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Comparison of the drift spectra generated using continuous and lumped-mass beam models

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Abstract. The drift spectrum, as a new measure of earthquake-induced demands in structures, was developed using a uniform-stiffness continuous beam model. Practical limitations in the procedures based on such models have caused structural engineers to pay little attention to drift spectra. In this paper, a method is proposed to estimate the seismic-induced inter-story drift demands, using the modal analysis technique of lumped mass, non-uniform beam models. The proposed approach is simple and can overcome many deficiencies of the previous methods. It can take into account the very important effect of height-wise stiffness reduction on drift demands, without the numerical difficulties usually encountered in computing the drift spectra. The drift spectra for high-rise shear buildings are computed using both continuous and lumped mass beam models. The comparisons indicate that the results of the continuous uniform-stiffness beam models in some cases are not acceptable. The effects of various parameters such as the structural system, the height-wise variation of structural stiffness, the number of stories, higher modes and damping ratios on inter-story drift demands are investigated. Also, the influences of the structural system and height-wise variation of structural stiffness on the height-wise distribution of maximum inter-story drift demands are evaluated using a number of building models.

1. Introduction

The response spectrum [1,2] is an important tool in seismic design of structures. Using modal analysis procedures, the peak elastic response of a Multi-Degree Of Freedom (MDOF) system during an earthquake excitation can be approximately determined from the earthquake response spectrum, without performing a dynamic time history analysis. The response spectrum analysis can also provide an estimate of the overall displacement demands of the building models. However, extensive research has shown that the inter-story drift ratio has the best correlation with the inflicted damages to buildings under earthquake excitations [3-5]. The distribution of the inter-story drift demands along the height of the buildings is not uniform, and therefore cannot be obtained directly from the displacement response spectrum.

The responses of the structures during near-field excitations cannot be characterized by a resonance buildup, so the underlying assumptions of the modal analysis method are not satisfied for structures subjected to these ground motions [6]. Iwan [6] introduced the so-called "drift spectrum" as a new measure of the induced seismic demands on structures for such situations. Iwan's drift spectrum was based on the solution of the problem for non-dispersive damped waves in a one-dimensional continuous medium, as treated by Courant and Hilbert [7]. He used the shear strain in a continuous shear beam as an analogy to the inter-story drift ratio in a building structure.

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Using modal analysis of a continuous shear beam, Chopra and Chintanapakdee [8] showed that the differences between drift and response spectra are not unique for near-field strong ground motions. These differences simply reflect the effects of higher modes on the response that are larger due to the characteristics of near-field ground motions [8]. Also, the existing modal combination rules are equally accurate for near-field and far-field ground motions, although their main assumptions are not satisfied by near-field excitations [8].

Miranda and Akkar [9] extended Iwan's drift spectrum for buildings with flexural or flexural-shear type of behavior. They proposed the so-called "generalized inter-story drift spectrum" that was based on shear-flexural continuous uniform stiffness beam. A modification factor was employed by Miranda and Akkar to consider any arbitrary combination of flexural and shear types of behavior for the structural system. The proposed generalized inter-story drift spectrum was based on modal analysis techniques. To compute the inter-story drift ratio, Miranda and Akkar used a code-based relationship to estimate the height of the structures from their fundamental periods.

As it was already mentioned, Iwan's drift spectrum was based on the propagation of non-dispersive damped waves, a concept not known by many structural engineers. Also, Kim and Collins [10] showed that the formulation used in the Iwan's drift spectrum could result in residual drifts for certain ground motions that are not consistent with the assumed linear elastic model. On the other hand, for shear beams or beams with predominant shear deformation, the computation of exact formulations of Miranda and Akkar [9] in most computers may include considerable amount of round off error. So, they provided some approximate relations for the problem in such beams. In this work, the importance of the error caused by the application of these approximate relations and their effects on the accuracy of the method will be discussed.

Since the introduction of the concept of drift spectra by Iwan [6] in 1997, not many engineers have paid attention to these new spectra due to the difficulties existed in treating the continuous beam models. In this paper, a new method is proposed to determine the drift spectra, using the well-established modal analysis technique for lumped mass, non-uniform beam models. This method, unlike the previous ones, can consider the effect of height-wise stiffness reduction on drift demands, which is shown to be very important. Application of Code-based approximate formulas, to relate the height of the structural models to their fundamental periods, which was among the shortcomings of previous methods, does not exist in the proposed approach.

The mean drift spectra for high-rise building models are computed for both continuous and lumped mass beam models, using ten near-field strong ground motions. The effects of various parameters such as the type of structural system, the ratio of the lateral stiffness at the top of the building model to the lateral stiffness at its base, number of stories, higher modes and damping ratio on the inter-story drift demands are investigated. Moreover, the effects of structural system and height-wise variation of structural stiffness on the distribution of maximum inter-story drift demands are evaluated using a number of building models.

2. Miranda and Akkar's "generalized drift spectrum"

Miranda and Akkar [9] used the Csonka's [11] flexural-shear beam model, shown in Figure 1, to obtain estimates of drift demands in linearly elastic buildings. It assumes that these beams are connected by an infinite number of axially rigid members that only transmit horizontal forces. They used the derivatives of the mode shapes of the continuous shear-flexural beam to approximate the drift ratios. When the undamped uniform model is subjected to a horizontal acceleration $\ddot{u}_g(t)$, the Inter-story Drift Ratio (IDR) at the jth story is estimated by [9]:

$$\mathrm{IDR}(j,t) \approx \frac{1}{H}\sum_{i=1}^{m}\Gamma_i\phi_i'(x)D_i(t), \tag{1}$$

where H is the total height of the building, Γ_i is the modal participation factor of the ith mode of vibration of the continuous beam model, and $\phi_i'(x)$ is the first derivative of the ith mode at a non-dimensional height x of the beam model which is the average height of the $j+1$ and j floors. Also, $D_i(t)$ is the relative

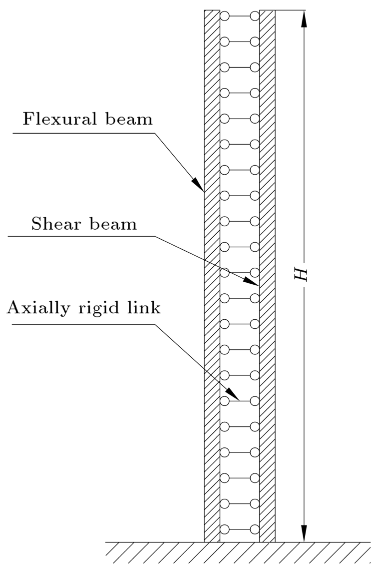


Figure 1. Flexural-shear beam used in Miranda and Akkar's generalized drift spectrum.

displacement response of a SDOF (Single-Degree of Freedom) system with period T_i and modal damping ratio ξ_i corresponding to those of the ith mode of vibration, subjected to a ground acceleration $\ddot{u}_g(t)$, and m is the number of modes considered in the analysis.

