Stochastic directional search: An efficient heuristic for structural optimization of building frames

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Abstract. Performance of an optimization heuristic highly depends on the method it employs to represent and decompose the search space. A class of optimization methods, including swarm intelligence, utilizes a special way of decomposing the search space via specified direction states. The present work expands the idea, providing a stochastic directional state search with tuned thresholds for selecting every direction state. The proposed meta-heuristic is then utilized for the problem of structural weight minimization in building frames under lateral and gravitational loading using a finite element analyzer. Treating a number of examples from the literature, the efficiency and effectiveness of the proposed method are shown to be superior to some other meta-heuristics, including an improved particle swarm optimizer.

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KEYWORDS
Stochastic directional search;
Structural design;
sizing optimization;
Building frame.

1. Introduction

Structural optimization is a class of problems dealing with numerous design variables, confined due to a variety of code-specific constraints. As a benchmark problem, discrete sizing optimization has been addressed by many investigators up to date.

The search space of structural optimization is usually discrete and non-convex, with a narrow feasible region, due to many limiting constraints. Hence, efficient and diverse approaches are required in this field. The class of traditional mathematical programming, MP [1], used to be applied to this field, providing a rapid local search, but its gradient-based nature is not suited to discrete spaces. Additionally, the performance of MP algorithms highly depends on the assumption of the initial starting point. Meta-heuristic algorithms, on the other hand, are more generalized and suitable for such discrete problems compared to MP techniques. They can provide effective solutions in the neighbourhood of the global optimum, via practical time, by extensive sampling of the design space, with no need for gradient information. Consequently, meta-heuristic applications have received considerable attention in the field of optimal structural design during recent decades [2,3]. Some of the interesting classes include evolutionary and genetic algorithms [4], ant colony optimization [5], harmony search [6], particle swarm optimization [7-10], big bang-big crunch [11], charged system search [12], colonial competition [13] and firefly algorithm [14].

Every meta-heuristic uses its special way of representing the search space, and aims to provide proper balance between intensification and diversification. Genetic algorithms alter coded genotypes resulting in diverse jumps in the phenotype space. Ant colony agents move through vertices on characteristic graphs of the discrete problem. Particle swarm optimization, PSO, uses vector-sum representation and a movement strategy among the search space, which has been shown to be effective in many engineering design problems [8-10].

The present work utilizes such a decomposition of search space in a different manner to provide an
efficient algorithm called Stochastic Directional Search, SDS. Its basics are first reviewed in the following sections. Then, it is utilized for structural design problem formulation and applied to a number of examples for sizing building frame members. The effectiveness and efficiency of SDS is finally compared with PSO via discussion on the results of the treated examples.

2. Basic components of particle swarm optimization

Swarm intelligence is assigned to a class of algorithms inspired by bird flocks or fish schools, providing a smart social behavior by the group action of simple individual agents. Simulation of such an action for numerical problems was introduced as particle swarm optimization by Kennedy and Eberhart in 1995 [7]. Since then, several variants of PSO have been successfully applied to engineering optimization problems [15-20].

Any particle movement in PSO is analogous to the variation of a current design vector to another. Such a design variation is performed via summation of three vectors, each one oriented towards a direction term denoted as a state [21]. The first vector directs in the same way as the previous movement and is thus called the inertia term. The second, called the cognitive term, models a simple memory for each particle to move towards its best experienced position up to the current iteration. The third one is a social term, directed toward the global best-so-far solution already found by the entire swarm. The standard PSO is thus formulated by the following relations:

\[ \Delta x_i^{k+1} = \Delta x_i^k + V_{i}^{k+1}, \]

\[ V_{i}^{k+1} = c_1 \Delta x_i^k + r_1 (P_i^k - x_i^k) + r_2 (g_i^k - x_i^k). \] (2)

For the artificial unit time interval, the velocity term, \( V_{i}^{k+1} \), in Eq. (1), denotes how the position of the \( i \)th particle (the corresponding design vector) is changed, moving into new position, \( \Delta x_i^{k+1} \), in the next iteration, \( k+1 \). The velocity, \( \Delta x_i^{k+1} \), in every next step is a vector-sum of the 3 terms in Eq. (2). \( P_i^k \) is the global best position found by the entire swarm, while \( g_i^k \) denotes the best position of every \( i \)th particle up to now. \( r \) is a uniformly random scalar between -1 and 1.

