Scientia Iranica A (2013) 20(6), 1685–1694

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions A: Civil Engineering
www.scientiairanica.com

Research Note

An analytical approach to reliability assessment of the shear wave velocity relationship

A. Johari[a,*], A.A. Javadi[b], M. Elmi[a] and S. Raei[a]

a. *Department of Civil and Environmental Engineering, Shiraz University of Technology, Shiraz, Iran.*
b. *Computational Geomechanics Group, College of Engineering, Mathematics and Physical Sciences, University of Exeter, Exeter, EX4 4QF, UK.*

Received 4 July 2012; received in revised form 3 February 2013; accepted 25 May 2013

KEYWORDS
Reliability;
Jointly distributed random variables method;
Monte Carlo simulation;
Point estimated method;
First order second moment method;
Shear wave velocity.

Abstract. During an earthquake, seismic waves propagate vibrations that carry energy from the source of the shaking outwards. Seismic waves can be distinguished by the velocity and shape of propagation. The velocity of waves depends on the elastic properties and density of the soil layers through which the waves pass. Probabilistic analysis of earthquake waves can be used as an effective tool to evaluate inherent uncertainty in the soil properties and the resulting uncertainty in site classification. In this research, the jointly distributed random variables method is used for probabilistic analysis and reliability assessment of the shear wave velocity relationship. The selected stochastic parameters are density, elastic modulus and Poisson's ratio which are modeled using truncated normal probability distribution functions. The results are compared with the Monte Carlo simulation, point estimated method and first order second moment method. Comparison of the results indicates very good performance of the proposed approach for assessment of reliability. It is shown that this method can correctly predict the influence of stochastic input parameters and capture the expected probability distribution of shear wave velocity correctly. It is also shown that the modulus of elasticity is the most effective parameter in shear wave velocity.

1. Introduction

The importance of shear wave velocity has been widely acknowledged in many fields. It has been generally recognized that shear wave velocity is a basic soil property. It has been used to obtain several soil parameters [1-5] and to study many aspects of soil behavior such as liquefaction [6-13], amplification of ground shaking [14], peak ground acceleration [15] and site classification [16,17]. There are two methods available for determining shear wave velocity for a particular soil. It may be determined directly in-situ by geophysical tests or indirectly based on soil parameters. In the direct method, the seismic wave propagates through the soil and the velocity of the wave is determined. In the indirect method, wave velocity can be obtained as a function of modulus of elasticity, Poisson's ratio and the bulk density of soil. In this method, the results are sensitive to changes in density, temperature, composition, water level, and volatile content of soil layers [18]. The inherent uncertainties in the characteristics which affect shear wave velocity dictate that determination of shear wave velocity by this method is of a probabilistic nature rather than being deterministic. Recent research on probabilistic analysis of shear wave velocity is documented in the literature [19-24].

In general, uncertainty is divided into three dis-

*. *corresponding author.*
Email-addresses:johari@sutech.ac.ir (A. Johari), a.a.javadi@ex.ac.uk (A.A. Javadi).

tinctive categories: uncertainty in soil parameters, model uncertainty and human uncertainty [25]. Parameter uncertainty is the uncertainty in the input parameters for analysis, model uncertainty is due to limitation of the theories and models used in the performance prediction [26], while human uncertainty is due to human error and mistakes [27]. Many probabilistic methods may be used for stochastic analysis of shear wave velocity. These methods can be grouped into three categories: analytical methods [28-30], approximate methods [31-35], Monte Carlo simulation [36] and artificial intelligence methods [37]. To the author's knowledge, there has been no analytical solution in the literature for the reliability assessment of shear wave velocity. In this research, the jointly distributed random variables method is used as an effective analytical method to assess the reliability of shear wave velocity considering uncertainty in the values of the parameters.

2. Shear wave velocity

The shear wave velocity is determined by Eq. (1) [5]. It can be seen that the shear wave velocity depends on three parameters.

$$V_s = \sqrt{\frac{E}{2\rho(1+\nu)}}, \tag{1}$$

where:

V_s : Shear wave velocity;
E : Modulus of elasticity;
ν : Poisson's ratio;
ρ : Bulk density of soil.

3. Stochastic parameters

To account for uncertainties in determination of shear wave velocity, three input parameters have been considered as stochastic variables. The selected parameters are modulus of elasticity (E), Poisson's ratio (ν), and bulk density (ρ). These stochastic parameters are modeled using truncated normal probability distribution functions (pdf). The distribution functions of the above mentioned stochastic parameters are as follows:

$$F_E(E) = \frac{1}{\sigma_E\sqrt{2\pi}} \exp\left(-0.5\left(\frac{E - E_{\text{mean}}}{\sigma_E}^2\right)\right)$$
$$E_{\min} \leq E \leq +\infty, \tag{2}$$

$$F_\nu(\nu) = \frac{1}{\sigma_\nu\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\nu - \nu_{\text{mean}}}{\sigma_\nu}\right)^2\right)$$
$$\nu_{\min} \leq \nu \leq +\infty, \tag{3}$$

$$F_\rho(\rho) = \frac{1}{\sigma_\rho\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\rho - \rho_{\text{mean}}}{\sigma_\rho}\right)^2\right)$$
$$\rho_{\min} \leq \rho \leq +\infty, \tag{4}$$

where:

$$\left\{ \begin{array}{c} E_{\min} = E_{\text{mean}} - 3\sigma_E \\ \nu_{\min} = \upsilon_{\text{mean}} - 3\sigma_\nu \\ \rho_{\min} = \rho_{\text{mean}} - 3\rho_\rho \end{array} \right\} \tag{5}$$

σ_E : Standard deviation of soil elastic modulus;
E_{mean} : Average value of soil elastic modulus;
$E_{\min}$ Minimum value of soil elastic modulus;
σ_ν : Standard deviation of soil Poisson's ratio;
ν_{mean} : Average value of soil Poisson's ratio;
$\nu_{\min}$: Minimum value of soil Poisson's ratio;
σ_p Standard deviation of soil bulk density;
ρ_{mean} : Average value of soil bulk density;
$\rho_{\min}$: Minimum value of soil bulk density.

