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Method of Fundamental Solution (MFS) coupled with Particle Swarm Optimization (PSO) to determine optimal phreatic line in unconfined seepage problem

Sh. Shahrokhabadi[a,1,*] **and A.R. Ahmadi**[b]

a. *Department of Civil Engineering, Vali-e-Asr University of Rafsanjan, Rafsanjan, P.O. Box 518, Iran.*
b. *Kerman Graduate University of Technology, Kerman, P.O. Box 76315-117, Iran.*

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Abstract. In this research, Method of Fundamental Solution (MFS) is coupled with Particle Swarm Optimization (PSO) technique to determine the optimal phreatic line in unconfined seepage problems. To model the unconfined boundary (phreatic line), a formulation with floating geometry is derived. Regarding the use of fundamental solution of the Laplace equation, expressed in the Radial Basis Functions (RBF), a boundary type of the mesh-free method can be established. In this research, an objective function, based on principle of minimum potential energy, is formed to control the position of unconfined boundary. MFS and PSO are utilized simultaneously to fit the phreatic line, using 4th degree polynomials, satisfying the flow continuity and energy principle. Efficiency and accuracy of the proposed method are verified through examples. The obtained results are in a good agreement with other numerical and experimental models.

1. Introduction

Seepage is one of the most important phenomena in geotechnical engineering which can be divided into confined and unconfined problems. The confined problems refer to problems with known boundaries and the unconfined ones refer to problems with unknown boundaries. Determination of the phreatic line is a fundamental step in solving unconfined seepage problems. Broad studies, mostly numerically-based, have been conducted in an effort to simulate this phenomenon in earth dams [1-4]. Advancements in computational technologies and data analysis have rendered numerical methods as valuable tools in almost all engineering fields. Finite Element Method (FEM) and Boundary Element Method (BEM) have been used extensively in modeling heat and mass transfer problems, especially in situations with moving boundaries (e.g. phreatic line in unconfined seepage problem) [5-7]. In this kind of problems, which is a complicated and time-consuming process, the major difficulty is the mesh generation and/or regeneration (as strategy may require). An excessively refined mesh is usually needed to simulate such problems, leading to large matrices and higher computational costs. Thus, special strategies for mesh generation must be adopted in FEM [7,8]. On the other hand, BEM is based on solving integral equations along the boundaries. Singularities on the boundaries are possible which render this approach as an inflexible method. Furthermore, BEM produces large matrices that makes this approach time-consuming in iterative solution procedures.

Over the last decade, a new group of numer-

1. *Present address: Department of Civil and Environmental Engineering, Mississippi State University, P.O. Box 9546, USA.*
*. *Corresponding author. Tel: +98 913 291 5015; Fax: +98 3913202233*
E-mail addresses: shahrokhabadi@vru.ac.ir (Sh. Shahrokhabadi), a.ahmadi@kgut.ac.ir (A.R. Ahmadi)

ical techniques, known as mesh-free methods, have been developed in computational mechanics. Most importantly, these methods offer a convenient approach in solving problems with variable geometry [9-12]. The basic idea in mesh-free methods is based on the Radial Basis Function (RBF), which introduces the relationship between two distinct nodes, useful in scattered data interpolation [13,14]. RBF is defined as a function that defines the Euclidean distance between two nodes [15,16].

The most popular RBF functions are:

$$r^{2m-2}\log(r), \tag{1a}$$

$$(r^2+c^2)^{\frac{m}{2}}, \tag{1b}$$

$$e^{-\beta r}, \tag{1c}$$

where $r=\|x-x_j\|$ is the Euclidean distance between any point, and node j in the problem domain m, c, β are constant values. Eq. (1a) is introduced as the generalized Thin Plate Spline (TPS). Eq. (1b) is defined as the generalized multiquadric and Eq. (1c) is the Gaussian function. In this research, Eq. (1a) is selected as the RBF; it has been shown that better results are to be expected when value of m is one [17].

Usually, mesh-free methods are divided into two main categories: domain type and boundary type. Since in domain type formulation any single RBF cannot satisfy the governing equations, obtaining a viable solution would require a large number of collocation points for both domain and boundary of the problem. On the other hand, boundary type formulation requires collocation nodes on the boundary of the problem only. In boundary type problems, fundamental solution of the governing linear differential equation can be selected as the RBF solution space [18]. This approach automatically satisfies the governing equation. Moreover, source nodes are set outside the problem domain, which contains no singularities, and only a few collocation points are needed on the boundary to solve the problem [19].

