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Simulation of flow over a side weir using simulink

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Simulation;
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Discharge coefficient.

Abstract. The present study focuses on the concept of an elementary discharge coefficient that is related to the discharge flowing through an elementary strip along the side weir length. Simulations of discharge and water depth elevation were done. It is shown that the predicted discharge was in agreement with the one observed, within an error not exceeding 10compared with other works, and was increased when the side weir was installed as oblique against the flow direction.

1. Introduction

Weirs are the simplest hydraulic structures used for flow measurement.

A side weir is a weir placed in the wall of a channel over which lateral outflow takes place when the water surface in the channel rises above the weir crest. Thus, it is used as a key structure in many hydraulic projects, including irrigation, flood regulation and others.

Chow [1] explains the different types of flow pattern passing over a side weir. Subramanya and Awasthy [2], EL-Khashab and Smith [3], Ranga Raju et al. [4], Hager [5] and Singh et al. [6] used experimental results to find the side weir equation.

Swamee et al. [7] dealt with different types of flow diversion structure, and established that the use of an expression for the variation of discharge coefficient, along with the spatially varied flow equations, is able to represent the diverted flows, satisfactorily.

Other investigators, such as Cheong [8], Uyumaz [9], Uyumaz and Smith [10] and Smith [11], have contributed both experimentally and analytically to the study of flow over side weirs for different flow conditions.

The inclined side weir discharge coefficient was studied, using a side weir with three different crest angles, by Mohammed, M.Y. and Mohammed A.Y. [12], and an equation for the discharge coefficient was obtained for an inclined side weir.

Paris et al. [13] studied application of the experimental observations to the generalized De Marchi [14] hypothesis (1934), which clearly shows that the functioning of side weirs on a movable bed can be modeled using this hypothesis. These findings could be instrumental in the design and verification of these structures.

In the present work, a methodology, based on the numerical solution of two ordinary differential equations for discharge and flow depth simulation using matlab simulink programming, is proposed.

2. Theory and basic equation

For the equations of spatially varied flow, the energy principle is considered more appropriate, with respect to the momentum approach. However, in the derivation of the energy solution, an estimate of the velocity of lateral outflow is necessary. So, it is assumed that this velocity is equal to the cross sectional mean flow velocity [1]. Figure 1 shows a definition sketch in the assumptions of one dimensional flow. The energy

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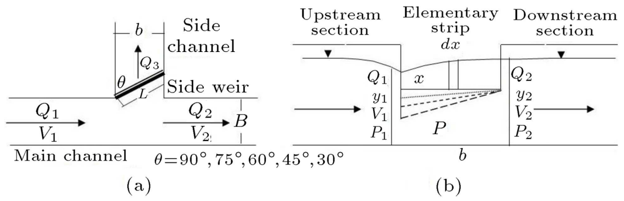

Figure 1. Definition sketch: (a) Plan; and (b) elevation.

equation is:

$$\frac{dE}{dx} = S_o - S_f, \tag{1}$$

where E = the specific energy; x = space coordinate; S_o = bottom slope and S_f = energy slope.

At section x for the hydrostatic pressure distribution and small bottom slope, the specific energy is:

$$E = y + \frac{\alpha V^2}{2g} = \frac{\alpha Q^2}{2gA^2}, \tag{2}$$

where y = depth of flow; α = velocity distribution coefficient; $V = Q/A$ = velocity; Q = discharge; A = cross sectional area and g = gravity acceleration.

Differentiating (2), given:

$$\frac{dE}{dx} = \frac{dy}{dx} + \frac{\alpha}{g}\left(\frac{Q}{A^2}\frac{dQ}{dx} - \frac{Q^2}{A^3}\frac{dA}{dx}\right). \tag{3}$$

And noting that for rectangular sections:

$$\frac{dA}{dx} = b\left(\frac{dy}{dx}\right), \tag{4}$$

where b = channel width, Eq. (3) becomes [15]:

$$\frac{dE}{dx} = \frac{dy}{dx}\left(1 - \frac{\alpha Q^2 b}{gA^3}\right) + \frac{\alpha Q}{gA^2}\frac{dQ}{dx}. \tag{5}$$

From Eqs. (1) and (5), Eq. (6) can be obtained:

$$\frac{dy}{dx} = \frac{S_o - s_f - \frac{\alpha Q}{gA^2}\frac{dQ}{dx}}{1 - \frac{\alpha Q^2 b}{gA^3}}. \tag{6}$$

S_f can be obtained from Eq. (6) as a valid Manning's formula:

$$S_f = \frac{n^2 q^2}{y^2 R^{4/3}}, \tag{7}$$

where n = roughness coefficient; $R = A/P_w$ = hydraulic radius; p_w = wetted parameter and $q = Q/b$ = discharge per width.