In an improved PSO variant, Eq. (2) is modified, adding the direction of a randomly generated design vector, \( R_i^k \), as the 4th term in the following relation:

\[ V_{i}^{k+1} = c_1 \Delta x_i^k + c_2 (P_i^k - x_i^k) + c_3 (g_i^k - x_i^k) + c_4 (R_i^k - x_i^k). \] (3)

In Eqs. (2) and (3), \( c_1, c_2, c_3 \) and \( c_4 \) stand for the inertial, cognitive, social and random vector coefficients that confine movement size towards the corresponding directions during any iteration of the algorithm.

3. The proposed stochastic directional search

In an iteration of PSO, the velocity of a particle is not necessarily parallel to any of the terms in Eq. (3), but is a vector sum of such randomly scaled vectors.

Here, a new algorithm is proposed, called the Stochastic Directional Search, SDS. According to SDS strategy, a move-step in any search iteration is oriented towards only one of the candidate states, i.e. inertial, cognitive, social or random directions, as depicted in Figure 1. The final path of every particle will be the vector-sum of such terms, but in consequent iterations. As a result, SDS allows explicit tuning of the probability threshold of any such state, in order to provide the desired balance between diversification and intensification by the algorithm. A roulette wheel procedure is employed for such a probability-based state selection.

4. Structural problem formulation

In order to reduce constructional cost, total structural weight is minimized under regulations of the Load and Resistant Factor Design method, due to the ASC- LRFD code [22]. The optimization problem is thus formulated, regarding the strength and serviceability constraints, as:

\[ \text{Minimize} \]

\[ W^t = \rho \sum_{i=1}^{M} A_i L_i. \] (4)

Subject to:

\[ \frac{\partial h}{\partial \tilde{a}} \leq 0, \] (5)

\[ \frac{\partial d_i}{\partial \tilde{a}} \leq 0, \] (6)

![Figure 1. Candidate states for the ith particle in directional search and a sample movement vector.](image-url)
\[ \left| \frac{P_u}{\varphi_i P_n} \right| + s \left| \frac{M_u}{\varphi_i M_n} \right| - 1 \leq 0, \quad (7) \]

\[ s = \begin{cases} \frac{\rho}{\beta} & \text{if } \left| \frac{P_u}{\varphi_i P_n} \right| > 0.2, \\ 1 & \text{otherwise} \end{cases}, \quad (8) \]

\[ \frac{b_{k+1,i}^{j+1}}{b_{k,i}^{j+1}} - 1 \leq 0, \quad (9) \]

\[ \frac{b_{k,i}^{j+1}}{b_{k,i}^{j}} - 1 \leq 0, \quad (10) \]

where \( \rho \) stands for the material density, \( L_i \) and \( A_i \) denote length and cross-sectional area of the \( i \)th member, so that \( W^i \) is the total structural weight. \( \theta^k \) stands for the \( k \)th story drift divided by its height, and \( \theta^{al} \) denotes the corresponding allowable drift ratio. \( b_{k,i}^{j} \) and \( h_{k,i}^{j} \) are the width and height of the column's W-section in the \( k \)th story, respectively, where \( b_{k,i} \) denotes the corresponding girder flange width. \( P_u \) and \( P_n \) are the applied column axial force and its nominal capacity, and \( \varphi_i \) is the corresponding resistant factor. Similarly, \( M_u \) stands for the applied moment, while \( M_n \) and \( \varphi_b \) denote the nominal flexural strength and its resistant factor of the \( i \)th member, respectively.

A fly-back strategy is applied for the last two constraints. For the others, a penalty function [23] is employed using the following relation:

\[ \text{Fitness} = -W \times (1 + K_p s \sum_i C_i)^\beta, \quad (11) \]

where \( C_i \) is the positive violation of the \( i \)th constraint and is taken 0 for non-violated constraints. \( K_p \) denotes the corresponding penalty factor and \( \beta \) is a prescribed amplifying coefficient.