Method of Fundamental Solution (MFS) is used in solving unconfined seepage problem. However, the flow continuity over the downstream slope is not considered [20].

In this research, the governing equation and boundary conditions of the problem are explained, followed by a brief description on MFS and RBF. Then the Particle Swarm Optimization (PSO) algorithm is briefly explained, and it is applied using a 4th degree polynomial as a phreatic line. At the end, numerical results are shown and compared to other numerical methods and indoor testing.

2. Governing equation and boundary conditions

Unconfined seepage problems can be considered as laminar steady state flow in a homogenous $x-y$-plane. These assumption leads to Laplace equation as the problem's mathematical model [21].

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0, \tag{2}$$

where $\phi(x,y)$ is the total energy.

Based on energy principle, the total energy on both domain and boundary of the problem is:

$$\phi=\mathrm{EH}+\mathrm{PH}, \tag{3}$$

where EH is the elevation head, and PH is the pressure head at every node of the problem's boundary.

Boundary conditions, regarding satisfaction of the flow continuity, in the computational domain are defined as [22] (see Figure 1):

1. Impermeable boundaries: Flow velocity is zero along these segments ($\nu=0$) or ($\frac{\partial\phi}{\partial n}=0$), these boundaries define streamlines (ψ_1).
2. Equipotential lines: Hydrostatic pressure is applied to these boundaries, and the total head (ϕ) is constant along these segments (ϕ'_1, ϕ'_2).
3. Phreatic line: This boundary is the upper streamline in the problem domain and pore pressure is zero on this surface (PH=0). In this paper, the phreatic line is modeled using a 4th degree polynomial function (ψ_2).
4. Seepage face: On this face, water seeps out of the dam and the total energy equals each node elevation, and pressure head equals zero. This boundary is neither an equipotential nor a streamline (η).

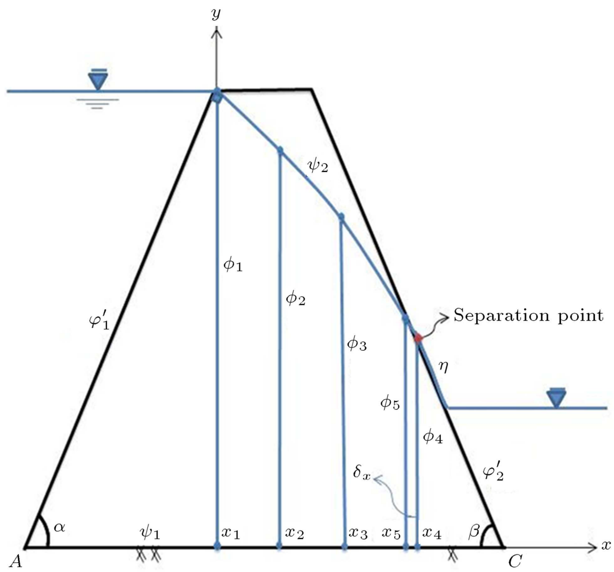

Figure 1. Flow through a 2-D homogenous dam.

Since equipotential lines and streamlines are perpendicular at upstream, the phreatic line is perpendicular to the upstream slope and the phreatic line must be tangent to downstream slope to satisfy flow continuity.

Generally, head loss leads to flow through a dam; consequently, the mathematical definition of head loss defines a path that descends along seepage flow. Eventually, total energy at ($x = x_1$, $y = \phi_1$) is constant and equal to upstream total head ($\phi = \phi_1'$).

In this research, the equation of the phreatic line is assumed as a fourth degree polynomial, which is introduced by Eq. (4):

$$P_4(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4. \tag{4}$$

Constraints depicted in Figure 1, are as follows:

$$P_4(x) = \phi_1'|_{x=x_1}, \tag{5a}$$

$$\nabla P_4(x) = -\tan^{-1}(\alpha)|_{x=x_1}, \tag{5b}$$

$$\nabla P_4(x) = \tan(\beta)|_{x=x_4}, \tag{5c}$$

$$\nabla P_4(x) < 0|_{x=x_i} \qquad (i = 1, 2, \cdots m), \tag{5d}$$

where ∇ is the gradient operator ($\frac{d}{dx}$) and m is the number of nodes on phreatic line. Eq. (5) should be considered in optimization procedure as constraints of objective function.

