicle moving velocity.

2. Vertically coupled vehicle-slab track simulation

Figure 1 illustrates the general configuration of a coupled vehicle-slab track system. The model consists of an upper Euler-Bernoulli beam accounting for the rail, and a lower one representing the concrete slab. Railpads are represented by a layer of springs with stiffness k_1 and a viscous damping factor, c_1. The subgrade is modelled by a layer of springs with stiffness, k_2, and a viscous damping factor, c_2.

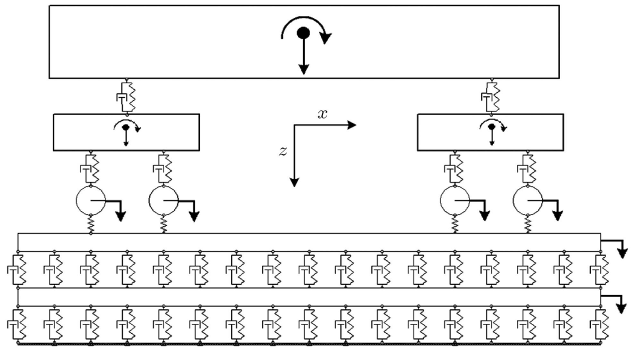

Figure 1. Configuration of vertical coupled vehicle-slab track model.

The moving vehicle is simulated as a multi-rigid-body system, which comprises one car body picked up on two bogies, four wheel sets, as well as four primary and two secondary suspensions. The primary and secondary suspensions are represented by parallel spring-damping elements. The secondary suspensions pin the car body and bogies. Each bogie is supported on two wheelsets, which are connected to the bogie through the primary suspensions.

The dynamic equations of motion of the vehicle in the coupling system can be described as:

$$M_v(\{\ddot{a}\}_v + C_v\{\dot{a}\}_v + K_v\{a\}_v = \{Q\}_v, \tag{1}$$

where M_v, C_v and K_v are mass, damping and stiffness matrixes of the vehicle, respectively, and $\ddot{a}_v, \dot{a}_v$ and a_v are the acceleration, velocity and displacement vectors of the vehicle model, respectively. Q_v indicates the external load vector of the vehicle model, which relies on both vehicle and rail displacement vectors.

The slab track system consists of the rail, slab and railpad and subgrade. The slab track is considered as two parallel beams, which are simulated by the Euler-Bernoulli beam. The rail and slab are discretely supported by the slab track bed, respectively, using a parallel spring and dashpot elements.

The equation of motion of the slab track can be written as follows:

$$\begin{bmatrix} M_{\text{Rail}} & 0 \\ 0 & M_{\text{Slab}} \end{bmatrix} \begin{Bmatrix} \ddot{a}_{r_t} \\ \ddot{a}_{s_t} \end{Bmatrix} + \begin{bmatrix} C_{\text{Rail}} & 0 \\ 0 & C_{\text{Slab}} \end{bmatrix} \begin{Bmatrix} \dot{a}_{r_t} \\ \dot{a}_{s_t} \end{Bmatrix} + \begin{bmatrix} K_{\text{Rail}} & 0 \\ 0 & K_{\text{Slab}} \end{bmatrix} \begin{Bmatrix} a_{r_t} \\ a_{s_t} \end{Bmatrix} = \{Q\}_t, \tag{2}$$

where M_{Rail}, C_{Rail} and K_{Rail} are mass, damping and stiffness matrixes of the rail, respectively and $\ddot{a}_{r_t}, \dot{a}_{r_t}$ and a_{r_t} correspond to the acceleration, velocity and displacement vectors of the rail. M_{Slab}, C_{Slab} and K_{Slab}

are mass, damping and stiffness matrixes of the slab, and $\ddot{a}_{s_t}, \dot{a}_{s_t}$ and a_{s_t} are the acceleration, velocity and displacement vectors of the slab, respectively. Q_t indicates the external load vector of the slab track model.

The vehicle and slab track models are jointed together throught a non-linear wheel-rail contact force with the following definition [7]:

$$
\begin{aligned}
&f_{wj} = C_H[a_{wj} - a_r(x_j, t)]^{1.5},\\
&f_{wj} = K_H[a_{wj} - a_r(x_j, t)],\\
&K_H = C_H[a_w - a_r]^{0.5} \quad \text{if} \quad a_w > a_r,\\
&K_H = 0, \qquad\qquad\qquad \text{if} \quad a_w < a_r,
\end{aligned}
\tag{3}
$$

where a_{wj} is wheel displacement, a_r is the vertical deflection of rail underneath the location (x_j) of the jth wheel, and C_H is the Hertzian contact coefficient.