5. Numerical examples

Three benchmark examples are selected from the literature [24-30] in order to evaluate performance of the proposed method. The AISC W-section list is considered as the discrete set for sizing the steel frame elements. In all the optimization examples, 30 particles are employed, with coefficients \( c_2, c_3 \) and \( c_4 \) taken as 2, where \( c_1 \) is linearly decreased from 0.9 to 0.4. In this study, the values of \( K_p \) and \( \beta \) are taken to be 1 and 3, respectively.

For the sake of true comparison, the initial randomly generated population of particles by the PSO algorithm is saved in the memory and identically used as the initial population of the SDS algorithm. Consequently, final fitness improvements are comparable between the algorithms as a measure of effectiveness. The probability thresholds are tuned after a number of trial runs as 2, 43, 54 and 1 percent for the inertial, cognitive, social and random direction states, respectively.

5.1. Ten-story one-bay frame

The 10-story frame in this example consists of 9 symmetric member groups under gravitational and lateral loading, as depicted in Figure 2 [30]. A list of W12 and W14, including 66 standard sections, are considered assignable to the columns, meanwhile all 267 standard W-sections can be chosen for the girders. Therefore, the search space cardinality in this example is \( 267^9 \times 66^9 \); that is of order \( 10^{21} \).

Maximum displacement (which occurs at the roof level) is restricted to 4.9 in, and allowable inter-story drift is 0.6 in for the 1st story and 0.48 in for the others.

The elastic modulus and yielding strength of constructional steel are taken as 29000 ksi and 36 ksi.

![Figure 2. One-bay ten-story frame, member groups and the applied loading.](image-url)
respectively. The effective length factors of a frame member, from typical nodes $A$ to $B$, are taken to be 1 for out-of-plane motion and computed according to the following relations [31] for the in-plane motion of the frame:

$$k = \sqrt{\frac{1.6G_AG_B + 4(G_A + G_B) + 7.5}{G_A + G_B}}$$

(12)

$$G = \sum \frac{EI}{Ei}$$

(13)

where $I_c$, $I_g$ are moment of inertia for the column and girder connected to the joint (A or B) and $l_c$, $l_g$ are the corresponding member unbraced lengths, respectively. The unbraced length of any girder is taken as $1/5$ of its total length.

The global best fitness achieved up to any iteration is saved and announced as the elitist fitness. The fitness history of such elitist solutions among all the search iterations is thus plotted in Figure 3 for both PSO and SDS. It is declared that, starting from the same initial point, the SDS results stand higher than the PSO. Additionally, SDS has more rapid convergence to the best solution than PSO. As declared in this example, the elitist fitness of PSO has remained constant for a number of final iterations, so it has become trapped in the local optima. In order to verify the SDS capability in over-passing local optima, it has been compared to the best results reported in the literature. According to Table 1, SDS can obtain the best structural weight of 60120 lb better than the least reported weight 61813 lb among PSO. Genetic Algorithm [24], Harmony Search [25], Ant Colony Optimization [26], Improved ACO [27] and Teaching And Learning Based Optimization, TALBO [28] belonging to the last method. In addition, Figure 4 shows that the proposed SDS has forced most of the inter-story drifts to get close to their allowable limit in the optimal result.
5.2. Fifteen-story three-bay frame
All standard W sections are considered as the section list in this example. The frame members are categorized in 12 groups; the first 2 belonging to the girders while the others are assigned to the column members as shown in Figure 5.

Allowable inter-story drift is confined to 0.52 in for the 1st story and 0.48 in for the others, while the maximum allowable absolute displacement of nodes at roof level is taken 6.95 in. Other assumptions, including material elastic modulus and strength, and effective and unbraced member lengths, are the same as the previous example.

In this example, the discrete search space cardinality or the number of searchable particle positions will be $26^{12}$, i.e. of order $10^{20}$, which is quite large. However, it is greater than the 1-bay 10-story frame. The convergence curve of Figure 6 again declares the superiority of the proposed SDS over PSO. It is notable that the SDS curve is also smoother than that of the PSO, which means more stable convergence toward the global optimum. The matter is confirmed by adding the previously reported best results of Genetic Algorithm and Simulated Annealing to a comparison with the present work of PSO and SDS. It is observed in Table 2 that the best structural weight of 37143 kg belongs to the proposed SDS, standing better than SA, GA [29] and PSO. In the SDS optimal solution, the inter-story drifts have gotten close to their limit, but have not overridden it. This achievement confirms the proper constraint handling capability of the algorithm (Figure 7).