For the coefficient α, Hager [16] shows that:

$$\alpha = \left[1 + \left(\frac{Q'}{q}\right)^2\right]^{4/3}, \tag{8}$$

where $Q' = dQ/dx$.

This quantity, numerically negative, may be expressed by:

$$\frac{dQ}{dx} = -\frac{2}{3}Cd\sqrt{2g}\,(y-P)^{3/2}, \tag{9}$$

where Cd = coefficient of discharge; P = weir height, and, from the specific energy equation, the following equation is obtained:

$$Q = b.y\sqrt{2g(E-y)}. \tag{10}$$

Eq. (6) can be written as when $\alpha = 1$:

$$\frac{dy}{dx} = \frac{S_o - \frac{Q^2 n^2}{b^2 y^{\frac{10}{3}}}\left(1+\frac{2y}{b}\right)^{\frac{4}{3}} + \frac{2\sqrt{2}}{3}Cd(y-P)^{\frac{3}{2}}\frac{Q}{b^2 y^2\sqrt{g}}}{1 - \frac{Q^2}{gb^2y^3}}. \tag{11}$$

Eqs. (9) and (11) can be solved with initial conditions (Figure 1), at $x = 0$, $y = y_1$ and $Q = Q_1$.

The coefficient of discharge, Cd, observed by De-Marchi, is [14,17]:

$$Cd = \frac{3b}{2L}\left[\frac{2E-3P}{E-P}\left(\sqrt{\frac{E-y}{y-P}} - 3\sin^{-1}\sqrt{\frac{E-y}{E-P}}\right)\right] + \text{Const}, \tag{12}$$

where E is specific energy given by:

$$E = y_1 + \frac{Q_1^2}{2gb^2y_1^2} \cong y_2 + \frac{Q_2^2}{2gb^2y_2^2}, \tag{13}$$

where Q_1 = upstream discharge; Q_2 = downstream discharge; y_1 = upstream water depth and y_2 = downstream water depth.

The side weir discharge, Q_3, can be obtained by:

$$Q_3 = Q_1 - Q_2. \tag{14}$$

3. Disharge coefficient

By using dimensional analysis, Cd can be expressed as:

$$Cd = f(F, L, b, P, y, S_o), \tag{15}$$

where F = approached Froude number.

Some of the proposed formulas for Cd with the upstream Froude number ($F_1 = Q_1/by_1\sqrt{gy_1}$) for various investigators are shown:

$$Cd = 0.864\left(\frac{1-F_1^2}{2+F_1^2}\right)^{0.5},$$

(Subramanya & Awasthy [2]), (16)

$$Cd = 0.81 - 0.6F_1, \qquad \text{(Ranga Raju et al. [4])}, \qquad (17)$$

$$Cd = 0.45 - 0.22F_1^2, \qquad \text{(Cheong [8])}, \qquad (18)$$

$$Cd = 0.485\left(\frac{2+F_1^2}{2+3F_1^2}\right)^{0.5}, \quad \text{for} \quad P=0,$$

(Hager [16]), (19)

$$Cd = 0.33 - 0.18F_1 + 0.49\left(\frac{P}{y_1}\right),$$

(Singh et al. [6]), (20)

$$Cd = 0.71 - 0.41F_1 - 0.22\left(\frac{P}{y_1}\right),$$

(Jalili & Borghei [18]). (21)

To solve Eqs. (9) and (11), Cd must be calculated and may be assumed to be a function of head to weir height ratio for a sharp crested rectangular side weir. Swamee [19] has given an equation for a discharge coefficient for sharp crested weirs:

$$Cd = k_1\left[\frac{k_2}{k_3+(y-P)/P}\right]^{k_4} + \left[\left(\frac{(y-P)/P}{(y-P)/P+1}\right)^{k_5}\right]^{-k_6}. \qquad (22)$$

4. Experimental verification

Data presented by Al-Talib [20] for water surface elevation were used to study the coefficient of discharge in a side weir.