According to Eqs. (1), (2) and (3), the equation of motion for the coupled vehicle-slab track system can be written as:

$$M_{vt}(\{\ddot{a}\}_{vt} + C_{vt}\{\dot{a}\}_{vt} + K_{vt}\{a\}_{vt} = \{Q\}_{vt}, \tag{4}$$

where M_{vt}, C_{vt} and K_{vt} are the mass, damping and stiffness matrixes of the coupled vehicle-slab track, respectively, and $\ddot{a}_{v_t}, \dot{a}_{v_t}$ and a_{v_t} are, correspondingly, the acceleration, velocity and displacement vectors of the coupled vehicle-slab track model. Q_{vt} indicates the external load vector.

The assembling of the whole system matrices, considering the vehicle and slab track components, can be shown in the matrix form as Eq. (5).

In the following system matrixes, $[C]$, $[B]$, $[W]$ are the car body, and the bogies and wheelset relevant matrices, respectively. The interaction between carbody, bogies and wheelsets is applied by $[C/B]$, $[B/W]$ and $[W/R]$ matrices.

$$
\begin{bmatrix}
[C] & [C/B] & 0 & 0 & 0\\
[C/B] & [B] & [B/W] & 0 & 0\\
0 & [B/W] & [W] & [W/R] & 0\\
0 & 0 & [W/R] & [R] & [R/S]\\
0 & 0 & 0 & [R/S] & [S]
\end{bmatrix}, \tag{5}
$$

$[R]$ and $[S]$ are the rail and slab matrices and these two elements are connected together via the $[R/S]$ matrix. Morover, the interaction between the rail and the vehicle is established by the $[W/R]$ matrix.

3. Solution procdure of equations of motion

For better understanding, the solution procdure of equations of motion can be demonstrated in a flowchart. Figure 2 shows the calculation process of the vehicle-slab track equations.

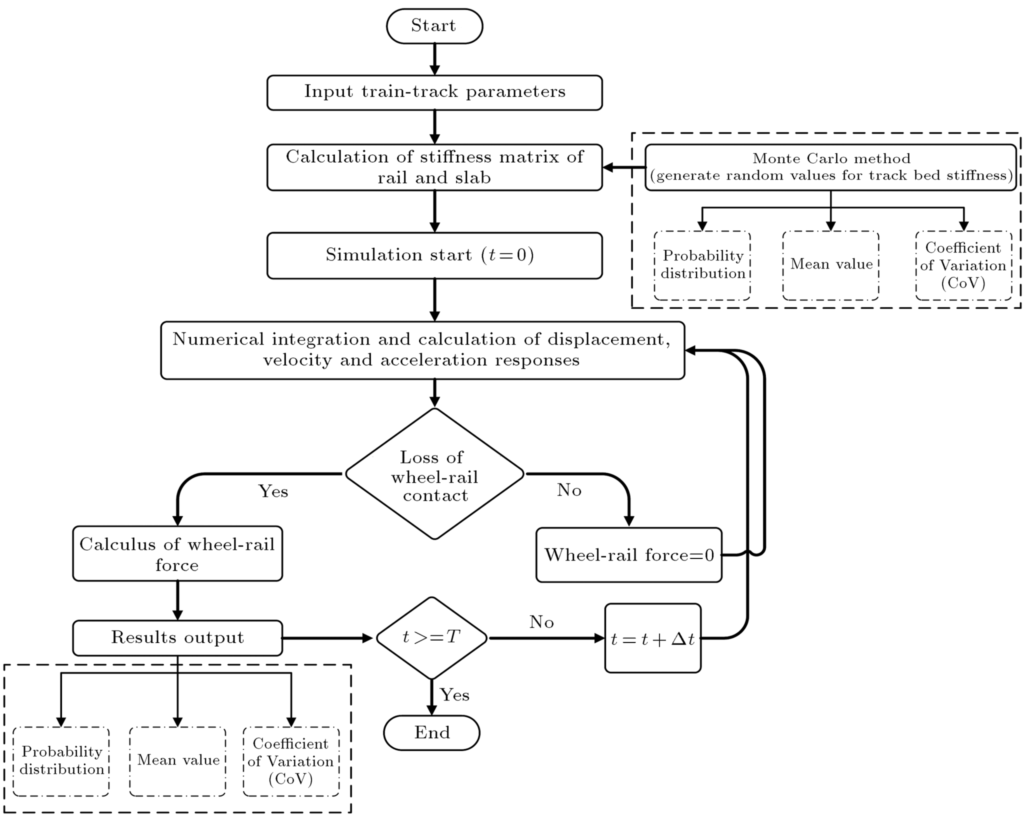

Figure 2. Flowchart of procedure of dynamic simulation of vehicle-slab track system.

