Relationships of yield-capacity-risk in a multiple reservoir system: The Munzur River Basin in Turkey

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Abstract. The aim of this study is to determine the relationships of yield-capacity-risk in a multiple reservoir system. In this study, yield, capacity and risk have been defined as the average energy production, storage volume of reservoir and obtaining the same yield with lower capacity, respectively. Then, an optimization model, using Dynamic Programming with Successive Approximations (DPSA) for a multi-reservoir system for energy production, has been developed. The objective function used in the model has the objective of maximization of the total energy. A multi-reservoir system in the Munzur River Basin of Turkey has been selected for the application. The results of the proposed approach have been evaluated with regard to the relationships of yield-capacity-risk. As a result, the capacity was increased when the yield was raised, and the same yield has been obtained in a different capacity by reducing the capacity under a certain yield risk. The yield risk has been raised in the direction of the yield coordinate of the yield-capacity-yield risk curve.

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1. Introduction

In a water resources system, stream flow has stochastic and statistical features in addition to the average amount. In the meantime, there has not been a relationship between the elements of the sequential flow series. These types of series are named as stochastic series. Changes in the flow at the effort situation by taking the advantage from the stream flow can be changed by the storage volumes of the reservoirs. These are defined as capacities in the water resources system. Thus, it can be said that reservoirs’ capacities have stochastic feature.

Different methods used for designing storage reservoir have advantages and/or disadvantages against each other [1]. Although synthetic data have been used for designing reservoir capacity, the relation between the storage reservoirs characteristics of capacity-risk-yield can be used directly [2,3].

A mathematical model of a multiple reservoir system with multiple objectives will be formulated for maximization of the energy production in this study. The optimal solution for this may be achieved by using Dynamic Programming with Successive Approximations (DPSA) technique [4,5]. Optimal operational policies of a multiple reservoir system may be obtained using this technique for a given flow series covering the planning horizon, usually on the basis of maximizing the firm energy or the total energy or both [6-12].

In the DPSA application, only one of the reservoirs is allowed to vary at a time, while all others are kept constant; thus, all the reservoirs are treated likewise to obtain better solutions by successive approximations, and this continues until no further improvement is achieved. In this way, a multi-dimensional problem is reduced to a series of one-dimensional problems, and provided that reasonable initial operational policies are adopted, the computational load

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is significantly reduced. The DPSA is also preferable since the energy production is a non-linear function of the storage (and therefore the release), which usually appears in the objective function [13-18].

In this study, relationships between yield, capacity, and risk have been determined. Thus, an optimization model based on DPSA for a multi-reservoir system for energy production has been used. The objective function in the model is about the energy production. An application has been performed in the Munzur River Basin of Turkey.

2. Definition of parameters in the system
   
   **Yield.** Yield is defined as total water amount taken from a dam as a result of arranging in the active storage reservoir of the dam in a year period. There is a certain capacity for any yield. However, obtaining the different capacities for any yield is possible as well as obtaining different yields from any capacity. In this situation, active storage capacity and yield of the reservoir should be defined under a certain risk. Chosen capacity can provide a certain yield under a certain risk, or certain yield can be obtained by a chosen capacity under a certain risk.

   **Capacity.** Active storage volume of dams can be taken into consideration as the dam capacity. There is an annual yield for every capacity amount. There are lots of methods for determining capacity. Generally, reservoir capacity is determined by using the relation between the amount of water arrived to a dam and the water amount taken from the dam. Taking into consideration the flood and drought conditions, the following steps are used for determining the capacity.

   1. The capacity which can provide the annual yield with the monthly average flow is determined, depending on the predicted management strategies.
   2. A certain flood/drought frequency period is chosen, and flood/drought control capacity is obtained by using the critical term flow of this period.
   3. Since flood control volume is discussed in the wet season and drought control volume is discussed in dry season, the total volume should not be achieved by the sum of these, but an appropriate combination is to be used.