By considering the stochastic variables within the range of their mean, plus or minus 3 times standard deviation (Eq. (5)), 99.8% of the area beneath the normal density curve is covered [37]. Thus, area correction will not be necessary. It should be noted that for choosing the initial data, the following conditions must be observed for modulus of elasticity (E), Poisson's ratio (ν), and bulk density (ρ):

$$\begin{cases} E_{\text{mean}} - 3\sigma_E > 0 \\ \nu_{\text{mean}} - 3\sigma_\upsilon > 0 \\ \rho_{\text{mean}} - 3\sigma_\rho > 0 \end{cases} \tag{6}$$

4. Jointly distributed random variables method

The Jointly Distributed Random Variables (JDRV) method is an analytical probabilistic method. In this method, density functions of input variables are expressed mathematically and jointed together by statistical relations. The jointly distributed random variables method has a number of advantages over other methods:

(i) It is a nearly exact method and can be used for stochastic parameters with any distribution curve (such as normal, exponential, gamma, uniform...), whereas some methods, like point estimate and first order second moment, require specific (e.g., normal) distribution functions.

(ii) The computational time of this method is significantly less than the Monte Carlo simulation, which requires a significant number of simulation runs.

The available statistical and probabilistic relations between parameters are given in this section [27-29].

If X is a random variable with the probability density of $f_X(x)$ and Y is a function of X in the form $Y = g(x)$, the probability density of Y can be determined as:

$$f_Y(y) = f_X(g^{-1}(y)) \times \left|\frac{dg^{-1}(y)}{dy}\right|. \tag{7}$$

If X and Y are two independent random variables with the probability densities $f_X(x)$ and $f_Y(y)$, and $Z = X + Y$, the probability density of Z will be:

$$f_{X+Y}(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y(z-x)dx, \quad -\infty < z < +\infty. \tag{8}$$

If X and Y are two independent random variables with the probability densities $f_X(x)$ and $f_Y(y)$, and $Z = Y/X$, the probability density of Z will be calculated as:

$$f_{Y/X}(z) = \int_{-\infty}^{+\infty} |X|\, f_X(x) f_Y(x.z)dx, \quad -\infty < z < +\infty. \tag{9}$$

This method has recently been used in a number of geotechnical applications [38-40].

5. Probabilistic assessment of shear wave velocity

In this research, the terms of the shear wave velocity equation (Eq. (1)) are grouped together in the following form (Eq. (10)) and the probability distribution equation of each group is derived separately using Eqs. (11) to (15). Derivations of these equations are given below:

$$\begin{cases} k_1 = \sqrt{2+2\nu} \\ k_2 = \frac{1}{\sqrt{\rho}} \\ k_3 = \frac{k_1}{k_2} \\ k_4 = \sqrt{E} \\ k_5 = \frac{k_4}{k_3} \end{cases} \tag{10}$$

On the other hand:

$$f_{K_1}(k_1) = \frac{1}{\sigma_\nu\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\frac{k_1^2-2}{2} - \nu_{\text{mean}}}{\sigma_\nu}\right)^2\right) \times k_1,$$

$$\sqrt{2+2\nu_{\min}} \leq k_1 \leq \sqrt{2+2\nu_{\max}}, \tag{11}$$

$$f_{K_2}(k_2) = \frac{1}{\sigma_\rho\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\frac{1}{k_2^2} - \rho_{\text{mean}}}{\sigma_\rho}\right)^2\right) \times \left|\frac{2}{k_2^3}\right|,$$

$$\frac{1}{\sqrt{\rho_{\max}}} \leq k_2 \leq \frac{1}{\sqrt{\rho_{\min}}}, \tag{12}$$

$$f_{K_3}(k_3) = \frac{b_{k_3} \times c_{k_3}}{a_{k_3}^3} \times \frac{1}{\sqrt{2\pi}\sigma_{k_1}\sigma_{k_2}}\left(2\Phi\left(\frac{b_{k_3}}{a_{k_3}}\right) - 1\right) + \frac{1}{a_{k_3}^2 \pi\sigma_{k_1}\sigma_{k_2}} \exp\left(\frac{-1}{2}\left(\frac{k_{1_{\text{mean}}}^2}{\sigma_{k_1}^2} + \frac{k_{2_{\text{mean}}}^2}{\sigma_{k_2}^2}\right)\right),$$

$$\frac{k_{1_{\min}}}{k_{2_{\max}}} \leq k_3 \leq \frac{k_{1_{\max}}}{k_{2_{\min}}} \rightarrow \sqrt{\rho_{\min}} \times \sqrt{2+2\upsilon_{\min}} \leq k_3 \leq \sqrt{\rho_{\max}} \times \sqrt{2+2\upsilon_{\max}}, \tag{13}$$

where:

$$a_{k_3} = \sqrt{\frac{1}{\sigma_{k_1}^2}k_3^2 + \frac{1}{\sigma_{k_2}^2}},$$

$$b_{k_3} = \frac{k_{1_{\text{mean}}}}{\sigma_{k_1}^2} + \frac{k_{2_{\text{mean}}}}{\sigma_{k_2}^2},$$

$$c_{k_3} = \exp\left(\frac{1}{2}\frac{b_{k_3}^2}{a_{k_3}^2} - \frac{1}{2}\left(\frac{k_{2_{\text{mean}}}^2}{\sigma_{k_2}^2} + \frac{k_{1_{\text{mean}}}^2}{\sigma_{k_1}^2}\right)\right),$$

$$\Phi_{k_3} = \int_{-\infty}^{k_3} \frac{1}{\sqrt{2\pi}} \exp\left(-0.5u^2\right) du,$$