The node at x_5 is dependent on the node at x_4 to guarantee that the phreatic line is tangent to the downstream slope:

$$x_5 = x_4 - \delta_x,$$

$$\phi_5 = \phi_4 + \delta_x . \tan(\beta), \tag{6}$$

where δ_x is introduced as a small distance between x_4 and x_5.

Since phreatic line is a 4th degree polynomial, it is necessary to evaluate this function at least at five nodes. Nodal coordinates are defined as follows:

$$P_4(x) = \phi_1'|_{x_1=0}, \tag{7a}$$

$$P_4(x) = \phi_2|_{x_2=\frac{1}{3}(x_4-x_1)}, \tag{7b}$$

$$P_4(x) = \phi_3|_{x_3=\frac{2}{3}(x_4-x_1)}, \tag{7c}$$

$$P_4(x) = \phi_4|_{x_4=C-\phi_4 . \cot(\beta)}, \tag{7d}$$

$$P_4(x) = \phi_5|_{x_5=\phi_5}. \tag{7e}$$

In Eqs. (7b) to (7d), $\phi_i (i = 2 - 4)$ is determined iteratively. ϕ_i' is a constant value and ϕ_5 depends on ϕ_4 in solution steps. C is a geometry value (toe of the dam) which is defined in Figure 1. Based on the third boundary condition (phreatic line), an objective function is defined as:

$$f = \sum_{i=1}^{m} \int (P_4(x_i) - \phi(x_i, y_i))^2, \tag{8}$$

where m is the number of nodes on phreatic line.

The main purpose of this research is to introduce a function to satisfy the imposed constraints (Eq. (5)) while minimizing the goal function given by Eq. (8).

3. Method of Fundamental Solution (MFS)

In mesh-free methods, the total energy during the nth iteration step is a linear combination of N Radial Basis Functions (RBFs) [23].

$$\phi^{(n)}(x_i, y_i) = \sum_{i=1}^{n} \alpha_i^{(n)} q_i(x, y), \tag{9}$$

where $q_i(x, y)$ is an RBF with its source node located at (x_i, y_i), and $\alpha_i^{(n)}$ is the RBF weight.

Fundamental solution of Laplace operator is selected as RBF, that is:

$$q_i(x, y) = \ln(d_i), \tag{10}$$

where d_i is Euclidean distance from a boundary node on the computational boundary to ith RBF center (source node) [19]. Source nodes are set outside of the computational domain. Thus, the solution form satisfies the governing equation automatically even at RBF centers. The main process is to find $\alpha_i^{(n)}$, which is accomplished by solving a system of equations formed from the boundary conditions.

Partial derivatives of the total energy define the flow velocity, i.e.:

$$\left(\frac{\partial \phi}{\partial x}\right)^n = \sum_{i=1}^{N} \alpha_i^{(n)} \frac{\partial q_i}{\partial x},$$

$$\left(\frac{\partial \phi}{\partial y}\right)^n = \sum_{i=1}^{N} \alpha_i^{(n)} \frac{\partial q_i}{\partial y}. \tag{11}$$

There are N unknowns in each iteration ($\alpha_i^{(n)}$, $i = 1, 2, \cdots N$). The constraint conditions yield the following system of simultaneous equations to solve:

$$\begin{bmatrix} \alpha_{1,1} & \cdots & \alpha_{1,N} \\ \vdots & \alpha_{i,j} & \vdots \\ \alpha_{N,1} & \cdots & \alpha_{N,N} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ . \\ \alpha_N \end{bmatrix} = \begin{bmatrix} b_1 \\ . \\ b_N \end{bmatrix}, \tag{12}$$

where $\alpha_{i,j} = q_j(x_i, y_i)$ and $b_i = \phi_(x_i, y_i)$. Solution of Eq. (12) yields RBF weights that can be used along with Eq. (9) to compute the total energy at every boundary point.

4. Particle Swarm Optimization (PSO)

Considering the non-linear objective function, which is in the form of multi criterion, will lead to solve nonlinear simultaneous equations. On the other hand, the objective function must be solved through free derivative optimization methods, because in the calculation procedure, only the numerical value of the objective function can be calculated [8]. Therefore, application of an optimization procedure which is not dependent on objective function derivative is useful for these kinds of problems. The proposed algorithm for optimization procedure is Particle Swarm Optimization (PSO).

There are several evolutionary methods that can be applied to solve the optimization problems. Among the available solution techniques, PSO is proved to be robust, effective and easy to apply [24]. This method is based on a very simple framework and can be used easily with primitive mathematical operators. This method does not need much computer memory, while the speed of computation is relatively fast [25].