The details of computing values of $\phi_i'(x)$ and Γ_i can be found in Miranda and Taghavi [12]. The mode shapes of vibration and their derivatives of continuous beams depend only on the lateral stiffness ratio α, defined as:

$$\alpha = H\sqrt{\frac{\mathrm{GA}}{\mathrm{EI}}}, \tag{2}$$

where GA and EI are the shear and flexural rigidities of the shear beam model, respectively.

Miranda and Akkar [9], assuming a $T_1 = 0.0853H^{0.75}$ relationship between fundamental period and height of steel moment-resisting frames, obtained the ordinates of the generalized inter-story drift spectrum. In a previous study, Miranda and Reyes [13] concluded that the maximum inter-story drift demands were not significantly influenced by reduction of stiffness along the height of the structural system. Based on this study, Miranda and Akkar [9] computed the drift spectra for uniform (stiffness) beams, and suggested the following equation describing the maximum inter-story drift ratio:

$$\mathrm{IDR}_{\max} \approx \max_{\forall t,x}\left|\frac{1}{H}\sum_{i=1}^{m}\Gamma_i\phi_i'(x)D_i(t).\right. \tag{3a}$$

Since application of any assumption for determining the height of the building models could reduce the accuracy of their proposed method, this assumption is not considered in this paper. Instead, a drift spectrum is generated using the value of the "inter-story drift ratio × total height", as its ordinates. Therefore, Eq. (3a) can be re-arranged as:

$$\mathrm{IDR}_{\max}\times H \approx \max_{\forall t,x}\left|\sum_{i=1}^{m}\Gamma_i\phi_i'(x)D_i(t).\right. \tag{3b}$$

3. Problem formulation

The response of an MDOF system with N degrees of freedom subjected to horizontal ground acceleration is governed by the following differential equation:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{1\}\ddot{u}_g(t), \tag{4}$$

where $[M]$, $[C]$, and $[K]$ are mass, damping, and stiffness matrices, respectively, $\{U\}$ is the relative displacement vector of the dynamic system, and $\ddot{u}_g(t)$ is the input ground acceleration. The response of elastic structures with classical damping can be obtained using the expansion theorem. The relative displacement vector of MDOF systems can be computed as:

$$\{U(t)\} = \sum_{i=1}^{N}\{U_i(t)\}, \tag{5}$$

in which $\{U_i(t)\}$ is the contribution of the ith mode to the displacement vector of the system $\{U(t)\}$, determined as:

$$\{U_i(t)\} = \Gamma_i\{\phi_i\}D_i(t), \qquad i = 1, 2, \cdots, N, \tag{6}$$

in which:

$$\Gamma_i = \frac{\{\phi_i\}^T[M]\{1\}}{\{\phi_i\}^T[M]\{\phi_i\}}, \qquad i = 1, 2, \cdots, N, \tag{7}$$

where Γ_i and $\{\phi_i\}$ are the modal participation factor and the ith mode shape, respectively. The relative displacement history of the ith mode shape of the system, $D_i(t)$, can be determined from:

$$\ddot{D}_i(t) + 2\xi_i\omega_i\dot{D}_i(t) + \omega_i^2 D_i(t) = -\ddot{u}_g(t),$$
$$i = 1, 2, \cdots, N, \tag{8}$$

in which ω_i and ξ_i are the ith mode un-damped natural frequency and damping ratios, respectively. The inter-story drift history of the jth story can be obtained as:

$$\begin{aligned}\mathrm{Drift}_j &= \sum_{i=1}^{N}\{U_i(j)\} - \{U_i(j-1)\}\\ &= \sum_{i=1}^{N}\Gamma_i[\phi_i(j) - \phi_i(j-1)]D_i(t),\end{aligned}$$
$$i = 1, 2, \cdots, N, \tag{9}$$

in which a smaller number of modes can be used for inter-story drift estimation.

Now, consider the lumped mass shear beam representing an N-story building shown in Figure 2. The masses of different floors that are considered to be equal, are lumped at each floor level. The mass matrix $[M]$ can therefore be written as:

$$[M] = m[I], \tag{10}$$

where m is the mass of each story and $[I]$ is the identity matrix. For a building with uniform mass, the modal participation factor given by Eq. (7) reduces to:

$$\Gamma_i = \frac{\sum\limits_{j=1}^{N}\phi_i(j)}{\sum\limits_{j=1}^{N}\phi_i^2(j)}, \qquad i = 1, 2, \cdots, N. \tag{11}$$

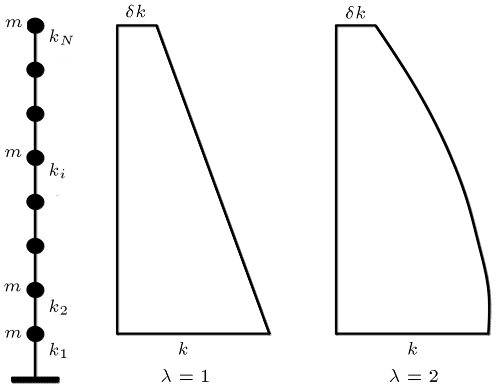


Figure 2. The lumped mass, non-uniform beam model and variation of stiffness along its height.