$$f_{K_4}(k_4) = \frac{1}{\sigma_E\sqrt{2\pi}} \exp\left(-0.5\left(\frac{k_4^2 - E_{\text{mean}}}{\sigma_E}\right)^2\right) \times 2k_4,$$

$$\sqrt{E_{\min}} \leq k_4 \leq \sqrt{E_{\max}}, \tag{14}$$

$$f_{K_5}(k_5) = \frac{b_{k_5} \times c_{k_5}}{a_{k_5}^3} \frac{1}{\sqrt{2\pi}\sigma_{k_4}\sigma_{k_3}}\left(2\Phi\left(\frac{b_{k_5}}{a_{k_5}}\right) - 1\right) + \frac{1}{a_{k_5}^2 \pi\sigma_{k_4}\sigma_{k_3}} \exp\left(\frac{-1}{2}\left(\frac{k_{4_{\text{mean}}}^2}{\sigma_{k_4}^2} + \frac{k_{3_{\text{mean}}}^2}{\sigma_{k_3}^2}\right)\right),$$

$$\frac{k_{4_{\min}}}{k_{3_{\max}}} \leq k_5 \leq \frac{k_{4_{\max}}}{k_{3_{\min}}} \rightarrow \frac{\sqrt{\rho_{\max}}\sqrt{E_{\min}}}{\sqrt{2+2\upsilon_{\max}}} \leq k_5 \leq \frac{\sqrt{\rho_{\min}}\sqrt{E_{\max}}}{\sqrt{2+2\upsilon_{\min}}}, \tag{15}$$

where:

$$a_{k_5}=\sqrt{\frac{1}{\sigma_{k_4}^2}k_5^2+\frac{1}{\sigma_{k_3}^2}},$$

$$b_{k_5}=\frac{k_{4_{\text{mean}}}}{\sigma_{k_4}^2}+\frac{k_{3_{\text{mean}}}}{\sigma_{k_3}^2},$$

$$c_{k_5}=\exp\left(\frac{1}{2}\frac{b_{k_5}^2}{a_{k_5}^2}-\frac{1}{2}\left(\frac{k_{4_{\text{mean}}}^2}{\sigma_{k_4}^2}+\frac{k_{3_{\text{mean}}}^2}{\sigma_{k_3}^2}\right)\right),$$

$$\Phi_{k_5}=\int_{-\infty}^{k_5}\frac{1}{\sqrt{2\pi}}\exp\left(-0.5u^2\right)du.$$

Using the above mathematical functions for k_1 to k_5 and $f_{K_1}(k_1)$ to $f_{K_5}(k_5)$, a computer program was developed (coded in MATLAB) to determine the probability density distribution curve for the shear wave velocity. In addition, for comparison, determination of shear wave velocity using the Monte Carlo simulation was also coded in the same computer program. To illustrate the capabilities of this method, an example with arbitrary data is given in the following sections.

6. Monte Carlo simulation

Monte Carlo simulation (MCs) can solve problems by generating suitable random numbers (or pseudo-random numbers) and assessing the dependent variables for a large number of possibilities. MCs involves the definition of the variables that generate uncertainty and probabilistic distribution function (pdf), determination of the value of the function using variable values randomly obtained considering the pdf, and repeating this procedure until a sufficient number of outputs are obtained to build the pdf of the function [36].

The statistics of the resulting set of values of the function can be computed and β (reliability index) or P_f (probability of failure) calculated directly. The method has the advantage of conceptual simplicity, but it can require a large set of values of the performance function to obtain adequate accuracy. Furthermore, the method does not give insight into the relative contributions of the uncertain parameters that are obtained from other methods. The computational effort can be reduced by using statistical techniques known as variance reduction schemes, and these should be employed whenever possible [41]. Generally, the number of MCs is determined through trial-and-error or based on the target failure probability [34,35].

7. Point estimate method

Rosenblueth [31] proposed the Point Estimate (PE) method, which uses a series of point estimates (point by point evaluations) of the response function at selected values (known as weighting points) of the input random variables to compute the moments of the response variable. This method applies appropriate weights to each of the point estimates of the response variable to compute moments.

In the PE method, all possible combinations are taken into account for two point estimates for each independent variable. One of the disadvantages of the original method is that it requires the performance function to be evaluated 2^n times, and this can become a very large number when the number of uncertain parameters is large. Recent modifications reduce the number of evaluations to the order of $2N$ but introduce their own complications [41]. If the probability density functions are symmetric (e.g., normal distribution), the estimated points are separated one standard deviation below and above the average. The mean and variance are given by the following equations:

$$E_{\text{Function}}\cong\frac{1}{2^n}\sum_{j=1}^{2n}g(x_j),\tag{16}$$

$$\sigma_{\text{Function}}^2=\frac{1}{2^n}(\sum_{j=1}^{2n}g(x_j))^2-E^2,\tag{17}$$

where:

$X_j =$ Input variables;

$g(x_j) =$ The function of input variables x_j;

$n =$ Number of variables;

$E_{\text{Function}} =$ Mean;

$\sigma^2_{\text{Function}} =$ Variance.

8. First order second moment method

The First Order Second Moment (FOSM) method [32,33] is an approximate approach based on the Taylor series expansion of the function to be evaluated. This expansion is truncated after the linear term (hence “first order”). The modified expansion is then used, along with the first two moments of the random variables, to determine the values of the first two moments of the dependent variable (hence “second moment”). If the number of uncertain variables is N, this method requires either evaluating n partial derivatives of the performance function or performing a numerical approximation using evaluations at $2n+1$ points. This method takes no account of the form of the probability density function, describing the random variables using only their mean and standard deviation. One of the great advantages of the FOSM method is that it reveals the relative contribution of each variable to the overall uncertainty in a clear and easily tabulated manner [41]. For uncorrelated input variables, the mean and variance of the function are given by the following equations:

$$E_{\text{Function}} \cong \sum_{j=1}^{n} g(E(x_i)), \tag{18}$$

$$\sigma^2_{\text{Function}} \cong \sum_{j=1}^{n} \left(\frac{\partial g}{\partial x_i}\right)^2 .\sigma^2(x_i). \tag{19}$$

9. Example

To demonstrate the efficiency and accuracy of the proposed approach in determining the probability density distribution curve of the shear wave velocity, an illustrative example is presented. For this purpose, the mean and variance of the stochastic parameters are selected as: E_{mean}=25000 kN/m^2, σ_E=5000 kN/m^2 for modulus of elasticity, ν_{mean}=0.25, σ_ν=0.05 for Poisson's ratio and ρ_{mean}= 1.8 Ton/m^3, σ_ρ= 0.1 Ton/m^3 for the bulk density of the soil. The probability density functions of the parameters are shown in Figures 1 to 3. In order to compare the results of the presented method with MCs, PE and FOSM methods, the final probability density distribution curves for the shear wave velocity are determined using the same data and the four methods.