Swarm behavior of bird societies has inspired a simple and highly effective optimization algorithm by Eberhart and Kennedy [26]. PSO includes a set of individuals for which their knowledge is improved iteratively in the search space. The position and velocity of the individuals define what is known as particles. PSO has no crossover or transformation between particles. Moreover, particles are never substituted by other individuals during the run. Conceptually, in the search domain, PSO tries to attract particles with high probability for fitness [27]. Each particle has a memory function which allows it to remember the best position it has experienced thus far. Also, the best global position is visited by the entire swarm. The information obtained thus far is divided into two parts; the first part includes the particle's memory of its past state and the second part entails memory of the society. PSO has a fitness evaluation measure that takes each particle's position, and returns a fitness value for it. Global Best is defined as the maximum fitness value that swarm has met. In addition, Local Best is defined as the maximum fitness value that each particle has individually experienced; hence, each particle remembers both the Global and the Local best.

The canonical PSO has been widely used in engineering and science [28-30]. Now, position (x_i) and velocity (v_i) for each particle form the set of all particles with population size of M ($i = 1, 2, \cdots, M$) are defined as:

$$\begin{aligned} v_i(t+1) =& w v_i(t) + c_1 r_1(t)(pbest_i(t) - x_i(t)) \\ &+ c_2 r_2(t)(gbest(t) - x_i(t)), \end{aligned} \tag{13}$$

$$x_i(t+1) = x_i(t) + v_i(t+1), \tag{14}$$

where c_1 and c_2 are acceleration factors (constant values); t shows the tth iteration; r_1 and r_2 are weights generated randomly with values between 0 and 1; w is an inertia weight that controls the impact of velocity from previous iteration on newly computed velocity; $gbest$ is the Global Best and $pbest$ is the Local Best.

Generally, PSO algorithm includes the following five main steps:

1. Randomly generation of an initial position vector x and the related velocity vector v for all particles in the population set.
2. Evaluating fitness value of each particle.
3. Comparing the fitness value of each particle to its local best.

   If the current value is better than the previous one, then it is replaced with the new value. When individual particle's fitness measure is compared to that of population's best, i.e. Global best, if the current value is better than the previous one, it is replaced as Global Best.
4. Changing velocity and position, using Eqs. (13) and (14).

   In conventional methods, usually a fixed number of trials are carried out with the minimum value from all the trails taken as the global best. In the current study, the limitation of conventional PSO is solved based on termination proposal, introduced by Cheng et al. [25].

   If $gbest$ gets stable after N_2 iterations, the algorithm will terminate by definition of the following criteria:

   $$|f_g - f_{sf}| \leq \varepsilon, \tag{15}$$

   where x_{sf} and f_{sf} mean the best solution in the current state and its related objective function value. ε is the tolerance of termination. PSO is not sensitive to optimization parameters in most problems. It is the brilliant specialty which ranks PSO as a high recommended method in Geotechnical problems [31].
5. Repeating steps 2 to 4 until the described termination criterion, based on Eq. (15).

   In optimization method design variables, variable bounds, constraints and penalty function are summarized in Table 1.

In this study, the external penalty function method as one of the most common forms of the penalty function in the structural optimization is employed to transform the constrained problem into the unconstrained one, as follows [24,32,33]:

$$\tilde{f}(x) = f(x)(1 + r_p pf), \tag{16}$$

Table 1. Design variables, variable bounds, constraints and penalty function in this research.

Case	Design Variables	Variable bounds (m)	Penalty function	Goal function	Constraints
Example 1	Eqs. (7b) to (7d)	0 to 24	Eq. (16)	Eq. (8)	Eq. (5)
Example 2	Eqs. (7b) to (7d)	0 to 3.2	Eq. (16)	Eq. (8)	Eq. (5)
Example 3	Eqs. (7b) to (7d)	0 to 0.46	Eq. (16)	Eq. (8)	Eq. (5)

where $\tilde{f}(x)$ and r_p are modified function (fitness function) and an adjusting coefficient, in a row, and pf is the penalization factor, which is defined as the sum of all active constraints violations.

After determination of variable designs in each iteration a 4th degree polynomial is formed, all the produced nodes on the curve should be considered if they satisfy the goal function (Eq. (8)) and the related constraints which are determined based on Eq. (5).