4. Modeling of random behavior of system

The behavior of a physical system should be predicted in the following steps:

1. Inspection of physical system;
2. Hypothesis formulation;
3. Prediction of the behavior of the system on the basis of the hypothesis;
4. Validiation of the hypothesis by experiment.

Sometimes it may be impractical to examine certain processes. It is obvious that there are many states which cannot be represented mathematically, due to the random nature of the problem. Even if a mathematical model can be formulated to describe some system of interest from the limited data available, it may not be possible to achieve a solution to the model by simple analytical methods and, consecutively, make predictions about the behavior of the system.

In a railway track, the real behavior of the substructure and superstructure parts are affected by many factors, such as foundation stiffness and rail irregularity. In many studies, the real behavior of a ballasted track assumes that the random behavior of rail irregularity has been investigated [8,9]. On the other hand, the track bed stiffness, in fact, has a variable nature with more or less random properties [10]. The random behavior of a ballasted track is the only case to have been investigated in the relevant literature.

One usual objective in using the Monte Carlo technique is to estimate certain parameters and probability distributions of random variables, whose values depend on interaction with random variables whose probability distributions are specified.

In the current study, the random behavior of track bed stiffness is simulated by the Monte Carlo method.

Vertical track stiffness can be defined as the ratio between vehicle wheel load and rail deflection at the contact point, as a function of time and location. The stiffness value usually depends on applied dynamic load amplitude and frequency, as well. This is an important parameter of wheel-rail interaction, which controls the mechanical behavior of the track as well as track degradation.

It is possible to measure the vertical track stiffness continuously along the track. Berrgren [11] developed a trolley by which the track can be loaded and track stiffness measured, while moving along a track at a speed of up to 30 km/h. As a case study, Berrgren (2009) [11] represented a real track stiffness measurement (Figure 3).

In this study, various values of vertical stiffness are derived digitally from the above mentioned measurement and used for probabilistic analyses. By analyzing the various probability distributions on these values, the normal probability distribution with the best fitness with derived values is found (Figure 4).The values of mean, variance and deviation of track bed stiffness are presented in Table 1.

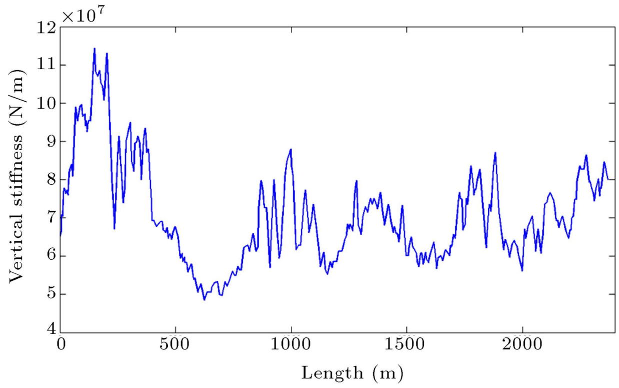

Figure 3. The measured vertical stiffness of track bed [13].

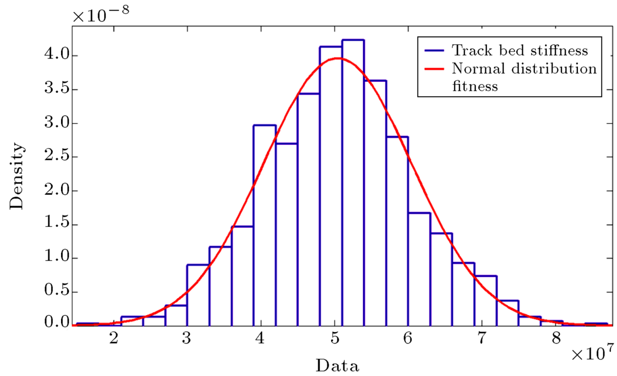

Figure 4. Fitting of normal distribution pattern to measured track stiffness value [11].

Table 1. Statistical parameters of track bed stiffness.

Statistical parameter	Mean value (N/m)	Variance	Deviation
Value	5.04×10^8	1.011×10^{14}	1.005×10^7

Before simulating track bed stiffness, the Coefficient of Variation (CoV) should be defined. In probability theory and statistics, the CoV is a normalized measure of dispersion of the probability distribution. The CoV is defined as the ratio of standard deviation, σ, and mean value, as follows [12]:

$$\text{CoV} = \frac{\sigma}{\mu}. \tag{6}$$

Based on the Monte Carlo method, track bed stiffness is simulated with 10000 spatial samples. The initial value for generating the spatial sample is based on the mean value and standard deviation of track bed stiffness shown in Table 1. As defined in Eq. (6), the mean value of track bed stiffness depends on the CoV value.