For this purpose, 5,000,000 generation points are used for the MCs. The adequacy of simulations was determined though trial-and-error. Table 1 shows the results. It can be seen that by increasing the number of simulations, the difference between the mean of V_s decreases.

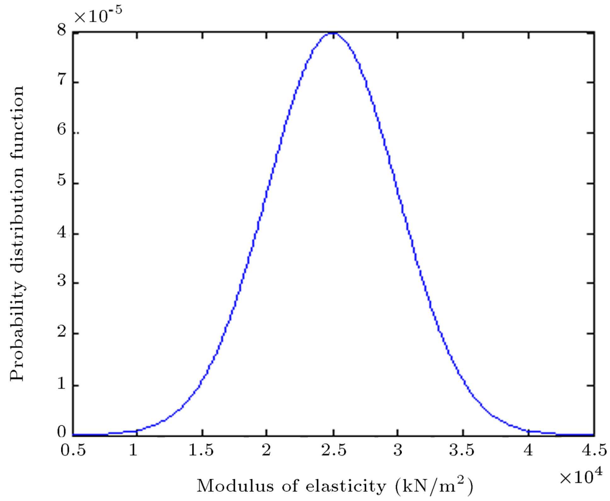

Figure 1. Probability density functions of the modulus of elasticity.

Table 1. The adequacy of Monte Carlo simulations.

Number of simulation	1,000,000	2,000,000	5,000,000
Mean of V_s	74.396	74.431	74.442

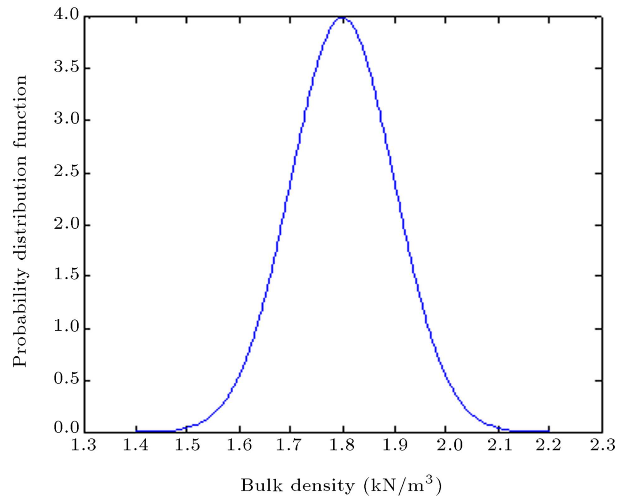

Figure 2. Probability density functions of bulk density.

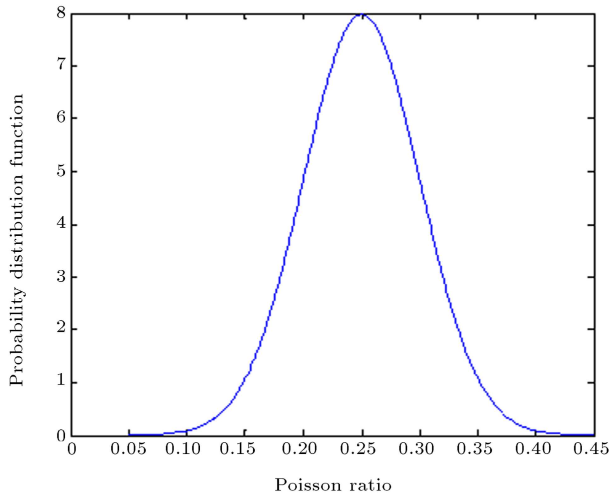

Figure 3. Probability distribution of Poisson's ratio.

In the PEM and FOSM methods, 8 points are selected. Tables 2 and 3 show calculations for determining the mean and variance of the shear wave velocity by PEM and FOSM methods, respectively, as defined in Eqs. (16) to (19).

Figure 4 shows the probability distribution functions of these methods. As seen in this figure, the probability of shear wave velocity, obtained using the developed method, has normal distribution and is very close to other methods' probability distributions. Figure 5 shows the cumulative density function of shear wave velocity obtained using the jointly distributed random variables method. The cumulative probability of shear wave velocity at an arbitrary value of shear wave velocity, V_s = 80 m/sec, is shown in this figure. It can be seen that with the selected data, the probability of shear wave velocity (V_s=80 m/sec) is about 75%. Also, with a probability of 100%, the shear wave

Table 2. Results from point estimate method for shear wave velocity (m/sec).

Point	E (kN/m^2)	ρ (Ton/m^3)	ν	V_s (m/sec)	w	$w.V_s$	$w.V_s^2$
1	30000	1.9	0.3	779.29	0.125	9.74	759.11
2	20000	1.9	0.3	636.28	0.125	7.95	506.07
3	30000	1.7	0.3	823.85	0.125	10.30	848.42
4	30000	1.9	0.2	811.11	0.125	10.14	822.37
5	20000	1.7	0.3	672.67	0.125	8.41	565.61
6	30000	1.7	0.2	857.49	0.125	10.72	919.12
7	20000	1.9	0.2	662.27	0.125	8.28	548.25
8	20000	1.7	0.2	700.14	0.125	8.75	612.75
		Sum				Mean of V_s = 74.29	5581.69
						Variance of V_s = 5581.69-(74.29)^2 = 62.68	

Table 3. Results from first order second moment method for shear wave velocity (m/sec).