These experiments were performed in the Hydraulic Laboratory of the Dams and Water Resources Engineering Department at the University of Mosul, Iraq, using a rectangular flume (10 m) long, (0.3 m) wide and (0.45 m) in height. The side channel was (2 m) long, (0.15 m) wide and (0.3 m) in height.

The discharge in the main channel was measured by a sharp crested weir of (0.3 m) distance from the end of the main channel with (10*30*0.1 cm) dimensions. The side weirs were made of wood with (12*15*0.15 cm) dimensions installed at the entrance of the side channel, inclined at angles of (30, 45, 75 and 90°), with respect to the side channel wall, with flow direction, with five different discharges of (7.3 to 16.5 l/sec). The range of various parameters is given in Table 1.

5. Constancy of energy

The assumption of constant energy for subcritical flow has to be checked to use the De-Marchi equation. Figure 2 shows that the average energy difference in the channel between the two ends of the weir ($\Delta E = E_1 - E_2$) is 4.5%. EL-Khashab and Smith [3] estimated a 5% value for subcritical flow. Thus, the assumption of constant energy is accepted for short sided weirs.

6. Results

For the data of each run, Eqs. (9) and (11) are solved, and the simulation for discharge and water depth was undertaken using the Simulink package of Matlab Ver. R2010a. Figures 3 and 4 show the discharge and water depth block diagram, respectively. This requires trial values of constants $K_1 - K_6$, which are estimated using statistical program SPSS V17 for all cases to be used in Eq. (22). Table 2 shows the constant estimating values; the solution yields y_2 and Q_2, then, Q_3 is obtained using Eq. (14).

Table 1. Range of parameters measured.

Variables	Value
Crest height, P (cm)	12
Q_1 (m^3/s)	0.00727 - 0.0165
Q_2 (m^3/s)	0.00687 - 0.01386
Q_3 (m^3/s)	0.0004 - 0.00258
F_1	0.121 - 0.191
Angles with wall	90, 75, 60, 45, 30
Length of side weir (distance) (cm)	15, 15.53, 17.32, 21.21, 30

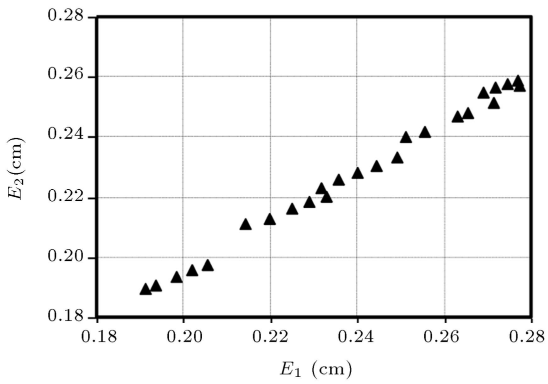

Figure 2. Average energy difference.

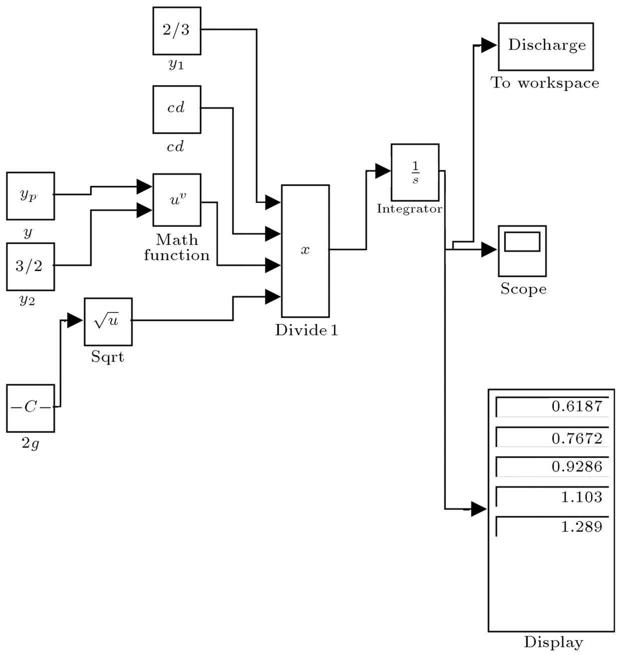

Figure 3. Discharge simulink block diagram.