5. Numerical studies

Here, some examples are solved using the presented methodology in order to verify the proposed method. First, a rectangular dam is analyzed and compared to available results [20,34,35]. Then, a trapezoidal dam is considered which is available in literature [7]. Next, the results are verified by a laboratory test [36].

Solution procedure starts with generation of random initial values for ϕ_1, ϕ_2, ϕ_3 (Eq. (7)); the mentioned items should satisfy geometrical conditions. Therefore, the variable bonds vary from 0 to 24, 3.2, and 0.46 m in Examples 1, 2, and 3, respectively. Thereby, forming individual set in PSO algorithm. Every individual line is solved in the seepage problem, using mesh-free concept. The obtained results for each node on the proposed phreatic line $((\phi_i\ (i = 1, 2, \cdots, N_s))$, where N_s is the number of nodes on phreatic line, is supplied to PSO to check the objective function and the related constraints. This procedure continues until the stop criterion is reached. The flowchart of the present method is illustrated in Figure 2.

Particle size, maximum velocity, maximum particle movement amplitude and maximum number of iteration are considered as important factors in PSO, that affect convergence rate and hence the cost of computation. In this research, the best values of mentioned parameters are achieved through several runs (Table 2).

MFS is based on the concept of RBF; hence the distance between source nodes and boundary nodes plays an important role in progression of the solution. Table 3 lists the parameters used in mesh-free analysis.

Table 2. PSO parameters used in solution procedure.

Particle size (n)	Max. velocity	Max. particle move	$N1$	$N2$
30	0.5	0.009	500	200

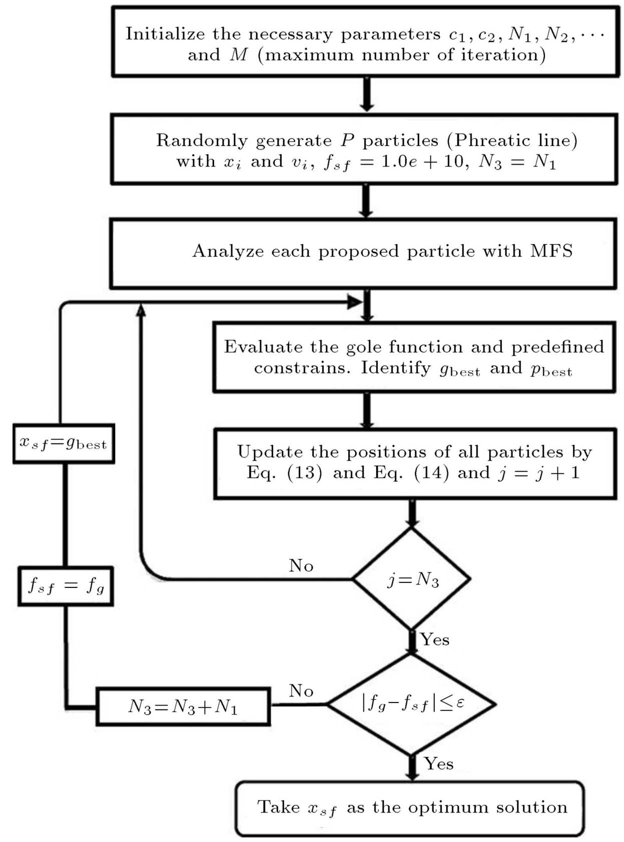

Figure 2. Flow chart for the particle swarm optimization and method of fundamental solution.

Table 3. MFS parameters used in this solution procedure.

Example	Source point-boundary point distance (cm)	Node distribution distance (cm)
1	20	50
2	30	50

Geometry of the rectangular dam is as follows:

1. Upstream water level (total energy-upstream) is 24 m. Thus $\phi_1' = \phi_1 = 24$ m.
2. The length of the dam is 16 m.
3. Downstream water level (total energy-downstream) is 4 m. Thus $\phi_2' = 4$ m.