In this paper, the CoV value is supposed to vary in the range of 0.1 to 0.5. Consequently, by assuming different values of CoV and a constant value for standard deviation, the mean values of track bed stiffness are calculated using Eq. (6). These obtained values are introduced to the developed FE code and, consequently, the effect of the randomness of the track bed stiffness on the dynamic behavior of the whole slab track system is investigated.

5. Numerical results and discussion

In the current section, many numerical examples are presented. Firstly, the validity of the developed model is investigated through comparison of the obtained results of the developed FE model with existing analytical results. Subsequently, some sensitivity analyses are performed to show some considerable aspects in the dynamic behavior of the slab track system with random bed stiffness.

5.1. Developed FE model verification

As the first numerical example, vertical deflection of the model, consisting of an infinite double beam resting on a visco-elastic foundation under a moving load with various speeds, is compared with that obtained by Hussein and Hunt [6]. As illustrated in Figure 5, acceptable agreement can be observed between the two groups of results, which confirms the validity of the developed FE model.

The slab track parameters used in the analysis are listed in Table 2.

5.2. Sensitivity analyses

Many parameters can affect the dynamic behavior of slab tracks. These parameters may be divided into two general categories:

1. Technical characteristics;
2. Operational characteristics.

Many sensitive analyses are performed to investigate the effects of technical parameters, such as slab height and the CoV of track bed stiffness, and operational parameters, such as vehicle speed, on the dynamic behavior of the slab track.

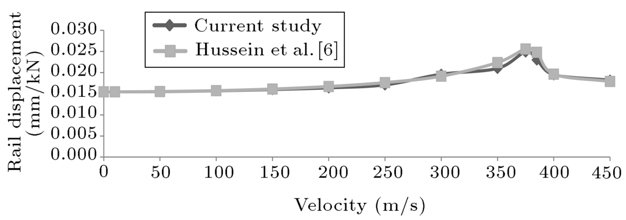

Figure 5. Comparing the results of the current study with analytical solution of Hossein and Hunt [6].

Table 2. Parameter values used for slab track.

Item	Notation	Value	Unit
Rail (UIC 60)			
Bending stiffness (steel)	EI_1	6.65×10^6	Pam^4
Mass per length	m_1	60.34	kgm^{-1}
Slab			
Bending stiffness (concrete)	EI_2	233.3×10^6	Pam^4
Mass per length	m_2	3500	kgm^{-1}
Rail pad			
Stiffness	k_1	40×10^6	Nm^{-2}
Viscous damping	c_1	6.3×10^3	Nsm^{-2}
Slab bearing			
Viscous damping	c_2	41.8×10^3	Nsm^{-2}

Table 3. Values of used parameters for sensitivity analyses.

Item	Notation	Value	Unit
Slab thickness	d	0.2, 0.3, 0.4	m
CoV of track bed stiffness	k_2	0.1, 0.2, 0.3, 0.4, 0.5	MPa
Vehicle speed	V	50, 80, 120, 160	km/h

It should be noted that in the FE model presented in this study, the vehicle starts moving at the left-hand side of a 50 m long slab track with a speed of 80 km/h, and the steady state response of the coupled vehicle-slab track system is obtained at the middle of the slab track, thereby ensuring that the end boundary conditions do not influence the obtained results [13]. Table 3 shows the used values of parameters for sensitivity analyses in the current study.

The results of sensitivity analyses are illustrated in Figures 6 to 9. It should be noted that the CoV of rail displacement is shown in the decibel scale (dB).

5.2.1. Effect of slab height and CoV of bed stiffness on rail and slab displacements

Figures 6 and 7 shows variations of the CoV of rail displacement with slab thickness, and the CoV of stiffness for various values of vehicle speed. As shown, the CoV of rail displacement increases by increasing the CoV of the track bed stiffness. This confirms that due to an increase in uncertainty in track bed stiffness values, the rail is confronted with more displacement variation. Moreover, by increasing slab thickness, the CoV of rail displacement is increased. It may be because of slab bending stiffness, which has a direct dependence on slab thickness. By increasing slab thickness, the slab bending stiffness is increased. On the other hand, the stiffness underneath the rail is increased. This means that more reaction force is