The non-uniform distribution of stiffnesses along the height of beam model is assumed to be estimated from:

$$k_i = \left(1 - (1-\delta)\left(\frac{i-1}{N-1}\right)^{\lambda}\right) k, \tag{12}$$

where k_i and k are the stiffness of the ith and the first stories, respectively. The parameter δ is the ratio of the lateral stiffness at the top story to that of the first story of the beam model, and λ is a non-dimensional parameter that controls variation of the lateral stiffness along the beam. This relationship is similar to the one used by Miranda and Reyes [13] for continuous beams. As they noted, values of λ equal to 1 or 2 correspond respectively to linear or parabolic variation of lateral stiffnesses along the height of the beam model, as shown in Figure 2. All elements of the stiffness matrix of the system can be determined in terms of the first story stiffness k, so this matrix can be written as:

$$[K] = k[S], \tag{13}$$

where non-dimensional matrix $[S]$ is a function of N, δ and λ.

Substituting Eqs. (10) and (13) into the characteristic equation of the system yields:

$$\det(-\omega_i^2[M] + [K]) = 0, \qquad i = 1, 2, \cdots, N, \tag{14}$$

or:

$$\det\left(-\omega_i^2 \frac{m}{k}[I] + [S]\right) = 0, \qquad i = 1, 2, \cdots, N. \tag{15}$$

The mode shapes (eigenvectors) can then be determined from:

$$\left(-\omega_i^2 \frac{m}{k}[I] + [S]\right)\{\phi_i\} = 0, \qquad i = 1, 2, \cdots, N. \tag{16}$$

Defining the non-dimensional parameter 'β_i' by:

$$\beta_i^2 = \omega_i^2 \frac{m}{k}. \tag{17}$$

Eqs. (15) and (16) can be written as:

$$\det(-\beta_i^2[I] + [S]) = 0, \tag{18}$$

and:

$$(-\beta_i^2[I] + [S])\{\phi_i\} = 0. \tag{19}$$

Eq. (19) shows that all N-story buildings with uniform mass and a similar variation pattern for stiffness, with no dependency on the value of stiffness, mass and height, would have identical mode shapes. Using Eq. (17) for the 1st and the ith modes of vibration produces:

$$\frac{\beta_i^2}{\beta_1^2} = \frac{\omega_i^2}{\omega_1^2}. \tag{20}$$

Therefore, the ith mode circular frequency or period can be obtained from the frequency or the period of the first mode as follows:

$$\omega_i = \frac{\beta_i}{\beta_1}\omega_1, \qquad T_i = \frac{\beta_1}{\beta_i}T_1. \tag{21}$$

This means that if the fundamental period of vibration of a building with particular stiffness variation is known, its higher mode natural periods can be calculated using Eq. (21).

For a given ground motion time history, the maximum inter-drift distribution for an N-story shear building with the height-wise stiffness variation described by Eq. (12) can be obtained from its fundamental period of vibration. The procedure is simple: first the matrix $[S]$ for the specified values of δ and λ is generated, and then Eqs. (18) and (19) can be used to find the non-dimensional parameters β_i and mode shapes $\{\phi_i\}$. The periods of the higher modes and the modal participation factors are calculated by Eqs. (21) and (11), respectively. If the period T_i of any mode is known, the response history of the equivalent SDOF systems, $D_i(t)$, for the selected modal damping ratios ξ_i, under any earthquake record can be determined by solving Eq. (8). The drift history of all stories can then be calculated from:

$$\begin{aligned}\mathrm{Drift}_j &= \sum_{i=1}^{N}\{U_i(j)\} - \{U_i(j-1)\} \\ &= \sum_{i=1}^{N}\Gamma_i[\phi_i(j) - \phi_i(j-1)]D_i(t), \\ j &= 1, 2, \cdots, N.\end{aligned} \tag{22}$$

Considering two buildings with n_1 and n_2 number of stories, and the same fundamental period T, one could write:

$$n_1 h_1 = H_1, \tag{23}$$

$$n_2 h_2 = H_2, \tag{24}$$

in which h_1 and h_2 are the story heights and H_1 and H_2 are the building heights, respectively. Thus, the maximum inter-story drift ratios (MIDR) of these buildings will be equal to:

$$\mathrm{MIDR}_1 = \frac{\Delta_{1,\max}}{h_1} = \frac{\Delta_{1,\max} n_1}{H_1}, \tag{25}$$

$$\mathrm{MIDR}_2 = \frac{\Delta_{2,\max}}{h_2} = \frac{\Delta_{2,\max} n_2}{H_2}, \tag{26}$$

where $\Delta_{i,\max}$ is the maximum inter-story drift demand of the ith building.

Tall buildings can be approximated by continuous beams. In a continuous beam model, the MIDR can be obtained using Eq. (3a). It is also shown in Eq. (19) that all N-story buildings with uniform mass and a similar variation pattern for stiffness, with no dependency on the value of stiffness, mass and height, would have identical mode shapes. It is therefore reasonable to assume that tall uniform-mass buildings with identical natural periods and similar variation pattern for stiffness have nearly the same mode shapes. Hence, from Eq. (3b) it can be concluded that:

$$\mathrm{MIDR}_1 \times H_1 \approx \mathrm{MIDR}_2 \times H_2, \tag{27}$$

or "maximum inter-story drift ratio × total height" for tall buildings with uniform mass and similar variation pattern for stiffness would be nearly the same if their fundamental periods of vibration be identical. Increasing the height of the buildings will improve this approximation. The maximum inter-story drift ratio × total height is an upper bound for the roof displacement of the building models. So, two tall buildings with uniform distribution of mass, same fundamental period and similar height-wise variation of stiffness, would have the same upper bound for roof displacement, which is consistent with the concept of displacement spectrum. This result will be numerically verified in this work, and then will be used in presentation of the drift spectra.

4. Near-field ground motions

As mentioned in Section 1, the drift spectrum was originally developed to provide a new measure of induced seismic demands in building models for situations in which the ground motion cannot be appropriately specified by the maximum spectral displacement. The fault-normal component of a near-field ground motion that is characterized by a forward directivity effect is such a case [14]. The forward directivity-related velocity pulse could impart a large amount of energy to the buildings at the onset of an earthquake episode [15]. Table 1 shows the 10 near-field records used in this study. The earthquake records with forward directivity have significant PGV, and exhibit long period velocity pulses due to directivity effects. To provide a rational basis for comparison, the ground motion records listed in Table 1 are scaled to 0.5 g. All these records were obtained from the PEER website [16].