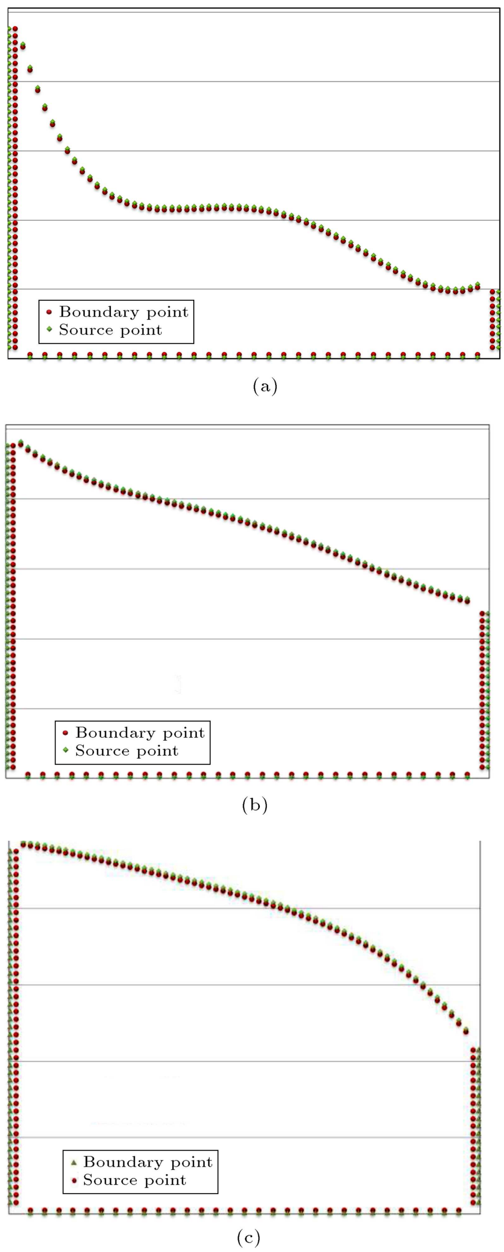

Figure 3. Instances of individual in PSO algorithm to solve Example 1. a) Individual 1; b) Individual 2; and c) Individual 3.

Some of PSO individuals, selected randomly, are shown in Figure 3.

The phreatic line, obtained from presented method is illustrated and compared with the previous studies (see Figure 4).

The node where the boundary condition number (3) and (4) meet is called separation node (see Figure 1). Different separation nodes which are the results of various methods are listed in Table 4.

Geometry of the trapezoidal dam is as follows:

1. Upstream water level (total energy-upstream) is 3.2 m. Thus $\phi_1' = \phi_1 = 3.2$ m.

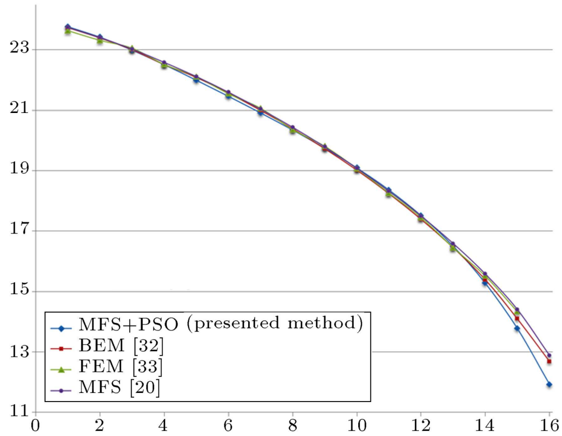

Figure 4. Comparison of phreatic line from previous studies and present method.

Table 4. Separation point achieved in different methods.

Method	Height (m)
MFS	12.88
FDM	12.79
FEM	Not presented
BEM	12.68
MFS+PSO (presented method)	12.86

Table 5. Separation point coordinates achieved in different methods.

Method	X (m)	Y (m)
Ouria & Toufigh (2009)	3.248	1.64
MFS+PSO (presented method)	3.382	1.537

2. The length of the dam is 9.6 m.
3. Downstream water level (total energy-downstream) is zero. Thus $\phi_2' = 0$.

Some of the PSO individuals, selected randomly, are shown in Figure 5.

The Phreatic line obtained from the present method is illustrated and compared to the results reported by other researchers (see Figure 6) [7]. Table 5 lists the separation node.

The outcomes of research are compared with the results of seepage flow through earth dam which is conducted in laboratory. The layout of the experimental facility is re-drawn in Figure 7. The dam is compacted with nature sedimentation with natural density of sand. The porosity is estimated to be 0.2, and the saturated hydraulic conductivity (K) is determined in the steady-state flow case, with $K = 3.5e^{-4}\ \frac{\text{m}}{\text{s}}$. The pressure in the dam is measured by piezometric tubes [36]. The results are illustrated and compared with experimental data in Figure 8.