The computed (Q_3) weir discharge is then compared with the observed weir discharge ($Q_{3\text{obs}}$) to yield the average percentage error (E) as:

$$E = \frac{100}{N} \sum_{i=1}^{N} \left| \frac{Q_{3\text{obs.}} - Q_3}{Q_{3\text{obs.}}} \right|. \tag{23}$$

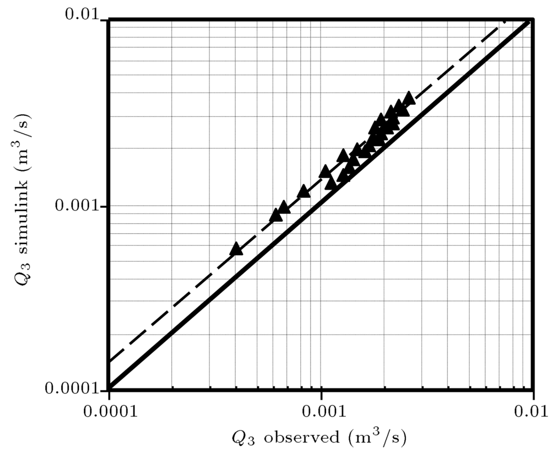

Figure 5. Observed and calculated discharge.

E was optimized in the weighted least squares sense to minimize it.

Figure 5 shows the observed side weir discharge and the discharge computed by Simulink. It can be seen that the majority of data falls in the error margin of +10%

The Cd values computed by Eq. (12) are depicted in Figure 6. It is evident from the scatter that some of the existing approaches are capable of predicting the side weir discharge accurately, especially when the side weir is installed perpendicular to the flow direction. But, when it is installed at an oblique direction with different angles, the Cd values increased and reached 0.88.

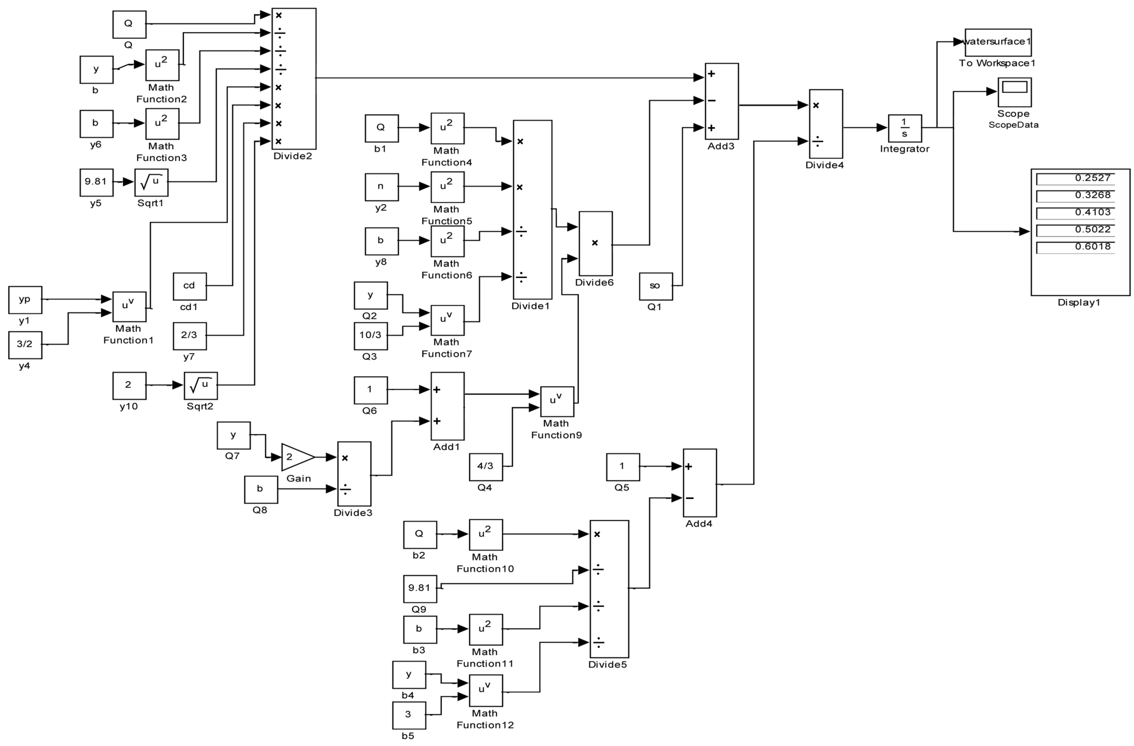

Figure 4. Water surface simulink block diagram.