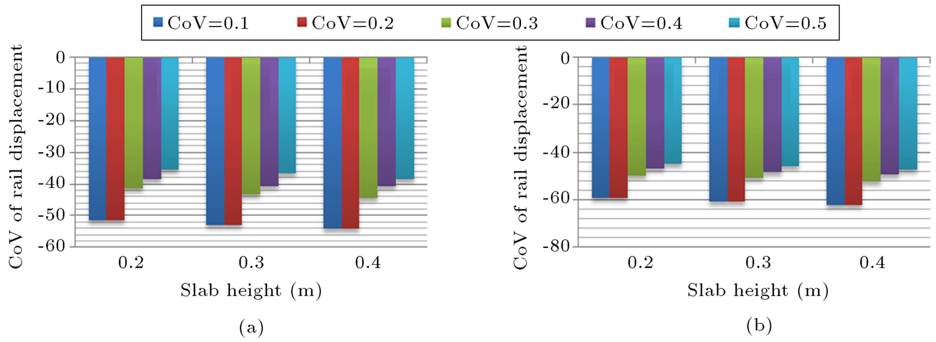

Figure 6. Variation of CoV of rail displacement with slab height and CoV of stiffness: (a) Vehicle speed of 50 km/h; and (b) vehicle speed of 80 km/h.

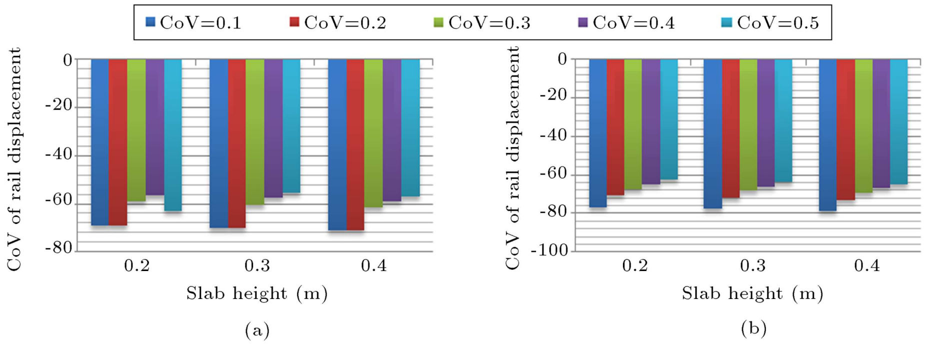

Figure 7. Variation of CoV of rail displacement with slab height and CoV of stiffness for (a) vehicle speed of 120 km/h, and (b) vehicle speed of 160 km/h.

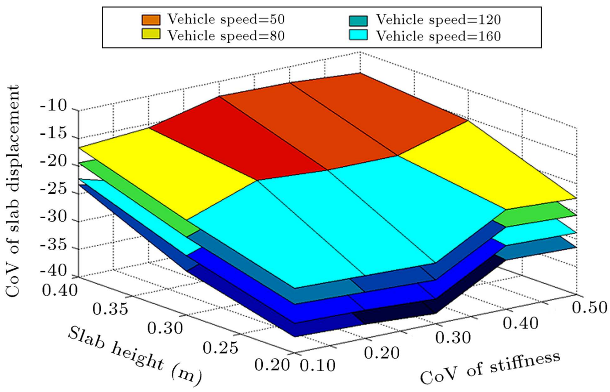

Figure 8. Variation of CoV of slab displacement with slab thickness and CoV of stiffness for various values of vehicle speed.

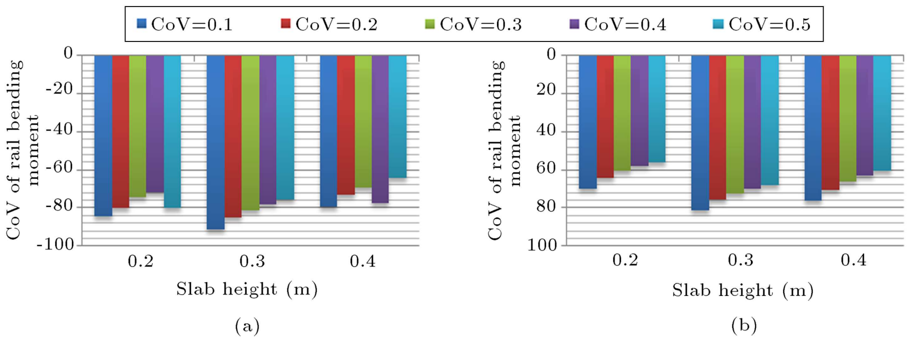

Figure 9. Relation of the CoV of rail bending moment with slab height for various bed stiffness CoV for (a) vehicle speed of 80 km/h, and (b) vehicle speed of 120 km/h.