Parameters x_i	Mean (μ_{x_i})	Standard deviation (σ^2)	$\left.\frac{\partial V_s}{\partial x_i}\right\vert_{\mu x_i}$	$\left(\frac{\partial V_s}{\partial x_i}\right)^2 \times \sigma^2_{x_i}$
E (kN/m^2)	25000	5000	1.49e-5	55.56
ρ (kN/m^3)	1.8	0.1	20.71	4.29
v	0.25	0.05	-29.81	2.22
Mean of $V_s = V_s(\mu_E, \mu_\rho, \mu_v)$ = 74.75				Sum=variance of V_s = 62.07

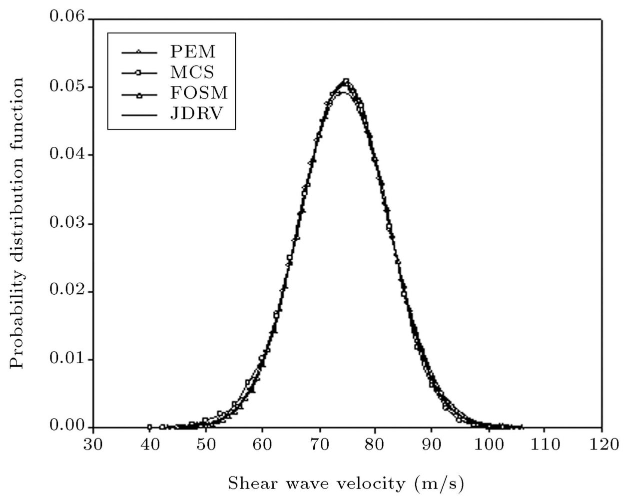

Figure 4. Shear wave velocity probability distributions function predicted by the four methods.

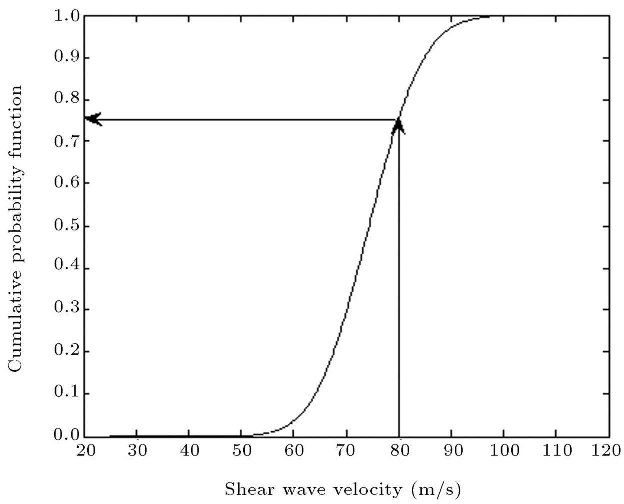

Figure 5. Cumulative probability function of shear wave velocity using jointly distributed random variables.

velocity of this site is less than 100 m/sec, hence, this site can be classified as grade "E" based on NEHRP and UBC codes. For comparison, the mean for MCs, PE and FOSM methods are given in Table 4. The results show that the jointly distributed random variables method has a mean and variance close to the other methods and is the closest method to Monte Carlo simulation.

Table 5 shows a beneath area comparative evaluation of the developed probability density function of shear wave velocity (V_s) using the JDRV method based on different variance. However, in this research, mean +/- 3*sigma is used for variance [37].

10. Sensitivity analysis

To evaluate the model response to changes in input parameters, a sensitivity analysis was carried out. For

Table 4. Methods comparing.

Method	Mean
Jointly distributed random variables	74.51
Monte Carlo	74.44
Point estimated	74.29
First order second moment	74.54

Table 5. Area beneath comparing of developed V_s pdf using JDRV method.

Variance	Area beneath of developed pdf using JDRV method
mean +/- 1*sigma	99.0866
mean +/- 2*sigma	99.8740
mean +/- 3*sigma	99.99882283
mean +/- 4*sigma	99.99999949

this purpose, the three stochastic input parameters were increased based on their standard deviation (new mean = old mean + 1.0×Std.). To evaluate the influence of changes in mean elastic modulus, this parameter was increased, while the ranges of the other stochastic input parameters were kept constant. The results are shown in Figure 6. It is shown, as expected, that with an increase in mean elastic modulus, the Cumulative Distribution Function (CDF) of the shear wave velocity shifts rightwards, indicating that for a given probability, the magnitude of shear wave velocity is increased by increasing the elastic modulus. Furthermore, this figure shows that with an increase in bulk density or Poisson's ratio, the CDF of the shear wave velocity shifts leftwards. Also, this figure shows the modulus of elasticity is the most effective parameter in shear wave velocity.

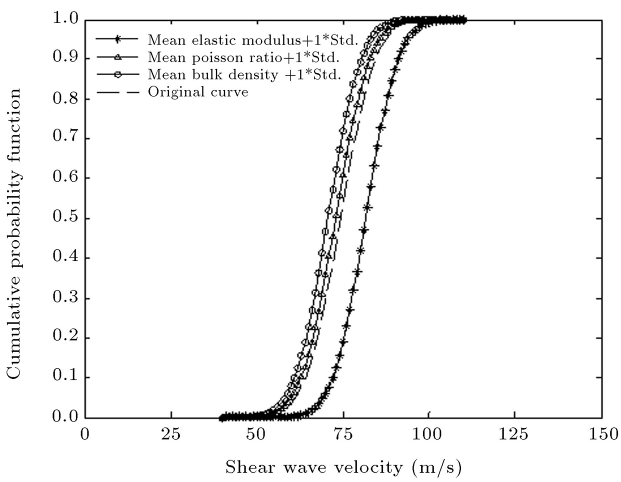

Figure 6. Sensitivity analysis to determine the most effective parameter based on change in mean of input parameters.