Table 2. Parameters estimated using Eq. (22).

No.	Angle	k_1	k_2	k_3	k_4	k_5	k_6	Q_{3obs} (l/s)	Error%
1	90	0.171	40.063	52.578	7.159	14.864	-0.685	0.450-1.951	10
2	75	0.286	39.866	46.037	7.23	13.681	-0.839	0..911-2.201	9.85
3	60	0.143	40.45	47.713	7.274	13.181	-1.331	0.955-2.312	9.5
4	45	0.308	39.702	50.413	7.101	13.395	-0.528	1.252-2.555	8.54
5	30	0.391	39.19	39.47	11.672	2.105	-6.55	1.532-2.822	8.443

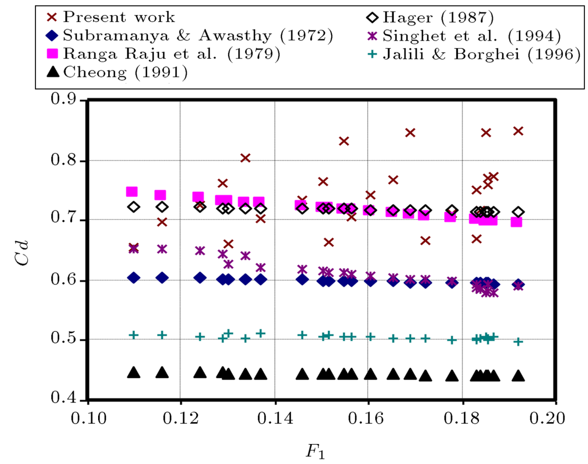

Figure 6. Compression of discharge coefficients equations.

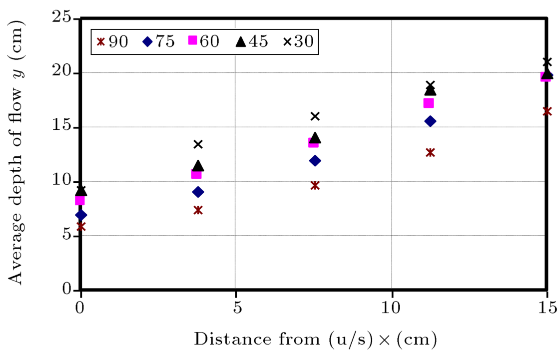

Figure 7. Average depth of flow variation with distance.

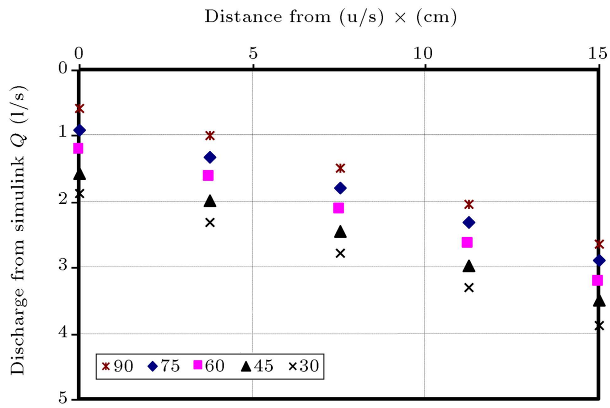

Figure 8. Average flow variation with distance.

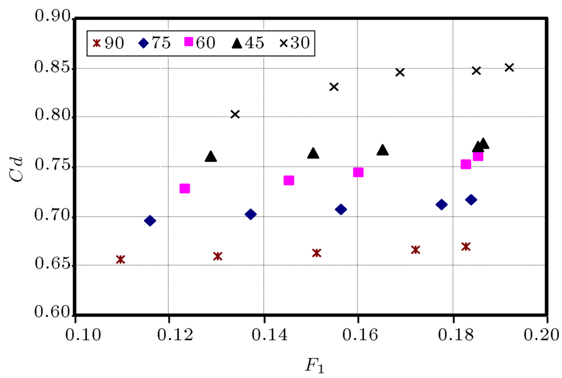

Figure 9. Discharge coefficient variation with Froude number.