Table 1. Details of the near-field ground motions.

No.	Earthquake	M_W[a]	Station	Date	$V_{s(30)}$[b] (m/s)	NEHRP site class	T_p[c] (sec)	PGA[d] (g)	Source Dist. R_{rupt}[e] (km)	Comp.
1	Cape Mendocino	7.01	Petrolia	92/04/25	712.8	C	3	0.662	8.2	090
2	Erzincan	6.69	Erzincan	92/03/13	274.5	D	2.7	0.496	4.4	EW
3	Imperial valley	6.53	EC Meloland Overpass	79/10/15	186.2	D	3.3	0.296	0.1	270
4	Kobe	6.90	JMA	95/01/17	312.0	D	1.0	0.821	1.0	000
5	Kobe	6.90	Takatori	95/01/17	256.0	D	1.6	0.611	1.5	000
6	Landers	7.28	Lucerne	92/06/28	684.9	C	5.1	0.721	2.2	275
7	Loma Prieta	6.93	LGPC	89/10/18	477.7	C	3.0	0.563	3.9	000
8	Northridge	6.69	Sylmar-Olive View	94/01/17	440.5	C	3.1	0.843	5.3	360
9	Northridge	6.69	Rinaldi	94/01/17	282.2	D	1.2	0.838	6.5	228
10	Tabas	7.35	Tabas	78/09/16	766.8	B	Backward	0.836	2.0	LN

[a] M_w: moment magnitude; [b] $V_{s(30)}$: 30 m divided by the travel time of an S-wave to 30 m depth;
[c] T_p: period of pulse; [d] PGA: peak ground acceleration.; [e] R_{rupt}: closest distance to rupture plane.

5. Drift spectrum based on lumped mass non-uniform beam model

5.1. Computation of drift spectrum

It was shown in Section 3 that if the stiffness variation along the height of a building model is represented by Eq. (12), it is possible to estimate its maximum inter-story drift under earthquake excitation, using its fundamental period of vibration. Using the described lumped mass beam model for a given earthquake record, the drift spectrum for N-story building models with specific values of δ, λ and stiffness variation given by Eq. (12), can be computed by simply repeating the procedure for the desired range of fundamental periods. Although a large number of drift spectra could be generated using the above approach, it will be shown that just a few spectra would be sufficient to address most practical cases. The drift spectra for building models with shear and flexural behavior can also be produced by the proposed method. However, only the results for shear beam models are presented here.

5.2. The influence of the number of stories on maximum inter-story drift demands

Figure 3(a) and (b) represent the effect of a building's number of stories on its maximum inter-story drift demands for a 0.02 damping ratio for a uniform stiffness beam and a beam with $\delta = 0.35$ and $\lambda = 2$, respectively. Clearly, the number of stories has an important effect on inter-story drift demands. For any given period, the inter-story drift demands are decreased by increasing the number of stories. This trend suggests that different presentations of the drift spectra, i.e. the product of maximum inter-story drift and number of stories, or product of Maximum Inter-story Drift Ratio (MIDR) and total height of building (H) (see Eqs. (25) and (26)) versus the fundamental period, as shown in Figure 4(a) and (b), may be more useful. These figures show that for the shear buildings with more than 20 Degrees Of Freedom (DOFs), the value of inter-story drift ratio multiplied by the total height is not very sensitive to the number of stories. To better quantify the difference between these spectra, Figure 5(a) and (b) show the same spectra normalized by the spectrum of 50-DOFs. These figures show that except for periods less than 0.5 sec (which are not common for high-rise buildings), the differences between drift spectra for shear buildings with more than 20 degrees of freedom are less than 4 percent. Therefore, for tall buildings with specific values of λ and δ, a single "maximum inter-story drift ratio $\times$ total height" spectrum can be used to approximately describe their behavior. This result is supported by Eq. (27). One can conclude that it is acceptable to

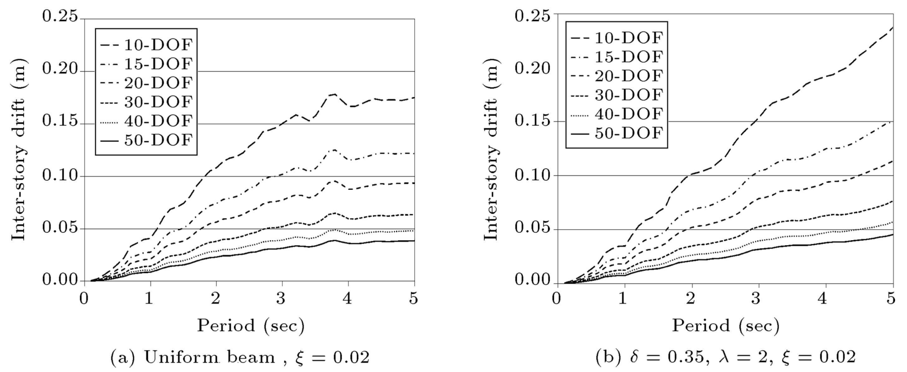


Figure 3. The effect of number of stories on the maximum inter-story drifts demands.

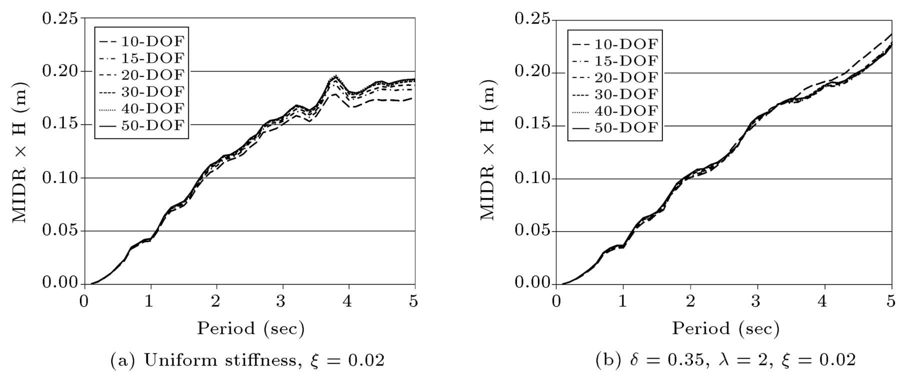


Figure 4. Alternative representation of the effect of number of stories on the maximum inter-story drifts demands.