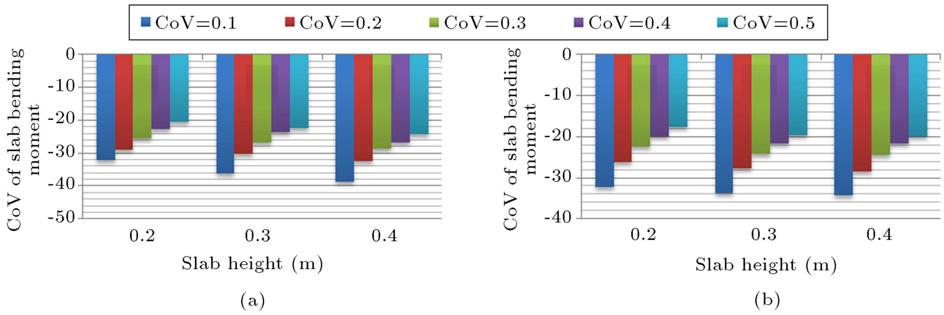

Figure 10. Relation of the CoV of slab bending moment with slab height for various bed stiffness CoV for (a) vehicle speed of 80 km/h, and (b) vehicle speed of 120 km/h.

applied to the rail, so, the CoV of the rail displacement is increased.

It is observed that due to the increase in vehicle speed from 50 km/h to 80 km/h, in the case of CoV of track bed stiffness equal to 0.1 and slab thickness equal to 0.2 m, a decrease of 15% in the CoV of rail displacement will take place. However, because of the increase in vehicle speed from 50 km/h to 120 km/h, a decrease of 34% in the CoV of rail displacement will arise. This procedure for an increasing vehicle speed of 50 km/h to 160 km/h leads to a 49% increase.

Figure 8 shows up a variation of the CoV of slab displacement with slab thickness, and the CoV of stiffness for various values of vehicle speed. As illustrated, the CoV of slab displacement increases by increasing the CoV of bed stiffness. As with the rail, by increasing the uncertainty in track bed stiffness, the slab is confronted with more displacement disparity.

It is seen from Figure 8 that an increase in vehicle speed from 50 km/h to 80 km/h in the case of the CoV of track bed stiffness equal to 0.1 and slab thickness equal to 0.2 m will create a decrease of 10% in the CoV of slab displacement, while an increase in vehicle speed from 50 km/h to 120 km/h will create a decrease of 23% in the CoV of rail displacement. This procedure for an increasing speed of 50 km/h to 160 km/h leads to 31% increase. Overall, the randomness of the bed stiffness has more effect on the slab than on the rail because the CoV of slab displacement is 45% more than the CoV of rail displacement.

5.2.2. Effect of slab height and CoV of bed stiffness on CoV of bending moment in rail and slab

In this section, the effects of slab thickness and the CoV of bed stiffness on the CoV of the bending moment in rail (CVRB) and slab (CVSB) are discussed for various vehicle moving speeds.

As illustrated in Figure 9, by increasing the CoV of track bed stiffness, the CVRB is increased. This confirms that uncertainty of track bed stiffness can directly affect rail bending moment. Also, by increasing the vehicle speed, CVRB shows a decrease (Figure 9(a) and (b)).

Figure 10 shows the variation of CVSB versus CoV of bed stiffness for various slab thickness at two different vehicle speeds.

It can be observed that due to the increase in the CoV of track bed stiffness, the CVSB is amplified. Moreover, due to the increase in vehicle speed, CVSB is decreased (Figure 10(a) and (b)).

5.3. The statistical interpretation of results

The simulation of vehicle-slab track system dynamic interaction can be separated into the following sections: 1) Input, 2) System configuration and 3) Output. The system configuration using the Finite Element Method (FEM) is partitioned into smaller elements. The input section and the characteristics of system elements can describe the behavior of the whole system, i.e. the output section. Figure 11 shows the whole system performance in each computational process in the current study. The loading condition is considered to be deterministic, so its input has a certain value. On the other hand, the elements of the system have random variation, i.e. track bed stiffness is supposed to be random throughout the simulation.

The track bed stiffness has a normal distribution, as shown in Figure 3. The one that must be investigated is the probability distribution of the system output. The results are shown in Figsures 12 and 13.

As Figure 12 shows, displacements of the rail and slab have an identical statistical probability to stiffness distribution. On the other hand, the form of distributions of bending moments in the rail and slab once again confirm this issue (Figure 13).

Table 4 provides the mean values and standard deviations of rail and slab responses.

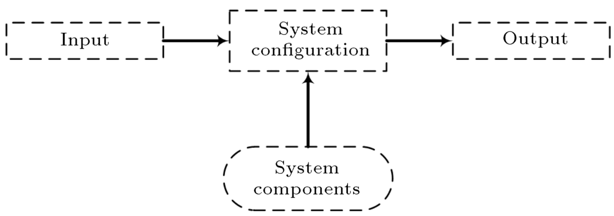

Figure 11. Flowchart of application of slab track system.