   **Yield and capacity risk.** There has been a linear relation between the capacity and the yield. It can be observed that there is a yield for every capacity value. Capacity risk can be defined as a case which enables the obtaining of the same yield with a different capacity, by reducing the capacity. Capacity risk results in the same yield with reducing the capacity under a certain risk. Then, the capacity risk \( C_R \) is:

   \[
   C_R = \frac{(C_F - C_L)}{C_F},
   \]

   where \( C_F \) is the first capacity and \( C_L \) is the last capacity. Similarly, the yield risk can be defined as a case that is possible to obtain the same capacity with a different yield, by reducing the yield. Yield risk also results in the same capacity with reducing the yield under a certain risk. Yield risk \( Y_R \) is also:

   \[
   Y_R = \frac{(Y_F - Y_L)}{Y_F},
   \]

   where \( Y_F \) is the first yield, and \( Y_L \) is the last yield.

3. Mathematical model of the system

   The optimization model is comprised of four parts:

   1. Statement of constituent equations;
   2. Formulation of constraints;
   3. Specification of objective function;
   4. Optimization technique.

   It may be noted that the model is for the long term planning and operation of a multi-reservoir system. Therefore, a monthly time scale is appropriate, and the monthly flows are used as inputs to the system. The system output to be optimized is represented in the objective function. Each part is now expressed in what follows.

   **Constituent equations.** A multi-reservoir system can be represented as a series of reservoirs, each with a storage capacity, a power production unit, inflows from upstream and releases downstream, as shown in Figure 1. Taking \( i = 1, 2, \ldots, N \), where \( N \) is the number of reservoirs, and \( t = 1, 2, \ldots, M \), where \( M \) is the number of the time intervals (months), the basic constituent equations of the system are the water balance relations of each reservoir for each time interval (for all \( i \) and \( t \)):

   \[
   S_{i,t+1} = S_{i,t} + F_{i,t} + Q_{i,t-1} + R_{i,t-1} - Q_{i,t} - R_{i,t} - L_{i,t},
   \]

   where \( S_{i,t} \) is the water stored in the reservoir; \( F_{i,t} \) is the inflow into the reservoir from its sub-drainage area; \( Q_{i,t} \) is the water released for energy production from the reservoir; \( R_{i,t} \) is the spilled water from the reservoir; and \( L_{i,t} \) is the water loss through evaporation and seepage from the reservoir.
The other constituent equations for power generation are expressed as:

\[ P_{i,t} = k_i Q_{i,t} h_{i,t}, \quad (4) \]

\[ h_{i,t} = h_{i,t} - h^i, \quad (5) \]

\[ h^i = \frac{8}{\pi^2 g} f^i l^i \frac{Q_{i,t}^2}{d^i}, \quad (6) \]

\[ \alpha^i = \frac{8}{\pi^2 g} f^i l^i \frac{1}{d^i}, \quad (7) \]

\[ l^i = l^i + \frac{d^i}{P^i} \sum K^i, \quad (8) \]

\[ h_{i,t} = h_{i,t} - \alpha^i Q_{i,t}^2, \quad (9) \]

where \( P_{i,t} \) is the power generation from the reservoir; \( h_{i,t} \) is the net turbine head for the reservoir; \( k_i \) is the average power generation coefficient for the reservoir; \( h_{i,t} \) is the water height in the reservoir (measured from turbine level); \( h^i \) is the total energy loss; \( f^i \) is the friction coefficient; \( l^i \) is the equivalence pipe length; \( d^i \) is the pipe diameter; \( P^i \) is the pipe length; \( K^i \) is the local coefficient of energy loss; \( \alpha^i \) is the average head loss coefficient for the reservoir, and \( h_{i,t} \) will be obtained as a function of the average storage volume (at the beginning and at the end of the time interval):

\[ h_{i,t} = h \left( \frac{S_{i,t} + S_{i,t+1}}{2} \right). \quad (10) \]

The total power generation of the system at time \( t \) is given as:

\[ \sum_{i=1}^{N} P_{i,t} = \sum_{i=1}^{N} k_i Q_{i,t} h_{i,t}. \quad (11) \]

The system constraints are related to the storage capacity, power generation capacity and water usage (for energy production, irrigation and other purposes).

**Specification of constraints.** The constraints to be expressed are for storage capacity, power generation, energy production, water spill and total water release.

1. The constraint on the storage capacity can be expressed as:

\[ S_{i,t}^{\min} \leq S_{i,t} \leq S_{i,t}^{\max}, \quad (12) \]

where \( S_{i,t}^{\min} \) and \( S_{i,t}^{\max} \) are the minimum and maximum storage capacities of the reservoir.