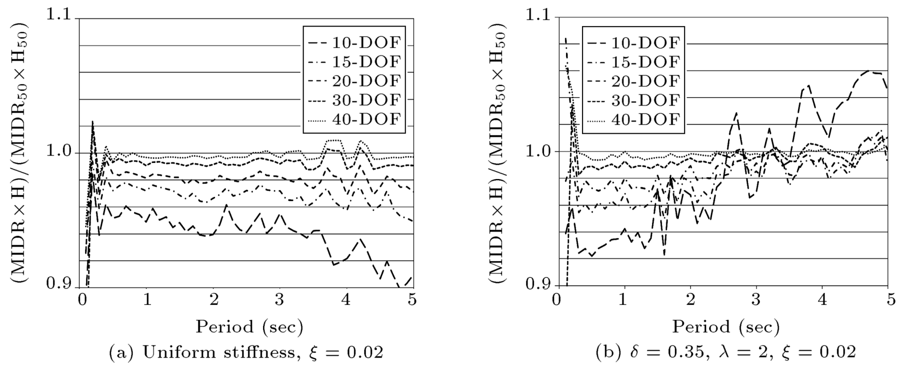


Figure 5. Quantifying the effect of number of stories on the maximum inter-story drifts demands.

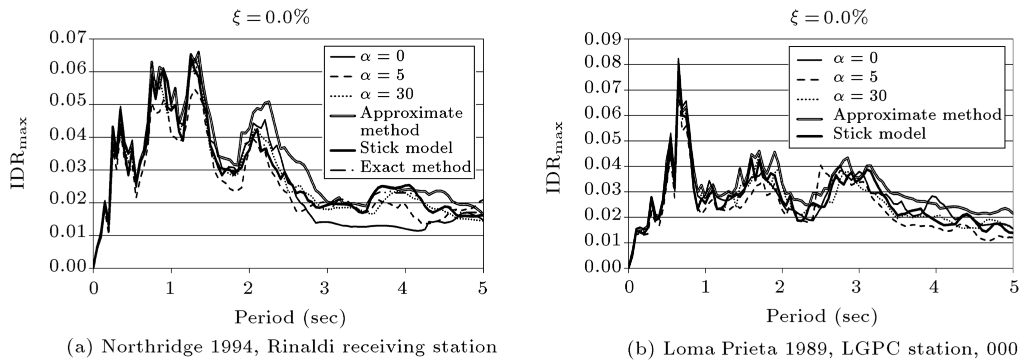


Figure 6. The effect of α on the maximum inter-story drifts demands with the assumed H-T relationship.

consider a 50-DOFs lumped mass shear beam as an approximation for a continuous shear beam. This result will be verified in Figure 6(a) by comparing the drift spectra of continuous model and 50-DOF stick model for the Rinaldi NS component of 1994 Northridge earthquake.

It should be noted that the proposed method has no assumption on the value of structural or story height. It is not possible to represent the inter-story drift ratio spectra without doing such assumptions. Therefore, from now on in this manuscript, the term "drift spectrum" means "maximum inter-story drift ratio × total height of building" spectrum.

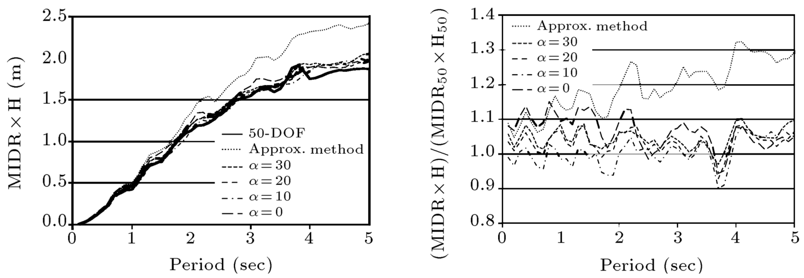
5.3. Comparison of drift spectra obtained using continuous and lumped mass beam models

The drift spectra obtained using the continuous and lumped mass beam models are compared to investigate their differences. For comparison purposes, the drift spectra obtained by the method of Miranda and Akkar (by using the code based relationship between the height and the fundamental period of structural models) for the Rinaldi NS component of 1994 Northridge earthquake is shown in Figure 6(a). This figure shows that the lumped mass beam with 50-DOFs very closely approximates values obtained by the exact continuous-beam method by Miranda and Akkar [9]. This figure also shows that the approximate method may lead to errors up to 33%.

Figure 6(b) shows the drift spectra obtained using the LGPC component of 1989 Loma Prieta earthquake. As this figure indicates, the spectra for small values of α are not necessarily very different from that of a shear beam model. Furthermore, the maximum inter-story drift demands are not necessarily very sensitive to the value of α. The next section discusses on this subject in more detail.