Figures 7 and 8 show the average water depth elevation and average flow discharge variation against distance along the longitudinal side weir (x) for all cases (all angles). The average water depth was increased towards the downstream side weir end when the side weir angle increased. While the average discharge decreased towards the downstream side weir end, maximum discharge and water depth occurred when the side weir was installed at an angle (30^o) oblique to the flow direction.

The values of Cd computed by Eq. (22) were plotted against F_1, as shown in Figure 9. From this figure, it is evident that Cd increased when F_1 and the oblique angle of the side weir increased.

To study the effect of parameter P/y_1 on the discharge coefficient, Cd values were plotted against P/y_1 in Figure 10. The figure reveals that Cd decreases with increased P/y_1, and increased when the side weir installed angle increased.

The results, as shown in Figures 11 and 12, represent the simulation of discharge and water depth. It appears that increasing discharge and water depth with time and side weir installed angles increases.

7. Conclusions

In this study, the elementary side weir discharge has been described with the coefficient of discharge model for an oblique side weir. Comparison of predicted and observed discharge shows the validation of the model

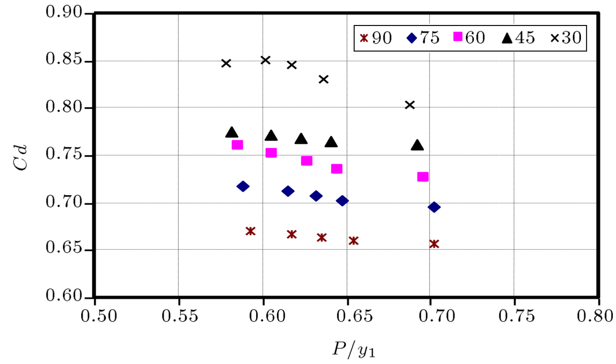

Figure 10. Discharge coefficient variation with P/y_1.

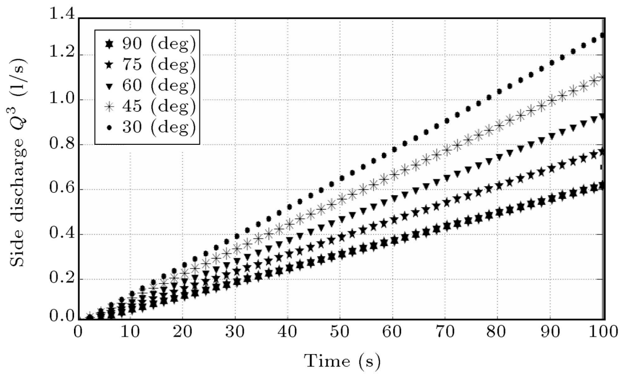

Figure 11. Side discharge simulink test results.

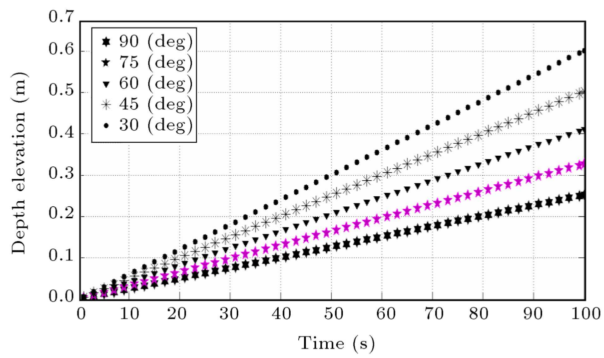

Figure 12. Water depth simulink test results.

in the simulation of discharge and water surface profile equations with Matlab Simulink.

The maximum discharge and water depth occurred when the side weir was installed at an angle (30°) oblique to the flow direction, with a majority error width of +10% for the observed side weir discharge and the discharge computed by Simulink.

The Cd values computed by Eq. (12) increased to 0.88 when the weir was installed at an oblique direction with different angles.

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