5.3. Twenty-four-story three-bay frame
This example consists of a 3-bay 24-story moment frame under lateral point loads of $w = 5761.8$ lb at each story level, and distributed gravitational loads of $w_1 = 300$ lb/ft, $w_2 = 436$ lb/ft, $w_3 = 474$ lb/ft and $w_4 = 408$ lb/ft, as depicted in Figure 8. Modulus of elasticity is 29732 ksi and the yielding strength of the material is assumed to be 33.4 ksi. The frame is designed under AISC-LRFD requirements, with maximum allowable

![Figure 5](image1.png)  
**Figure 5.** Three-bay fifteen-story frame, member groups and the applied loading.

![Figure 6](image2.png)  
**Figure 6.** Convergence history of the 3-bay 15-story frame.

![Figure 7](image3.png)  
**Figure 7.** The optimal inter-story drift ratios obtained by SDS for the 2nd example.
to the girders, the resulting search space will be of order \(10^{34}\). Figure 9 shows that, even in such a large search space, the proposed algorithm is again more efficient and effective than PSO. Table 3 declares the least frame weight is obtained by SDS as 20216 lb, while it is 24968 lb for PSO. Using even more fitness evaluations by other methods, such as 13924 analyses by HS, has not revealed better results than the SDS result, which is achieved only via 6000 fitness evaluations. The matter shows the efficiency of the proposed method in addition to its effectiveness in searching the global optimum of the problem. Figure 10 also illustrates that in this higher story example, more inter-story drift responses have tended towards their constraint boundary.

6. Conclusion
In this study, directional decomposition of the search space in swarm algorithms has been concerned. Consequently, the idea of the stochastic explicit selection of direction states is utilized in the proposed SDS method. Despite vector-sum movements in any PSO
Table 2. Comparison of optimal results by different methods for the 3-bay 15-story frame.

<table>
<thead>
<tr>
<th>Element group</th>
<th>Saka [29]</th>
<th>Present work</th>
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<td></td>
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</tr>
<tr>
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<td>W21×50</td>
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<tr>
<td>2</td>
<td>W24×55</td>
<td>W21×57</td>
</tr>
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<td>W10×39</td>
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<td>7</td>
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<td>10</td>
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<td>11</td>
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<td>Weight (kg)</td>
<td>409.49</td>
<td>393.92</td>
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Table 3. Comparison of optimal results by different methods for the 3-bay 24-story frame.

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iteration, the SDS strategy enables application of a roulette wheel selection, guided by fine tuned probability thresholds, in selecting each direction state. Hence, SDS is expected to show a superior performance by finer balancing the intensification and diversification features of the search process.

Such an expected search improvement is then evaluated using three literature benchmarks of steel moment frames. PSO results in the examples proves it can lead to premature convergence into local optima, while the smoother convergence curves of SDS exhibit its effectiveness and higher stability search toward the global optimum.

The achieved solutions by PSO and SDS in the treated examples were further compared with those reported in literature by GA, ACO, SA, HS and TALBO algorithms.

Consequently, SDS led to the highest quality solutions among all these algorithms, even with less computational effort in function evaluations. It again confirmed the importance of the better tuning capabilities in SDS.

It is worth mentioning that in the optimal frame design by SDS, inter-story drifts tend to their allowable limits, showing the proper constraint handling performance of this algorithm.

Finally, in view of the obtained results, the developed stochastic directional search is an effective and efficient search method in weight minimization of moment frames under behavioral and section-size constraints. Based on the revealed theoretical discussion, further investigation into the performance of the algorithms in other fields of optimization is recommended as a future scope of research.

References


**Biographies**

Mohsen Shahrouzi was born in 1975. He obtained his BS degree in Civil Engineering and his MS degree in Earthquake Engineering from Sharif University of Technology, in 1997 and 2000, respectively, and, subsequently, received his PhD degree from the International Institute of Earthquake Engineering and Seismology, in 2006. Dr. Shahrouzi is currently Assistant Professor of Earthquake Engineering at Kharazmi University, Iran. He is author of 50 papers, 17 of which have been published in international journals and 7 presented at international conferences.

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