5.4. Influence of the structural system on the value and height-wise location of maximum inter-story drift demands

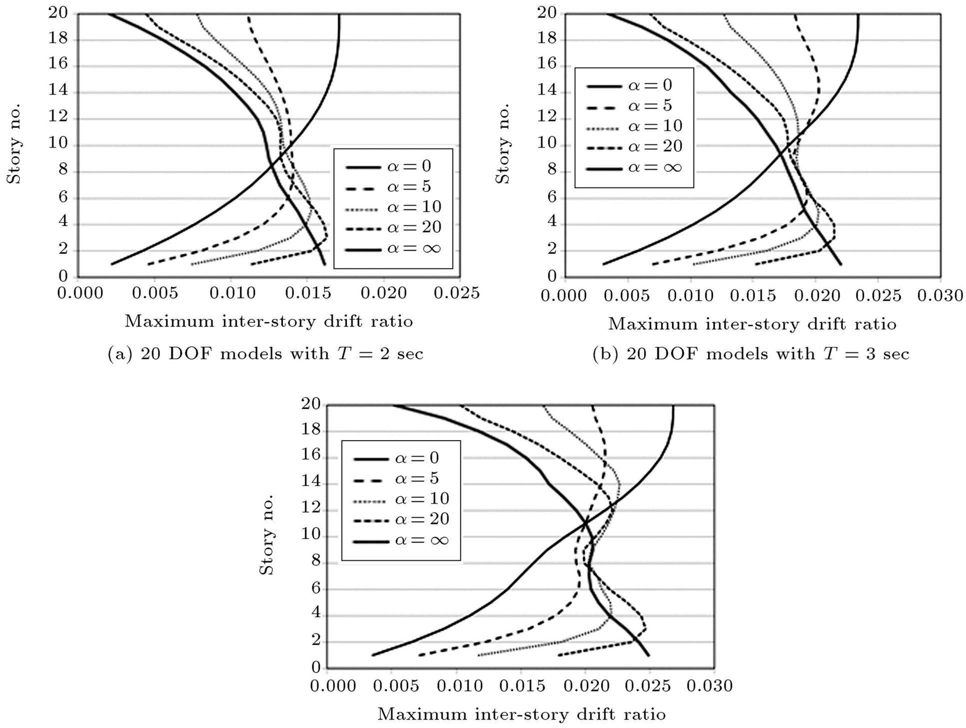
In Section 5.3, it is shown that the amount of maximum inter-story drift ratio is not necessarily dependent on the structural system, but the drift spectra do not provide any information about the height-wise location of the maximum inter-story drift ratio. Thus, the effect of structural system on the position of maximum inter-story drift demands needs to be investigated. The drift spectra of continuous shear-flexural beams for various values of α and drift spectrum of 50-DOFs lumped mass uniform beams are presented in Figure 7(a). The drift spectra for values of α less than 30 are calculated using the exact relations derived by Miranda and Akkar [9]. However, as mentioned in Section 1, for large values of α, the computation of these exact



(a) Drift spectra for high rise buildings $\xi = 0.02$

(b) Quantifying the effect of α for high rise buildings $\xi = 0.02$

Figure 7. The effect of α on the maximum inter-story drifts demands.



(a) 20 DOF models with T = 2 sec

(b) 20 DOF models with T = 3 sec

(c) 20 DOF models with T = 4 sec

Figure 8. The effect of α on the height-wise variation of the maximum inter-story drifts demands for 20-DOF uniform-stiffness beam models with $\xi = 0.02$.

relations in most computers may include considerable amount of round off error. Thus, a lumped mass shear beam with 50-DOFs is considered as an approximation for a continuous beam. Also, Figure 7(a) represents the results of the approximate relations of Miranda and Akkar for large values of α that corresponds to a shear beam model. Figure 7(b) shows the same drift spectra normalized by the spectrum of a 50-DOFs beam models. This figure shows that the average drift spectra obtained, using the 10 earthquake records of Table 1, are not very sensitive to the value of α. Moreover, Figure 7(b) shows that the error caused by using the approximate relations is not negligible. These results are not consistent with the results obtained by Miranda and Akkar, as they examined only two ground motion records. It is observed that by increasing the number of records, the sensitivity of the inter-story drift ratios to the value of α is reduced and the error, due to using approximate relations, becomes important. The presented results are obtained using the strong ground motions of Table 1, and more earthquake records should be studied before generalizing this finding.

To investigate the effect of structural system on the location of maximum inter-story drift demands, 15 uniform-stiffness structural models that are representatives of 5 different structural systems are investigated. The 70 meters tall structural models have 20 DOFs with fundamental periods of vibration of 2, 3 and 4 seconds. They exhibit pure flexural behavior with $\alpha = 0$, pure shear behavior with $\alpha = \infty$ and shear-flexural behaviors with $\alpha = 5$, $\alpha = 10$ and $\alpha = 20$. The damping ratio is assumed to be $\xi = 0.02$. The distributions of inter-story drift ratio demands over the height of these structural models are shown in

Figure 8. From this figure it is concluded that the location of maximum inter-story drift ratio is very sensitive to the structural system, while its value is not strongly dependent on the structural system, for the considered ground motions. Figure 8 shows that the location of maximum inter-story drift ratio for uniform-stiffness structures with flexural behavior is at roof level, while for structures with shear behavior is at first story, and for systems with combined shear-flexural behavior is somewhere in the mid-height of the structural models.

5.5. *Influence of the structural fundamental period on maximum inter-story drift demands*

Figure 9 shows the effect of the fundamental period of vibration of the structural models on maximum inter-story drift demands for uniform-stiffness 20-DOF models with damping ratio $\xi = 0.02$. This figure demonstrates the distribution of inter-story drift demands over the height of structural models for five values of α. As this figure indicates, by increasing the period of vibration, the inter-story drift demands usually increase, but as Figure 7(a) shows, small decreases are also possible. Figure 9 also shows that the height-wise distribution of maximum inter-story drift demands is mostly dependent on the structural system. For certain values of α, uniform height-wise changes in the stiffness do not significantly affect the distribution pattern of maximum inter-story drift demands. These obtained results are due to the earthquake records listed in Table 1.

5.6. *Influence of the stiffness reduction pattern on maximum inter-story drift demands*

Figure 10(a) shows the effect of the stiffness reduction pattern λ on maximum inter-story drift demands for 20-DOFs beam models with $\delta = 0.35$ and damping ratio $\xi = 0.02$. The sensitivity of maximum inter-story drift demands to changes in λ is negligible, thus making it possible to assign it a fixed value. Since, for most of the building models, λ lies between 1 and 3, for the remainder of this work, this parameter is assumed to be equal to 2. Figure 10(b) presents the same spectra normalized by the spectrum for $\lambda = 2$. It is seen that the error due to ignoring λ is in general less than 10%, while for a wide range of periods, it is less than 5%.