Additionally, a sensitivity analysis was carried out based on change in variance of input parameters [34]. For this purpose, variances of the three stochastic input parameters (σ) were changed based on initial variances (σ_i). To evaluate the influence of changes in variance of each parameter, the selected parameter was changed, while the range of variances of the other stochastic input parameters was kept constant. The results are shown in Figure 7. This figure shows the modulus of elasticity has a larger slope and, therefore, is the most effective parameter in shear wave velocity. This is consistent with the results obtained from another form of sensitive analysis assessment (Figure 6).

11. Parametric analysis

For further verification of the proposed method, a parametric analysis was performed. The main goal was to find the effect of each parameter on the probability distributions function of shear wave velocity. Figures 8

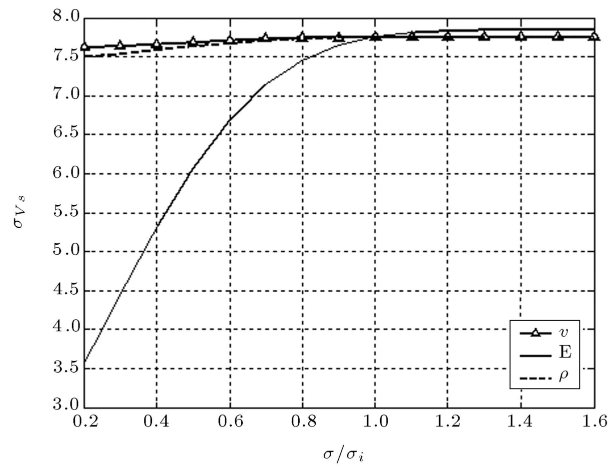

Figure 7. Sensitivity analysis to determine the most effective parameter based on change in variance of input parameters.

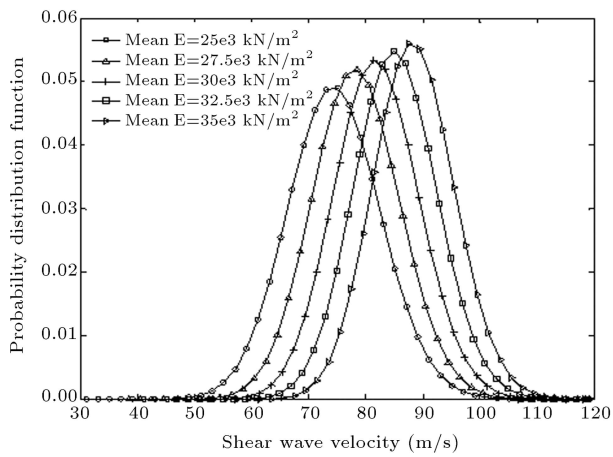

Figure 8. Parametric analysis of output model with respect to soil elastic modulus.

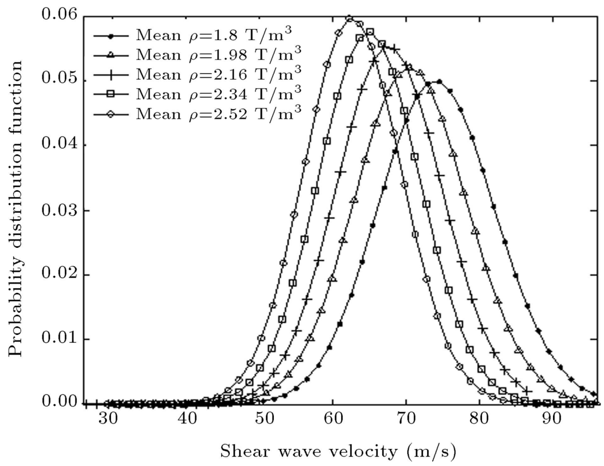

Figure 9. Parametric analysis of output model with respect to soil bulk density.

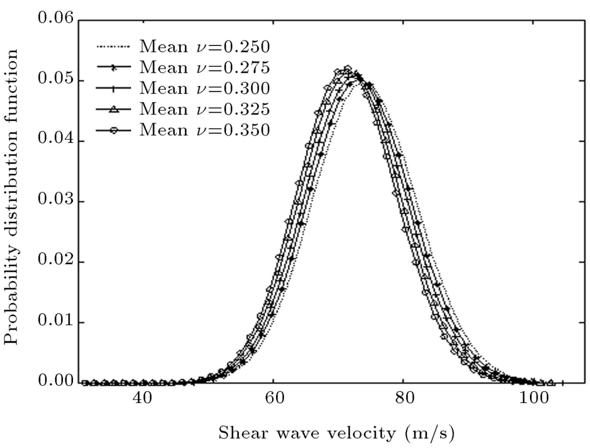

Figure 10. Parametric analysis of output model with respect to soil Poisson's ratio.

to 10 present the predicted values of the probability distributions function of shear wave velocity as a function of each parameter, where others were kept constant.

Figure 8 shows that, as expected, the mean of probability distributions function of shear wave velocity continuously increases with an increase in soil elastic modulus. Furthermore, this figure indicates that the height of the probability density function increases with an increase in soil elastic modulus. As a result, the variance decreases because the area underneath the normal density curve should be 1.

Figure 9 shows the parametric analysis of the model with respect to soil bulk density. It can be seen that the mean of probability distributions function of shear wave velocity continuously decreases with an increase in soil bulk density. Furthermore, this figure indicates that the height of the probability density function increases with an increase in soil bulk density. As a result, the variance decreases, as the area underneath the normal density curve should be 1.

Finally, parametric analysis of the model, with respect to the Poisson's ratio of soil, is given in Figure 10. It can be seen that by changing the Poisson's ratio of the soil, the change in the trend of variations of mean of probability distributions function of shear wave velocity is the same as that of shear wave velocity when the bulk density changes. It continuously decreases (shifts leftward) with an increase in the Poisson's ratio of the soil, but the effect of this parameter is less than that of bulk density on shear wave velocity.