Note that the results obtained from the present

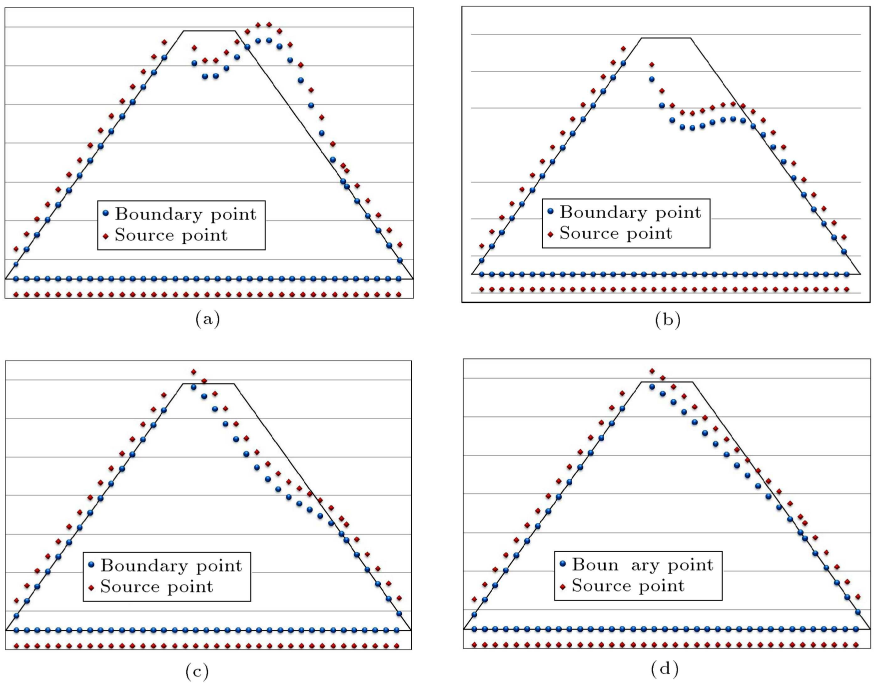

Figure 5. Instances of individual in PSO algorithm to solve Example 2. a) Individual 1; b) Individual 2; c) Individual 3; and d) Individual 4.

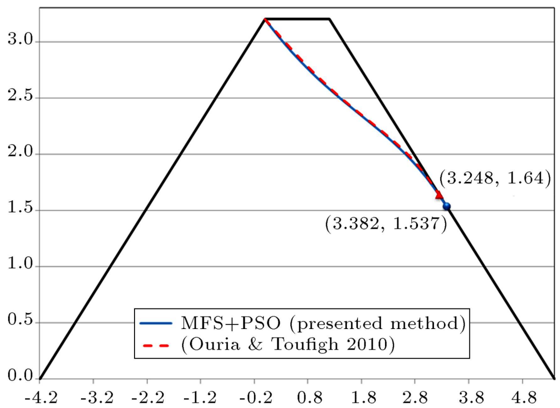

Figure 6. Resulted phreatic line from present method and finite element method.

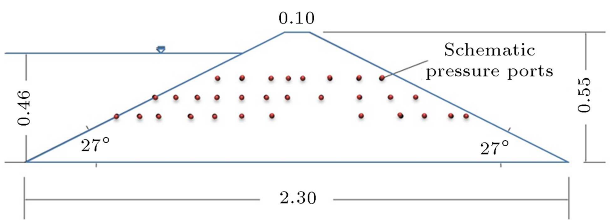

Figure 7. Redrew layout of experimental facility [36].

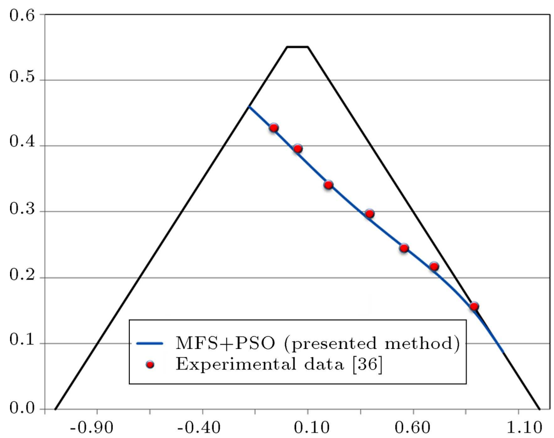

Figure 8. Resulted phreatic line from present method and indoor testing.

method are flexible and can accurately model the unconfined boundary, which renders a larger range of selection for individuals (phreatic line) as compared to previous studies; furthermore, the computed results satisfy flow continuity.