5.7. *The effect of top to bottom story stiffness ratio on the maximum inter-story drift demands*

Figure 11(a) shows the effect of the lateral stiffness ratio of the top story to that of the first story, δ, on maximum inter-story drift demands for 20-DOFs models with $\lambda = 2$ and $\xi = 0.02$, while Figure 11(b) shows the same spectra normalized by the spectrum for $\delta = 1$. These figures demonstrate that the effect of δ on maximum inter-story drift demands for values of $\delta < 0.5$ is not small. Reducing the lateral stiffness ratio causes inter-story drifts to increase significantly. This effect for values of δ greater than 0.5 is insignificant. A value of $\delta = 0.5$ can be used for values of $\delta \geq 0.5$ with an error below 10%, while for values of $\delta < 0.5$, different spectra should be computed. These figures show that

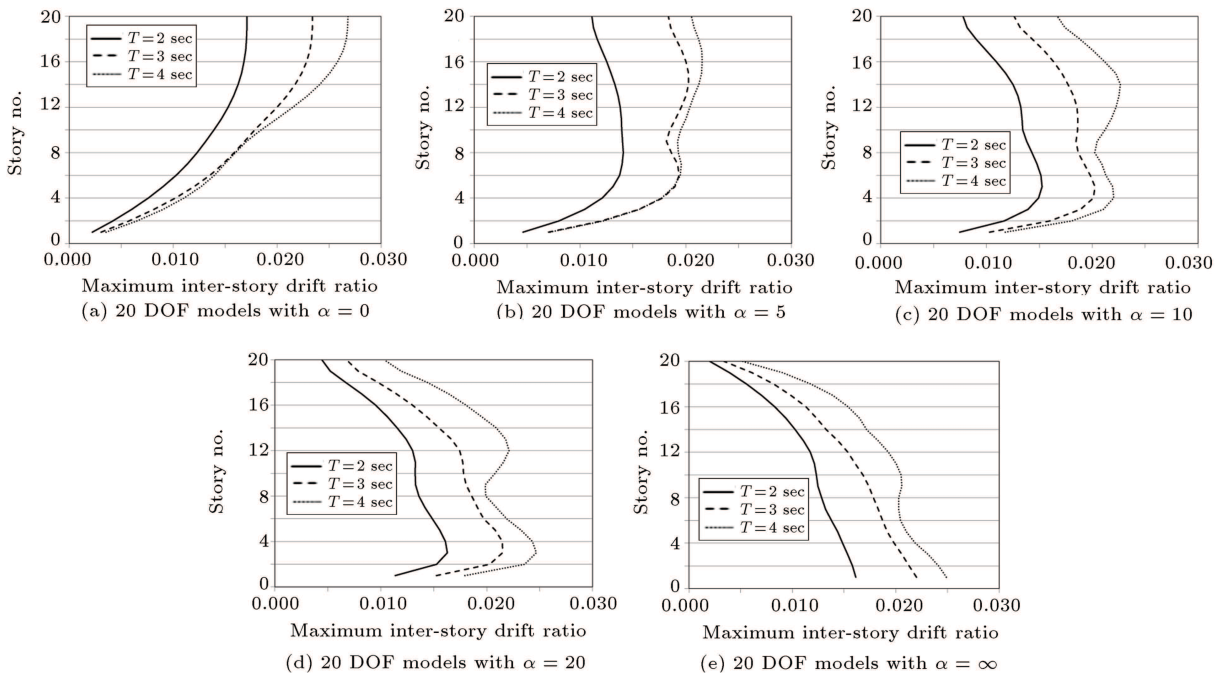
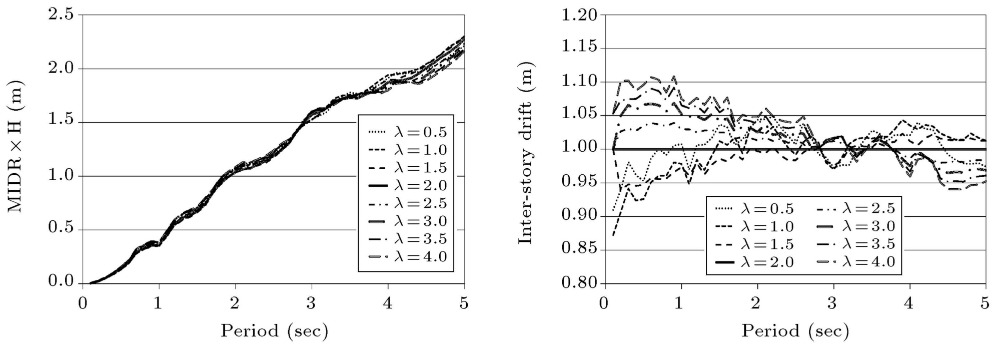


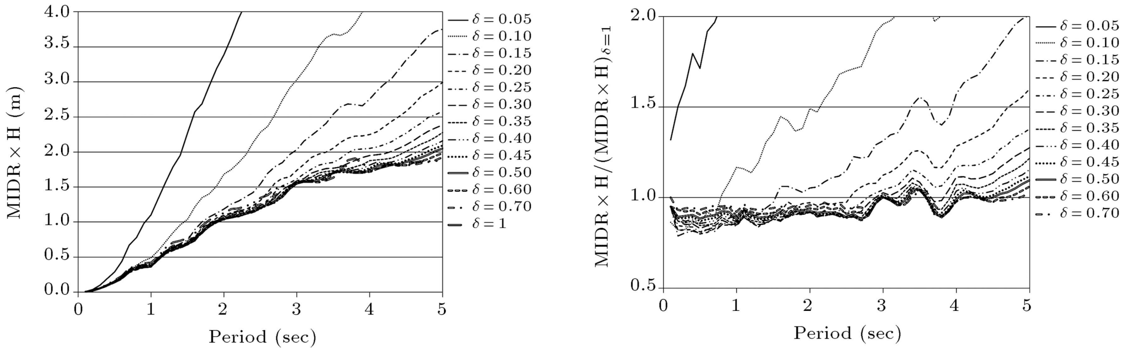
Figure 9. The effect of structural fundamental period on the height-wise variation of the maximum inter-story drifts demands for 20-DOF uniform-stiffness beam models with $\xi = 0.02$.



(a) Drift spectra for 20-DOFs with $\delta = 0.35$ and $\xi = 0.02$

(b) Quantifying the effect of λ

Figure 10. The sensitivity of the maximum inter-story drift demands to the stiffness reduction pattern.