12. Conclusion

Shear wave is an important wave that propagates from an earthquake. Determination of this parameter for a site can be used for many purposes such as site classification. The shear wave velocity can be obtained as a function of in situ soil parameters. The determination of shear wave velocity using this approach is a probabilistic problem due to the inherent uncertainties in the geotechnical parameters, model performance, and human uncertainty. In this paper, an analytical method; Jointly Distributed Random Variables (JDRV) method, was used to assess the reliability of this type of wave, based on the uncertainty in geotechnical properties. The selected stochastic parameters were bulk density, elastic modulus and the Poisson's ratio of soil, which were modeled using truncated normal probability distribution functions. The results showed that the probability distribution of shear wave velocity has a near normal distribution and compares with the output of other methods of analysis such as Monte Carlo simulation, point estimated and first order second moment methods. In addition, a sensitivity analysis of the selected method indicated that this method correctly predicted the influence of stochastic input parameters. The results also indicated that the jointly distributed random variables method was able to capture the expected probability distribution of shear wave velocity correctly. The sensitivity analysis also showed that the modulus of elasticity is the most effective parameter in shear wave velocity.

References

1. Foti, S. and Lancellotta, R. "Soil porosities from seismic velocities", *Geotechnique*, **51**, pp. 551-554 (2004).

2. Horikawa, H., Mizuno, K., Satake, K., Sekiguchi, H., Kase, Y., Sugiyama, Y., Yokota, H., Suehiro, M. and Pitarka, A. "Three-dimensional subsurface structure model beneath the Osaka Plain", *Annual Report on Active Fault and Pale Earthquake Researches*, **2**, pp. 291-324 (2002).

3. SW-AJA "Soil behavior under earthquake loading conditions, state of the art evaluation of soil characteristics for seismic response analyses", Prepared Under

Subcontract No. 3354, Union Carbide Corp., for U.S. Atomic Energy Commission, Contract No. W-7405-eng (1972).

4. Prakash, S. *Soil Dynamics*, McGraw Hill Book Co., New York, NY, Reprint, 1991, S.P. Foundation, Rolla, MO (1981).

5. Kramer S.L., *Geotechnical Earthquake Engineering*, Prentice Hall (1996).

6. Andrus, R.D. and Stokoe, K.H. II "Liquefaction resistance of soils from shear-wave velocity", *J Geot. Geoenv. Eng., ASCE,* **126**(11), pp. 1015-1025 (2000).

7. Andrus, R.D., Piratheepan, P., Ellis, B.S., Zhang, J. and Juang, H.C. "Comparing liquefaction evaluation methods using penetration vs relationship", *J. Soil Dyn. Earthq. Eng.*, **24**, pp. 713-721 (2004).

8. Baxter, C.D.P., Bradshaw, A.S., Green, R.A. and Wang, J. "Correlation between cyclic resistance and shear wave velocity for providence silts", *J. Geot. Geoenv. Eng.*, **134**(1), pp. 37-46 (2008).

9. Zhou, Y.G. and Chen, Y.M. "Laboratory investigation on assessing liquefaction resistance of sandy soils by shear wave velocity", *J. Geot. Geoenv. Eng.*, **133**(8), pp. 959-927 (2007).

10. Ning Liu, S.M., Mitchell, J.K. and Hon, M. "Influence of nonplastic fines on shear wave velocity-based assessment of liquefaction", *J. Geot. Geoenv. Eng.*, **132**(8), pp. 1091-1097 (2006).

11. Chen, Y.M., Ke, H. and Chen, R.P. "Correlation of shear wave velocity with liquefaction resistance based on laboratory tests", *J. Soil Dyn. Earthq. Eng.*, **25**(6), pp. 461-469 (2005).

12. Rouch, A.F., Duffy, M. and Stokoe, K. "Laboratory correlation of liquefaction resistance with shear wave velocity", *J. Geot. Geoenv. Eng., Geotechnical Special Publication*, **101**, pp. 66-80 (2000).

13. Tokimatsu, K. and Uchida, A. "Correlation between liquefaction resistance and shear wave velocity", *Soils Found., Japanese Soci. of Soil Mech. Found. Eng.*, **30**(2), pp. 33-42 (1990).

14. Tamura, I., Yamazaki, F. and Shabestrai, K. "Estimation of the site amplification ratio from the average swave velocity using K-NET data", *CD-ROM Proceedings of 55th JSCE Annual Meeting*, I-B357, Japan (2000).

15. Suetomi, I., Ishida, E., Isoyama, R. and Goto, Y. "Amplification factor of peak ground motion using average shear wave velocity of shallow soil deposits", *13th World Conference on Earthquake Engineering*, Canada (2004).

16. *International Conference of Building Officials (ICBO)*, "Uniform building code (UBC)", Whittier, Calif (1997).

17. NEHRP "Recommended provisions for seismic regulations for new buildings", Federal Emergency Management Agency, Washington, D.C. (1997).

18. Goes, S., Govers, R. and Vacher, P. "Shallow mantle temperatures under Europe from P and S wave tomography", *J. Geoph. Res.*, **105**(11), pp. 153-169 (2000).

19. Toprak, S., Holzer, T.L., Bennett, M.J. and Tinsley, J.C. "CPT- and SPT-based probabilistic assessment of liquefaction", *Proc. 7th U.S.-Japan Workshop on Earthquake Resistance Des. of Lifeline Fac. and Counter Measures Against Liquefaction*, Multidisciplinary Center for Earthquake Engineering Research, Buffalo (1999).

20. Juang, C.H., Chen, C.J. and Jiang, T. "Probabilistic framework for liquefaction potential by shear wave velocity", *J. Geot. Geoenv. Eng., ASCE*, **127**(8), pp. 670-678 (2001).