2. The constraints for power generation can be expressed as:

\[ 0 \leq P_{i,t} \leq P_{i,t}^h, \quad (13) \]

where \( P_{i,t}^h \) is the installed power generation capacity for the reservoir.

3. The constraint on releases for energy production can be expressed as:

\[ Q_{i,t}^{\min} \leq Q_{i,t} \leq Q_{i,t}^{\max}, \quad (14) \]
where $Q_i^{\text{min}}$ and $Q_i^{\text{max}}$ are the minimum and maximum releases for the reservoir. Obviously, $Q_i^{\text{max}}$ is related to the installed power generation capacity $P_k$, while $Q_i^{\text{min}}$ is related to the minimum required release downstream $D_i^{\text{min}}$ such that:
\[
Q_i^{\text{min}} = D_i^{\text{min}},
\]
when:
\[
R_i,t = 0.
\]

1. The constraint on the spill of water can be written as:
\[
0 \leq R_i,t \leq R_i^{\text{max}},
\]
where $R_i^{\text{max}}$ is the maximum spillway capacity for the reservoir.

2. The constraints on the total water release can be written as:
\[
D_i^{\text{min}} \leq (Q_i,t + R_i,t) \leq D_i^{\text{max}},
\]
where $D_i^{\text{min}}$ is the minimum release for the pollution control or navigation, and $D_i^{\text{max}}$ is the maximum safe discharge for the downstream of reservoir.

**Objective function.** The primary objective is to maximize energy production, encompassing the maximization of secondary energy (or the total energy production). For the determination of the firm power, the critical dry period within the observed monthly flow series must be selected, and using the critical period flow series [10], the firm power may be stated as:
\[
P_F = \max \sum_{i=1}^{N} \left[ \min \sum_{i=1}^{N} P_{i,t} \right],
\]
where $NN$ is the total number of months in the critical period.

For the maximization of the total energy of the system, average monthly flows may be used, and the already obtained firm power $P_F$ is to be imposed as a constraint:
\[
\sum_{i=1}^{N} P_{i,t} \geq P_F, \quad (19)
\]
\[
M \sum_{i=1}^{N} (P_{i,t} - P_F), \quad (20)
\]
which is equivalent to maximizing the secondary energy, since $P_F$ is a constant.

**Optimization method.** In DPSA, Eqs. (3) to (20) represent the “stage transformation equations”, where time periods (t) are “stages”, and the storage levels in each reservoir ($S_{i,t}$) are “states.” Thus, the releases from a reservoir ($Q_{i,t}$, $R_{i,t}$) appear as the basic decision variables. However, it must be noted that the spilled water release $R_{i,t}$ will only take place when the storage and turbine release capacity constraints of the reservoir are violated, otherwise it will be zero. Thus, $R_{i,t}$ is a dependent variable, and the real decision variable is $Q_{i,t}$.

In general, the objective function describes the benefit functions that depend upon the water stored in each reservoir and the releases from the reservoir. These functions are usually non-linear relations and the solution by optimization becomes complicated when more than one expected benefit of storage or release is taken into account at a given time. The optimization is done using a DPSA technique, which divides the problem with multi-decision variables into sub-problems with only one decision variable, and then solve the problem while taking decision variables one by one. The DPSA technique has an advantage over the other types of the dynamic programming in terms of the reduced calculated time and computer memory requirements.

There are three variables in DPSA: state, decision and stage. The group of their values related to some constrains is called system politic. The criterion which determines the effect of this system politic is also expressed as an objective function. A schematic view of the state-decision-stage variables in DPSA of a reservoir is shown in Figure 2 [9].

Optimization by DPSA was programmed using MATLAB (Mathematic laboratory, the language of technical computing). This program has one main program and six sub-programs, as shown in Figure 3. In the main program, first, the beginning policy of the optimization process is designated. This beginning policy is important to reach the optimal solution and to reduce the computer operation time. Second, in sub-programs, the optimization process with this politic is
Table 1. Basic characteristics of reservoirs in the system.