(a) Drift spectra for high-rise buildings with $\lambda = 2$ and $\xi = 0.02$

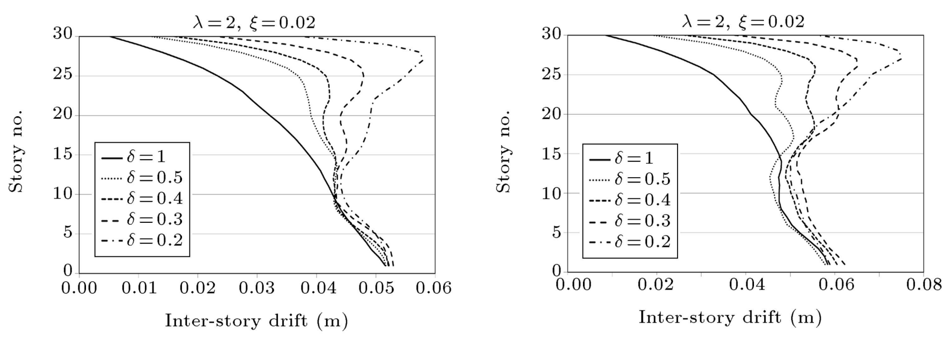
(b) Quantifying the effect of δ for high rise buildings $\xi = 0.02$

Figure 11. The sensitivity of the maximum inter-story drift demands to the parameter, δ.

the effect of lateral stiffness ratio of the top stories, with respect to the first ones, is considerable. For further investigation, the distribution of inter-story drift ratio demands over height of two shear building models with 30 stories and fundamental periods of vibration of 3 sec and 4 sec, is illustrated in Figure 12(a) and (b), respectively. In these figures, drift demands are computed for various values of δ for $\lambda = 2$ and $\xi = 0.02$. These figures show that lateral stiffness ratio of the top story to that of the first story not only affects the value of maximum inter-story drift demand, but also influences its location along the height of the building.

5.8. *The effects of damping ratio and higher modes on the maximum inter-story drift demands*

Figure 13 shows the maximum inter-story drift spectrum obtained using different numbers of modes for the 20-story building models with $\delta = 0.35$, $\lambda = 2$, and $\xi = 0.02$. As this figure shows, for the near-field ground motions of Table 1, the spectral values obtained, using



(a) 30-story shear building model, $T = 3$ sec

(b) 30-story shear building model, $T = 4$ sec

Figure 12. Effect of height-wise distribution of stiffness on height-wise location of maximum inter-story drift demands.

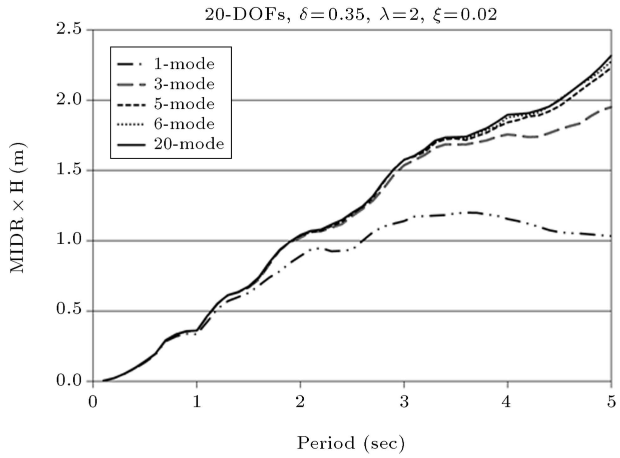


Figure 13. The effect of higher modes on the maximum inter-story drift demands.

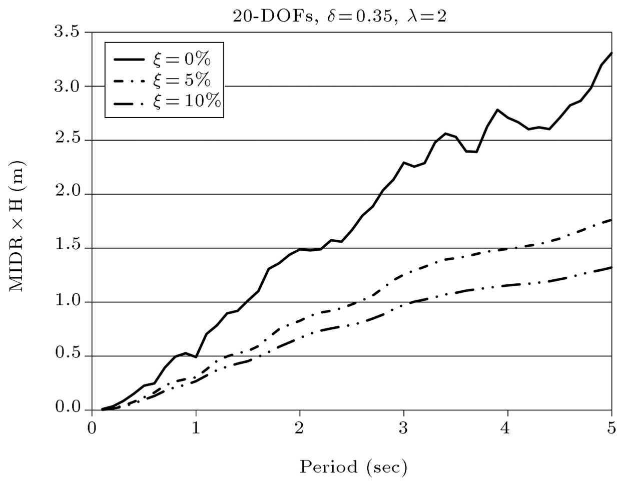


Figure 14. The effect of damping ratio on the maximum inter-story drift demands.

only one or two modes, are highly under-estimated, while those using the first 6 modes of vibration for 20-story buildings can produce acceptable estimates. Also, Figure 14 shows the effect of damping on maximum inter-story drift demands for 20-story buildings with $\delta = 0.35$ and $\lambda = 2$. As expected, an increase in the damping ratio leads to a decrease in the maximum inter-story drift demand.

6. Conclusions

A new method based on a lumped mass beam model for estimating the inter-story drift demands of buildings is proposed. It is shown that for the considered near field earthquake records, the average of the maximum inter-story drift ratios for high-rise buildings is mostly dependent on their fundamental periods, and its sensitivity to the structural system is not considerable, although the location of its occurrence is strongly dependent on the structural system. The drift spectra for shear buildings with different numbers of stories and non-uniform distribution of stiffness along the height are presented. It is observed that the maximum inter-story drift is not strongly dependent on the stiffness variation pattern. However, its dependency on the ratio of lateral stiffness of the top story to that of the first story of the building models, that was not referred to in previous methods, is very important. For shear buildings in which the lateral, top to bottom story stiffness ratio is greater than 0.5, a single drift spectrum can be used, while for ratios less than 0.5, different spectra would be needed. Moreover, it is deduced that the product of maximum inter-story drift ratio and total height for tall buildings with similar variation pattern for stiffness are mostly dependent on their natural fundamental periods. Based on this finding, for each value of lateral stiffness ratio of top to base story, for buildings taller than 20 stories, a single spectrum is presented. In contrast to other available methods, no code-based relationship is used in this approach to estimate the height of the structural models from their fundamental periods. The proposed method is simple because it uses well-established lumped mass beam concept rather than complicated continuous shear-flexural beam representation with due computational difficulties. The proposed drift spectra are powerful tools for structures with regular height-wise stiffness distribution. They can be used to study the effect of various parameters on inter-story drift demands and preliminary assessment of drift demands, providing guidelines to optimum design of structures.

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