21. Kayen, R., Seed, R.B., Moss, R.E., Cetin K.O., Tanaka, Y. and Tokimatsu, K. "Global shear wave velocity database for probabilistic assessment of the initiation of Seismic-Soil Liquefaction", *11th Int. Conference on Soil Dyn. and Earthq. Eng.*, Berkeley (2004).

22. Juang, H., Yang, S.H. and Yuan, H. "Model uncertainty of shear wave velocity-based method for liquefaction potential evaluation", *J. Geot. Geoenv. Eng., ASCE*, **131**(10), pp. 1274-1282 (2005).

23. Dabiri, R., Askari, F., Shafiee, A. and Jafari, M.K. "Shear wave velocity-based liquefaction resistance of sand-silt mixtures: Deterministic versus probabilistic approach", *IJST, Transactions of Civil Engineering*, **35**(C2), pp. 199-215 (2011).

24. Comina, C., Foti, S., Boiero, D. and Socco, L.V. "Reliability of VS,30 evaluation from surface-wave tests", *J. Geot. Geoenv. Eng., ASCE*, **137**(6), pp. 579-586 (2011).

25. Morgenstern, N.R. "Managing risk in geotechncial engineering", *Proceeding of the 10th Pan American Conf. on Soil Mech. Found. Eng.*, **4**, pp. 102-126 (1995).

26. Whitman, V.W. "Organizing and evaluating uncertainty in geotechnical engineering, in uncertainty in geologic environment: From theory to practice", *Proceedings of Uncertainty '96, Geotechnical Special Publication No 58, ASCE*, **1**, pp. 1-38 (1996).

27. Sowers, G.S. "The human factor in failure civil engineering", *ASCE*, pp. 72-73 (1991).

28. Hoel, P.G., Port, S.C. and Stone, C.J., *Introduction to Probability Theory*, Houghton Mifflin, Boston, MA, (1971).

29. Stirzaker, D., *Probability and Random Variables: A Beginner's Guide*, Cambridge University Press (1997).

30. Tijms, H., *Understanding Probability*, 2nd Ed. Cambridge University Press (2007).

31. Rosenblueth, E. "Point estimates for probability moments", *Proceedings of the National Academy of Science*, **72**(10), pp. 3812-3814 (1975).

32. Ang, A.H.S. and Tang, W., *Probability Concepts in Engineering Planning and Design, Basic Principles*,

Volume I, John Wiley and Sons, New York, USA (1984).

33. Dettinger, M.D. and Wilson, J.L. "First order analysis of uncertainty in numerical models of groundwater flow", 1, *Mathematical Development, Water Resour. Res.*, **17**(1), pp. 149-161 (1981).

34. Luo, Z., Atamturktur, S., Cai, Y. and Juang, C.H. "Simplified approach for reliability-based design against basal-heave failure in braced excavations considering spatial effect", *J. Geot. Geoenv. Eng.*, **138**(4), pp. 441-450 (2012a).

35. Luo, Z., Atamturktur, S., Cai, Y. and Juang, C.H. "Reliability analysis of basal-heave in a braced excavation in a 2-D random field", *Comput. Geotech.*, **39**, pp. 27-37 (2012b).

36. Metropolis, N. and Ulam, S. "The monte carlo method", *Journal of the American Statistical Association*, **44**, pp. 335-341 (1949).

37. Luo, Z., Atamturktur, S., Juang, C.H., Huang, H. and Lin, P.S. "Probability of serviceability failure in a braced excavation in a spatially random field: Fuzzy finite element approach", *Comput. Geotech.*, **38**(8), pp.1031-1040 (2011).

38. Johari, A. and Javadi, A.A. "Reliability assessment of infinite slope stability using the jointly distributed random variables method", *Scientia Iranica,* A, **19**(3), pp. 423-429 (2012).

39. Johari, A., Javadi, A.A., Makiabadi, M.H. and Khodaparast, A.R. "Reliability assessment of liquefaction potential using the jointly distributed random variables method", *J. Soil Dyn. .Earthq. Eng.*, **38**, pp. 81-87 (2012).

40. Johari, A., Fazeli, A. and Javadi, A.A. "An investigation into application of jointly distributed random variables method in reliability assessment of rock slope stability", *Comput. Geotech.*, **47**, pp. 42-47 (2013).

41. Baecher, G.B. and Christian, J.T., *Reliability and Statistics in Geotechnical Engineering*, John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex PO19 8SQ, England (2003).

Biographies

Ali Johari obtained his BS, MS and PhD degrees in 1995, 1999 and 2006, respectively, from Shiraz University, Iran, where he is currently Assistant Professor in the Civil and Environmental Engineering Department. He was a Post-Doctoral Researcher at Exeter University in 2008, where he is also a member of the research staff of the Computational Geomechanics Group. His research interests include unsaturated soil mechanics, application of intelligent systems in geotechnical engineering, probabilistic analysis and reliability assessment. He has also consulted and supervised numerous geotechnical projects.

Akbar Javadi obtained his BS and MS degrees from Tabriz University, Iran, and a PhD degree from Bradford University, UK, where he is Associate Professor of Geotechnical and Geoenvironmental Engineering and Head of the Computational Geomechanics Group. He teaches geotechnical courses at undergraduate and graduate levels, and has also supervised over 20 PhD students and 3 postdoctoral researchers. He is a member of the British Geotechnical Society; member of the International Society of Soil Mechanics and Geotechnical Engineering (ISSMGE); a graduate member of the Institution of Civil Engineers; and committee member of the UK Association of Computational Mechanics in Engineering (ACME). The results of his research have been presented in over 200 publications in international journals, book chapters, conference proceedings and research reports.

Mohsen Elmi was born in Karaj, Iran, in 1986. He obtained an MS degree in Earthquake Engineering from Shiraz University of Technology, Iran, in 2012. His research interests include soil-structure interaction and reliability analysis.

Saeed Raei obtained his MS degree in Geotechnical Engineering from Shiraz University of Technology, Iran, in 2012. His research interests include numerical modeling and reliability analysis.