<table>
<thead>
<tr>
<th>Reservoirs</th>
<th>Y. Konaktepe</th>
<th>Güllyayla</th>
<th>Kaletepe</th>
<th>Tunceci</th>
<th>Uzunçayır</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage area (km²)</td>
<td>1086</td>
<td>1151</td>
<td>1586</td>
<td>1630</td>
<td>3335</td>
</tr>
<tr>
<td>Elevations (m)</td>
<td>1240-1070</td>
<td>1070-900</td>
<td>900-925</td>
<td>925-900</td>
<td>900-840</td>
</tr>
<tr>
<td>Installed power (MW) (Upper limits)</td>
<td>105</td>
<td>50</td>
<td>53</td>
<td>21</td>
<td>117</td>
</tr>
<tr>
<td>Dam height (m)</td>
<td>110</td>
<td>25</td>
<td>65</td>
<td>25</td>
<td>58</td>
</tr>
<tr>
<td>Maximum operational level (m)</td>
<td>1240</td>
<td>1070</td>
<td>900</td>
<td>925</td>
<td>900</td>
</tr>
<tr>
<td>Minimum operational level (m)</td>
<td>1170</td>
<td>1070</td>
<td>900.3</td>
<td>925</td>
<td>873.8</td>
</tr>
<tr>
<td>Maximum volume (10⁶ m³)</td>
<td>456</td>
<td>6</td>
<td>47.5</td>
<td>5</td>
<td>303</td>
</tr>
<tr>
<td>Minimum volume (10⁶ m³)</td>
<td>20</td>
<td>6</td>
<td>10</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>HPP* elevation (m)</td>
<td>1070</td>
<td>900</td>
<td>925</td>
<td>900</td>
<td>840</td>
</tr>
</tbody>
</table>

*HPP: Hydroelectric Power Plant

Figure 3. View of the relations between the main program and the sub-programs.

started as the operational level in the current reservoir is taken as variable successively, the objective function of the system is implemented to be realized by using the values of the beginning policy in the operational levels of the others, and these solutions obtained are kept in the memory of the model. Third, the values of the system parameters generated from these solutions are continuously controlled by taking into consideration of the system constraints at each stage of the optimization process. Finally, the operational process, to integrate these solutions, is started and the optimal solution is reached. In this optimization process, using the beginning policy, the optimal solution can be reached. Solutions should be sought using other beginning policies. These sub-programs are DYN AU, FEAS U, MFIRM U, HDATU and BUHAR U, as explained below:

DY NAU is a sub-program which evaluates the values of the state and decision variables in the objective function one by one for each stage variable. FEASU is a sub-program which decides whether double of the state-decision variables is possible or not, for each stage variable. MFIRMU is a sub-program which helps selecting an optimal solution among the optimal solutions for each stage-state-decision variable.

HDATU is a sub-program which is used to calculate the reservoir operation level with the selected volume for each stage-state-decision variable. BUHARU is a sub-program which is used to calculate the amount of evaporation with the operation level in each stage-state-decision variable.

4. Application for optimization model

An application for the multi-objective optimization was presented for the five sequential reservoirs situated on the Munzur River (at the upper reaches of the Euphrates) with an energy production objective. The system was planned originally for energy production purposes only. Table 1 shows the basic characteristics of five reservoirs in the system. The variation of the reservoir storage volume with the turbine head is given in Table 2.

Inflows into the reservoirs have been obtained from the flow gauging stations data with correlation of the drainage areas of the reservoirs and the gauging stations. Then, the inflow into each reservoir from its own sub-drainage area is evaluated. Monthly mean flows (July 1964 - June 1978) into the reservoirs from the sub-drainage areas are shown in Table 3.
drought period monthly inflows into the reservoirs from
sub-drainage areas (July 1972 - June 1973) are given in
Table 4. For the DPSA solution technique 10.10^6 m³
discrete volumes both for storages and releases have
been used, with operations starting at the beginning
of the drought season (July) when the reservoirs were
full.

5. Results
To determine the average powers of the system, an
optimization model, based on a DPSA developed for
multiple reservoirs, was used. The optimization results
have been obtained in two steps, as described in the
mathematical model of the system by using the DPSA
model. In the first step, the primary objective defined
in the objective function was the determination of the
firm power (with critical period monthly flows of 12
months from July 1972 to June 1973). In the second
step, the primary objective was used to maximize the
total energy (using the average monthly flows), and in
this case, the average power (yield) for the system was
obtained.