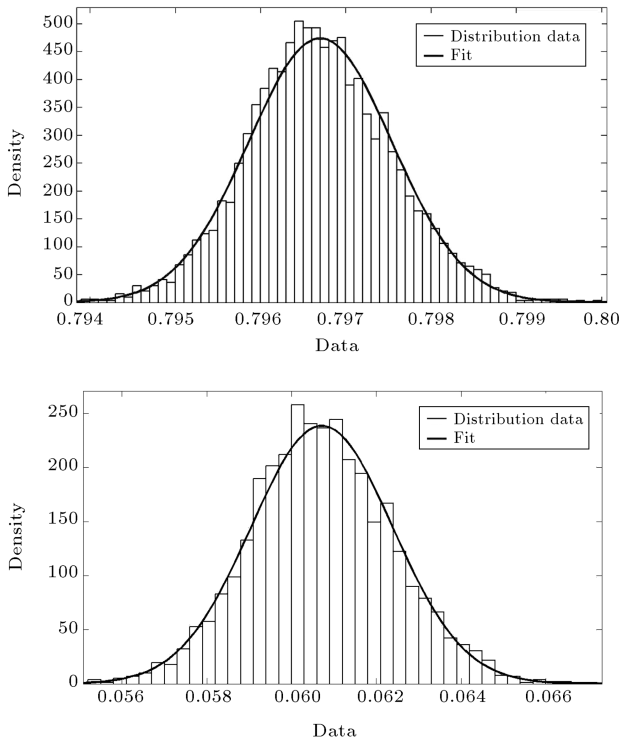

Figure 12. Normal distribution of the analysis outputs: (a) Rail displacement; and (b) slab displacement.

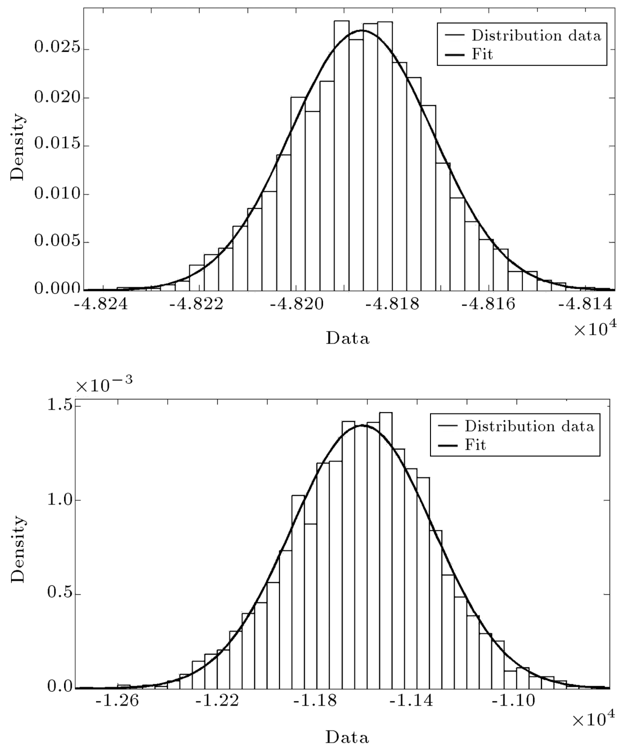

Figure 13. Normal probability distribution of (a) Rail; and (b) slab bending moment.

Table 4. Probabilistic parameters of simulation output.

Element	Displacement		Bending moment	
	Mean	Standard deviation	Mean	Standard deviation
Rail	0.79	0.0008	48186×10^6	14.8
Slab	0.06	0.002	11619×10^4	285

6. Conclusion

In the present study, the dynamic behavior of a vertically coupled vehicle-slab track has been investigated using the finite element method. The vertical stiffness of the track bed is assumed to be a random variable using Monte Carlo simulation. The rail and slab have been modelled using the double Euler-Bernoulli beam theory. The railpads and track bed also consider springs and dashpots. The loading condition has been induced by a moving wagon, which was modelled by a series of lumped mass systems with ten degrees of freedom. Consequently, the effects of random track bed stiffness on slab and rail internal forces and displacements were investigated. Even though rail irregularity assumptions are important factors in vehicle-track interaction, the omission of the effect will yield scientifically reasonable results. However, consideration of this factor will yield much more accurate results. Due to complications involved in analysis, this could be the topic of another research paper. By giving variety to the CoV of track bed stiffness, the CoV of the output, as well as rail and slab displacements and bending moments, were achieved. Some important findings of the research can be summarized as follows:

- The CoV of the rail and slab displacement has a direct dependency on the CoV of the track bed stiffness.
- By increasing vehicle speed, the CoV of rail displacement is decreased.
- By increasing the uncertainty in track bed stiffness, the rail and slab are confronted by more displacement variations.
- Overall, the CoV of slab displacement is 45% more than the CoV of rail displacement.
- By increasing the CoV of track bed stiffness, the CVRB and CVSB are increased.
- The outputs of the system, such as rail and slab displacement and bending moment, have a normal distribution, as well as foundation stiffness distribution.