In this research, only boundary of the problem is modeled, and hence the number of collocation nodes and source nodes are much less than similar solution procedures that are based on domain type formulations, whether they are mesh-free or finite element methods. Totally, all the mentioned efforts translate to time saving in both modeling and computing. It should also be noted that the present method is sensitive to the distance between source nodes and collocation nodes; hence suitable results are achieved through trial and error in node positioning.

6. Discussion

The advantage of current method is the usage of a fully mesh-free method, dealing only with the floating boundary of the problem. In optimization procedure, each individual proposes a different curve which should be analyzed. Ouria and Toufigh used an optimization procedure which was based on Finite Element Method (FEM) analysis, but the procedure needs a special strategy to establish a convergent method, because the regularity of generated meshes has profound effects on convergence of the results [7]. A study done by Shahrokhabadi and Toufigh introduced the use of Natural Element Method (NEM) instead of FEM. Despite much less sensitivity on mesh generation, the mentioned procedure should consider the problem domain to be tessellated by Delaunay triangles. Accordingly, the problem domain should be analyzed as well, which is not the aim in finding optimum phreatic line. Moreover, Delaunay tessellation is a kind of mesh generation in this approach. Subsequently, it is not a fully mesh-free method [37].

The recent study proposes a method which uses only the boundaries of the problem, and by using a simple algorithm, results in the same achievements in much less time-consuming process. The method also considers the flow continuity which was not considered in the previous studies. Specially, Figure 4 shows that the previous studies did not consider flow continuity, which means that phreatic line should be tangent on downstream slope. It could not easily be obtained because the slope of rectangular dam is $\frac{\pi}{2}$ at downstream (in the case of rectangular dam). However, introducing suitable constraints in optimization procedure of the presented method fulfills this goal.

Eventually, the proposed method benefits from Method of Fundamental Solution (MFS). This leads to the analysis of all proposed individuals (curves) by PSO. In the similar way, if FEM was used, some randomly individuals which led to inappropriate mesh generation would cause diverge. Therefore, this kind of individuals should be omitted prior to analysis, but in the current method, all individuals can be analyzed.

Regarding the 15 times of running the algorithm, the stability of the proposed method is observed in Table 6. It is noteworthy that the stability of the presented method is acceptable based on the shown standard deviation.

Table 6. Stability of presented method with respect to worst, mean, best, and standard deviation.

Case	Worst	Mean	Best	Standard deviation
Example 1	1.65	0.7984	0.039	0.5389
Example 2	1.2	0.1447	0.0029	0.3275
Example 3	0.1892	0.0269	0.0003	0.0704

7. Conclusion

Here, the Method of Fundamental Solution (MFS) is coupled with the particle swarm algorithm to simulate unconfined seepage problem. Fundamental solution of Laplace operator satisfies the governing equation. In addition, the entire source nodes are placed outside the computational domain, and thus singularities do not appear. The computational procedure is the boundary type of Mesh-free methods. The mesh generation is not an issue here; in addition, only few collocation points are needed along the boundary with associated source nodes to model and solve the seepage problem successfully without numerical integration, all of which translates into time saving in modeling and computations, as well as low requirement on computing resources.

Particle Swarm Optimization (PSO) is adapted to find the best possible 4th degree polynomial, thereby minimizing the error function and satisfying flow continuity constraints.

PSO is an optimization procedure that does not require the computation of error function's derivative. Simplicity of PSO, as well as MFS, leads to the the introduction of a new robust procedure that is not limited by the mesh generation or the usual optimization issues.

Accuracy of the present method is confirmed via solving a standard rectangular homogeneous dam that has been studied by other researchers. A trapezoidal dam is also investigated and accurate results have been obtained with satisfaction of flow continuity constrains, and the accuracy of the present method is also evaluated through experimental data, which is available in other researches.

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Biographies

Shahriar Shahrokhabadi received his MSc from Shahid Bahonar University of Kerman, Iran, in 2009. He was a faculty member in Civil Engineering Department of Vali-e-Asr University of Rafsanjan. He is currently doing his PhD program in Mississippi State University, Mississippi, USA. He has over 7 years experience in both computational mechanics. His major research includes mesh-free methods, especially Natural Element Method (NEM).

Ali Reza Ahmadi is a Research Associate Professor at International Center of Science, High Technology & environmental Science. He received his PhD in Mechanical Engineering from University of Kansas in 2003. He received his D.E. in Civil Engineering from University of Kansas in 1988. His areas of research includes computational mechanics, computational mathematics and Finite Element Method (FEM).