Different yields were obtained by changing the

<table>
<thead>
<tr>
<th>Table 3. Monthly mean flows into reservoirs from sub-drainage areas (10^6 m³).</th>
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</thead>
<tbody>
<tr>
<td>Reservoirs</td>
</tr>
<tr>
<td>July</td>
</tr>
<tr>
<td>August</td>
</tr>
<tr>
<td>September</td>
</tr>
<tr>
<td>October</td>
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<td>April</td>
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<tr>
<td>May</td>
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<tr>
<td>June</td>
</tr>
</tbody>
</table>

(July 1964 - June 1978)

<table>
<thead>
<tr>
<th>Table 4. Critical period monthly inflows into reservoirs from sub-drainage areas (10^6 m³).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoirs</td>
</tr>
<tr>
<td>July</td>
</tr>
<tr>
<td>August</td>
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<tr>
<td>September</td>
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<tr>
<td>May</td>
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<tr>
<td>June</td>
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</tbody>
</table>

(July 1972 - June 1973)
capacities (storage volumes) of the reservoirs in the optimization model. Variation of the yield (average power) to the capacities of the reservoirs are given in Table 5. Variation of the yield to capacity given in Figure 4 is obtained data in Table 5. In Figure 4, the yield is increased when the capacity is raised. Variations of the yield risk to yield and the capacity risk to capacity are shown in Figures 5 and 6, by using Eqs. (2) and (3), respectively. In Figures 5 and 6, the data of the yield and the capacity are reduced when the risks are raised. The relationship of the capacity risk to yield risk is shown in Figure 7. It is shown that there is a linear relation. Figure 8 is obtained from the yield-capacity-yield risk curve by using Eq. (2). Here, when yield is raised, the capacity is increased, and the same yield has been obtained in a different capacity by reducing the capacity under a certain yield risk. It was observed that the yield risk has been raising on the direction of the yield coordinate.

<table>
<thead>
<tr>
<th>Yield of system (MW)</th>
<th>Total capacity (10⁶ m³)</th>
<th>Y. Konaktepe + Kaletepe + Uzunçayır (10⁶ m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>162.48</td>
<td>81</td>
<td>46+5+30</td>
</tr>
<tr>
<td>159.27</td>
<td>73</td>
<td>42+5+26</td>
</tr>
<tr>
<td>155.89</td>
<td>65</td>
<td>38+5+22</td>
</tr>
<tr>
<td>151.99</td>
<td>57</td>
<td>34+5+18</td>
</tr>
<tr>
<td>147.67</td>
<td>49</td>
<td>30+5+14</td>
</tr>
<tr>
<td>142.83</td>
<td>41</td>
<td>26+5+10</td>
</tr>
<tr>
<td>133.92</td>
<td>33</td>
<td>22+5+6</td>
</tr>
</tbody>
</table>

Figure 5. Variation of yield risk to yield.

Figure 4. Variation of yield to capacity.

Figure 6. Variation of capacity risk to capacity.
6. Conclusions

In this study, an optimization model, based on the DPSA developed for multiple reservoirs, was used to determine average powers (yields) of a system. Different yields were obtained by changing the capacities (storage volumes) of the reservoirs in this model. Yield and capacity risks were obtained by using Eqs. (1) and (2). As a result:

- It was observed that the yield of the reservoir was increased when the capacity was raised.
- There has been a linear relation between the yield risk and the capacity risk.
- Yield and capacity were reduced with raising risks.
- When the yield was raised, the capacity was increased, and the same yield has been obtained in a different capacity by reducing the capacity under a certain yield risk.
- The yield risk has been raised in the direction of the yield coordinate of the yield-capacity-yield risk curve.

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Biography

Miçahit Opan was born in 1974, in Sivas of Turkey. He received his BS degree at Selçuk University in Konya, his MS degree at Yıldız Technical University in Istanbul, and his PhD at Kocaeli University, respectively, in Civil Engineering. Currently, he is working as associate professor in the Department of Civil Engineering at Kocaeli University.