References

1. Lei, X.Y. and Zhang, B. "Analysis of dynamic behavior for slab track of high-speed railway based on vehicle and track elements", *Journal of Transport Engineering*, **137**(4), pp. 227-240 (2011).

2. Galvin, P., Romero, A. and Dominguez, J. "Vibrations induced by HST passage on ballast and non-ballast tracks", *Journal of Soil Dynamic Earthquake Engineering*, **30**(9), pp. 862-873 (2010).

3. Xiang, J. He, D. and Zeng, Q.Y. "Analysis theory of spatial vibration of high-speed train and slab track system", *Journal of Central South University Technology*, **15**(1), pp. 121-126 (2008).

4. Mazilu, T. "Interaction between a moving two-mass oscillator and an infinite homogeneous structure: Green's functions method", *Journal of Achieve of Applied Mechanic*, **80**(8), pp. 909-927 (2010).

5. Bitzenbauer, J. and Dinkel, J. "Dynamic interaction between a moving vehicle and an infinite structure excited by irregularities-Fourier transforms solution", *Journal of Achieve of Applied Mechanic*, **72**(2-3), pp. 199-211 (2002).

6. Hussein, M.F.M. and Hunt, H.E.M. "Modelling of floating-slab tracks with continuous slabs under oscillating moving loads", *Journal of Sound and Vibrations*, **297**, pp. 37-54 (2006).

7. Zakeri, J.A., Ghorbani, V. "Investigation on dynamic behavior of railway track in transition zone", *Journal of Mechanical Science and Technology*, **2**, pp. 287-292 (2011).

8. Zhai, W., Wang K. and Chai, C. "Fundamentals of vehicle-slab coupled dynamic", *Journal of Vehicle System Dynamic*, **47**, pp. 1349-1376 (2009).

9. Lei, X. and Noda, N.A. "Analyses of dynamic response of vehicle and track coupling system with random irregularity of track vertical profile", *Journal of Sound and Vibration*, **258**, pp. 147-165 (2002).

10. Anderson, L. and Neilsen, S.R.K. "Vibrations of a track caused by variation of the foundation stiffness", *Journal of Probabilistic Engineering Mechanics*, **18**, pp. 171-184 (2003).

11. Berggren, E. "Railway track stiffness dynamic measurements and evaluation for efficient maintenance", Doctoral Thesis, Royal Institute of Technology (KTH), Aeronautical and Vehicle Engineering, Div. of Rail Vehicles (2009).

12. Lutes, L.D. and Sarkani S. *Random Vibrations, Analysis of Structural and Mechanical Systems*, Elsevier Butterworth-Heinemann (2004).

13. Zakeri, J.A., Xia, H. and Fan, J.J. "Dynamic responses of train-track system to single rail irregularity" *Latin American Journal of Solids and Structure*, **6**, pp. 89-104 (2009).

Biographies

Mohammad Mehrali received BS and MS degrees in Railway Engineering from Iran University of Science and Technology (IUST), in 2010 and 2012, respectively. His research interests include dynamic analysis of structural systems, random vibration, and railroad design.

Saeed Mohammadzadeh obtained a BS degree in Civil Engineering from Sharif University of Technology, Tehran, Iran, and MS and PhD degrees from Iran University of Science and Technology, where he is currently Assistant Professor in the School of Railway Engineering. He has conducted numerous research projects in his field and had several papers, as well as six books, published in his areas of interest. His research interests include design and analysis of railway tracks and structures, optimization and reliability analysis, behavior and deterioration analysis, and maintenance and repair management.

Morteza Esmaeili obtained a BS degree in Civil Engineering from Iran University of Science and Technology (IUST), and MS and PhD degrees in Geotechnical Engineering from Tehran University (UT), Iran. He is currently Associate Professor in the School of Railway Engineering at Iran University of Science and Technology (IUST). He has published numerous papers in his fields of interest, including five books. His research interests include track geotechnology, mechanics of porous media, train induced vibration and design of ballast-less tracks.

Mehrdad Nouri received BS and MS degrees in Railway Engineering from Iran University of Science and Technology (IUST), in 2009 and 2011, respectively. He is currently PhD Student in the School of Railway Engineering at Iran University of Science and Technology. His research interests include dynamic analysis of structural systems, optimization methods (mathematical and heuristic methods), reliability of structural systems, neural networks, signal processing, image processing and computer programming.